Engineering Optimal Network Effects via Social Media Features and Seeding in Markets for Digital Goods and Services

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Firms nowadays are increasingly proactive in trying to strategically capitalize on consumer networks and social interactions. In this paper, we complement an emerging body of research on the engineering of word-of-mouth (WOM) effects by exploring a different angle through which firms can strategically exploit the value-generation potential of the user network. Namely, we consider how software firms should optimize the strength of network effects at utility level by adjusting the level of embedded social media features in tandem with the right market seeding and pricing strategies, in the presence of seeding disutility. We explore two opposing seeding cost models where seeding-induced disutility can be either *positively* or *negatively* correlated with customer type. We consider both complete and incomplete information scenarios for the firm. Under complete information, we uncover a complementarity relationship between seeding and building social media features which holds for both disutility models. When the cost of any of these action increases, rather than compensating by a stronger action on the other dimension in order to restore the overall level of network effects, the firm will actually scale back on the other initiative as well. Under incomplete information, this complementarity holds for seeding disutility is negatively correlated with customer type but may not always hold in the other disutility model, potentially leading to fundamentally different optimal strategies. We also discuss how our insights apply to asymmetric networks.

Key words: social commerce and social media; network effects; social interaction; seeding; adoption process; digital goods

1. Introduction

For many categories of products, it has been widely known in the industry and documented in a rich research literature that the existing network of users can impact in many ways the adoption process. First, it may induce word-of-mouth (WOM) effects, leading to faster or more efficient propagation of information about the product, helping consumers in the valuation learning process.

Second, increased adoption within certain social groups, even in the absence of informative interpersonal communication, may lead to herding behavior, i.e., imitation effects, especially when the intended market exhibits homophilic tendencies. Third, if the product is susceptible to network effects at individual utility level, a larger network may boost the value of the product to each user and, implicitly, increase the willingness-to-pay (WTP) of potential adopters.

The relatively recent advent of social media allowed firms a greater ability to capitalize on the network of users. Many businesses with an online storefront (e.g., Amazon, Newegg, Best Buy, Target, Beach Camera, Apple App Store, etc.) introduced features and channels to allow users to rate products. Similarly, many practices and businesses (from car dealers to medical doctors) are now rated online by customers. Two-sided platform providers (e.g., eBay, eLance, Airbnb) introduced feedback mechanisms through which participants can build reputation. Many content creation and productivity software (e.g., Adobe, Google Docs, Microsoft Word, wikis) introduced collaboration tools which made these products or services more appealing to various users. Along the same lines, several cloud storage services (e.g., Dropbox, Mozy) allow users to share documents. Some companies also support online forums through which customers can interact with each other and start discussion threads about product-related topics (e.g., Dell, Amazon Web Services). Moreover, the value of social and professional networks (e.g., Facebook, LinkedIn, Salesforce Chatter), massive multiplayer online games (e.g., Blizzard's World of Warcraft), video, voice, and text chatting tools (e.g., Google Talk, Yahoo Messenger, Skype), or blogging and microblogging tools (e.g., Twitter) is predicated on social interactions. Many other examples can be mentioned where social media features have facilitated growth in value or acceleration of information dissemination in association with products or services by enhancing the potential benefit of user interactions.

Most of the literature capturing the influence of consumer networks on the product adoption traditionally considers the manifestation, strength, and impact of such effects outside the influence reach of the firm (e.g., Bass 1969, Robinson and Lakhani 1975, Kalish 1983). In other words, while social interactions do occur and have been shown to influence consumer behavior, very few studies actually explored how the firms should manage and strategically influence these interactions (Godes et al. 2005). Recent studies began opening this path, focussing primarily on firm's strategies and opportunities to engineer WOM effects. Biyalogorsky et al. (2001) study how consumer referral actions should be incentivized. Dellarocas (2006) inspects how strategic manipulation of online forums can shift the information value of online reviews for customers. Chen and Xie (2008) explore the firm's benefit from establishing an online community where consumers can post reviews. Forman et al. (2008) empirically show that the prevalence of reviewer disclosure of identity information can be associated with an increase in consumer trust in the reviews with impact on subsequent online product sales. Thus, firms might benefit from strategically creating online review platforms and incentive mechanisms that encourage reviewers to share more information. Godes and Mayzlin (2009) empirically study how firms should strategically recruit customers for WOM campaigns based on loyalty considerations in order to drive sales. Aral and Walker (2011) highlight the effectiveness of viral product features in generating social contagion. Aral et al. (2011) analyze the performance of seeding and referral incentive programs as two popular methods to engineer social contagion.

We extend this literature by considering how the firm can strategically engineer the *strength* of network effects at utility level. On this dimension, we focus on deriving the optimal level of social media functionality that increases the value of social interactions to each user. Such functionality includes features and environments that facilitate communication between users (e.g., chat capabilities, virtual reality environments where avatars can interact), collaboration on and co-creation of content (e.g., wikis, content editing and tagging), peer referral (e.g., on professional networks), building of reputation, etc.

Our study focuses on paid digital goods and services where the value is mostly induced by the network. Some examples include (but are not restricted to) massive multiplayer online games (e.g., World of Warcraft), social dating sites (e.g., eHarmony, Match.com), professional social networks (Salesforce's Chatter Plus), specialized online forums with paid memberships (e.g., Naturescapes.net). We point out that our results go beyond digital goods and services, and apply to other products and services (e.g., voice communication services) where marginal costs are negligible and the bulk of value is derived from the network. When network effects strongly dominate stand-alone benefits from the product (i.e., benefits in the absence of the network), firms may find it profitable to spark adoption by giving away some consumption for free. In this paper, we focus on *seeding* strategies, whereby the firms give the products with full functionality and perpetual license to a few customers in order to boost the Willingness-to-Pay (WTP) of other customers and catalyze adoption (Lehmann and Esteban-Bravo 2006, Jiang and Sarkar 2009, Galeotti and Goyal 2009). Alternative strategies that are also employed in the industry to spark adoption involve freemium approaches (limited-time free trials or free versions with stripped-down functionality - Niculescu and Wu 2012).

In addition to potential demand cannibalization and boost in network effects, in the context of paid products, seeding may induce a separate effect on the paying customers. If some customers are charged for the product, then seeding implies price discrimination in the market: seeded customers pay less than unseeded individuals who end up buying the product. If seeding is extensive and paying customers observe it, then they might consider the price scheme unfair. Extensive experimental studies have been conducted to illustrate that price discrimination could potentially lower customers' WTP. Oliver and Shor (2003) show evidence of strong negative effects on fairness perception, satisfaction, and purchase completion among online shoppers that are prompted to enter a coupon code towards the conclusion of the checkout process for such customers that did not receive a code in advance. Novemsky and Schweitzer (2004) compare the internal social comparisons (between buyer and seller) and external social comparisons (between buyer and buyer). They find that buyer's satisfaction will be increased if other buyers have a smaller surplus. Xia et al. (2004) provide a literature review and conceptual framework to understand the fairness of pricing. When price comparisons are perceived as unfavorable for similar transactions, they predict that customers will have an adverse response to the seller's strategy, which may involve negative emotions (such as anger or outrage), negative WOM, and reduced demand. According to Hinz et al. (2011), when price discrimination is observed, it is often the case that customers feel unhappy about the unfair pricing. Also, it is not uncommon in the industry for adopters to question pricing practices of the firms and exert pressure on them. For example, Amazon offered a public apology and refunds to over 6,000 customers in response to public backlash over a series price tests through which different online shoppers were quoted different prices for various DVDs (Weiss 2000). In another example, after introducing first generation iPhone in June 2007, within just three months Apple dropped the price by \$200 for the entire market. Faced with a flood of complaints from early adopters, Apple decided to refund each of them \$100 in the form of store credit (Wingfield 2007).

Building on prior literature and industry observations, in the context of digital goods and services with an associated price tag, we formalize the negative effects of seeding associated with price discrimination via a disutility incurred by paying customers. In our model, the more customers are seeded, the higher is the backlash and valuation downgrading from paying customers who question the fairness of the pricing scheme. To capture various potential market scenarios, we consider two contrasting seeding disutility models. Under the first seeding disutility model, SDU^{+}, for each customer, the seeding-induced disutility is *positively* correlated with her type. Thus, the highest type customers are experiencing the highest seeding-induced disutility. Under the second model, SDU^{-}, every customer experiences a seeding disutility that is *negatively* correlated to her type. In this case, high type customers do not experience much disutility due to seeding. Various examples justifying each setup have been included in the main text.

In this paper we explore the tradeoff between benefits and costs associated with seeding and building social media features into the digital product. On one hand seeding boosts WTP of potential customers because it leads to larger user networks. On the other hand, seeding induces disutility for paying customers and, contingent on seed allocation, can cannibalize demand. Similarly, building social media features that boost the strength of network effects at the utility level (i.e., by allowing more value extraction from social interactions) would lead to increased WTP for the customers but involves building efforts. Taking these tradeoffs into consideration, we seek out to find what are the optimal seeding, pricing, and social media strategies for the firm.

Depending on firm access to market information, we explore two scenarios: complete information on firm side (the firm knows enough about the customers such that it can perform targeted seeding) and incomplete information (the firm does not know much about the customers besides the consumer distribution and cannot resort to targetted seeding). First, under complete information, for each of the disutility models, we solve completely the market equilibrium, discuss market coverage, and investigate the interaction effects between seeding and building social media features. Under both disutility scenarios, we find that if the marginal cost/penalty associated with one of these initiatives increases, the firm will scale back (or sometimes leaves unchanged) its efforts on the other dimension as well. This is interesting because, at the utility level, both actions increase the impact of network effects. However, if the investment required to build more social media features in the product is higher, while the firm scales back on such features, it will not try to compensate by seeding more. Similarly, if the seeding disutility rate is higher, while the firm scales back on seeding, it does not try to boost the strength of network effects.

Under incomplete information, while the firm decides strategically on the seeding volume, we assume the seeds end up being spread *uniformly* in the market. Under disutility model SDU^{-}, complementarity between seeding and building social media features continues to hold. However, under SDU^{+}, some new patterns emerge. When the seeding penalty is small, it may be actually possible for both seeding and building social media features to be both increasing in the seeding penalty. This is a different kind of complementarity, where actions adjust in the same direction but in the opposite direction to the penalty. When seeding penalty is intermediate, the two actions act as substitutes to each other with respect to changes in penalty rate. When the seeding penalty increases, the firm reduces seeding and builds at the same time stronger network effects. Once seeding disutility is large, we encounter the same complementarity effects as in all the other settings. Also, for small social media building costs, the actions act as substitutes to each other with respect to changes in social media costs. If it is more expensive to build social media features but not too expensive, the firm increases the seeding ratio and decreases the level of social media features. However, once the social media cost becomes large enough, the outcome reverts to the previously uncovered complementarity in actions.

We also explore how our frameworks and results extend to asymmetric networks where connections between users activate only in one direction. We show that some of the previous insights continue to remain robust, and we also inspect various seeding patterns induced by the network structure. A discussion of the value of information is included in the conclusion.

At a high level, in the industry, firms put a lot of effort in harnessing network effects and this paper provides a host of novel and managerially relevant insights into how to optimally use *in* *tandem* two levers (seeding and building social media features) aimed at boosting network effects taking into account how they can jointly impact the market outcome.

The rest of the paper is structured as follows. In §2, we introduce our general modeling framework. In §3, we present the analysis of the complete information case. In §4, we extend our discussion to incomplete information settings. In §5, we extend our analysis to asymmetric networks and provide further discussions on robustness of key findings. We conclude in §6. All proofs of our results can be found in the Appendix.

2. General Model

Consider a software market with a monopolistic firm and a heterogeneous and stationary pool of potential customers of mass normalized to 1 and types θ distributed uniformly in the interval $[0, 1]$. For simplicity, we focus on a symmetric, fully connected consumer network and assume the software exhibits *heterogeneous* network effects. Thus, if the current installed base has size δ, then a customer of type θ will get a direct benefit $b\delta\theta$ from the software, where b captures the strength of network effects. Apart from the network-generated value, we assume the product/service carries negligible stand-alone value. Our model is consistent with setups in Dhebar and Oren (1985, 1986).

In order to boost paying customers' product valuation, the software firm seeds $\alpha\%$ of the market. While the seeding process in itself jumpstarts adoption, it also generates *disutility* at the individual level for paying customers as discussed in the Introduction. For each paying customer θ, we model this disutility as a function $\Delta(\alpha, \theta)$ that is nonnegative, convex, and increasing in α, and captures heterogeneity of this effect as experienced by each customer. The nonlinear dependence on the size of the seeded pool captures the fact that disutility is very limited for small seeded pools but rapidly increases as more customers are seeded.

