Competition and Integration of Digital Content Distribution Channels

Abstract

In this paper, we investigate the interactions between two digital content distribution channels: central server (CS) based and peer-to-peer (P2P) based. A central server based channel (such as iTunes) forms a network that displays negative externality, while P2P (such as Vuze) is a network with positive externality. When presented with the option of these two channels to obtain digital contents, users self select to join one channel based on its comparative utility. The equilibrium demands of both channels are derived when they are in a competition setting as well as in an integration setting. In a competition setting, the equilibrium demand of CS channel increases along with the increase of service capacity. It is also higher than that in the P2P channel. In an integration setting, it is observed that the equilibrium demand for CS based channel is much lower than that in a competition setting. This is actually induced by setting a higher price on the CS based channel. In other words, majority of demands are directed to P2P channel to better utilize its positive externality. At the same time, allowing only a small amount of demand in the CS based channel can also alleviate its negative externality. Finally, the gross utility in the integration setting is higher than that in the competition setting for both channels. These findings provide guidance on understanding the network externalities of digital content distribution channels. In order to fully utilize both channels, social policies and incentive mechanism need to be designed to align competition demand with the coordinated channel utilization.

Keywords: digital distribution channels, central server, peer-to-peer, competition, integration

1. Introduction

Together with globalization, information technology has dramatically changed today's business. It enables business to obtain competitive advantages through technological innovations and new formats of information distribution. The wide spread of internet utilization is one of such enablers. Nowadays, a huge amount of products and services can be delivered directly online, such as software, music, video, games, voice communication, or even professional consultation. Online delivery can be implemented either through a central server (CS) based channel where the content is obtained from a designated server or server farm; or through a peer-to-peer (P2P) channel where the content is assembled from pieces provided by computing resources of peers in the network. For example, Apple's iTunes is set up as a CS based architecture with content offered at positive prices and downloadable from iTunes store, while the popular movie download site Vuze.com (originally known as Azureus) is set up as a P2P based architecture for music, video, and other content sharing. Although the study of either CS or P2P based channel itself is of great interest, it is also believed that the analysis of interactions between these two channels will provide important insights for the entire digital content market (Casadesus-Masanell and Hervas-Drane 2010). In this paper, we are interested in investigating such interactions of these two channels in both competition and integration settings.

A P2P network is a network for pooling resources, such as computing cycle, hard disk storage, network bandwidth, and contents, as well as communication and collaboration (instant messaging) at numerous edge nodes of the Internet. The members of a P2P community can exchange information and resources directly with each other. Among the various P2P applications, the first popular and prevalent one is file sharing, especially music file sharing (such as KaZaA, Gnutella, and Grokster). P2P file sharing applications generated 4.5 million downloads in 2002, an increase of more than 300 percent over a 12-month period (Lee 2003). A more recent report shows that P2P file sharing still accounted for 25.0% of global broadband traffic in 2010, although down from 38.0% in 2009 (Cisco 2010). One popular P2P file sharing application is Vuze (vuze.com), which allows users to view, publish and share original DVD and HD quality video content with an annual membership fee. Content is presented through channels and categories containing TV shows, music videos, movies, video games and others. Another popular P2P applications developed recently is P2P video streaming. According to the Chinese company, PPTV's official information, its product, PPLive, has more than 200 million user installations and its active monthly user base is 104 million as of December 2010. Further, the average viewing time per person per day has reached over 2.5 hours, the highest stickiness among all Chinese websites. These observations are in line with the technical report by Envisional, which attributes P2P network traffic for 20.0% of all internet traffic (Envisional 2011). Nonetheless, P2P network has been an indispensable channel for digital content distribution over Internet, where individual peer nodes can request and share contents in a variety of formats.

On the other hand, there are also numerous applications of CS based channels dedicated to digital content distribution. Apple's iTunes store has always been a centralized distribution channel for music, video, and mobile applications since its inception in 2001. PDBox (pdbox.co.kr) is another example of a CS based file sharing community, which distributes legalized digital contents such as movies and music for a fee. Online video providers such as YouTube (YouTube.com), Netflix (Netflix.com), and Hulu (hulu.com) also utilize a variety of proprietary or commercial CS channels or cloud services (e.g. Amazon EC2) to distribute contents.

According to Cisco's network traffic report of 2010, P2P traffic has been surpassed by online video, most of which are central-server based traffic, as the largest category. More specifically, the subset of online video that includes streaming video, flash, and Internet TV represented 26.0% of the internet traffic, compared to 25.0% for P2P in 2010 (Cisco 2010). While still growing in absolute terms, P2P traffic growth is relatively stagnant now comparing with the CS based traffic. Intrigued by this phenomenon that CS based content delivery seems to take over the dominate position used to be assumed by P2P channel, we start to analyze the economic aspect of interactions between two channels. The following research questions will be addressed in this study:

- Will the central server based channel eventually attract all users? Is it still possible to have both channels coexist in the digital content delivery market?
- If both channels keep competing head on head for the same market, will the competition equilibrium socially optimal as if they are coordinated by a social planner? If not, what kind of the market alignment needs to be made?

Before we present our attempts in capturing characteristics of the two channels, it is important to note that these channels are distinctly different in terms of both technical implementation and business operation model. However, from a user's perspective, either channel would work as long as it delivers requested contents with desired quality and reasonable waiting time. Therefore, we here focus on the difference between two channels in terms of user's valuation of obtained content as well as the associated cost. A CS based channel stores digital contents on servers and all users' requests are processed through these servers (or a server farm); on the other hand, a P2P channel allows users to obtain requested content from peer nodes' computing devices. Because each peer node can modify its content freely, and can decide whether to share the content, there is no guarantee to obtain the desired content. Thus, user's utility of obtaining content from a P2P channel is based on the content availability and the waiting time of obtaining the content. A P2P channel has this inherent drawback due to the decentralized structure, though it is this structure that makes it more scalable than a CS based channel. On the other hand, users can always rely on the availability of content from a CS based channel while traffic delay in the channel depends upon the aggregated number of requests. If both of these channels are presented, users will self-select one to obtain contents based on the comparative utilities.

In consideration of the above mentioned differences between two channels, we characterize the CS based channel with negative externality in terms of the network size, and the P2P channel with positive externality. In other words, more users in the CS channel lead to more disutility; while more users in the P2P channel result in higher utility. In a competition setting, both channels set prices to maximize their respective revenues, while in an integration setting, the channels are coordinated together to maximize the total revenue. It is found that, in an integration setting, the equilibrium demand of CS based channel is much lower than that of P2P based channel. In other words, the coordinated equilibrium features a full utilization of P2P channel in light of its positive externality. In contrast, in the competition setting, the equilibrium demand of CS based channel is not lower than that of P2P based channel. Further, in a competition setting, only when the service capacity of CS based channel is moderate, can two channels co-exist with a stable equilibrium. Comparing the characteristics of delivering content through two different channels, the integration setting can always utilize both channels more efficiently with a higher total utility. Therefore, from a social planner's perspective, a market mechanism of encouraging utilization of P2P channel should be established. This study thus sheds managerial insights on the necessity of aligning competition behavior with coordinated allocations.

The paper proceeds as follows. We introduce related works in Section 2. Section 3 proposes the basic utility models of user to obtain digital contents from two distribution channels. The competition and integration equilibria are derived and discussed in Section 4. Section 5 offers some extended models. We conclude and discuss the directions for future research in Section 6.

2. Related Works

There are abundant prior works in channel coordination as well as related works studying digital content distribution channel. However, there are very few studies exploring current digital content market with regards to both central server based and peer-to-peer based channels.

