Dynamic Pricing of Enterprise Software with Value Uncertainty: Motivation for Selling Software as a Service

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Abstract

This paper studies a software vendor's decision to offer perpetual or lease-based licensing when customers are uncertain about their valuation for the software prior to adoption, and adoption requires an upfront implementation cost. First, in a discrete-time overlapping-generations model, I show that perpetual licensing can be more profitable than lease-based licensing when consumer valuation uncertainty is present. This result sheds light on the historical prevalence of perpetual licensing in the enterprise software market and contrasts with the conventional wisdom of durable-goods theories. Second, I model the new lease-based Software-as-a-Service (SaaS) as a type of leasing that requires a lower implementation cost, and I study the monopolist's motivation to offer SaaS. Offering SaaS is likely to be optimal for applications that feature lower levels of customer valuation uncertainty and require less customer-specific implementation. These two findings offer insights on the variation of SaaS adoption across different applications.

Keywords: dynamic pricing; Software as a Service; software licensing; value uncertainty

1 Introduction

Software has been extensively studied as a durable good (Waldman 2003). A well-known result in the literature on durable-goods pricing is that leasing is at least as profitable as, if not more profitable than, selling (Coase 1972; Bulow 1982). Nonetheless, perpetual licensing, the equivalent of selling for software, has been the predominant pricing model for business software for decades, accounting for more than 92% of packaged software licenses held by client firms in North America and Europe according to a Forrester 2005 survey (Wang 2006). In contrast, lease-based licensing is rarely offered, and its revenue contribution is negligible for many software applications (Cusumano 2007).

This is especially puzzling since software, as compared to other durable goods, has many features that make it suitable for leasing. For instance, since the value of software is not affected by users' maintenance efforts, it is no longer necessary to incentivize maintenance by transferring the asset ownership to its user (Smith and Wakeman 1985). Leasing is relatively easy to implement, as it is easy to turn software functions on or off, and the length of a lease can be factored into vendors' pricing metrics to improve profits (Jian and Kannan 2002). Leasing also gives vendors more control over customers' leapfrogging software upgrades. Finally, the abundant demand for extended payment schedules (such as in leasing) for enterprise software is evidenced by the offering of financing services by many software vendors, such as SAP, IBM, Oracle and Microsoft. So, why is leasing uncommon in the enterprise software market?

Advances in web-based technologies enable the new Software-as-a-Service (SaaS) model, in which software is owned and managed remotely by the vendor and delivered as a service to customers on a subscription basis over the Internet. Thus, SaaS can be seen as a special case of software leasing that, from customers' perspectives, requires lower upfront implementation and, sometimes, lower ongoing management costs compared to traditional perpetual licensing. Indeed, lower implementation costs are repeatedly cited as a key driver for SaaS adoption (Mertz et al. 2008). Despite the apparent efficiency benefits from using shared services, SaaS adoption varies widely across different software applications, contributing as little as 3-4% in revenue in the Enterprise Resource Planning (ERP) software market and more than 75% in the web-conferencing software market (Mertz et al. 2007).

The objective of this paper is to bridge this gap between theory and practice and to investigate two issues: first, the predominance of perpetual licensing over lease-based licensing prior to SaaS;

second, a software vendor's motivation to offer either perpetual licensing or the lease-based SaaS model and how that motivation may vary depending on the software application. I demonstrate that two important features of enterprise software adoption can explain the observed pricing practice:

1. Software has features of an experience good; that is, customers are unable to fully evaluate it before using it (Shapiro and Varian 1998). Requirements-engineering methods have been widely adopted for evaluating alternative software systems, but extensive evidence shows that these methods cannot fully capture many contingencies that heavily influence the value of an enterprise software adoption, and, hence, large uncertainty remains (Hitt et al. 2002). According to some studies, approximately one half of all ERP projects fail to achieve the anticipated benefits (Robey et al. 2002). The uncertainty in the value and cost of packaged enterprise software has drawn so many complaints that a group of software vendors and integrators (including Adobe, Borland, IBM, Novell, Oracle, and SAP) launched the Software Economics Council, which addresses concerns regarding the efficiency, effectiveness, and value of software products (Chabrow 2005).