In this paper we focus on the adoption *sequence*, whereby customers progressively join the network towards a market equilibrium. Seeding occurs immediately before the product is released for sale. Customers do not know the overall type distribution in the market and act in a myopic fashion, making their adoption decision on the perceived utility from the product computed based on the current observed installed base. In general, in the software industry (and others), there are many cases where firms make public the information regarding installed base or such information is estimated and reported with regularity by market research firms.[1] Consistent with such observations, we assume in our model that customers have access to this information. If at a given moment the installed base is of size δ (including seeded customers), then a paying customer of type θ would momentarily perceive the utility from buying the product as:

$$u(\theta|\alpha, b, p, \delta) = b\delta\theta - \Delta(\alpha, \theta) - p. \tag{1}$$

A customer of type θ adopts as soon as she perceives $u(\theta|\cdot) \geq 0$. This adoption sequence is consistent with setups inRohlfs (1974) and Dhebar and Oren (1985), among others.

The firm is proactive in managing the strength of network effects via choosing the right amount of social media features that boost the value of social interactions to each user. As such, we assume that the firm will incur a convex cost cb^2 to induce network effects at marginal strength b, with $c > 0$. If we denote by $N(\alpha, b, p)$ the number of *paying* customers *at the conclusion* of the sequential adoption process (in equilibrium), then the firm's optimization problem becomes:

$$\max_{\alpha \in (0,1), b>0, p>0} \pi(\alpha, b, p) = pN(\alpha, b, p) - cb^2. \tag{2}$$

We consider all development efforts with the exception of the building of social media features sunk. Moreover, as we are focusing on digital goods and services, the reproduction costs are assumed negligible.

While, as discussed in the Introduction, it has been documented that price discrimination can lead to a decrease in willingness to pay due to perceived unfairness, there is very little research connecting this disutility to consumer characteristics. Related literature offers various insights as to how various customer groups react to negative firm actions (e.g., price increases or service failures). For example, under conditions of high price inequality, consumers that shop with higher frequency perceive price increases as less fair compared to customers that shop with lower frequency (Huppertz et al. 1978). A different study by Martin et al. (2009) takes a somewhat opposite stance

[1] For example VG Chartz (www.vgchartz.com) reports industry-wide weekly sales numbers (in terms of number of units sold) for both software games and console hardware.

by showing that loyal customers do not necessarily perceive major price increases less fair than non-loyal customers (and, in the case of small price increases, actually the opposite might occur). The same study proposes that, under conditions of a price increase, post customer loyalty is greater for previously loyal customers than non-loyal customers. Such findings are consistent also with Hess et al. (2003), whereby it is argued that more loyal customers invest in maintaining the relationship with the vendor and, thus, are more forgiving towards minor negative actions compared to non-loyal customers. However, some customers may come to expect certain relational benefits in exchange for their loyalty. For example, customer service quality expectations may be positively correlated to the longevity of the customer-firm relationship duration (Heilman et al. 2000).

Thus, the above literature hints at evidence that disutility from firm's actions may be different for distinct customer groups. However, in the absence of a clear consensus regarding how seeding-induced disutility is related to customer type (or WTP), for completeness of the analysis we choose to explore two opposing models in order to account for various market peculiarities. The seeding disutility functions under the two models are parameterized as follows:

$$\text{Model } SDU^{+}: \qquad \Delta(\alpha,\theta) = s\alpha^2\theta,$$
$$\text{Model } SDU^{-}: \qquad \Delta(\alpha,\theta) = s\alpha^2(1-\theta).$$

Under model SDU^+, for each customer, the seeding-induced disutility is *positively* correlated with her type. Thus, the highest type customers are experiencing the highest seeding-induced disutility. Under model SDU^-, every customer experiences a seeding disutility that is *negatively* correlated to her type. In this case, high type customers do not experience much disutility due to seeding.

If adoption starts, then, at any subsequent moment, the instantaneous utility is increasing in type under both SDU^+ and SDU^-.[2] Consequently, our model is consistent with the extant literature on vertical differentiation in the sense that if a customer adopts, all higher type customers must

[2] Under SDU^+, at a given point along the adoption process, if installed base is δ, then $u(\theta|\cdot) = (b\delta - s\alpha^2)\theta - p$. If the adoption starts, it means that $b\alpha - s\alpha^2 > 0$ such that the firm can charge a positive price. Then, at later stages, $\delta \geq \alpha$ because later-stages installed base includes both seeded and paying customers. Under SDU^-, $u(\theta|\cdot) = (b\delta + s\alpha^2)\theta - s\alpha^2 - p$. Thus, for any $\delta \geq \alpha$, if adoption starts, instantaneous utility is increasing in customer type at any given point.

adopt as well. As such, customer type is positively correlated with instantaneous consumer WTP. If there are different types that perceive at a given time non-negative utility from the product and they have not adopted yet, for simplicity we assume the higher type moves first. In that sense, we assume that type (higher utility) is positively correlated with the urgency to use the product for whatever mission-critical needs that customer has.

3. Complete Information

We first consider the case where the firm has complete information about customer types and, thus, can perform seeding targeted towards specific individual types. For example, such scenarios may correspond to markets where consumers leave a considerable and relevant informational footprint after (online) activities which is made available to the vendor. Such information may be collected perhaps in association with the consumption of a related product/service offered by the same vendor or by a partner of the vendor.

3.1. Optimal Strategy Under Model SDU^+

We start by exploring necessary conditions for optimality:

Lemma 1. *Under SDU^+ and complete information, if the firm stays in the market (i.e., it can make profit) then its optimal strategy $\{\alpha^*, b^*, p^*\}$ and optimal seed allocation must satisfy the following:*

(i) $b^*\alpha^* - s\alpha^{*2} \geq p^*$;

(ii) all customers with types $\theta \in [0, \alpha^*)$ *are seeded;*

(iii) all customers with types $\theta \in [\alpha^*, 1]$ *purchase the product.*

Part (i) of Lemma 1 states that seeding cannot be effective at sparking adoption unless it is coupled with strong enough network effects. In other words, the optimal strategy has to be chosen in such a way that at least the highest type customer wants to adopt at the very beginning. Parts (ii) and (iii) basically capture the fact that, under optimal seeds allocation, there is no segment of the market left without a product. While it seems more or less intuitive that seeds should go to the low type customers to prevent sales cannibalization, what is interesting is that, under the optimal strategy, if any unseeded customer purchases the product, then all unseeded customers purchase the product. Seeding more induces two opposing effects at the instantaneous utility level: it increases the network benefits and it also increases the seeding disutility. Nevertheless, the firm will choose to manipulate the three controls (seeding, pricing, and social media features) in such a way as to seed right up to the lowest-type paying customer. Adoption occurs in decreasing order of types for customers with type $\theta \in [\alpha, 1]$. The following result characterizes the optimal strategy of the firm.

Proposition 1. *Under SDU$^+$ and complete information, the following hold:*
(i) if $cs \geq \frac{1}{4}$, then the firm exits the market;
(ii) if $cs < \frac{1}{4}$, then the firm's optimal strategy $\{\alpha^*, b^*, p^*\}$ is given by:

$$\alpha^* = \frac{3(1-2cs) - \sqrt{1-4cs+36c^2s^2}}{4} \leq \frac{1}{2}, \quad b^* = \frac{\alpha^*(1-\alpha^*)}{2c}, \quad p^* = b^*\alpha^* - s{\alpha^*}^2. \tag{3}$$

As it turns out, the individual rationality (IR) constraint at adoption time is binding for the highest type (the first paying adopter). Thus, under an optimal strategy, the firm will choose a price such that seeding just jumpstarts adoption and this minimal push is enough for the adoption to gain momentum and not stall until every unseeded consumer has a product.

We focus our attention on the interaction between building more social media features to increasing the strength of network effects and seeding the market. Technically each of these actions on the firm's behalf is aimed at boosting WTP but they both come at a cost. So a natural question arises: are these actions complementary or in substitution of each other? In other words, if the cost/penalty associated with being more proactive on one of these two dimensions increases, would the firm increase or decrease its activity (even by a marginal amount) on the other dimension? The following result addresses this question:

Proposition 2. *Under SDU$^+$ and complete information, when the software firm stays in the market $\left(cs < \frac{1}{4}\right)$, seeding and embedding more social media features are complementary actions. If any of the costs associated with these actions (s or c) increases, the firm scales back on both dimensions (i.e., $\frac{\partial \alpha^*}{\partial s} \leq 0$, $\frac{\partial \alpha^*}{\partial c} \leq 0$, $\frac{\partial b^*}{\partial s} \leq 0$, $\frac{\partial b^*}{\partial c} \leq 0$).*

The fact that the firm scales back on a particular action if the associated cost/penalty with that respective action increases (i.e. $\frac{\partial \alpha^*}{\partial s} \leq 0$, $\frac{\partial b^*}{\partial c} \leq 0$) is to be expected. However, the interesting results in Proposition 2 characterize the interaction between the two actions. At first glance, one might expect that as the cost of boosting the strength of network effects via more sophisticated social media features increases, the firm might turn to the other lever it has access to in order to increases WTP (and vice-versa). However, seeding is valuable to the firm as long as it does not cannibalize too much demand and does not induce paying customers to downgrade their valuation of the product too much. When it is more costly to generate strong marginal network effects via social media, the firm reduces its investment along that dimension. In turn, if it were to compensate such an action by a boost in seeding, in order to reach the same overall network effect the firm would have to seed more customers. Thus, for the same network effect, the firm would actually see an increase in seeding penalty and a decrease in the number of paying customers. As it turns out, these two effects dominate the benefits from the boost in overall network benefits from seeding, and, consequently, the firm prefers to downsize the seeding pool as well.

A similar argument goes in the other direction. If the seeding-induced downgrading of WTP of paying customers is more intense, then the firm first scales back on seeding. Again, at first glance, it might seem like a good idea in such a case to simultaneously boost the strength of network effects so that the firm does not have to rely on seeding that much. Nevertheless, once operating at optimal network effect strength levels, it is costly to further upgrade b, and this cost is not recovered by the benefit of an upwards shift in WTP due to stronger network effects.

When seeding does not induce any disutility for the paying customers ($s = 0$), then $\alpha^* = \frac{1}{2}$, $b^* = \frac{1}{8c}$, and $p^* = \frac{1}{16c}$. In such cases, the optimal seeding ratio and, implicitly, the ratio of paying customers are independent from the strength of network effects embedded in the product. If the cost of adding social media features is increasing, the firm settles for a lower strength of network effects and, at the same time, charges customers less such that (IR) constraint remains binding for the highest type while the seeding ratio is kept unaltered.

As seen from Proposition 1, $\frac{1}{2}$ actually represents the upper bound for α^*. As such, under the optimal strategy, the software firm prefers an outcome where the majority of customers are paying customers whose WTP is influenced by a well balanced combination of seeding and social media features that boost network effects.

Last, we mention that the results in this section can be extended to more general type distribution functions and utility structures. This discussion has been included in Appendix B.

3.2. Optimal Strategy Under Model SDU^-

In this section, we explore a setting where customer type is negatively correlated with seeding-induced disutility. As argued in §2, in some instances, more loyal customers may react with less jealousy to seeding procedures compared to less loyal customers. It may be the case that more loyal customers also have higher willingness to pay. Customers with higher WTP may be big clients such as corporations who developed a relationship with the vendor over time and for whom switching costs and employee retraining would be too high. Such clients might be less likely to fret much over some other customers receiving the product for free. At the other end of the type spectrum, customers who do not derive much value of the product and might operate on a tight budget might be more upset if other got it for free. The following lemma characterize the market segmentation under the vendor's optimal strategy.

Lemma 2. *Under SDU^- and complete information, if the firm stays in the market (i.e., it can make profit) then its optimal strategy $\{\alpha^*, b^*, p^*\}$ and optimal seed allocation must satisfy the following:*

(i) $b^*\alpha^* \geq p^*$;

(ii) there exists a marginal paying customer $\theta_m > \alpha^*$ such that all customers with type $\theta \in [\theta_m, 1]$ purchase the product, no customers of type $\theta < \theta_m$ purchase the product, and all seeds come from the interval $[0, \theta_m)$ (though they may not need to be grouped at the very low end).

Condition (i) illustrates the fact that adoption has to start with the highest type customer. Condition (ii) states that, similar to the SDU^+ case, paid adoption occurs among the top tier customers. Nevertheless, unlike in the case of SDU^+, full market coverage may not be optimal. We will revisit this point later. Firm's optimal strategy is presented below:

Proposition 3. *Under SDU^- and full information, the firm always enters the market. Let θ_m^* be the marginal paying customer under optimality. Then firm's optimal strategy is as follows:*

Region 1: $0 < cs \le \frac{1}{8}$. *Then:*

$$\alpha^* = \frac{1}{2}, \quad b^* = \frac{1}{8c}, \quad \theta_m^* = \frac{1}{2}.$$

Region 2: $\frac{1}{8} < cs < \frac{31-7\sqrt{17}}{16}$. *Then:*

$$\alpha^* = 3\theta_m^* - 1, \quad b^* = \frac{(1-\theta_m^*){\theta_m^*}^2}{c}, \quad \theta_m^* = \tilde{x},$$

where $\tilde{x}$ is the unique real solution to the equation $cs(2-6x)+x^3 = 0$ over the interval $[-1+\sqrt{1+8cs}, \sqrt{2cs}]$.