2.1. Channel Competition and Coordination

Competition among multiple distribution channels has always attracted interest in marketing literature. Jeuland and Shugan (1983) focus on the vertical channel coordination problem in the context of a single manufacturer, single retailer channel. Regarding the pricing strategy of distribution channels of information goods, Dewan *et al*. (2000) develop a competition model between Internet Service Providers (ISPs) and proprietary content providers (PCPs), and explain the interplay between pricing decisions. Their results show that PCPs should benefit by free content available at the ISPs' sites to induce more customers to join the Internet.

With the increase in business opportunities of different channels, more firms are operating on multiple channels. For example, a firm can market or even sell its products either via the Internet, offline retail shops, or mail catalogue. Intuitively, multiple channels within a same firm can eventually complement one another to increase the overall profit generation. While at the same time, there is also an erosion effect from competition among multiple channels. Gulati *et al*. (2000) focus on whether and how different sales channels should be integrated. Steingfeld (2002) argues that the integration of online and offline sales channels may support the acquisition of new customers, thereby generating increased revenues and reduced costs. Coughlan (1985) discusses the problem of choosing a vertical marketing channel in a product differentiated duopolistic market in an empirical model, and finds out integration of the marketing function resulting in greater price competition and lower prices than the use of independent marketing middlemen.

In this paper, it is assumed that a user is facing a choice between a CS based channel and a P2P channel to obtain digital content. This assumption shares the key characteristic of consumer behavior that motivates the research interests in multi-channel marketing. Reardon and McCorkle (2002) emphasize the switching behavior of consumers in multi-channel market based on time allocation. Consumers face a tradeoff when deciding where to buy goods and services, and a decision between alternative distribution channels is made on the basis of the relative opportunity costs of time, costs of goods, pleasure derived from shopping, perceived value of goods, and relative risk of each channel. Schoenbachler and Gordon (2002) outline the key issues facing multi-channel marketers, and encourage multi-channel businesses to take a customer-centric view rather than a channel focused view, addressing the aspects which are considered as risks of decision from the consumers' side. Rangaswamy and van Bruggen (2005) provide a summary of the extant literature and open areas for further research, in the multi-channel setting.

2.2. CS- and P2P-based Channels

Based on the internet traffic report, even about 3 years ago, P2P traffic was still the largest category. Now, central server based delivery, especially for online video content, has gained the popularity and exceeded the traffic volume through P2P channel. This phenomenon ignites the research motive for understanding the interactions between two channels. Casadesus-Masanell and Hervas-Drane (2010) characterize the size of the P2P network as a function of a CS based channel's pricing strategy, and show that the firm which adopts a CS content distribution channel may be better off by setting high prices and allowing the P2P network to survive.

With respect to content provision in the context of P2P networks, several studies on disincentive (Feldman *et al.* 2003) and incentive (Lai *et al.* 2003) of sharing behaviors and free-riding phenomena in P2P networks have been conducted recently. For example, empirical evidence provided by Adar and Huberman (2000) and Asvanund *et al.* (2004) indicate that there is a very high degree of free riding behaviors in P2P networks. Krishnan *et al*. (2002, 2004) provide a plausible explanation for the existence of free-riding behaviors in P2P file sharing networks. Utilizing P2P technologies as content distribution channels, Gayer and Shy (2003) analyze the profit incentives of publishers to utilize the Internet and P2P for distributing versions of digital product that compete with product sold in stores, and suggest if the externality of digital version is significant, it will increase the sales in stores. Recently, Lang and Vargov (2005) develop a monopolistic pricing model to compare the profitability between P2P and client server architecture.

In all, majority of previous literature might either cover channel coordination in a traditional product market or study the economic viability of a single digital content distribution channel. The main objective of this paper differs from those by analyzing competition and integration between the two channels.

3. User Utility Model

Two digital content distribution channels, central server based and peer-to-peer based, are investigated in this study, both of which are operated by providing digital contents upon user's requests. This assumption of the coexistence of two channels is realistic based on the current internet content provision market (Cisco 2010). For a specific digital content, e.g., music, video, or game, the CS based channel serves a group of users at the demand rate of Λ_1, while the P2P based channel has a demand rate of Λ_2. The total demand rate for this content can simply be normalized to 1 for the entire market, that is,

$\Lambda_1 + \Lambda_2 = 1$. In order to focus on channel behavior, it is assumed that users are homogeneous with a constant value of v for the content requested. Possible extension on modeling heterogeneity of user valuation is discussed in Section 5.

If a user is to obtain the content through a CS based channel (such as iTunes), the utility she receives is:

$$U_{\text{CS}} = v - d\frac{\Lambda_1}{\mu_0},$$

where the second term refers to the disutility from delay in the CS channel. Here, μ_0 is the capacity of the central server, defined as the service rate (number of requests serviced per time unit); and d is the user's time-sensitivity cost parameter. The disutility formulation is a close representation of the time-shared queueing model which best describes computer systems (MacKie-Mason and Varian 1995). Implicitly, we make the assumption that the users are atomistic, that is, the traffic each user generates is insignificant to impact the system performance, but rather, the utility each user receives is determined by the aggregated demand rate, Λ_1. Also, note that U_{CS} decreases as Λ_1 increases. In other words, with a given service rate, μ_0, the higher the aggregated demand rate of the content, the larger the waiting cost for the user. Therefore, this is a network that displays *negative* network externality. Further, in order to generate non-negative utility for users to join, the demand of the CS channel is bounded by:

$$\Lambda_1 \le \Lambda_1^c \equiv \frac{v\mu_0}{d}.$$

On the other hand, if a user chooses to obtain the content via a P2P channel (such as Vuze or PPLive), the utility she gets is:

$$U_{\text{P2P}} = H \cdot v - \delta,$$

where H refers to the hit rate for a requested content. In a P2P network, the requested content may not exist; or, may not be easily found as there is a limit on the number of peer nodes that are searched. Therefore, H is simply interpreted as the probability of finding the content. δ here is the P2P disutility parameter that includes the associated cost such as search delay, download delay, sharing cost. It is important to understand that this disutility is assumed to be independent of the P2P community size, as a P2P network is often considered to be *scalable*. That is, an increase in demand is accompanied by an increase in capacity. In fact, the main advantage of using P2P is to avoid the possible network congestion when demand increases. The disutility here is more associated with P2P technology rather than the P2P community size.

Let us take a closer look at this hit rate. Intuitively, H increases with the size of P2P community. We use the demand rate through the P2P channel, Λ_2, as a proxy of the P2P community size. In general the larger the P2P community size, the higher the demand rate for this specific content. For the sake of analytical tractability, we choose the following expression to model the hit rate, H,

$$H = h\Lambda_2 .$$

The above form correctly captures the characteristics of H, i.e., it should increase with Λ_2. h is the coefficient to reflect the impact of P2P demand rate on the hit rate. Without loss of generality, we can assume $h \leq 1$ to ensure that H is bounded by 1. It is also necessary to note that there are always free-riding behaviors in a P2P network and they can affect both h and δ. Since free-riding is not the focus of this study, we assume that h and δ are given by an equilibrium free riding ratio.

On the contrary to U_{CS}, the utility U_{P2P} reflects that P2P is a network with *positive* externality. Users obtain higher utility from a P2P network when the demand rate (community size) is larger. Further, in order for a P2P community to emerge (with non-negative utility for P2P channel users), there is a critical mass defined as:

$$\Lambda_2 \geq \Lambda_2^c \equiv \frac{\delta}{vh} .$$

4. Channel Competition and Integration

As we have discussed in the introduction section, it is interesting to investigate the interactions between two digital content distribution channels. According to Cisco's 2010 report, both channels are ranked at the top two categories, each accounting for over 20% of the total internet traffic (Cisco 2010). Given the current status of increasing popularity of CS channel (such as Netflix, iTunes Store, Hulu), will P2P channel eventually lose its own market? From a social planner's perspective, is the coordination between two channels necessary to improve the overall utility?