2. Implementation of enterprise software requires substantial upfront investments, such as buying infrastructure hardware and software, customizing the software, reengineering business processes, training users, hiring outside consultants and so forth (Hitt et al. 2002). A typical ERP installation costs about $15 million and takes one to three years to finish (O'Leary 2000). It is common for firms to spend over $100 million to implement an ERP system (Robey et al. 2002).

This paper analyzes a software vendor's pricing problem when clients' adoption is described by these two features. I consider a standard overlapping-generations model with an infinitely-lived monopoly software vendor and overlapping generations of customers. Customers do not know their true valuation for the software prior to adoption but receive an informative signal about it (e.g., through reading product descriptions, looking at demos, talking with consultants, etc.). They learn their true valuation after using the software. There is an upfront implementation cost required to use the software. The monopolist can sell perpetual or lease-based licenses for his software in each period. Perpetual licenses allow customers to use the software in perpetuity, while leasing allows customers to use the software for only one period.

I focus on a vendor's revenue from software sales only and model SaaS as a special case of software leasing that requires a lower implementation cost. To answer the proposed research questions, I investigate the profitability of perpetual and lease-based licensing under two conditions: 1) when both require the same upfront implementation cost (representing the scenario prior to SaaS); 2) when lease-based licensing requires a lower implementation cost compared to perpetual licensing (i.e., perpetual licensing vs. SaaS).

This paper makes three main contributions. First, I propose a formal model for customers' valuation uncertainty, which is intended to more closely represent the demand characteristics in the enterprise software market. It differs from the approach taken in the literature on quality uncertainty: the latter usually assumes that consumers agree to a common ordering of quality levels, while in my model, customers' valuation is best seen as depending on the fit between customers' needs and the software's features (i.e., some features are valuable to certain customers but not to others). In addition, the quality-uncertainty literature often assumes that all consumers have the same prior belief about product quality before using the product. In this paper, customers' perceived valuation for the software prior to use is heterogeneous and depends on their private knowledge of their own needs and the software prior to using it. This knowledge is informative of their true valuation of the software.

Second, the analysis demonstrates how customers' valuation uncertainty and upfront implementation costs may impact the profitability of perpetual and lease-based licensing. It is shown that under a broad range of parameter values, perpetual licensing (or selling) is more profitable than lease-based licensing. This result sheds light on the historical prevalence of perpetual licensing in the software market and is in sharp contrast to the conventional wisdom of durable-goods theories. The intuition is as follows. Leasing allows customers to make sequential adoption decisions. Therefore, a customer who is uncertain about her true valuation of the software can try out the software by leasing it for one period and decide later whether or not to continue using it. This option may not benefit the monopolist, however. Although some customers are willing to pay more for the software after learning their true valuation, others are only willing to pay less. When the lease renewal price is discriminative, those returning lessees who find the software to be a misfit will not renew their leases. These customers are willing to pay more for the software before learning their true valuation, and perpetual licensing allows the monopolist to benefit somewhat from such a perception. In the meantime, customers who perceive the software favorably before using it also prefer perpetual licensing to leasing, because the software is likely to be highly valuable, and buying it outright allows them to avoid a higher lease renewal price in the future. Finally, high implementation costs make leasing more costly to the monopolist because he has to subsidize more heavily in the initial lease, while only a fraction of the initial adopters will be willing to renew their leases at a higher price later on.

Finally, I add to the growing research on SaaS adoption by proposing new demand-side factors and unpacking how these factors may impact the intention to adopt SaaS as an alternative to perpetual licensing. The findings show that although SaaS requires lower implementation costs, it is not always optimal for a software vendor to offer this alternative. Offering SaaS is likely to be optimal for software applications that feature a low level of customer valuation uncertainty, such as small-scale applications that support relatively simple business functions that are straightforward to evaluate upfront (e.g., online office suites). SaaS is also likely to be optimal if the implementation cost saving is substantial. Software implementation costs have several components. SaaS reduces costs mainly through shared IT infrastructure (software and hardware) and technical services. Clients still incur the costs associated with customization, internal process transformation, user training, etc. (Xin and Levina 2008). The result suggests that SaaS is likely to be popular for applications that require few such customer-specific implementation investments, such as software that supports common business processes and requires little or no customization (e.g., payroll applications). These theoretical findings offer insights on the variation in SaaS adoption across different software applications.