Region 3: $\frac{31-7\sqrt{17}}{16} \le cs$. *Then:*

$$\alpha^* = 3\theta_m^* - 1, \quad b^* = \frac{1-24cs-36c^2s^2+(1+6cs)\sqrt{1+36cs+36c^2s^2}}{16c}, \quad \theta_m^* = \frac{-b^*+3s+\sqrt{b^*(b^*+3s)}}{9s}.$$

Under all regions, the firm sets optimal price $p^* = b^*(1-\theta_m^*+\alpha^*)\theta_m^* - s{\alpha^*}^2(1-\theta_m^*)$.

One immediate difference from SDU^+ is that under SDU^- the firm will always prefer to enter the market. That is because the highest type customers experience very small seeding disutility and thus their WTP is more or less dictated by network-generated value. If c is very high, low b can induce positive WTP at top tier, which, coupled with low (but not too low) prices would induce revenues that would dominate associated social media costs.

Another difference from SDU^+ is that, under SDU^-, the individual rationality (IR) constraint at adoption time is binding for the marginal paying customer rather than the highest paying customer. Thus, in the very beginning, many customers may be willing to adopt solely based on the network value generated by the seeds. One of the reasons leading to this outcome is the fact that seeding disutility is low for top tier customers under model SDU^-.

One interesting aspect of the optimal strategy under SDU^- is that, when seeding penalties and costs associated with social media features are relatively low ($cs \le \frac{1}{8}$), then the firm responds to the disutility solely through adjusting the price downwards by the biggest seeding disutility a paying customer can experience (i.e. the one experienced by the lowest paying type θ_m). Its seeding and social media engineering strategies do not change under small fluctuations in seeding penalty. Also, in such regions, seeding is not influenced by changes in the cost of adding more social media features. However, in markets characterized by higher costs ($cs > \frac{1}{8}$), the optimal seeding and social media engineering strategies will depend on both s and c. As it turns out, differently from SDU^+, under SDU^-, full market coverage does not always hold. The following corollary to Proposition 3 captures this:

Corollary 1. *Under SDU^- and complete information, full market coverage ($\alpha^* = \theta_m^*$) is attained only when $cs \leq \frac{1}{8}$. When $cs > \frac{1}{8}$, then $\alpha^* < \theta_m^*$, and there are always customers that are not purchasing the product and are not seeded.*

When the costs associated with seeding and/or building more social media features into the product are relatively high, it is not optimal to lower the price to a level where all unseeded customers adopt or seed more. Unseeded customers of low type actually have high disutility from seeding so their WTP would be rather low. The firm is better off keeping the price higher and extracting more consumer surplus from the high tier. Seeding these customers also would also generate a decrease in WTP for customers in the mid-type range (where there is significant seeding disutility) which would shrink or completely eliminate any benefits from the increased WTP at the high end (where there is little seeding disutility).

In Regions 2 and 3, as $\alpha^* < \theta_m^*$ and $\alpha^* = 3\theta_m^* - 1$, it follows immediately that $\alpha^* < \frac{1}{2}$. Thus, again, it is never optimal to seed more than half of the market.

In spite of the differences in both model and optimal strategy, we find that the complementarity result between seeding and building social media features for model SDU^+ (Proposition 2) extends to SDU^-.

Proposition 4. *Under SDU^- and full information, optimal seeding ratio α^* and optimal strength of network effects b^* are both non-increasing in c and s.*

For Region 1, b^* is independent of s and α^* is independent of both c and s. Strict monotonicity is experienced in Regions 2 and 3. As discussed below, under SDU^-, under optimality, (IR) constraint is binding for the lowest paying type since highest types have negligible seeding disutility. If adding more social media features becomes costlier, the firm will reduce its investment in social media and thus, decrease the marginal network-generated value of the product. This, in turn, would lower WTP for all customers. If the firm would respond by increasing seeding to boost network effects, actually the WTP of the low end of the paying group would decrease a lot and that dictates price. Thus, the firm would see either a smaller paying segment or would have to charge a lower price, and those actions would lead in this model to a lower profit. As a result, it is better off for the firm to decrease seeding as well. Alternatively, if s increases, the low end gets affected most. In order to compensate for that decrease, the firm would have to invest a lot in social media features. Granted that at the top tier this increases WTP for the customers, however, in Regions 2 and 3, where c was also relatively high, associated social media costs would increase steeply for such a process and wipe out other benefits. Therefore, the firm finds it more profitable to actually decrease social media features and manipulate demand more through price.

4. Incomplete Information

When the firm has incomplete market information, i.e., it knows the type distribution but not the exact type of each customer, it may not be able to target individual customers by type. In this scenario, when a firm attempts to seed the market, one of the inherent downsides is the potential to cannibalize some of the demand from high-type customers since the firm cannot ensure that the seeds go to the lowest end (Niculescu and Wu 2012). For example, in the app markets for iOS and Android devices, it is not uncommon for some of the developers to offer their apps for free for a limited period of time in order to boost adoption in the market. In such cases, any customer who comes across the app during this limited time window can download it for free. We consider the case where the firm decides strategically on the seeding volume but the seeds end up being spread *uniformly* in the market. We again explore both models SDU^+ and SDU^- for a complete picture of how the firm adjusts seeding and the level of social media features in products in response to fluctuations in costs depending on market specifics.

Before we discuss each model, we would like to point out some other differences between uniform seeding and seeding under complete information. Under complete information, we saw in §3 that full market coverage always occurs under SDU^+ and it may occur also for SDU^- contingent on small costs. However, under uniform seeding, full market coverage never occurs, regardless of the seeding disutility model. This is because for any positive price, there will always be unseeded customers with low types for whom benefits fall below that price. Moreover, for both seeding disutility models, the (IR) constraint at the adoption time is binding for the lowest paying customer. This is not the case under complete information for SDU^+.

4.1. Uniform Seeding Under Model SDU^+

In this case, firm's optimal solution is as follows:

Proposition 5. *Under uniform seeding and model SDU^+, when the firm decides to enter the market, its optimal strategy is given by:*

$$\begin{aligned} b^* &= \arg\max_{b \geq 0} \left\{ \frac{b^2(3-16cs) + 6b(-2b+s)z(b) + (11b-8s)sz(b)^2}{16s} \right\}, \\ \alpha^* &= z(b^*) \in \left[0, \frac{5-\sqrt{5}}{10}\right], \\ p^* &= \alpha^*(b^* - s\alpha^*), \end{aligned}$$

where $z(b)$ is the unique solution to the equation $b^2(1-4z) - 4s^2z^3 + bsz(-2+9z) = 0$ over the interval $\left[0, \frac{5-\sqrt{5}}{10}\right]$. The marginal paying customer has type $\theta_m^* = \frac{\alpha^*(b^*-s\alpha^*)}{b^*(1-\alpha^*)}$. There exists a bound $\xi \leq \frac{1}{4}$ such that when $cs > \xi$ then the firm does not enter the market.

First, we observe that the upper bound on seeding has decreased dramatically $\left(\frac{5-\sqrt{5}}{10} \approx 0.2764\right)$ compared to the complete information case $\left(\frac{1}{2}\right)$. The fact that uniform seeding effectively reduces the available market at the high end deters the firms from pursuing overly aggressive seeding campaigns. Revisiting the app market example, this observation is consistent with the practice to limit the free promotion to a short window in order not to overdo the seeding. Moreover, obviously, profit under full information seeding dominates the profit under uniform seeding. Thus, when the firm would not enter the market under the former case, it will also no do so under the latter.

Given the complexity of the solution, we perform a numerical sensitivity analysis on the optimal seeding volume α^* and level of social media features b^*. This analysis yields very interesting and, in certain regions, different results, compared to the ones under complete information. We capture

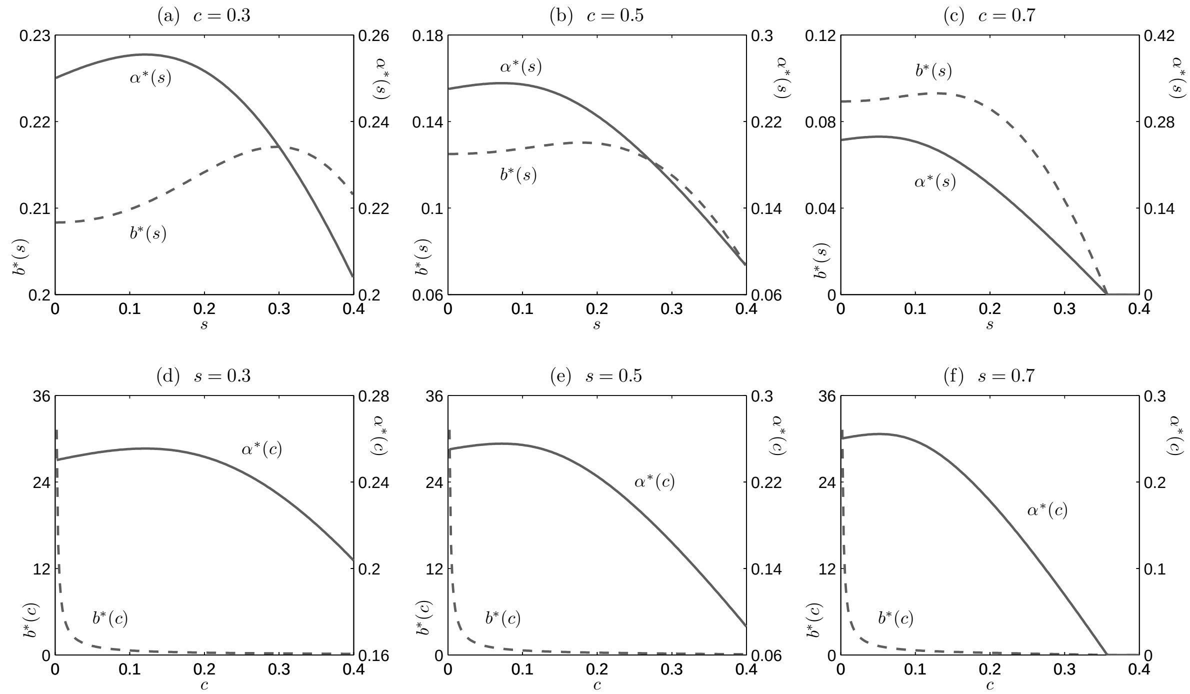

Figure 1 **Optimal α^* and b^* under uniform seeding and seeding disutility model SDU^+. The yy-plots have b^* on the left-hand y-axis and α^* on the right-hand y-axis. Panels (a-c) consider sensitivity with respect to s, while panels (d-e) consider sensitivity with respect to c.**

these results in Figure 1, where panels (a-c) illustrate how α^* and b^* change with respect to s, while panels (d-f) explore sensitivity with respect to c. We see that the complementarity results now hold only when costs are sufficiently large (cs high enough). In such a case, similar insights compared to the ones in §3.1 apply and we omit this discussion for brevity. When c is small, we can see that it is optimal to build a high level of social media features in the product, but such a practice would not be optimal for high c. Thus, similar to the complete information case and quite intuitive in nature, b^* will tend to decrease in c.

However, this is where similarities stop. First, under small cs we notice that α^* is increasing in both c and s. Moreover, b^* tends to also increase in s when s is small. To get a better understanding of the sensitivity with respect to s, we illustrate in Figure 2 several properties of the equilibrium outcome corresponding to panel (a) in Figure 1. Note that under complete information we have full market coverage and an increase in α^* would always result in a shrinking of the paying group of customers. When new seeds are given away, all of them are actually cannibalizing paying customers.