In order to answer these questions, we carry out the analysis on two distribution channels in both a competition and an integration setting. In the competition setting, each channel operates separately. We start with formulating an organic competition, which is purely a competition of the channel characteristics. Then, pricing schema is introduced to allow each channel make decisions to maximize its own revenue. Each channel can set a price rate at the first stage, and allows the demand distribution to reach equilibrium at the second stage. It is found that only when the service rate of CS channel is moderate, can both channel coexist in the market. In the integration setting, both channels are coordinated by a social planner. In order to maximize the overall utility, optimal pricing scheme is derived to adjust demand distribution between two channels. It is found that the coordinated demand distribution encourages more utilization of the P2P channel, which is not observed in the competition setting. We elaborate findings in the following discussion.

4.1. Channel Competition

First of all, it is necessary to examine an organic competition purely based the channel characteristics. The competition here doesn't refer to competition of the provided content quality. Instead, we focus our discussion on the intrinsic difference between two channels: negative externality in CS channel and positive externality in P2P channel.

Facing two channels, users self-select into a channel with a higher utility. For example, a user will choose the CS channel if $U_{\mathrm{CS}} \geq U_{\mathrm{P2P}}$, or in terms of the demand rates,

$$\Lambda_2 \leq \frac{1}{hv}\left(v+\delta\right)-\frac{d}{hv\mu_0}\Lambda_1 . \qquad (1)$$

When more and more users join the CS channel, their utilities drop due to the negative externality from a larger demand rate. To a certain extent, a marginal user finds that the P2P channel may indeed offer a higher utility. This would induce users to switch between channels, and eventually equilibrium may be reached. At equilibrium, the users are indifferent to either channel, that is, $U_{\mathrm{CS}} = U_{\mathrm{P2P}}$.

Figure 1 depicts the possible equilibria. The left figure illustrates a stable equilibrium, while the figure in the right shows an unstable equilibrium. In the region above (below) the solid line boundary, which is described by taking the equal sign of Equation (1), P2P (CS) is preferred over CS (P2P). Users live on the dotted line ($\Lambda_1 + \Lambda_2 = 1$). If the entire boundary is above this dotted line, the CS channel is strictly better; On the contrary, if the boundary is below the dotted line, the P2P channel is strictly better. When the boundary line crosses the dotted line where the users live, Pareto demand distribution may be found by characterizing the intersection point. It can be easily shown that the unstable equilibrium actually requires the condition of $v + \delta \le hv$, since we have already constrained $h \le 1$, this condition cannot hold true. Therefore, only the stable equilibrium is considered. The stable equilibrium of demand distribution is summarized in the Proposition 1.

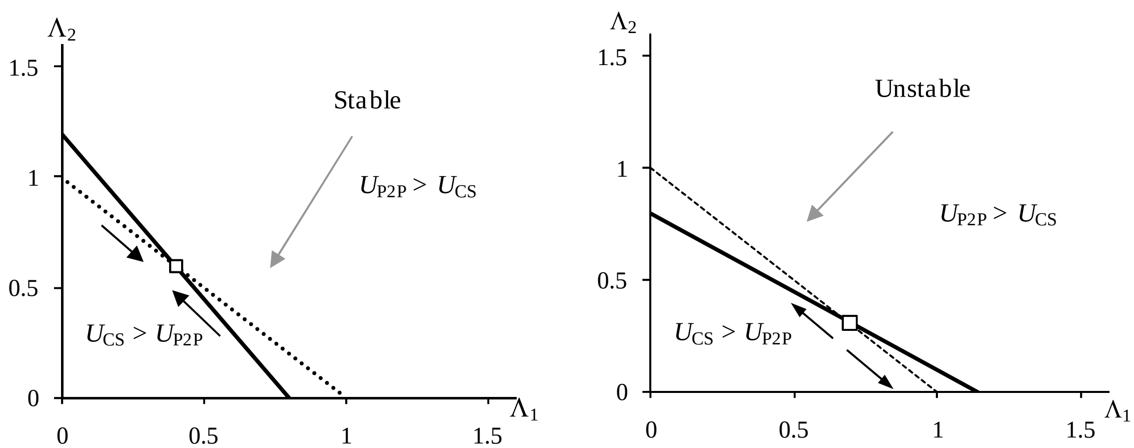

Figure 1. Possible equilibria

PROPOSITION 1. *The stable equilibrium distribution between a central-server based and a peer-to-peer based channel is given as follows:*

i. *When* $\mu_0 \le \Gamma^{\min} \equiv \dfrac{d\left(hv - \delta\right)}{hv^2}$, *the stable equilibrium distribution is* $(\Lambda_1^*, \Lambda_2^*) = (\Lambda_1^{eq}, 1 - \Lambda_1^{eq})$ *where*

$$\Lambda_1^{eq} \equiv \frac{(v+\delta)-hv}{d/\mu_0 - hv}.$$

ii. When $\mu_0 > \Gamma^{\min}$*, the equilibrium distribution is* $(\Lambda_1^*, \Lambda_2^*) = (\Lambda^{cs}, 0)$ *, where*

$$\Lambda^{cs} = \min\left(\frac{v\mu_0}{d}, 1\right).$$

The following numerical demonstrations help to interpret these results. Let us use the following set of parameter values: $v = 1$, $d = 0.8$, $h = 0.8$, and $\delta = 0.08$. We vary μ_0 to demonstrate how the equilibrium evolves as the CS channel increases its service rate.

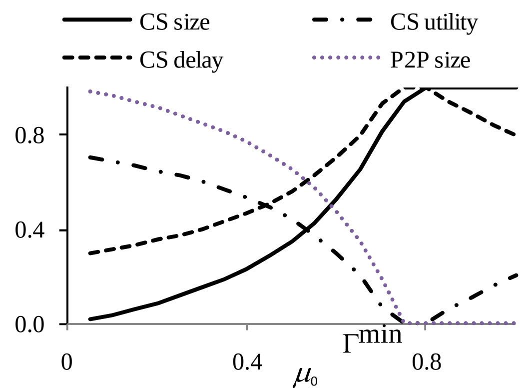

Figure 2. Equilibrium demand Λ_1^*, CS user utility, and CS QoS against μ_0 for $\delta = 0.4$

Figure 2 shows the scenario where the users have low P2P cost. If the CS channel has a very low service rate, it can only draw a small amount of the demand. Majority of the users join P2P network. As the service rate of CS channel increases, more and more users switch to the CS channel. At a point, $\mu_0 = \Gamma_1^{\min} = 0.7$, the entire user base is only captured by the CS channel. When more users switch to the CS channel, their utility drops, as shown in Figure 2. This is due to the fact that the increase in capacity cannot catch up with that in demand. As a result, Figure 2 also displays that the Quality of Service (QoS) of the CS channel, measured by the delay, $d\Lambda_1 / \mu_0$, deteriorating as the service rate increases. At $\mu_0 = \Gamma_1^{\min}$, there is even a drop in utility to zero, because of the discrete increase of demand. After this point, users are better off with a higher service rate, as the demand is stabilized at 1.

Let us further examine the numerical results for the same set of parameters except for a higher P2P cost, $\delta = 0.4$. First, it can be noted that, when users have higher P2P cost (δ), they are more likely to choose the CS channel. This is reflected by the fact that the value of $\Gamma^{\min} = 0.4$ is much lower in Figure 3 than that in Figure 2. Second, it can be seen that the equilibrium utility U_{CS}^* also drops with an increased service rate. Finally, there exists a range of the service rate, where the CS channel might only be able to attract a portion of the entire user demand; while the rest of users are not enough to form their P2P network.