Pricing of software and services has been studied quite extensively, most often with a focus on usage-based pricing (see, for example, Sundararajan 2004; Gupta et al. 2001). In a related context, Huang and Sundararajan (2005) study optimal pricing of usage-based on-demand computing, taking into account clients' outside options and the vendor's infrastructure cost structure. Jain and Kannan (2002) study the search-based versus subscription-fee pricing of online services. Jiang et al. (2007) examine the impact of piracy and network externality on a software vendor's motivation to offer usage-based licensing. Ma and Seidmann (2008) model the competition between a conventional software vendor that offers fixed-price licensing and an ASP that offers usage-based pricing. In contrast, term-based pricing (i.e., leasing vs. perpetual licensing) has not received much attention.

There are emerging interests in understanding the strategic implications of the new SaaS model. Choudhary (2007) investigates how a software vendor's licensing schemes (i.e., perpetual licensing or the SaaS model) influence his incentives to invest in software quality. Zhang et al. (2009) study the strategic choices of pricing and service levels by competing web service providers. Susarla et al. (2003) assess the determinants of customer satisfaction with the ASP model. This paper adds to this growing literature by suggesting new demand-side factors and demonstrating how these factors may impact the intention to adopt SaaS.

2 The Model Setup

Consider a standard discrete-time overlapping-generations model. At the beginning of each period t $(t = 0, 1, 2, ...)$, a unit mass of new consumers enters the market and lives for two periods. Assume that there are no age-2 consumers in the initial period (i.e., $t = 0$). A monopoly software vendor offers a software via perpetual or lease-based licensing in every period. I use the terminology of the durable-goods literature and refer to perpetual licensing as selling and lease-based licensing as leasing. Selling allows a consumer to use the software for the rest of her life, while leasing allows a consumer to use it for only one period. The monopolist is able to recognize his prior customers and, hence, is able to distinguish returning lessees from new customers.

Consumers are indexed by a type parameter v, which represents the gross utility that a consumer gains from using the software for one period. Within each generation, there are two types of consumers: a fraction β of the consumers are high types with value $v = v_H$, and a fraction $1 - \beta$ are low types with value $v = v_L$ ($0 < v_L < v_H$). I choose to follow the well-known two-type model as closely as possible to make the results comparable to the ones of the standard durable-goods literature (e.g., Biehl 2001; Conlisk et al. 1984; Hart and Tirole 1988; Sobel 1991). This model captures the basic forces generating the results of durable-goods theories while allowing for relatively simple analytic strategies (Biehl 2001).[1]

A consumer's true type is unknown to her before she adopts the software. Nevertheless, she receives a noisy signal y of her true type (e.g., from reading the product description or consulting experts) when she enters the market, where $y = v_H$ or v_L. With probability α, the signal is the same as her type, and with probability $1 - \alpha$, the signal is wrong. A sufficient condition that the signal be informative is:

$$\Pr\{v = v_H | y = v_H\} = \frac{\beta\alpha}{\beta\alpha + (1-\beta)(1-\alpha)} > \beta,$$

which gives $\alpha > 1/2$[2]. A consumer learns her true type only after using the product for one period.

[1] The number of periods that customers "live" beyond two periods is immaterial since it has been shown that the more realistic overlapping-generations models with an arbitrary number of periods, or in which time flows continuously rather than discretely, yield similar economic insights, although they are much harder to handle (Weil 2008).

[2] If $\alpha < 1/2$, consumers can simply reverse the signal, and all results still hold.