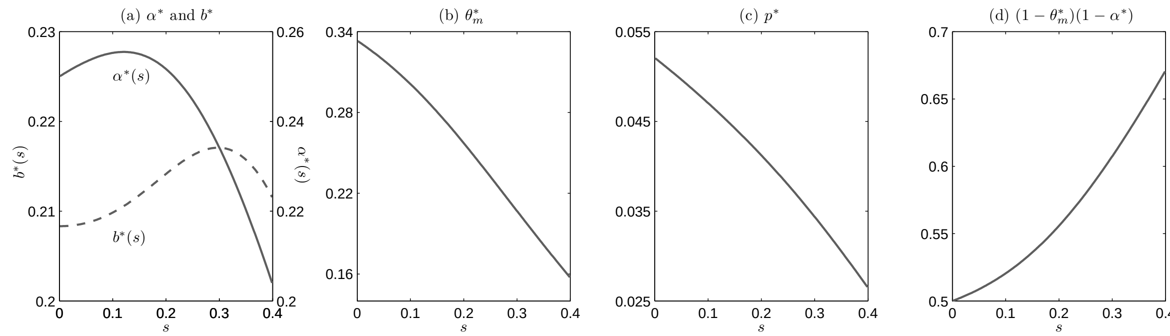

Figure 2 **Details of equilibrium when $c = 0.3$ under uniform seeding and model SDU^+.**

However, as it turns out, under uniform seeding, since we do not have full market coverage (due to low types not adopting), new seeds only cannibalize a fraction of the paying customers as they get distributed uniformly. Thus, the firm can actually increase at the same time both the seeding ratio α^* and the size of the paying group $(1-\alpha^*)(1-\theta_m^*)$, as can be seen from panels (a) and (d) of Figure 2. Given that increasing seeding involves more seeds to the high types as well, the high-type paying group shrinks. However, the firm responds by lowering price and expanding adoption towards the lower end of market, as can be seen from panels (b) and (c). When s is small, increasing seeding comes at a low penalty and thus, it results mostly in increased WTP. Additional increase in b also further adds to the increase in WTP. In such a case, the newly added revenue from the low end of the market (due to lower price and higher WTP, both of which induce more customers to join) may actually cut the double losses at the high end (due to fewer unseeded customers and lower price).

As s increases, for intermediate ranges we see that the firm will switch towards using social media engineering as a substitute for seeding. Once s is not too small, the benefits from extra seeding vanish as the WTP cannot be boosted that high without substantial investments in boosting b due to the increasing disutility. This effect, together with the reduced pool of paying high-type customers, make increasing the seeding ratio suboptimal. In such a region, in parallel with a decrease in α^* we see an increase in b^*. The firm finds it optimal to continue to expand the market into lower types by decreasing price, thus absorbing the increase in the seeding disutility. At the same time, the decrease in α^* also expands the group of high-type paying customers. While the seeding disutility is not too high, the firm will still keep α^* at relatively high levels, adjusting upwards b^* in parallel with the decrease in α^* to reverse a decrease in network value. Still maintaining b^* at high levels is worth it given that higher types would still pay a good amount of money without other adoption having taken place. Nevertheless, once s becomes too high, it is too costly for the firm to use this approach because it would be necessary to invest a lot in b^* to maintain WTP at high levels. Thus, in such regions, the firm will resort to decreasing all three controls $\{b^*, \alpha^*, p^*\}$.

For c, we see that under small values, seeding ratio is increasing in c, which, again, hints at seeding and social media engineering being substitutes to each other in generating profit. An increase in c will induce a decrease in the level of social media features embedded in the product. Similar evolutions of p^*, θ_m^*, and $(1-\alpha^*)(1-\theta_m^*)$ are observed as in the case of changes in s. For small ranges of c, even if b^* decreases, it remains moderate in value, providing the potential for a substantial network value if installed base is robust. Small upward adjustments in seeding ratio might be profitable as they help retain network value and allow the firm to profitably expand in the lower type segment of the market by lowering the price. However, the boost in seeding is taken advantage of exactly through network effects and, thus, once c gets really high, the firm would not find it optimal to further invest in b^* which, in turn, would expose customers to a potentially high seeding disutility without high benefits from the network. To compensate for this, the firm will reduce the seeding ratio, thus increasing the number of paying customers among the higher types and also reducing the disutility at all levels. Together with an associated price decrease, the firm strategically extends the group of paying customers also towards more of the lower-type customers.

4.2. Uniform Seeding Under Model SDU^-

Under uniform seeding and model SDU^-, firm's optimal strategy is as follows:

Proposition 6. *Under uniform seeding and model SDU^-, the firm always enters the market and its optimal strategy is given by:*

$$b^* = \frac{1 - 12cs - 9c^2s^2 + \sqrt{(1+3cs)^2(1+18cs+9c^2s^2)}}{32c},$$
$$\alpha^* = \frac{-2b^* + \sqrt{4{b^*}^2 + 3b^*s}}{3s} \in \left[0, \frac{5-\sqrt{5}}{10}\right],$$

$$p^* = \frac{2b^* + b^*\alpha^* - 6s{\alpha^*}^2}{9}.$$

The type of the marginal paying customer is $\theta_m^* = \frac{1}{3}$.

Under SDU^- and uniform seeding, the type of the lowest paying customer is always the same ($\theta_m^* = \frac{1}{3}$), in contrast with most of the cases previously studied, with the exception of a low cost region under SDU^- and full information. Even compared to the latter, there is one important difference, namely that in the current case, while θ_m^* is constant, the firm adjusts all its controls α^*, b^*, p^* in response to fluctuations in c and s. Moreover, as α^* can be rewritten as $\alpha^* = \frac{1}{3}\left(\frac{-2b^*}{s} + \sqrt{\frac{b^*}{s}\left(\frac{b^*}{s}+3\right)}\right)$, given that $\frac{b^*}{s}$ is a function of cs, it follows that α^* is a function of cs, and opposite-direction cost fluctuations that leave cs unchanged also leave the seeding ratio unchanged. Moreover, we notice that, compared to the full information case, the upper bound on the seeding ratio also decreased significantly from $\frac{1}{2}$ to $\frac{5-\sqrt{5}}{10} \approx 0.2764$. Similar to the discussion in §4.1, given that seeding strategies reduce the top-tier paying segment under uniform seeding, the firms will not engage in overly subsidizing adoption.

We next explore the sensitivity of α^* and b^* with respect to costs. An increase in s would result in a decrease in WTP pronounced more significantly at lower type levels. As such, unlike under SDU^+, the firm does not see it beneficial to expand into the lower market by inducing lower θ_m^* and more paid adoption. In turn, it shifts towards increasing the paying customer segment towards the top tier by lowering α^*. In tandem, it also lowers b^* as there is less pressure to induce strong network effects to compensate for seeding disutility and this allows the firm to ease down on costs of building a high level of social media features into the product. On the other hand, when c increases, given convexity of social media costs, insights remain robust behind b^* being decreasing. Once the network-induced value decreases for all types, if the firm would try to compensate with an increase in seeding, it will induce a higher disutility and reduce the number of paying customers at the top tier. The firm would then have to operate at suboptimal price levels to sustain a robust adoption. As such, the firm also prefers to reduce the seeding ratio, preventing a strong drop in WTP and, at the same time, increasing the size of the paying segment. Thus, the complementarity results uncovered under full information continue to hold. The above insights are formalized in the next result.

Proposition 7. *Under uniform seeding and model SDU^-, optimal seeding ratio α^* and optimal strength of network effects b^* are both non-increasing in c and s.*

Unlike in the case of complete information, the two seeding disutility models may lead to optimal strategies that are different not only in specific value but in more fundamental behavior. As such, when there is seeding disutility in the market, it is very important for firms to account for its proper form when detailed consumer information is lacking.

5. Extension: Asymmetric Networks

In this section, we present an illustration of how our results can be extended to asymmetric networks. Among others, network asymmetry may be induced by lack of full connectivity (e.g, Zubcsek and Sarvary 2011) and/or by single-direction links (e.g., Lehmann and Esteban-Bravo 2006). As such, there are a vast number of possible asymmetric network scenarios. We will explore one case that pertains to the latter category.

Suppose the market is segmented into two type groups: low-value group L ($\theta \in [0, r]$) and high-value group H ($\theta \in (r, 1]$). Suppose the network is fully connected but asymmetric in nature. Within each group, all links are bi-directional. Across groups, links are uni-directional, going from high-value group to low-value group but not vice versa. In a sense, the high-value and low-value groups correspond to the innovators and imitators in Lehmann and Esteban-Bravo (2006). We assume that the network-induced value for each customer is given by the volume of incoming links. For each segment, the seeding ratio is always upper bounded by the size of the segment.

In this extension, we focus on SDU^+ seeding disutility model. We further assume that seeding disutility is only manifesting within the high valuation group (and only with respect to seeds in that group), while in the low-value group it is negligible ($s_L = 0$, $s_H > 0$). Of course, this is just one example of how seeding disutility can manifest in the market. We consider the case of full information. While the market is segmented into two groups, we still assume that the seller only approaches it with a unique price. Seller's optimization problem consists of how to choose p^*, b^*, and seeding levels $\{\alpha_L^*, \alpha_H^*\}$ corresponding to each segment.

It is straightforward to establish the following properties in equilibrium under optimal strategy. First, all seeds should go the lowest end in each group. Second, both groups will be fully covered. Let us focus on the high end group first. If the group H is not completely seeded (that might be an option), and price is low enough such that paid adoption can start in that group, given that group H can be treated in isolation (it is not influenced by group L), similar to our previous results it turns out that adoption goes all the way until the high group is covered without stalling. It is irrelevant if at some point along the process adoption also started at the low level. Note that if there is any paid adoption in group H, the highest type customer must always be willing to adopt from the start. It cannot be optimal for some customers to be left without seeds in the high end group and the price be set above the WTP of the highest type. This is because low group adoption cannot increase WTP at the high end (due to uni-directional links) and thus, those unseeded customers in group H will never buy. However, seeding them would increase WTP in the low end group even further (since group L does not exhibit seeding disutility). As such, in optimality, either the entire group H is seeded or all unseeded customers purchase the product.

Next, in terms of equilibrium, we point out that it is irrelevant whether paid adoption starts in group L before being complete in group H. If it starts after, basically the WTP of the first adopters in group L will be higher. Nevertheless, if it starts before, if at any point there is any stalling in group L, adoption will pick up again once adoption from above picks up. As such, any equilibrium outcome where paid adoption in the low group is starting before paid adoption is complete in the high group can be replicated under a strategy where the low group market is open after high group market is fully covered. If it is optimal to set the price very high such that there is no adoption in the low-value group, then it is irrelevant whether customers are seeded in that group or not due to asymmetry. As such, for simplicity, we can assume in these settings full seeding of the low-value group. However, if it is optimal to have paid adoption in the low end group as well, then, once adoption starts, it can again be shown that it will not stall until everyone is covered (due

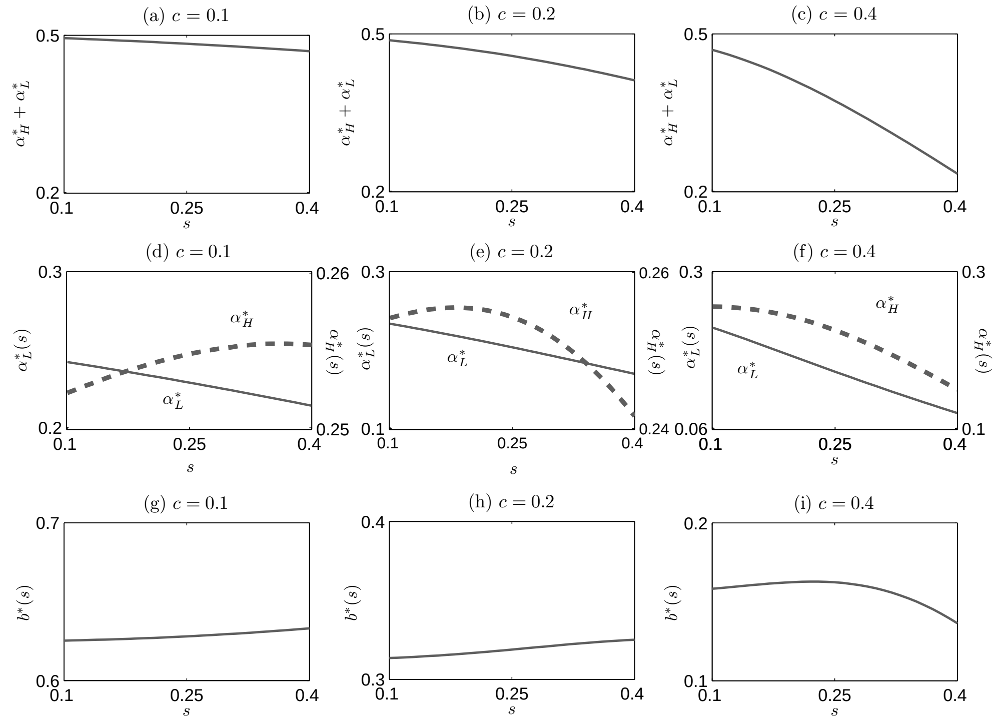

Figure 3 **Sensitivity of optimal strategies with respect to s under asymmetric network structure and seeding disutility model SDU^+ when $r = 0.4$.**

to the concavity of the instantaneous willingness to pay, via a similar argument as in the case of symmetric networks).

Under optimality, it can be easily shown that when r close to 1, it is optimal to seed the entire high end segment, and when r is close to 0, it is optimal to price in such a way that there will be no paid adoption anyways in the low end segment. When r is in intermediate ranges, it is optimal to seed a fraction of each group and have paid adoption in each group as well.