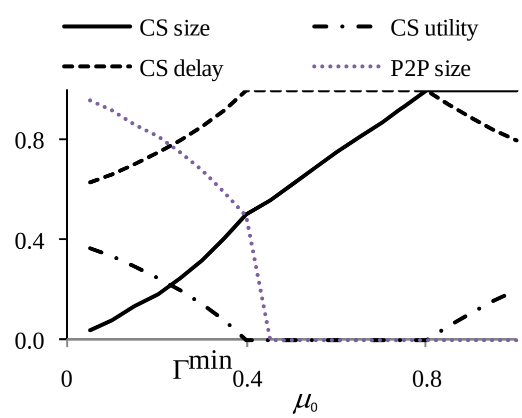

Figure 3. Equilibrium demand Λ_1^* , CS user utility, and CS QoS against μ_0 for $\delta = 0.4$

Proposition 1 provides a simplified overview of the organic competition between two channels. It points out when the service rate of CS channel is lower that a threshold, $\Gamma^{\min}$, it is possible for both channels to co-exist. However, if the service rate is larger than this threshold, CS channel may dominate the market. This proposition helps to understand what happened in the digital content delivery market in the past few years. When the service rate of CS channel is not so large, which was a reality for the digital content market in earlier days, both channels play important roles. With recent technology advancement in enhancing the service rate of CS channel, such as utilizing distributed server farms, serving content request through powerful cloud resources, more and more users can be accommodated in the CS channel. It is predicted in an organic competition setting, with the increase of service rate of CS channel, P2P channel can be diminishing.

4.2 Channle Competition with Pricing

Although Section 4.1 provides a straightforward insight on the competition purely based on channel characteristics, it is necessary to extend the discussion in the business operation mode. Two channels might operate on very different business models. For example, some content providers focus on boosting the volume of channel traffic because generating advertisement revenue is the goal (such as YouTube, PPLive). Other content providers focus on fee-based content provision because that is the source of revenue (such as Netflix, Vuze). Here in this study, the fee-based content provision model is analyzed. Suppose users pay a flat service fee similar to the monthly or yearly subscription fee charged by Netflix or Vuze. Factoring an average content request rate from users, we can simply incorporate a normalized price for each request, *P*, in the utility model. In the CS channel, we have:

$$U_{CS} = v - \frac{d}{\mu_0}\Lambda_1 - P_1, \text{ and } \Lambda_1 \le \Lambda_1^c \equiv \frac{(v - P_1)\mu_0}{d};$$

while in the P2P channel, we have:

$$U_{P2P} = h\Lambda_2 \cdot v - \delta - P_2 \text{ , and } \Lambda_2 \ge \Lambda_2^c \equiv \frac{\delta + P_2}{hv}.$$

The competition between two channels now can be set up in a two-stage game. At the first stage two channels set corresponding prices, and at the second stage users self-select the channel based on the net utility, which in equilibrium determines the demand distribution. The derivation of Pareto optimal equilibrium demand distribution in the second stage is similar to that in Section 4.1 with the consideration of two prices: $\left(\Lambda_1^*, \Lambda_2^*\right) = \left(\Lambda_1^{eq}, 1 - \Lambda_1^{eq}\right)$, where

$$\Lambda_1^{eq}\left(P_2, P_1\right) \equiv \frac{v + \delta - hv + P_2 - P_1}{d / \mu_0 - hv} \quad .$$

It can be observed that the price difference of two channels is playing a critical role now in characterizing the equilibrium demand distribution. The complete results are presented in Proposition 2.

PROPOSITION 2. *The stable equilibrium is given as follows:*

i. *if* $\Delta \ge 0$ *and* $\mu_L \le \mu_0 \le \mu_U$ *, the stable equilibrium demands are*

$$\Lambda_1^* = \frac{d/\mu_0 + v + \delta - 2hv}{3(d/\mu_0 - hv)}, \textit{ and } \Lambda_2^* = \frac{2d/\mu_0 - v - \delta - hv}{3(d/\mu_0 - hv)}$$

and the equilibrium prices are

$$P_1^* = (d/\mu_0 + v + \delta - 2hv)/3, \textit{ and } P_2^* = (2d/\mu_0 - v - \delta - hv)/3 \textit{ respectively;}$$

ii. if ($\Delta < 0$ and $\mu_0 \le \mu_H$), or ($\Delta \ge 0$ and ($\mu_0 \le \mu_L$ or $\mu_U \le \mu_0 \le \mu_H$)), the equilibrium demands are

$$\Lambda_1^* = \frac{v}{2d/\mu_0 - hv}, \textit{ and } \Lambda_2^* = 1 - \frac{v}{2d/\mu_0 - hv}$$

and the equilibrium prices are

$$P_1^* = v\frac{d/\mu_0 - hv}{2d/\mu_0 - hv}, \textit{ and } P_2^* = hv\left(1 - \frac{v}{2d/\mu_0 - hv}\right) - \delta \textit{ respectively;}$$

iii. if $\mu_0 > \mu_H$, P2P channel exits the market, the equilibrium demand and price for CS channel are

$$\Lambda_1^* = \min\left(\frac{v}{2d/\mu_0 - hv}, 1\right), \textit{ and } P_1^* = (d/\mu_0 - hv)\Lambda_1^* \textit{ respectively;}$$

In the above expressions, we have $\Delta = (1-3h)^2 v^2 - 4\delta(v + 3hv - \delta)$, and

$$\mu_L = d\frac{5hv + v - 2\delta - \sqrt{\Delta}}{2hv(2hv + 2v - \delta)}, \ \mu_U = d\frac{5hv + v - 2\delta + \sqrt{\Delta}}{2hv(2hv + 2v - \delta)}, \ \mu_H = d\frac{2(hv - \delta)}{hv(hv + v - \delta)}.$$

If the conditions in Part i are satisfied, that is, if the CS capacity is in the medium range, users in each channel receive positive net utility. In Parts ii and iii, users receive zero net utility and this changes the nature of competition. This happens when the CS server capacity is either low or high. The competition behavior described in Part i will lead to $\Lambda_1 = 1/3$ when μ_0 goes to zero, resulting in a negative net utility. On the other hand, when μ_0 increases, Λ_2 decreases, and if it falls bellows the critical mass, users will receive negative utility. Therefore, we focus on the part i equilibrium for the following discussion.

To better illustrate the Part i of Proposition 2 (where users have positive utility), we use the same set of parameters in Figure 2 to calculate the equilibrium. Figure 4 first shows the equilibrium demand distribution of two channels, the CS user utility, and the CS quality of service. The equilibrium prices of the both channels are indicated in the right hand side figure with regards to the service rate of CS channel.