A consumer has to pay a fixed cost to implement and use the software for the first time. It is assumed that there are no further adjustment costs in future periods after the software is implemented, if applicable. Denote by c_s the implementation cost required when the software is sold via perpetual licensing, and by c_l the implementation cost required when the software is leased, and let $0 \leqslant c_l, c_s \leqslant v_L$[3]. The monopolist and consumers have the same discount factor δ. The marginal production cost of the software is constant at zero. To avoid discussing trivial cases, I assume that $\beta v_H > v_L$, which implies that the number of high-type consumers is not too low.

The following simplifying notation is used throughout the paper: Denote the conditional probabilities $\Pr(v = v_i | y = v_j)$ by ψ_{ij}, $i, j = H$ or L; and the probability that an age-1 consumer receives the high-type (or low-type) signal $\Pr(y = v_H)$ by ψ^H (or $\Pr(y = v_L)$ by ψ^L). Accordingly, in each period, the measure of age-1 consumers who receive the high-type (or low-type) signal is ψ^H (or ψ^L)[4]. A consumer's expected utility from using the software for one period given signal y is denoted by $E[v|H] = E[v|y = v_H] = v_H \psi_{HH} + v_L \psi_{LH}$ and $E[v|L] = E[v|y = v_L] = v_H \psi_{HL} + v_L \psi_{LL}$.

The timing of the game is as follows. At the beginning of each period t ($t = 0, 1, 2, ...$), one unit mass of new consumers enters the market and receives private signals on their valuation for the software, which is unknown to the monopolist. A consumer of age 2 who has leased or bought the software previously has learned her true type. The monopolist announces three prices for his software: a selling price $p_{s,t}$, a leasing price $p_{l,t}$ for first-time adopters, and a lease renewal price r_t. Consumers observe the prices and make a purchase decision anticipating their total payoff. Leasing gives a consumer access to the software for one period, while selling gives her access for 2 periods (the length of a consumer's lifetime). Trade takes place. Consumers who decide to buy or lease the software then invest a fixed cost (i.e., c_s or c_l) to implement the software and learn their true valuation for the software through using it. At the end of the period, the monopolist observes the realized sales from selling, leasing and lease renewal, although he does not know each consumer's private signal/type or age unless the consumer has leased the software in the past. Note that pure selling and pure leasing are special cases of this setup in which the prices for the alternative option are prohibitively high. For simplicity, denote these high prices by $\overline{p_l}$ (the high leasing price in the

[3]Note that it is also assumed that the monopolist does not gain from consumers' implementation expense. In practice, some software vendors are able to capture part of this surplus by offering implementation consulting services. Nonetheless, costs related to hardware infrastructure, organizational change, user training and so forth are still borne by clients and, hence, impact their willingness to pay for the software.

[4]Judd (1985) shows that there exists a probability measure on the realization of a continuum of *i.i.d.* draws, such that the realization function is measurable with probability 1, and the law of large numbers holds with probability 1.

cases of pure selling) and $\overline{p_s}$ (the high selling price in the cases of pure leasing), respectively. I am interested in characterizing the *Subgame Perfect Nash Equilibria* (SPNE) of the game.

A history in period t includes all past prices and realized sales from leasing and selling. Both the monopolist and consumers remember the history of the game and their own actions. Consumers know the history, possibly through trade reports or their own observations prior to entering the market. A consumer's strategy specifies an adoption decision in each period given her history of observations and the current prices.

The monopolist's strategy specifies a set of selling and leasing prices in each period given the history of the game. Note that the monopolist's continuation profit from period t on is influenced only by the residual demand from age-2 consumers at the beginning of period t. This residual demand can be described by a vector of four variables: the measure of age-2 consumers who receive the high-type signal and have not adopted (i.e., bought or leased) or have leased the software in period $t-1$, denoted by $x_{H,n}(t)$, $x_{H,l}(t)$, respectively; and the measure of age-2 consumers who receive the low-type signal and have not adopted or have leased the software in period $t-1$, denoted by $x_{L,n}(t)$, $x_{L,l}(t)$, respectively. By definition, $x_{i,j}(t) \geqslant 0$ ($i = H$ or L, $j = n$ or l) and $x_{H,n}(t) + x_{H,l}(t) \leqslant \psi^H$, $x_{L,n}(t) + x_{L,l}(t) \leqslant \psi^L$.