We focus the remaining part of our discussion on the interesting regions for r, c, and s, where it is optimal to have paid adoption in both groups. Figures 3 and 4 illustrate sensitivity of α_L^*, α_H^*, $\alpha_L^* + \alpha_H^*$, and b^* with respect to cost parameters s and c. In the plots, parameter r is chosen at 0.4 and we consider $c, s \in [0.1, 0.4]$, ranges which ensure optimality of paid adoption in both segments.

First, as it can be seen from plots (a-c) in Figure 3, aggregate seeding ratio $\alpha^* = \alpha_L^* + \alpha_H^*$ is decreasing in s. Thus, we uncover a similar pattern as in the case of the symmetric network. For low ranges of s, if s increases, the seeding disutility in group H increases but there is no impact on group L. All else equal, the decrease in WTP in group H puts downward pressure on price. This

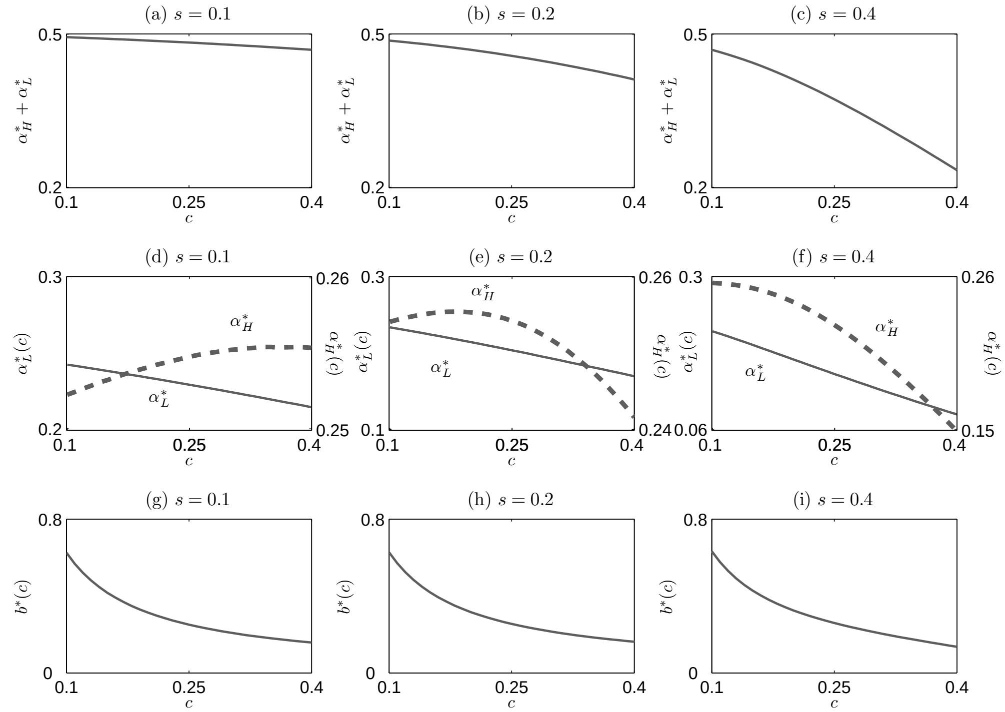

Figure 4 **Sensitivity of optimal strategies with respect to** c **under asymmetric network structure and seeding disutility model** SDU^+ **when** $r = 0.4$**.**

impacts not only revenue from high-end group but also that from low-end group. To prevent too much revenue loss, the firm will react first by releasing some of the pressure on price in the high end buy boosting b^*. When c is low, as in panels (d,g) or (e,h), the firm will embed a relatively higher level of social media features in the product. As b^* is pushed even higher, while s is still small, in may be beneficial to increase α_H^* in response to an increase in s since in both groups seeding is taken advantage of via network effects. In the low-end group, the boost in b^* eliminates some of the need for a high level of seeding and, thus, the firm can actually decrease α_L^* without affecting too much the WTP of consumers in that group. For high c, as in panels (f,i), in small ranges of s, given that it is too expensive to operate at high levels of b^*, the boost in b^* allows the firm to reduce its reliance on seeding in the low end group. It will also result to the same practice in the high end group for different reasons. Since b^* is small, for small s, as s increases the small boost in b^* cannot compensate for the increase in seeding disutility. As such, we see in panel (e,f) that α_H^* is decreasing in s. This reasoning applies pretty much in all cases once s gets large enough because seeding disutility simply grows too big and the firm will curb seeding in the high end group, since it cannot boost b^* too much due to the associated convex costs. This is also accompanied by an expected decrease in b^*, as can be seen in panel (i) of Figure 3.

Figure 4 captures sensitivity with respect to c. Again, from panels (a-c) we see that aggregate seeding ratio is decreasing in c. At the same time, convexity of social media costs induces b^* to decrease in c. Thus, insights from the symmetric network case carry through. As c increases, b^* decreases, and, as a result, in both groups, the network value of the product decreases. When c is low and s is not too high, in order to avoid pressure on price, the firm might try to boost α_H^*. This can be seen in panels (d-e). However, once seeding disutility is too high, this is not optimal anymore, as can be seen in panel (h). Once c is too high, b^* is pushed too low and seeding penalty would be too high if the firm would try to push seeding further. As such, it will decrease α_H^*. Looking at the low end group, as the firm makes adjustments in the high end group to prevent too much pressure on price, it will expand its sales in the low end group by reducing seeding ratio, this having more paying customers.

6. Conclusion

Firms nowadays are increasingly proactive in trying to strategically capitalize on consumer networks and social interactions. In this paper, we complement a recently emergent and growing body of research on the engineering of WOM effects by exploring a different angle through which firms can strategically exploit the value-generation potential of the user network. Namely, we consider how software firms should optimize the strength of network effects at the utility level in tandem with the right seeding and pricing strategies in the presence of seeding disutilities. To the best of our knowledge, our study is one of the first explorations of this research path. Our results have important managerial implications for practitioners in the industry who are increasingly trying to capitalize on network effects in a more effective way. Moreover, the applicability of our results is augmented by the fact that we consider two potential seeding disutility scenarios, thus covering a wide range of plausible consumer reactions in the market.

Under complete information when the firm can target individual customer types, after deriving the optimal firm strategy and the associated market structure, we uncover counterintuitive complementarities between seeding and the building of social media features in the presence of disutility associated with seeding. While both initiatives contribute to a direct increase in WTP of consumers, each of them comes at a cost. The inherent tradeoffs induce the firm to scale back on any of these initiatives if the cost associated with the other increases. Thus, markets with high seeding disutility due to price discrimination are also exhibiting low network effects embedded in the products. Alternatively, markets for products where significant investment in social media features is necessary in order to make an impact on the network value also experience low levels of market seeding. We further show that our results are robust to both seeding disutility models.

Under incomplete information, while complementarity between seeding and building social media features continues to hold for SDU^- everywhere, under disutility model SDU^+ we uncover a host of new insights. In particular, a more peculiar complementarity can occur as well, where, for small seeding penalties, it many be optimal for both seeding and level of social media features to be increasing in the seeding penalty. Moreover, for intermediate seeding penalties or low costs we uncovered the potential for complementarity between the two actions. Thus, different disutility models may actually lead to very different optimal strategies under incomplete information. This highlights the importance of not overlooking the form of the seeding disutility in various markets, especially in the absence of consumer information.

In an extension, we also explore how our framework and insights can be extended to certain classes of asymmetric networks. At a high level, some of the previous results remain robust, while new insights also emerge. Moreover, we explored how the firm jointly approaches the seeding process for different groups in the market by allocating seeds in each group.

As expected, the value of information computed by taking the difference (or percent difference) between profits under complete and incomplete information is decreasing in both c and s. We only briefly mention here this point as it follows common intuition. As it is becoming increasingly costly to induce high WTP, the gap between profits also decreases and the firms are less willing to pay high premiums for low returns.

Future research opportunities are abundant. For example, alternative forms of network effects (e.g., additive) can be considered in association with products that also carry an intrinsic value that is independent of the user network. Also, as an alternative to seeding, firms can consider freemium strategies in order to jump-start adoption. Implications of seeding disutility can be further explored in the context of asymmetric networks. Moreover, it would be an interesting exercise to empirically test our model predictions.

Appendix

A. Proofs

Proof of Lemma 1. (i) If $b^*\alpha^* - s\alpha^{*2} < p^*$, then paid adoption cannot start regardless of how seeds are assigned. Immediately after seeding, there are only α customers in the market. Given that we assume customers are myopic, even the highest type customer ($\theta = 1$) cannot perceive a positive momentary utility at that stage. Thus, no unseeded customer is willing to be the first to pay for the software.

(ii, iii) We prove (ii) and (iii) simultaneously. Note that if the firm chooses to enter the market (i.e., it can make profit), then $\alpha^* > 0$ and $b^* > 0$. Via simple interchange arguments, it can be easily shown that the lowest-type paying customer must have type greater or equal to the highest-type seeded customer. Basically all that we have to show is that there are no unseeded customers that are left without a product.

Let $\bar{\theta}_s$ be the highest-type seeded customer and $\underline{\theta}_p$ be the lowest-type paying customer. Then $0 \le \alpha^* \le \bar{\theta}_s \le \underline{\theta}_p \le 1$. It is trivial to see that the highest type customers (with $\theta = 1$) will be purchasing customers under any optimal seeding policy and seeds allocation.

Consider the function:

$$h(\theta) = \left[b^*(1 - \theta + \alpha^*) - s\alpha^{*2}\right]\theta - p^*.$$

Then h is concave in θ. If all customers with types above θ adopt, then $h(\theta)$ would represent the perceived utility for type θ at the moment this type is considering adoption. Since consumers of type $\theta = 1$ are adopters, it must be the case that $h(1) \ge 0$. Moreover, the following holds:

$$\left(b^* - s\alpha^{*2}\right)\alpha = b^*\alpha^* - s\alpha^{*3} \ge b^*\alpha^* - s\alpha^{*2}.$$

Consequently,

$$h(\alpha^*) \geq h(1) \geq 0.$$

Since h is concave in θ, thus, $h(\theta) \geq 0$ for all $\theta \in [\alpha^*, 1]$. Since $\bar{\theta}_s \geq \alpha^*$, then it immediately follows that adoption cannot stall before $\bar{\theta}_s$ adopts. Consequently, we have $\bar{\theta}_s = \underline{\theta}_p$ and all customers above $\bar{\theta}_s$ adopt.

The only thing left to prove is that $\bar{\theta}_s = \alpha^*$, i.e., there are no unseeded customers below $\bar{\theta}_s$. We prove the argument by contradiction. Suppose that under an optimal seeding allocation and optimal parameter values, $\bar{\theta}_s > \alpha^*$. Then, by seeding in interval $[0, \alpha^*)$, the customers in the interval $[\alpha^*, \bar{\theta}_s)$ will also adopt given that $h(\theta) \geq 0$ for $\theta \geq \alpha^*$, and thus profit will increase. Contradiction. Such a seeding allocation cannot be optimal. Thus, $\bar{\theta}_s = \alpha^*$. Customers in the interval $[0, \alpha^*)$ are seeded and all other customers end up buying the product. □

Proof of Proposition 1. According to Lemma 1, the software firm will only consider scenarios where $b\alpha - s\alpha^2 \geq p$. Then, customers with types $\theta \in [0, \alpha)$ are seeded and customers with types $\theta \in [\alpha, 1]$ are purchasing the product. For each paying customer θ, at purchase time the installed base is $\delta(\theta, \alpha) = 1 - \theta + \alpha$ since, in addition to all seeded customers, all customers with higher types would have already purchased the product and no customer with lower types is moving ahead of current type. Therefore, the utility of the customer of type θ at purchase time is:

$$u(\theta|\alpha, b, p, \delta(\theta, \alpha)) = \left[b(1 - \theta + \alpha) - s\alpha^2\right] \theta - p. \tag{A.1}$$

The utility function is concave in θ. Thus, for adoption to start and not to stall, it is necessary and sufficient that the utilities of the first paying customer and last paying customer are non-negative, i.e.:

$$\begin{aligned} u(\theta = 1|\alpha, b, p, \delta(1, \alpha)) &= \left(b\alpha - s\alpha^2\right) - p \geq 0 \\ u(\theta = \alpha|\alpha, b, p, \delta(\alpha, \alpha)) &= \left(b - s\alpha^2\right) \alpha - p \geq 0. \end{aligned}$$

Given that $\alpha \in [0, 1]$ we have $b\alpha - s\alpha^2 < b\alpha - s\alpha^3$. Since the firm is profit maximizing, the (IR) constraint will be binding for the highest type and thus:

$$p^*(\alpha, b) = b\alpha - s\alpha^2. \tag{A.2}$$

Consequently:

$$\pi(\alpha, b) = (b\alpha - s\alpha^2)(1-\alpha) - cb^2.$$

$\pi(\alpha, b)$ is quadratic and concave in b. Note that for the constraint $b\alpha - s\alpha^2 \geq p$ to hold, it is necessary that $b \geq s\alpha$. Solving this constrained optimization problem, it immediately follows that:

$$b^*(\alpha) = \max\left\{s\alpha, \frac{\alpha(1-\alpha)}{2c}\right\}.$$

Note that:

$$s\alpha \geq \frac{\alpha(1-\alpha)}{2c} \quad \Leftrightarrow \quad \alpha \geq 1 - 2cs.$$

Then:

$$\pi(\alpha) = \begin{cases} \frac{1}{4c} \times \alpha^2 (1-\alpha)(1 - 4cs - \alpha) & \text{if } \alpha < 1 - 2cs, \\ -cb^2 & \text{otherwise.} \end{cases}$$

Thus, if $\alpha \geq 1 - 2cs$ the firm cannot make any profit and would exit the market. Therefore, in order to make profit, the firm would choose $\alpha < 1 - 2cs$. In this case, note that $\pi(\alpha) = 0$ has four roots $\alpha_1 = \alpha_2 = 0$, $\alpha_3 = 1$, and $\alpha_4 = 1 - 4cs$. We distinguish 2 cases:

(i) $\frac{1}{4} \leq cs$. Then $\alpha_4 \leq \alpha_1 = \alpha_2 = 0 < \alpha_3$. Then $\pi(\alpha)$ is decreasing on $(-\infty, \alpha_4]$, decreasing and then increasing on $[\alpha_4, 0]$, bouncing off at 0 since that is a double root, decreasing and then increasing on $[0, 1]$, and increasing on $[1, \infty)$. Given that the firm considers $\alpha \in [0, 1 - 2cs)$ consequently we must have $\alpha^* = 0$. The optimal price and optimal network effects are 0 and the firm exits the market since it cannot make any profit.