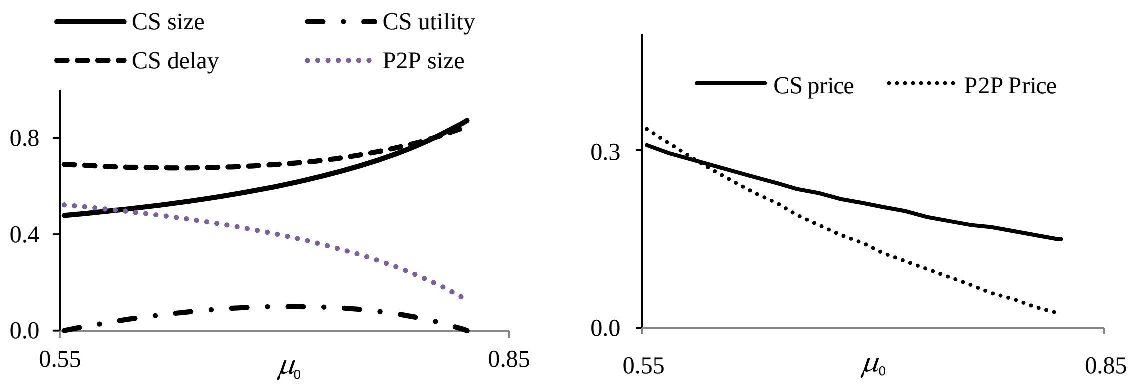

Figure 4. Equilibrium demand, user utility, CS QoS, and prices against μ_0 for $\delta = 0.08$

In the feasible region of the service rate, the equilibrium demand of CS channel increases with the increase of service rate, while the equilibrium price decreases. However, user's utility first increases with a larger service rate and then decreases as more and more users join the CS channel to deteriorate the quality of service. On the contrary, opposite effects can be observed in P2P channel: the equilibrium demand of P2P channel decreases with the increase of CS service rate. The equilibrium price of P2P channel also decreases and it decreases much faster than that of the CS channel. Intuitively, this is resulting from P2P channel using a deeper decrease in price to compete with the CS channel with an increasing service rate. Similar results are found for the case when the P2P channel has a relatively higher cost.

Comparing the results here with those in the organic competition setting, it is observed that with the pricing schema, both channels can use the price to adjust user's utility, which shifts the feasible region of stable equilibrium. As the service rate of CS channel increases, changes of demand distribution and user utility are not as big as those in Section 4.1. The flexibility of price adjustment partially soothes the difference between two channel characteristics.

4.3. Channel Selection

Given the prior competition analysis, it is necessary to investigate two firms' choices of the two channels. We still assume channel parameters are given exogenously based on available technology on the market. Section 5 extends some discussion of endogenizing both CS service rate and P2P hit rate.

First, we already know that the equilibrium result when two channels compete. As derived in Section 4.2, the CS channel has an equilibrium revenue of $P_1^* \Lambda_1^{eq}$, and the P2P channel has an equilibrium revenue of $P_2^* \left(1 - \Lambda_1^{eq}\right)$. Second, let us consider the situation that both firms choose the P2P channel to deliver content. In this case, a price competition leads to revenue deterioration. Because of the positive externality, if a firm lowers price to attract more traffic, it is indeed beneficial as a positive feedback to encourage more price reduction. The equilibrium result for such price competition ends up with zero revenue. Third, let us consider the situation that both firms choose the CS channel. In this case, a price deviation still leads to traffic redirection, however, due to the negative externality; it doesn't encourage further price deviation and has a pullback force. Equilibrium might be generated with positive revenue for firms.

It is now necessary to elaborate on the third scenario where two CS channels compete. Suppose in equilibrium we have the two firms with $\left(P_1^{cs}, \Lambda_1^{cs}\right)$ and $\left(P_2^{cs}, \Lambda_2^{cs}\right)$, which should satisfy the following condition because users are indifferent,

$$x\Lambda_1^{cs} + P_1^{cs} = x\Lambda_2^{cs} + P_2^{cs} \ .$$

Here, $x = d / \mu_0$. Now suppose firm 1 wants to deviate by lowering the price with a small scalar ε. Although lowering the price can bring in more traffic, it also deteriorates the user utility with a larger market size due to the negative externality. Let us denote the market demand rate increased by η, In fact, we can simply calculate this size increase by solving:

$$x\left(\Lambda_1^{cs} + \eta\right) + \left(P_1^{cs} - \varepsilon\right) = x\left(\Lambda_2^{cs} - \eta\right) + P_2^{cs} \ ,$$

which leads to $\eta = \varepsilon / 2x$. In other words, for any price decrease (increase) ε, the demand rate for this channel will be increased (decreased) by $\varepsilon / 2x$.

Since this is an equilibrium assumption, , we need to guarantee there is no incentive for either firm to deviate. With analyzing this price deviation and demand rate change, it is necessary that the following no-change-on-revenue condition holds, $P_1^{cs}\Lambda_1^{cs} = \left(P_1^{cs} - \varepsilon\right)\left(\Lambda_1^{cs} + \eta\right)$, which leads to:

$$P_1^{cs} = 2x\Lambda_1^{cs}.$$

Since the user utility needs to be non-negative, $v - x\Lambda_1^{cs} - P_1^{cs} \geq 0$, we have $\Lambda_1^{cs} \leq v / 3x$. In order to maximize the revenue, $P_1^{cs}\Lambda_1^{cs}$, it naturally leads to $\Lambda_1^{cs} = v / 3x$, $P_1^{cs} = 2v / 3$, and $R_1^{cs} = 2v^2 / 9x$.

This discussion also applies to firm 2 due to symmetry in firms. Therefore, in equilibrium, we have both firms with a market demand rate of $v / 3x$, and the revenue at $2v^2 / 9x$. Note that we need to have the total demand limited with 1, i.e., $2v / 3x < 1$, or $\mu_0 \leq 3d / 2v$.

When parameters in the feasible region (conditions in Proposition 2 part i hold and $\mu_0 \leq 3d / 2v$), the corresponding utility matrix can be written:

		Firm 2	
		P2P	**CS**
Firm 1	**P2P**	$0, 0$	$\frac{\left(v+\delta-2x+hv\right)^2}{9\left(x-hv\right)}, \frac{\left(v+\delta+x-2hv\right)^2}{9\left(x-hv\right)}$
	CS	$\frac{\left(v+\delta+x-2hv\right)^2}{9\left(x-hv\right)}, \frac{\left(v+\delta-2x+hv\right)^2}{9\left(x-hv\right)}$	$\frac{2v^2}{9x}, \frac{2v^2}{9x}$

It is obvious that the firms prefer choice of differentiating channels when

$$\frac{\left(v+\delta-2x+hv\right)^2}{\left(x-hv\right)} > \frac{2v^2}{x}.$$

Otherwise, both firms will use the CS channel. This partially explains the possibility in the digital content delivery market where there is no P2P channel and only CS channel is utilized.

4.4. Channel Integration

The competition setting described earlier is an uncoordinated setting. In other words, the equilibrium derived in the decentralized setting might not be an efficient outcome from the perspective of a social planner. In this section, we analyze an integration setting where two channels can be combined together for digital content distribution. We are interested to see if coordination allows, whether channel behavior may differ from that in competition.

If two channels are coordinated by a social planner, then a proper pricing scheme can be set to maximize the overall revenue. The setting is very similar to the competition setting with pricing, except that in the first stage, the social planner sets both prices of CS and P2P channel. Therefore, from a user's perspective, nothing changes at the second stage. Demand distribution equilibrium still holds as: $\left(\Lambda_1^*, \Lambda_2^*\right) = \left(\Lambda_1^{eq}, 1-\Lambda_1^{eq}\right)$, where

$$\Lambda_1^{eq}\left(P_2, P_1\right) \equiv \frac{v+\delta-hv+P_2-P_1}{d/\mu_0-hv}.$$

However, the social planner may make different decisions on the channel prices at the first stage. The following proposition depicts the results in this integration setting.

PROPOSITION 3. *The stable equilibrium distribution is given as follows: if* $v+\delta-2hv \geq 0$, *then when* $\mu_0 \leq \frac{d\left(hv-\delta\right)}{hv\left(v-\delta\right)}$, *the stable equilibrium distribution can be reached as* $(\Lambda_1^*, \Lambda_2^*) = (\Lambda_1^{eq}, 1-\Lambda_1^{eq})$ *where* $\Lambda_1^{eq} = \frac{v+\delta-2hv}{2\left(d/\mu_0-hv\right)}$.

To give a numerical example of Proposition 3, we keep using the set of parameters in Figure 3, except for adjusting P2P shape parameter *h* to 0.6. The adjusting is necessary to keep the conditions in Proposition 3 satisfied. Figure 5 shows the equilibrium demand distribution of channels, CS user utility, and CS service of quality. The equilibrium prices of the both channels are indicated in the right hand side figure with regards to the service rate of CS channel.

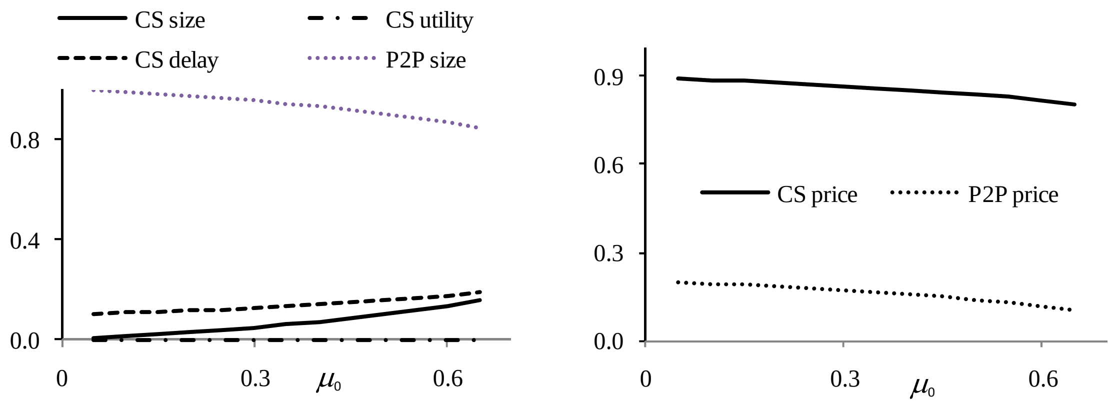

Figure 5. Equilibrium demand, user utility, CS QoS, and prices against μ_0 for $\delta = 0.4$

It can be seen that, in the feasible parameter region, the equilibrium demand of CS channel increases with the increase of CS service rate, and the equilibrium price decreases. Further, as more and more users join the CS channel, the quality of service deteriorates. User's net utility always stays at zero because of centralized price extraction. Again, the opposite effects are observed in the P2P channel: the equilibrium demand of P2P channel decreases with the increase of CS service rate. The equilibrium price of P2P channel also decreases.

It is further observed that, in this integration setting, the equilibrium demand of P2P channel is much larger than that of CS channel. Also, the equilibrium price of CS channel is always higher than that of P2P channel, which means the gross utility before pricing is higher for user in the CS channel. Nonetheless, both prices decrease with the increase of CS service rate. The difference between equilibrium prices is optimally set, and always kept at the level of $P_1^* - P_2^* = (v + \delta)/2$, regardless of the improvements in CS service rate. This coordinated equilibrium draws a different picture on how to utilize the two channels. It simply imposes a high premium price on the CS channel to partially cancel out the utility improvement from high service rate. At the same time, this equilibrium fixates an optimal price gap between the two channels, and uses a relatively low price on the P2P channel to redirect traffic to the P2P community. An ultimate explanation for equilibrium in the integration setting may still trace back to the channel characteristics: CS with negative externality and P2P with positive externality. The coordinated equilibrium efficiently allocate demand (by adjusting the content pricing) to achieve an overall maximized revenue, where P2P channel has its preferred more demand and CS channel has it preferred less demand. In fact, this overall maximized revenue is indeed the maximized user gross utility, because both equilibrium prices represent the gross channel utility. This result thus demonstration a socially optimal channel integration.

4.5 Equilibrium Comparisons

To further compare the equilibrium in various settings, we plot these equilibrium comparisons in Figure 6 with $\delta = 0.01$ and $h = 0.5$, while all the other parameters stay the same as in Figure 2. This parameter set fulfills the equilibrium conditions stated in both Proposition 2 and 3.

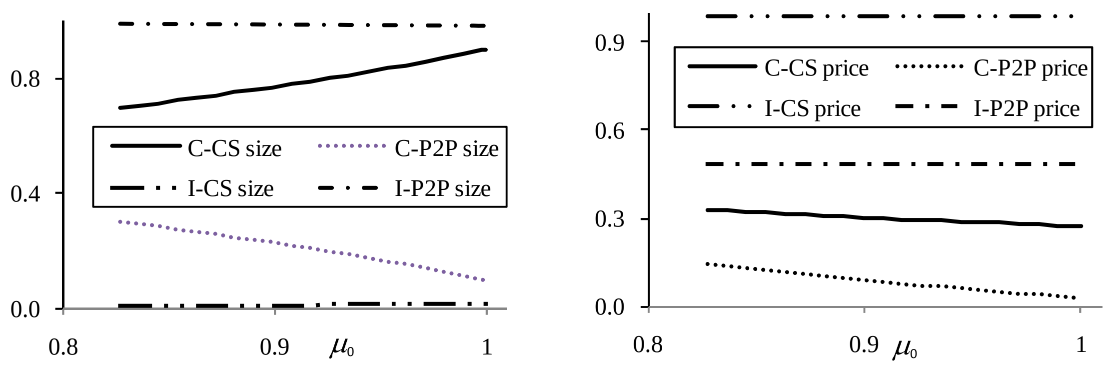

Figure 6. Competition and integration equilibrium comparison

First, Figure 6 illustrates the equilibrium demand distributions in both the competition and integration settings. It is shown at the same level of service rate, P2P channel attracts majority of the demand as a socially efficient practice. However, in the competition setting, P2P demand is not only less than that of the CS channel, but also decreases quickly when CS service rate improves. It can also be noted that the demand change in the integration setting is relatively stable with regards to the continuous improvement of CS service rate.

Second, Figure 6 also shows the phenomenon of price dispersion in both settings. In the integration setting, the optimal price is essentially the gross utility derived from either CS or P2P channel. The equilibrium suggests a differentiation pricing strategy which limits the demand of CS channel to highlight a premium service: a very small group of users and a very high price. Further, more demands are directed to the P2P channel with a relatively low price due to its scalability and positive externality. In the competition setting, the price of CS channel is also higher than that of P2P channel. But the price of P2P drops sharply when the CS service rate improves. Further, the equilibrium prices in the competition setting are lower than those in the integration setting.

Finally, it is noted that with this parameter set, there will be no equilibrium reached for an organic competition. The service rate is relatively high here, and all the users join the CS channel with no utilization of P2P channel. When competition with pricing is allowed, both P2P and CS channels are utilized and the equilibrium gross utility is higher than that without pricing mechanism. However, CS demand rate is still larger than that in the P2P channel, and the gross utility of CS channel is higher than that in the P2P channel. Finally, in an integration setting, not only both channels exist, but P2P channel attracts majority of the demand, and CS is priced as a premium services for a limited number of users. The gross utility for user is the highest in this integrated setting. Further, user gross utility in the CS channel is higher than that in the P2P channel. We summarize these observations in the following table.

Table 1. Summary of Equilibrium Comparison

	Demand rate of CS channel	**Demand rate of P2P channel**	**Gross utility before price of CS channel**	**Gross utility before price of P2P channel**
Organic competition	1	0	Lowest	0
Competition with pricing	Large	Small	Medium	Small
Integration	Small	Large	High	High

Overall, the integration setting has a socially efficient allocation of demand by factoring both the negative externality from CS channel and the positive externality from P2P channel. Users enjoy a higher gross utility either in CS or P2P channel than in the competition setting. Corresponding incentive mechanism needs to be designed to encourage more utilization of P2P channel in the digital content distribution market.