(ii) $\frac{1}{4} > cs$. In this case, $\alpha_1 = \alpha_2 = 0 < \alpha_4 = 1 - 4cs < \alpha_3 = 1$. Then $\pi(\alpha)$ is decreasing on $(-\infty, 0]$, bouncing off at 0 since that is a double root, increasing and then decreasing on $[0, 1 - 4cs]$, decreasing and increasing on $[1 - 4cs, 1]$, and increasing on $[1, \infty)$. Therefore, α^* is the unique root of the FOC that falls in the interval $(0, 1 - 4cs)$. That root is not 0 (which is one of the roots of the FOC). Computing the FOC, we get:

$$\frac{\partial \pi}{\partial \alpha} = \frac{\alpha}{2c} \times \left[1 - 4cs - \alpha(3 - 6cs) + 2\alpha^2\right].$$

The first order condition gives optimal $\alpha^* = \frac{3(1-2cs)}{4} - \frac{\sqrt{1-4cs+36c^2s^2}}{4}$. It is trivial to verify that $\alpha^* \leq \frac{1}{2}$. $\square$

Proof of Proposition 2. Follows directly from computing the derivatives. □

Proof of Lemma 2 (i) At the very beginning, right after the seeding process and before any paid adoption has occurred, a customer of type θ perceives an instantaneous utility $u(\theta|\cdot) = (b\alpha\theta - s\alpha^2(1-\theta)) - p = (b\alpha + s\alpha^2)\theta - s\alpha^2 - p$. Thus, at the very beginning (and actually also at every moment afterwards), the instantaneous utility is increasing in type. Therefore, at least the highest type customer must want to adopt. Thus, we need $u(1|\cdot) \geq 0$, or $b\alpha \geq p$.

(ii) Given that at any moment utility will be increasing in type (disutility is the same once seeding has occurred and the network benefits increase more for the higher types), it can be easily observed (via swapping and interchanging) that seeding should not be at the high end. No seeded customer should have a higher type than a paying customer. Moreover, given monotonicity of utility in type at any given time, everyone above the marginal customer should adopt as well. □

Proof of Proposition 3. We present here a sketch of the proof. Some portions are omitted for brevity but available from the authors upon request. First, for the firm to make any profit, it is necessary to charge a positive price. That means that the highest-type adopter ($\theta = 1$) must have positive benefit from the product before any paid adoption occurs. Since $b\alpha > 0$, highest-type adopter always has a positive WTP for the product. For a paying customer of type θ, at the moment of purchase (after all the higher types have already adopted and before any other lower type adopts), her perceived utility is:

$$u(\theta|\cdot) = [b(1-\theta+\alpha)\theta - s\alpha^2(1-\theta)] - p.$$

The utility function is concave in θ. Given that $u(0|\cdot) = -s\alpha^2 - p \leq 0$, adoption stops at a certain marginal type θ_m where $u(\theta_m|\cdot) = 0 \leq u(1|\cdot)$, or, equivalently, $\theta_m \leq \alpha + \frac{s\alpha^2}{b}$. Optimal price is given by $p^* = b(1-\theta_m+\alpha)\theta_m - s\alpha^2(1-\theta_m)$. Also, $\theta_m \geq \alpha$. Then the profit is given by

$$\begin{aligned} \pi(\theta_m, \alpha, b) &= p(1-\theta_m) - cb^2 \\ &= [b(1-\theta_m+\alpha)\theta_m - s\alpha^2(1-\theta_m)](1-\theta_m) - cb^2. \end{aligned} \tag{A.3}$$

We optimize first in α under the constraint $0 \leq \alpha \leq \theta_m \leq \min\left\{\alpha + \frac{s\alpha^2}{b}, 1\right\}$. Moreover, we verify within each region that the profit is positive. Note that the profit function is concave in α. Solving unconstrained $\frac{\partial\pi}{\partial\alpha}=0$, we obtain root $\alpha=\frac{b\theta_m}{2s(1-\theta_m)}$. Moreover, given that $\alpha\geq 0$, it can be shown that $\alpha+\frac{s\alpha^2}{b}\geq\theta_m$ is equivalent to $\alpha\geq\frac{-b+\sqrt{b(b+4s\theta_m)}}{2s}$. Thus, we have

$$\alpha^*=\max\left\{\frac{-b+\sqrt{b(b+4s\theta_m)}}{2s},\min\left\{\theta_m,\frac{b\theta_m}{2s(1-\theta_m)}\right\}\right\}.$$

After considering and comparing all the feasible cases (analysis omitted for brevity but available from the authors), it can be shown that the optimal solution is the following:

- Region 1: $0<cs\leq\frac{1}{8}$. In this case $\alpha^*=\theta_m^*=\frac{1}{2}$, $b^*=\frac{1}{8c}$, $\pi^*=\frac{1-4cs}{64c}$.
- Region 2: $\frac{1}{8}<cs<\frac{31-7\sqrt{17}}{16}$. In this case, $\alpha^*=\frac{b^*\theta_m^*}{2s(1-\theta_m^*)}$, θ_m^* is the unique real solution to the equation $cs(2-6\theta_m)+\theta_m^3=0$ over the interval $[-1+\sqrt{1+8cs},\sqrt{2cs}]$, $b^*=\frac{2s(1-\theta_m^*)^2\theta_m^*}{4cs-{\theta_m^*}^2}$, and $\pi^*=\frac{s(1-\theta_m^*)^4{\theta_m^*}^2}{4cs-{\theta_m^*}^2}$. Replacing $cs=\frac{{\theta_m^*}^3}{6\theta_m^*-2}$, it can be shown that $\alpha^*=3\theta_m^*-1$, $b^*=\frac{(1-\theta_m^*){\theta_m^*}^2}{c}$, and $\pi^*=s(1-\theta_m^*)^3(3\theta_m^*-1)$.
- Region 3: $\frac{31-7\sqrt{17}}{16}\leq cs$. Then $\alpha^*=\frac{-b^*+\sqrt{b^*(b^*+4s\theta_m^*)}}{2s}$, $\theta_m^*=\frac{-b^*+3s+\sqrt{b^*(b^*+3s)}}{9s}$, and $b^*=\frac{1-24cs-36c^2s^2+(1+6cs)\sqrt{1+36cs+36c^2s^2}}{16c}$, and $\pi^*=b^*(-b^*c+\alpha^*-\alpha^*\theta_m^*)$. Replacing θ_m^* into α^* and π^*, we obtain $\alpha^*=\frac{-b^*+\sqrt{b^*(b^*+3s)}}{3s}$ and $\pi^*=\frac{b^*(6s\alpha^*-\alpha^*\sqrt{b^*(b^*+3s)}-9csb^*+b^*\alpha^*)}{9s}$. Then, we see that $-b^*\sqrt{b^*(b^*+3s)}=3\alpha^*s$. Replacing in θ_m^*, we obtain $\theta_m^*=\frac{1+\alpha^*}{3}$, or $\alpha^*=3\theta_m^*-1$.

It can be shown that the optimal profit is positive in all three regions. $\square$

Proof of Corollary 1. Follows immediately from the proof of Proposition 3. $\square$

Proof of Proposition 4. As Proposition 3 suggests, the optimal strategy is characterized over three distinct regions for the value of cs. We explore each region separately.

Region 1: $0<cs\leq\frac{1}{8}$. This case is straightforward. Results hold in weak form with respect to changes in s and only b responds to changes in c.

Region 2: $\frac{1}{8}<cs<\frac{31-7\sqrt{17}}{16}$. Note first that the optimal values α^* and b^* depend on θ_m^*. Thus, we need to first understand the monotonicity of θ_m^* with respect to c and s. θ_m^* is defined in implicit form as the unique solution to equation $cs(2-6\theta_m)+\theta_m^3=0$ over the interval $[-1+\sqrt{1+8cs},\sqrt{2cs}]$. Let us denote $z=cs$ and define function $\tau(\theta_m,z)=z(2-6\theta_m)+\theta_m^3$. Then $\theta_m^*(z)$ is the unique function such that $\tau(z,\theta_m^*(z))=0$ for every $z\in\left(\frac{1}{8},\frac{31-7\sqrt{17}}{16}\right)$. Therefore, over this interval,

$$\frac{\partial\theta_m^*(z)}{\partial z}=-\frac{\frac{\partial\tau}{\partial z}}{\frac{\partial\tau}{\partial\theta_m}}=-\frac{2-6\theta_m^*}{3({\theta_m^*}^2-2z)}.$$

As discussed in the solution to Region 2, $\theta_m^* < \sqrt{2z}$. Moreover, $\theta_m^* > \frac{1}{3}$. Thus, clearly, $\frac{\partial \theta_m^*(z)}{\partial z} < 0$. Consequently, since $z = cs$, θ_m^* is decreasing in both c and s.

Since $\alpha^* = 3\theta_m^* - 1$, it immediately follows that $\frac{\partial \alpha^*}{\partial s} < 0$ and $\frac{\partial \alpha^*}{\partial c} < 0$.

Next, we consider the derivatives of b^* with respect to c and s:

$$\frac{\partial b^*}{\partial c} = \frac{\theta_m^* \left[c(2-3\theta_m^*)\frac{\partial \theta_m^*}{\partial c} - (1-\theta_m^*)\theta_m^*\right]}{c^2} \quad \text{and} \quad \frac{\partial b^*}{\partial s} = \frac{\theta_m^*(2-3\theta_m^*)}{c} \times \frac{\partial \theta_m^*}{\partial s}.$$

It can be easily seen that $\theta_m^* < \frac{1}{2}$. Given that $2 - 3\theta_m^* > 0$, $\frac{\partial \theta_m^*}{\partial s} < 0$, $\frac{\partial \theta_m^*}{\partial c} < 0$, then it immediately follows that $\frac{\partial b^*}{\partial s} < 0$ and $\frac{\partial b^*}{\partial c} < 0$.

Region 3: $\frac{31-7\sqrt{17}}{16} \leq cs$. Since we have the formula for b^* only in terms of c and s, it can be easily verified that $\frac{\partial b^*}{\partial s} < 0$ and $\frac{\partial b^*}{\partial c} < 0$.