5. Extended Models

We study the competition and integration of two digital content distribution channels: central-server based and peer-to-peer based. In order to capture the major channel characteristics: positive externality in the P2P channel and negative externality in the CS channel. We choose a basic user utility model to conduct the analysis. It is found that the integration setting promotes the utilization of P2P channel in accordance to its positive externality, while the competition setting still witnesses a heavy utilization of the CS channel regardless of its negative externality. Before applying this finding to current digital content market, we need to elaborate on several extensions of the basic model.

First, in Section 3, user valuation toward the content is homogenous in this study. In a more comprehensive model, it will be necessary to incorporate the heterogeneity among users. Intuitively, when user's valuation upon the content varies, it is more likely that the feasible equilibrium can be changed. To show this effect, we assume that user's valuation $\tilde{v}$ is a random variable which is uniformly distributed between 0 and 2*v*. Here we keep the mean valuation to be *v*. The corresponding utilities for CS and P2P channels are $\tilde{v}-(d/\mu_0)\Lambda_1$ and $h\Lambda_2\tilde{v}-\delta$, respectively. This yields, for users whose valuation is above $\tilde{v}\geq\delta/h\Lambda_2$ will select P2P or CS channel; while for those whose valuation satisfies $\tilde{v}-(d/\mu_0)\Lambda_1\geq h\Lambda_2\tilde{v}-\delta$, or

$$\tilde{v}\geq\frac{(d/\mu_0)\Lambda_1-\delta}{1-h\Lambda_2},$$

will select CS channel. Hence, we can write the equilibrium equations:

$$2v\Lambda_1=2v-\frac{(d/\mu_0)\Lambda_1-\delta}{1-h\Lambda_2} \text{ and } 2v(\Lambda_1+\Lambda_2)=2v-\frac{\delta}{h\Lambda_2}.$$

Solving these two equations simultaneously yields the equilibrium demand to each channel. The explicit expression involves a cubic equation whose solution is cumbersome. For a special case, we set $\delta=0$. We have:

$$\Lambda_2^* = \frac{2v + d / \mu_0 - \sqrt{(2v + d / \mu_0)^2 - 8hvd / \mu_0}}{4hv}.$$

It can be easily shown that $\partial\Lambda_2^* / \partial\mu_0 < 0$. This is consistent with our finding in Section 4.1, indicating our results are robust with respect to user heterogeneity.

It is always possible to argue that channels should be able to choose its service rate (CS channel) or the hit rate (P2P channel) to compete in the market. In this study, both the service rate and hit rate are exogenous before the pricing decision. In fact, a content channel provider, either CS or P2P, does choose its service rate or the hit rate first through available technology investment. This infrastructure decision or long term decision needs to be made first at the channel set up period. Only after then, the content pricing decision and market competition or integration can come into play. Therefore, if we want to extend the model with a longer time horizon to incorporate the infrastructure decision, the result in this paper will still hold and serve as a sub-game equilibrium to feedback to the setup period. More specifically, given the anticipated results derived here, channels can make decisions in the setup period to balance the recurring revenue, infrastructure set up cost, and other factors. An optimal level of service rate or hit rate can be derived consequently with more infrastructure level information. It can be easily shown that the CS profit $\pi_{CS} = P_1\Lambda_1$ increases with its capacity μ_0, that is, $\partial\pi_{CS} / \partial\mu_0 > 0$. If we set $\partial\pi_{CS} / \partial\mu_0$ to the marginal cost of μ_0, we can derive an optimal server capacity level. Similarly, the P2P channel can optimize its hit rate *h*, given its marginal cost.

6. Conclusions and Future Directions

Digital content distribution has become one of the most popular applications over internet (Cisco 2010). Users listen to music online, watch movies online, play games online, and even get professional consultation online. The two major distribution channels of digital contents can be categorized as central server based channel and peer-to-peer based channel. Users can obtain digital content from either channel as long as the valuation of content is no less than the cost of obtaining the content. Given a market where both channels compete, a user simply self-selects one channel that provides a higher comparable utility.

Recently, this content distribution market witnesses a dramatic increase of demand through the CS channel and a decline of P2P community size. In order to investigate this phenomenon and provide deeper understanding of both channels, we use a basic user utility model to capture a key distinction between the two channels: positive externality in the P2P channel and negative externality in the CS channel. More specifically, a larger demand size in the CS channel leads to congestion and higher delay; while a larger demand size in the P2P channel is favorable because the possibility of locating the content increases with a larger community size.

We start with the analysis of organic channel competition purely based on exogenous channel characteristics. It is found that there exists a threshold for the service rate of CS channel. P2P channel can compete in the market only when the service rate of CS channel is lower than the threshold. Then, we extend the discussion to competition with pricing. Incorporating the pricing decision for channels allow the flexibility for either channel to sooth the difference between channels. It is found that, only when the service rate of CS channel is in a moderate range, a stable market equilibrium with positive user utility can be reached, featuring a higher demand rate in the CS channel. In order to compete, the normalized price of P2P channel has to be reduced significantly when the service rate of CS channel increases. It can also be shown that the equilibrium utility first increases and then decreases with the increase of service rate of CS channel, despite an increasing demand rate for CS channel. In other words, the demand in CS channel quickly outgrows the capacity, therefore, the quality of service provided by the CS channel worsens with an increased capacity. These results in the competition setting match with the status of digital content market, where content distribution through the CS channel becomes more popular than P2P in the past few years, thanks to technology advancement in improving CS service capacity.

Although the competition equilibrium reflects the reality, in order to address the second research question raised in the introduction section, we are also interested in identifying whether there are more efficient demand allocations between the two channels. We set up an integration setting to let two channels coordinated by a social planner. By maximizing the total revenue of two channels together, it is found that an optimal solution suggests heavy utilization of the P2P channel. In fact, the CS channel is priced as a premium service and only serves a very small amount of users. This coordinated allocation indeed aligns the market equilibrium with the channel characteristics: utilizing more P2P in light of its positive externality and limit the demand in CS channel to alleviate its negative externality. Finally, the gross utilities for users are the highest in the equilibrium derived from the integration setting.

Our study pinpoints a unique distinction between a CS based and P2P based channel. With the comparison of these equilibrium outcomes, this study provides guidance of understanding the different network externalities in two channels. In order to fully utilize both channels, social policies and incentive mechanisms need to be designed to align the competition equilibrium with the efficient demand allocation in the integration setting.

When competition with pricing is introduced, we adopt a common business model of charging user with a flat channel service fee. Pricing decision can be used to adjust demand rate for both CS and P2P channels.

There are several directions to extend this paper. We extend the basic model to accommodate heterogeneous users with respect to their valuations for a specific digital content. However, the content to be distributed is still assumed to be homogeneous. Current market practice involves differentiating user's request by the content popularity where only the most popular content can be distributed by the P2P channel. In fact, it would be interesting to investigate the impact of content popularity on channel utilization. Furthermore, we have already noticed the geographic difference of channel utilization. Currently, P2P channel is still very popular in Asian countries, while CS channel is taking the dominating position in North America. It will be interesting to find theoretical support for this geographic difference.