From the proof of the optimal strategy in Region 3, we know that $\alpha^* = \frac{-b^* + \sqrt{b^*(b^*+3s)}}{3s}$. It immediately follows that:

$$\frac{\partial \alpha^*}{\partial c} = \frac{\left(\sqrt{b^*+3s} - \sqrt{b^*}\right)^2}{6s\sqrt{b^*(b^*+3s)}} \times \frac{\partial b^*}{\partial c} \quad \text{and} \quad \frac{\partial \alpha^*}{\partial s} = \frac{-\left(\sqrt{b^*+3s} - \sqrt{b^*}\right)^2 \left(b^* - s\frac{\partial b^*}{\partial s}\right)}{6s^2\sqrt{b^*(b^*+3s)}}.$$

It can be easily seen that $\frac{\partial \alpha^*}{\partial c} < 0$. Furthermore, note that $b^* = s\gamma(cs)$ where $\gamma(x) = \frac{1-24x-36x^2+(1+6x)\sqrt{1+36x+36x^2}}{16x}$. It can be shown that $\gamma(x)$ is decreasing when $x \geq \frac{31-7\sqrt{17}}{16}$. Then $b^* - s\frac{\partial b^*}{\partial s} = -cs^2\gamma'(cs) > 0$. Then, it immediately follows that $\frac{\partial \alpha^*}{\partial s} < 0$. □

Proof of Proposition 5. First, for the firm to make any profit, it is necessary to charge a positive price. That means that the highest-type adopter ($\theta = 1$) must have positive benefit from the product before any paid adoption occurs. Thus, it is necessary to have $b \geq s\alpha$. For any (paying or non-paying) individual with type θ, there are $\alpha\theta$ seeded customers with type smaller than θ. Therefore, when a paying customer of type θ decides to purchase the product, at the moment of purchase (after all the higher types already adopted and before any other lower type adopts) her perceived utility is:

$$u(\theta|\cdot) = [b(1-\theta+\alpha\theta) - s\alpha^2]\theta - p.$$

The utility function is concave in θ. Given that $u(0|\cdot) = -p \leq 0$, adoption stops at a certain marginal type θ_m where $u(\theta_m|\cdot) = 0 \le u(1|\cdot)$, or, equivalently, $\theta_m \le \frac{\alpha(b-s\alpha)}{b(1-\alpha)}$.[3] Then, it follows that $p = [b(1-\theta_m + \alpha\theta_m) - s\alpha^2]\theta_m$, and the profit is given by:

$$\begin{aligned} \pi(\theta_m, \alpha, b) &= p(1-\alpha)(1-\theta_m) - cb^2 \\ &= [b(1-\theta_m+\alpha\theta_m) - s\alpha^2]\theta_m(1-\alpha)(1-\theta_m) - cb^2. \end{aligned} \tag{A.4}$$

We optimize first in θ_m under constraint $\theta_m \le \min\left\{\frac{\alpha(b-s\alpha)}{b(1-\alpha)}, 1\right\}$. Note that the profit is cubic in θ_m with a positive coefficient for θ_m^3. Solving for $\frac{\partial \pi}{\partial \theta_m} = 0$, we obtain the following two local extremes:

$$\theta_{m,1} = \frac{2b - b\alpha - s\alpha^2 - \sqrt{b^2 - b^2\alpha + b^2\alpha^2 - bs\alpha^2 - bs\alpha^3 + s^2\alpha^4}}{3b(1-\alpha)}, \quad \theta_{m,2} = \frac{2b - b\alpha - s\alpha^2 + \sqrt{b^2 - b^2\alpha + b^2\alpha^2 - bs\alpha^2 - bs\alpha^3 + s^2\alpha^4}}{3b(1-\alpha)}.$$

It can be shown that $0 \le \theta_{m,1} \le 1 \le \theta_{m,2}$. Thus, the profit is increasing in θ_m over $[0, \theta_{m,1}]$ and decreasing over $[\theta_{m,1}, 1]$. Thus:

$$\theta_m^* = \min\left\{\theta_{m,1}, \frac{\alpha(b-s\alpha)}{b(1-\alpha)}\right\}. \tag{A.5}$$

We will split the analysis into ten cases, based on when $\theta_{m,1} < (\ge) \frac{\alpha(b-s\alpha)}{b(1-\alpha)}$.

Case 1. $\frac{1}{2} < \alpha < 1$ and $b \ge \frac{s\alpha^2}{2\alpha-1}$.

Case 2. $\frac{5+\sqrt{5}}{10} < \alpha < 1$ and $\frac{-3s\alpha^2+5s\alpha^3-\sqrt{5}s\alpha^2(1-\alpha)}{2(1-5\alpha+5\alpha^2)} \le b < \frac{s\alpha^2}{2\alpha-1}$.

Case 3. $\frac{5+\sqrt{5}}{10} < \alpha < 1$ and $s\alpha \le b < \frac{-3s\alpha^2+5s\alpha^3-\sqrt{5}s\alpha^2(1-\alpha)}{2(1-5\alpha+5\alpha^2)}$.

Case 4. $\alpha = \frac{5+\sqrt{5}}{10}$ and $\frac{(3+\sqrt{5})s}{5(\sqrt{5}-1)} \le b < \frac{s\alpha^2}{2\alpha-1}$.

Case 5. $\alpha = \frac{5+\sqrt{5}}{10}$ and $s\alpha \le b < \frac{(3+\sqrt{5})s}{5(\sqrt{5}-1)}$.

Case 6. $\frac{1}{2} < \alpha < \frac{5+\sqrt{5}}{10}$ and $\frac{-3s\alpha^2+5s\alpha^3-\sqrt{5}s\alpha^2(1-\alpha)}{2(1-5\alpha+5\alpha^2)} \le b < \frac{s\alpha^2}{2\alpha-1}$.

Case 7. $\frac{1}{2} < \alpha < \frac{5+\sqrt{5}}{10}$ and $s\alpha \le b < \frac{-3s\alpha^2+5s\alpha^3-\sqrt{5}s\alpha^2(1-\alpha)}{2(1-5\alpha+5\alpha^2)}$.

Case 8. $\frac{5-\sqrt{5}}{10} < \alpha \le \frac{1}{2}$ and $\frac{-3s\alpha^2+5s\alpha^3-\sqrt{5}s\alpha^2(1-\alpha)}{2(1-5\alpha+5\alpha^2)} \le b$.

Case 9. $\frac{5-\sqrt{5}}{10} < \alpha \le \frac{1}{2}$ and $s\alpha \le b < \frac{-3s\alpha^2+5s\alpha^3-\sqrt{5}s\alpha^2(1-\alpha)}{2(1-5\alpha+5\alpha^2)}$.

Case 10. $0 < \alpha \le \frac{5-\sqrt{5}}{10}$.

For cases 1, 2, 4, 7, and 9, we have $\theta_m^* = \theta_{m,1} < \frac{\alpha(b-s\alpha)}{b(1-\alpha)}$. For all the other cases (3, 5, 6, 8, 10), we have $\theta_m^* = \frac{\alpha(b-s\alpha)}{b(1-\alpha)} \le \theta_{m,1}$. Replacing θ_m^* in (A.4), we obtain an expression for profit in terms of b and α. It can be shown that when $\alpha > \frac{5-\sqrt{5}}{10}$, i.e., in cases 1-9, for any given feasible b, profit

[3] We dismiss the case $\theta_m = 1$ because in that case the firm cannot make profit.

is decreasing in α. Thus, none of these cases is possible under optimality. Consequently, *under optimality, the firm will choose α and b such that case 10 occurs.* Thus, under optimality:

$$\theta_m^* = \frac{\alpha^*(b^* - s\alpha^*)}{b^*(1-\alpha^*)}, \quad 0 < \alpha^* \leq \frac{5-\sqrt{5}}{10}, \quad b^* \geq s\alpha^*. \tag{A.6}$$

Replacing θ_m^* in (A.4), we obtain:

$$\pi(\alpha, b) = b(1-2\alpha)\alpha - \frac{s^2\alpha^4}{b} - s\alpha^2(1-3\alpha) - cb^2. \tag{A.7}$$

It can be shown that when $\alpha < \frac{5-\sqrt{5}}{10}$, then $\frac{\partial^2\pi(\alpha,b)}{\partial\alpha^2} \leq 0$ for any feasible b, i.e., profit is concave in the seeding ratio. Moreover, $\frac{\partial\pi(\alpha,b)}{\partial\alpha}|_{\alpha=0} > 0$, $\frac{\partial\pi(\alpha,b)}{\partial\alpha}\Big|_{\alpha=\frac{5-\sqrt{5}}{10}} < 0$. Moreover, when $b < s$, it can also be shown that $\frac{\partial\pi(\alpha,b)}{\partial\alpha}\Big|_{\alpha=\frac{b}{s}} < 0$. Thus, the optimal seeding ratio is interior, unique, and satisfies FOC. Moreover, $\alpha^*(b) < \frac{b}{s}$ for any b, so the only constraint on b that we imposed will be satisfied. For any b, the seeding ratio is given in implicit form as the unique solution to $\frac{\partial\pi(\alpha,b)}{\partial\alpha} = 0$ over the interval $[0, \frac{5-\sqrt{5}}{10})$. Simplifying FOC, for any b, $\alpha^*(b)$ satisfies:

$$b^2[1-4\alpha^*(b)] - 4s^2\alpha^*(b)^3 + bs\alpha^*(b)[-2+9\alpha^*(b)] = 0 \quad \text{and} \quad \alpha^*(b) < \frac{5-\sqrt{5}}{10}. \tag{A.8}$$

Using (A.8) to simplify (A.7), we obtain:

$$\pi(b) = \frac{b^2(3-16cs) + 6b(-2b+s)\alpha^*(b) + (11b-8s)s\alpha^*(b)^2}{16s}. \tag{A.9}$$

Consequently b^* maximizes the above expression over $[0, \infty)$. It can be shown that $\lim_{b\to\infty} \alpha^*(b) = \frac{1}{4}$. It follows immediately that for large b, the profit will be decreasing in b. Therefore, there exists a maximum.

Note that the profit in this model cannot exceed the profit under full information, where the seeds are allocated optimally. Since, according to Proposition 5, the firm does not enter the market, this holds as well in the uniform case. Thus, there exists $\xi \leq \frac{1}{4}$, such that, when $cs > \xi$ then the firm chooses not to enter the market. $\square$

Proof of Proposition 6. Note that in this model, for any $b > 0$ and $\alpha > 0$, since $b\alpha > 0$, the firm can get the adoption started under a positive price since the highest type customers have no seeding-induced disutility. The firm will never choose $\alpha = 0$ or $b = 0$. For any (paying or non-paying) individual with type θ, there are $\alpha\theta$ seeded customers with type smaller than θ. Therefore, when a paying customer of type θ decides to purchase the product, at the moment of purchase (after all the higher types already adopted and before any other lower type adopts) her perceived utility is:

$$u(\theta|\cdot) = [b(1-\theta+\alpha\theta)\theta - s\alpha^2(1-\theta)] - p.$$

The utility function is concave in θ. Given that $u(0|\cdot) = -s\alpha^2 - p \leq 0$, adoption stops at a certain marginal type θ_m where $u(\theta_m|\cdot) = 0 \leq u(1|\cdot)$, or, equivalently, $\theta_m \leq \frac{\alpha(b+s\alpha)}{b(1-\alpha)}$.[4] Then, it follows that the optimal price is $p = b(1-\theta_m+\alpha\theta_m)\theta_m - s\alpha^2(1-\theta_m)$, and the profit is given by:

$$\begin{aligned}\pi(\theta_m,\alpha,b) &= p(1-\alpha)(1-\theta_m) - cb^2 \\ &= [b(1-\theta_m+\alpha\theta_m)\theta_m - s\alpha^2(1-\theta_m)](1-\alpha)(1-\theta_m) - cb^2. \qquad \text{(A.10)}\end{aligned}$$

We optimize first in θ_m under constraint $\theta_m \leq \min\left\{\frac{\alpha(b+s\alpha)}{b(1-\alpha)}, 1\right\}$. Note that the profit is cubic in θ_m with a positive coefficient for θ_m^3. Solving for $\frac{\partial\pi}{\partial\theta_m} = 0$, we obtain the following two local extremes:

$$\theta_{m,1} = \frac{2b - b\alpha + s\alpha^2 - \sqrt{b^2 - b^2\alpha + b^2\alpha^2 - 2bs\alpha^2 + 4bs\alpha^3 + s^2\alpha^4}}{3b(1-\alpha)}, \quad \theta_{m,2} = \frac{2b - b\alpha + s\alpha^2 + \sqrt{b^2 - b^2\alpha + b^2\alpha^2 - 2bs\alpha^2 + 4bs\alpha^3 + s^2\alpha^4}}{3b(1-\alpha)}.$$

It can be shown that $0 \leq \theta_{m,1} \leq 1 \leq \theta_{m,2}$. Therefore, the profit is increasing in θ_m over $[0,\theta_{m,1}]$ and decreasing over $[\theta_{m,1},1]$. Thus:

$$\theta_m^* = \min\left\{\theta_{m,1}, \frac{\alpha(b+s\alpha)}{b(1-\alpha)}\right\}. \qquad \text{(A.11)}$$

We will split the analysis into five cases, based on when $\theta_{m,1} \leq (\geq) \frac{\alpha(b+s\alpha)}{b(1-\alpha)}$.

Case 1. $\frac{1}{2} \leq \alpha$.

Case 2. $0 < \alpha < \frac{1}{2}$ and $b \leq \frac{s\alpha^2}{1-2\alpha}$.

Case 3. $\frac{5-\sqrt{5}}{10} \leq \alpha < \frac{1}{2}$ and $\frac{s\alpha^2}{1-2\alpha} < b$.