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Appendix

Proof of Proposition 1

In order to derive the equilibrium distribution as indicated in Figure 1, the intersection point can be calculated as $(\Lambda_1^*, \Lambda_2^*) = (\Lambda_1^{eq}, 1-\Lambda_1^{eq})$, where

$$\Lambda_1^{eq} \equiv \frac{(v+\delta)-hv}{d/\mu_0 - hv}.$$

Now we need to discuss the feasibility of this equilibrium. The solid line boundary intersects with the axis at $\left(0, \frac{v+\delta}{hv}\right)$ and $\left(\frac{\mu_0(v+\delta)}{d}, 0\right)$. Comparing these two points with the dotted line, $\Lambda_1 + \Lambda_2 = 1$, we have the following scenarios to analyze.

If $\frac{\mu_0(v+\delta)}{d} \le 1$, and since $\frac{v+\delta}{hv} > 1$, there should exist a stable equilibrium as shown in Figure 1. Further, the non-negative user utility constraints conditions also need to be satisfied. Because at the equilibrium, $U_{\text{CS}} = U_{\text{P2P}}$, only one of them needs to be checked:

$$U_{\text{CS}} \geq 0 \Rightarrow \Lambda_1^{eq} \leq \Lambda_1^{c} \Rightarrow \mu_0 \leq \Gamma^{\min} \equiv \frac{d\left(hv-\delta\right)}{hv^2} < \frac{d}{v+\delta}.$$

This simply means, when $\mu_0 \leq \Gamma^{\min}$, we can have a stable equilibrium of the demand distribution.

Further, if $\Gamma^{\min} < \mu_0 \leq \frac{d}{v+\delta}$, there is still an intersection point, but the user utility realized at this point is negative, which means the equilibrium demand rate for CS channel is larger than the allowed upper bound. Therefore, the demand rate of CS channel has to be decreased to the upper bound along the dotted line, which results in zero CS user utility. At the same time, P2P channel still cannot provide positive utility and no user will join P2P channel.

If $\frac{\mu_0\left(v+\delta\right)}{d} \geq 1$, and since $\frac{v+\delta}{hv} > 1$, there is no intersection point, and all users live below the boundary line, so CS channel dominates the market.

Proof of Proposition 2

Suppose that an equilibrium is reached at $\left(\Lambda_1, \Lambda_2; P_1, P_2\right)$. Users in each channel receive the utility:

$$U_{CS} = v - \frac{d}{\mu_0}\Lambda_1 - P_1 = u, \text{ and } U_{P2P} = hv\Lambda_2 - \delta - P_2 = u.$$

Now, let's perturb this equilibrium. Suppose P2P provider lowers its price by an infinitesimal amount of ΔP_2, that is, $P_2 \to P_2 - \Delta P_2$. Due to the increased user utility in P2P channel, a portion of users ($\Delta\Lambda_2$) will shift from CS channel to P2P. $\Delta\Lambda_2$ should yields the same utility for both channels, that is:

$$\frac{d}{\mu_0}\Delta\Lambda_2 = hv\Delta\Lambda_2 + \Delta P_2.$$

Therefore,

$$\Delta\Lambda_2 = \frac{\Delta P_2}{d/\mu_0 - hv}.$$

The P2P channel's new profit is $\left(P_2 - \Delta P_2\right)\left(\Lambda_2 + \Delta\Lambda_2\right)$, which if is below that previous value of $P_2\Lambda_2$, there if no incentive for P2P provider to make the move. We have the condition:

$$\Lambda_2 \geq \frac{P_2}{d / \mu_0 - hv}.$$

Similarly we can derive the condition for CS provider not to move, explicitly,

$$\Lambda_1 \geq \frac{P_1}{d / \mu_0 - hv}.$$

It can verify that the inequality for CS channel is always binding. While if the inequality for P2P channel is binding, we can substitute them into the first two equations in this proof. This gives:

$$\Lambda_1 = \frac{v-u}{2(d / \mu_0) - hv} \text{ and } \Lambda_2 = \frac{\delta + u}{2hv - (d / \mu_0)}.$$

To solve for *u*, we use $\Lambda_1 + \Lambda_2 = 1$. We have:

$$u = \frac{-2(d / \mu_0)^2 + (5hv + v - 2\delta)(d / \mu_0) - hv(2hv + 2v - \delta)}{3((d / \mu_0) - hv)}.$$

If $u \geq 0$, we plug it to Λ_1 and Λ_2. Then we obtain the results stated in Part i. If we solve $u = 0$, we find the two thresholds of capacity μ_L and μ_U, such that if $\mu_L \leq \mu_0 \leq \mu_U$, $u \geq 0$.

Outside of this region, we set $u = 0$ and the inequality for P2P is not binding. So, we solve for CS channel and get:

$$\Lambda_1 = \frac{v}{2(d / \mu_0) - hv},$$

While the price is $P_1 = ((d / \mu_0) - hv)\Lambda_1$. For P2P, the demand $\Lambda_2 = 1 - \Lambda_1$, and its price is set to make $U_{P2P} = hv\Lambda_2 - \delta - P_2 = 0$.

Proof of Proposition 3

Proof: first, it is noted that the difference of the two prices actually determines the equilibrium demand at the second stage. Since now the pricing decision can be centralized, we can simply denote $dP = P_2 - P_1$. The total revenue maximization problem for the social planner is

$$\max_{P_1,P_2} R = P_1 \cdot \Lambda_1^{eq}\left(P_1,P_2\right) + P_2 \cdot \left(1-\Lambda_1^{eq}\left(P_1,P_2\right)\right)$$
$$s.t. \quad U_{cs} = U_{p2p} \ge 0,\ P_1 \ge 0, P_2 \ge 0$$
,

which is equivalent to

$$\max_{P_1,dP} R = dP\left(1-\Lambda_1^{eq}\left(dP\right)\right) + P_1$$
$$s.t. \quad U_{cs} \ge 0,\ P_1 \ge 0$$
.

Here, in order to capture non-negative utility constraint, it can be seen that if there exists an optimal pair of $\left(P_1^*, dP^*\right)$, the optimal price of CS channel is always able to extract all the user utility, that is, $P_1^* = v - \frac{d}{\mu_0}\Lambda^{eq}\left(dP^*\right)$. Therefore, we have an equivalent unconstrained maximization problem as:

$$\max_{dP} R = dP\left(1-\Lambda_1^{eq}\left(dP\right)\right) + v - \frac{d}{\mu_0}\Lambda^{eq}\left(dP\right).$$

Solving the first order condition, and checking the second order condition ($d/\mu_0 - hv > 0$), we have

$$dP^* = -\frac{v+\delta}{2},\ \Lambda_1^{eq} = \frac{v+\delta-2hv}{2\left(d/\mu_0 - hv\right)},$$

$$P_1^* = \frac{\left(v-\delta\right)d/\mu_0 + 2hv\left(d/\mu_0 - v\right)}{2\left(d/\mu_0 - hv\right)},\ P_2^* = \frac{\left(hv - 2d/\mu_0\right)\delta + hv\left(2d/\mu_0 - v\right)}{2\left(d/\mu_0 - hv\right)}, \text{ and}$$

$$R^* = \frac{\delta^2 + 2\delta\left(v - 2d/\mu_0\right) + v\left(4hd/\mu_0 + v - 4hv\right)}{4\left(d/\mu_0 - hv\right)}.$$

With this equilibrium result, it is necessary to validate all the constraints. First, in order to satisfy $0 \le \Lambda_1^{eq} \le 1$, it is required that $\mu_0 < \frac{d}{hv}$ *and* $0 \le v + \delta - 2hv \le 2\frac{d}{\mu_0} - 2hv$. Second, the non-negative constraint of prices still needs to be enforced. Since $dP^* < 0$, we only need to guarantee $P_2^* \ge 0$, which leads to $\mu_0 \le \frac{2d\left(hv-\delta\right)}{hv\left(v-\delta\right)}$. Combining these conditions together, we summarize the feasible region of the equilibrium as the proposition stated.