Case 4. $0 < \alpha < \frac{5-\sqrt{5}}{10}$ and $\frac{s\alpha^2}{1-2\alpha} < b \leq \frac{s\alpha^2\left(1-2\alpha+\sqrt{\alpha(1-\alpha)}\right)}{1-5\alpha+5\alpha^2}$.

Case 5. $0 < \alpha < \frac{5-\sqrt{5}}{10}$ and $\frac{s\alpha^2\left(1-2\alpha+\sqrt{\alpha(1-\alpha)}\right)}{1-5\alpha+5\alpha^2} < b$.

For cases 1, 2, 3, and 4, we have $\theta_m^* = \theta_{m,1} \leq \frac{\alpha(b+s\alpha)}{b(1-\alpha)}$. For case 5, we have $\theta_m^* = \frac{\alpha(b+s\alpha)}{b(1-\alpha)} \leq \theta_{m,1}$. Replacing θ_m^* in (A.4), we obtain an expression for profit in terms of b and α. It can be shown that in all cases 1-4, for any given feasible b, profit is decreasing in α. Thus, none of these cases

[4] We dismiss the case $\theta_m = 1$ because in that case the firm cannot make profit.

is possible under optimality. Case $\alpha = \frac{5-\sqrt{5}}{10}$ can be easily ruled out as well (to make sure that the optimal solution is not at the boundary of Case 3). Consequently, *under optimality, the firm will choose* α *and* b *such that case 5 occurs*. Thus, under optimality:

$$\theta_m^* = \frac{\alpha^*(b^* + s\alpha^*)}{b^*(1-\alpha^*)}, \quad 0 < \alpha^* < \frac{5-\sqrt{5}}{10}, \quad b^* > \frac{s\alpha^{*2}\left(1 - 2\alpha^* + \sqrt{\alpha^*(1-\alpha^*)}\right)}{1 - 5\alpha^* + 5\alpha^{*2}}. \tag{A.12}$$

Replacing θ_m^* in (A.10), we obtain:

$$\pi(\alpha, b) = b(1-2\alpha)\alpha - s\alpha^3 - cb^2. \tag{A.13}$$

From the above expression, for any b, we see that profit is cubic in α with negative coefficient for the degree 3 term. The solutions to $\frac{\pi(\alpha,b)}{\partial\alpha} = 0$ are:

$$\alpha_1 = \frac{-2b - \sqrt{4b^2 + 3bs}}{3s} < 0 < \alpha_2 = \frac{-2b + \sqrt{4b^2 + 3bs}}{3s} < \frac{5-\sqrt{5}}{10}.$$

It immediately follows that:

$$\alpha^*(b) = \frac{-2b + \sqrt{4b^2 + 3bs}}{3s}. \tag{A.14}$$

It can be verified that $\frac{s\alpha^*(b)^2\left(1-2\alpha^*(b)+\sqrt{\alpha^*(b)(1-\alpha^*(b))}\right)}{1-5\alpha^*(b)+5\alpha^*(b)^2} < b$ for any $b > 0$. Thus, the constraint in Case 5 holds for any b once the firm seeds $\alpha^*(b)$ of the market. Then, inserting $\alpha^*(b)$ in (A.13), we obtain:

$$\pi(b) = \frac{b[(8b+6s)\sqrt{b(4b+3s)} - 16b^2 - 9bs(2+3cs)]}{27s^2}. \tag{A.15}$$

Then, it can be shown that $\frac{\partial\pi(b)}{\partial b} = 0$ has three roots

$$\begin{aligned} b_1 &= \frac{1 - 12cs - 9c^2s^2 - \sqrt{(1+3cs)^2(1+18cs+9c^2s^2)}}{32c} \\ < b_2 &= 0 \\ < b_3 &= \frac{1 - 12cs - 9c^2s^2 + \sqrt{(1+3cs)^2(1+18cs+9c^2s^2)}}{32c}, \end{aligned}$$

and $\frac{\partial\pi(b)}{\partial b} < 0$ on $(b_1, 0) \cup (b_3, \infty)$ and $\frac{\partial\pi(b)}{\partial b} > 0$ on $(-\infty, b_1) \cup (0, b_3)$. Consequently:

$$b^* = b_3 = \frac{1 - 12cs - 9c^2s^2 + \sqrt{(1+3cs)^2(1+18cs+9c^2s^2)}}{32c}. \tag{A.16}$$

Then, it immediately follows that $\theta_m^* = \frac{1}{3}$. Plugging in all the optimal parameters in the profit function, it can be shown that the optimal profit is positive for any $s > 0$ and $c > 0$. $\square$

B. Generalization of Model SDU^+ Under Complete Information

The analysis of firm's strategies under seeding disutility model SDU^+ and full information (on the seller side) can be extended to general consumer utility structures and type distributions as detailed in the next two sub-sections.

B.1. General Utility Structures

In this section we explore a more general form of the utility function, where the link between customer types and WTP is moderated by a function w that is twice differentiable, with $\frac{\partial w}{\partial \theta} > 0$, $w(0) = 0$, and $w(1) = 1$. Thus, if at the current moment the installed base has size δ, the utility perceived momentarily by a paying customer of type θ is:

$$u(\theta|\alpha, b, p, \delta) = (b\delta - s\alpha^2)w(\theta) - p. \tag{B.1}$$

The following result captures firm's optimal strategies when $w(\theta)$ is concave.

Proposition B1. *Under full information and SDU^+, when w is concave, then the optimal strategy is the same as in Proposition 1.*

Proof of Proposition B1. When w is concave, it can be shown that Lemma 1 still applies. Since the proof of this statement follows similar steps as the proof of Lemma 1, we omit it for brevity. Proof is available from the authors upon request. If the firm stays in the market, every customer ends up with the product and seeds go to the lowest valuation customers. Nevertheless, unlike in Lemma 1, depending on the properties of moderating function w, (IR) need not always be binding for the highest type customers at adoption time. Similar to the argument in the proof of Proposition 1, for any paying customer of type θ, at the moment of adoption the installed base is $\delta(\theta, \alpha) = 1 - \theta + \alpha$. Thus, for any paying customer of type $\theta \in [\alpha, 1]$, the utility at the adoption moment is given by:

$$u(\theta|\alpha, b, p, \delta(\theta, \alpha)) = [b(1 - \theta + \alpha) - s\alpha^2]w(\theta) - p. \tag{B.2}$$

Since w is concave, given the boundary conditions $w(0) = 0$ and $w(1) = 1$, then it immediately follows that $w(\theta) \geq \theta$ for all $\theta \in [0, 1]$. Then:

$$\frac{\partial u(\theta|\alpha, b, p, \delta(\theta, \alpha))}{\partial \theta} = -bw(\theta) + [b(1 - \theta + \alpha) - s\alpha^2]\frac{\partial w(\theta)}{\partial \theta},$$

$$\frac{\partial^2 u(\theta|\alpha,b,p,\delta(\theta,\alpha))}{\partial\theta^2} = -2b\frac{\partial w(\theta)}{\partial\theta} + [b(1-\theta+\alpha)-s\alpha^2]\frac{\partial^2 w(\theta)}{\partial\theta^2} \leq 0.$$

Thus, perceived utility at adoption time $u(\theta|\alpha,b,p,\delta(\theta,\alpha))$ is concave in θ. Thus, for adoption to start and not stall at all, it is necessary and sufficient that the (IR) constraints hold for the extreme adopting types $\theta=1$ and $\theta=\alpha$. These(IR) constraints are:

$$\begin{aligned} u(\theta=1|\alpha,b,p,\delta(1,\alpha)) &= (b\alpha-s\alpha^2)-p \geq 0 \\ u(\theta=\alpha|\alpha,b,p,\delta(\alpha,\alpha)) &= (b-s\alpha^2)w(\alpha)-p \geq 0. \end{aligned}$$

Since $w(\theta)\geq\theta$ for all $\theta\in[0,1]$ and $\alpha\in[0,1]$, then it follows that $(b-s\alpha^2)w(\alpha)\geq(b-s\alpha^2)\alpha\geq b\alpha-s\alpha^2$. Since the firm is profit maximizing, the (IR) constraint will be binding for the highest type. We retrieve exactly the same solution as in Proposition 1. □

In Proposition B1, Lemma 1 continues to apply: seeds go to the lowest end of the valuation spectrum (i.e., $\theta\in[0,\alpha^*)$) and all other customers purchase the software. Proposition B1 extends our findings under the baseline model where the utility function was linear in type to more general non-linear cases. Proposition B1 shows that under a concave moderating function w, the optimal strategy is the same as in the baseline case. Thus, the complementarity interaction between seeding and increasing the strength of network effects via social media features characterized in Proposition 2 extends to this setting as well and same insights apply. Similar to the analysis in §3.1, (IR) constraint at adoption time will be binding for the highest type customers. Also, the majority of customers will be paying customers ($\alpha^*\leq\frac{1}{2}$).

B.2. General Distribution Functions

So far we have assumed that customers are uniformly distributed. In this section we relax this assumption and consider a general customer type cdf F that is continuous, strictly increasing, and twice differentiable, with boundary conditions $F(0)=0$ and $F(1)=1$ (i.e., no atom mass concentrated at any customer types). Similar to the baseline case in §3.1, we consider $w(\theta)=\theta$.

While again, for the very general case the optimal solution is not tractable in closed form, we are able to derive firm's strategy for certain distribution classes as illustrated in the following result:

Proposition B2. *Suppose F satisfies the following two constraints for all $\theta \in [0,1]$: (1) $2F'(\theta)+\theta F''(\theta) \geq 0$, and (2) $F(\theta) \leq \theta$. Then the optimal strategy $\{\alpha^*, b^*, p^*\}$ is the same as in Proposition 1, with customers with types $\theta \in [0, F^{-1}(\alpha^*))$ being seeded and all other customers purchasing the product.*

Proof of Proposition B2. Again, under the very specific conditions in this proposition, results similar to the ones in Lemma 1 hold in the sense that when the firm chooses to stay in the market, it will choose a strategy such that all customers get the product (all seeds go to the lowest end of the type distribution and all other customers end up purchasing the product). Proof is omitted for brevity but available from the authors upon request. Since α denotes the seeding ratio, then under optimal strategy customers in the interval $[0, \theta_\alpha)$ are seeded where $F(\theta_\alpha) = \alpha$.

If $\alpha\%$ of the market is seeded, then seeds go to types $[0, \theta_\alpha)$ with $F(\theta_\alpha) = \alpha$. For a paying customer of type $\theta \in [\theta_\alpha, 1]$, at adoption time there exists an installed base of mass $\delta(\theta, \alpha) = 1 - F(\theta) + \alpha$. Thus, the perceived utility at adoption time is given by:

$$u(\theta|\alpha, b, p, \delta(\theta, \alpha)) = [b(1 - F(\theta) + \alpha) - s\alpha^2]\theta - p. \tag{B.3}$$

Then, using the first constraint on F, we obtain:

$$\frac{\partial u(\theta|\alpha, b, p, \delta(\theta, \alpha))}{\partial \theta} = -bF'(\theta)\theta + [b(1 - F(\theta) + \alpha) - s\alpha^2],$$

$$\frac{\partial^2 u(\theta|\alpha, b, p, \delta(\theta, \alpha))}{\partial \theta^2} = -b\,[2F'(\theta) + \theta F''(\theta)] \leq 0.$$

Thus, perceived utility at adoption time is concave in consumer type and, consequently, in order for paid adoption to start and not stall, it is necessary and sufficient that the (IR) constraint holds for the extreme paying customer types $\theta = 1$ and $\theta = \theta_\alpha$:

$$\begin{aligned} u(\theta = 1|\alpha, b, p, \delta(1, \alpha)) &= (b\alpha - s\alpha^2) - p \geq 0, \\ u(\theta = \theta_\alpha|\alpha, b, p, \delta(\theta_\alpha, \alpha)) &= (b - s\alpha^2)\theta_\alpha - p \geq 0. \end{aligned}$$

Using the second constraint on F, we have $\alpha = F(\theta_\alpha) \leq \theta_\alpha \leq 1$. Then: $b\alpha - s\alpha^2 \leq b\theta_\alpha - s\alpha^2 \leq b\alpha - s\alpha^2\theta_\alpha$. Consequently, when the firm maximizes profit, the (IR) constraint at adoption time must be binding for the highest type $\theta = 1$. The rest of the proof is identical to the proof of Proposition 1. $\square$

In general, convex cdf functions F satisfy the required criteria. Such functions describe markets where there are more high type customers. Thus, once a few low type customers are seeded, paid adoption can sustain momentum at the high valuation level given the distribution skewness. Also distribution functions that are sublinear, increasing, and not extremely concave on any type interval satisfy the criteria. For all such distributions, the complementarity results in Proposition 2 continue to hold as well.

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