I shall assume that the monopolist knows the demand state (as described by the vector of four variables) in each period in addition to the prices and realized sales of each kind (i.e., selling and leasing) in history. This may be a reasonable assumption since in a Nash equilibrium of the non-cooperative game, the monopolist knows the strategy profile of the consumers and can calculate the demand state in each period precisely given the transaction history of the game. If individual consumers deviate (i.e., off the equilibrium path), the demand state does not change since there is a continuum of consumers. The monopolist's strategy, therefore, specifies a set of prices given each demand state.

This assumption is typical in the durable-goods literature and, thus, allows my results to be comparable to those in that literature. For instance, it is often assumed that after trade happens in each period, the seller knows the residual demand curve precisely, even though he cannot observe the willingness to pay of those consumers who have bought the product (Bulow 1982; Biehl 2001). Of course, it may be desirable to consider the possibility that an exogenous shock might cause a positive measure of consumers to deviate collaboratively from their equilibrium strategy. At the end of the paper, I propose a refinement of NE, using undominated strategies, that would allow one to dispense with this assumption.

In the following, I first focus on understanding how consumers' valuation uncertainty impacts the profitability of perpetual and lease-based licensing and assume that $c_l = c_s = c$. This represents the scenario before SaaS was available, in which the same implementation is required whether customers buy or lease the software. I start with a basic 2-period model in which only one unit mass of consumers enters the market at the beginning of period 0, and no new consumers enter the market afterwards (Section 3). I compare the monopolist's optimal pricing policy in a benchmark model in which consumers know their true valuation for the software prior to adoption (Section 3.1) with that in a model featuring consumers' valuation uncertainty (Section 3.2). Without the complexity of an infinite-period game, I am able to characterize all equilibria of the game and demonstrate the tradeoff between perpetual and lease-based licensing.

While the 2-period model is a helpful exposition, the pricing and adoption results are driven by the existence of a final period. In the subsequent Section 4, I extend the basic model to allow entry of new consumers in an infinite-period model, which affords a closer representation of the actual business environment. That section also considers the impact of the new lease-based SaaS model, assuming that $0 \leqslant c_l < c_s$. The economic implications of the results are discussed in Section 5.

3 A Basic Two-Period Model

The results of this section show that while pure selling and pure leasing are equally profitable in the absence of consumer valuation uncertainty, selling can be more profitable than leasing when such uncertainty exists. Moreover, concurrent selling and leasing can be more profitable than either pure selling or pure leasing when consumers have valuation uncertainty.

3.1 A Benchmark Model without Valuation Uncertainty

This benchmark model can be seen as a special case of the original model in which $\alpha = 1$. For consistency, the demand state is still described by $(x_{H,n}(t), x_{H,l}(t), x_{L,n}(t), x_{L,l}(t))$; only in this case, $x_{i,n}(t)$ (or $x_{i,l}(t)$) represents the measure of age-2 consumers who are type-i and have never bought or leased (or have leased) the software in period $t-1$, $i = H$ or L. I first solve for the optimal pure selling and pure leasing prices separately, and then compare their profitability and discuss the option of concurrently selling and leasing the software. Note that leasing is equivalent to selling in period 1, the final period. Thus, let $p_{s,1} = p_{l,1} = p_1$.

When the software is only for sale, a consumer with valuation v who does not buy the software in period 0 buys the software in period 1 if the benefit from adoption is higher than the cost, or $v - c \geqslant p_1$. In period 0, a consumer buys the software if the payoff of buying in period 0 is no less than that of buying for the first time in period 1 or of not buying the software at all, or

$$(1+\delta) v - c - p_{s,0} \geqslant \max \{\delta (v - c - Ep_1), 0\},$$

where Ep_1 represents the expected price in period 1.

The monopolist's optimal pricing strategy given consumers' decision can be solved through backward induction. In period 1, a measure $x_{H,n}(1)$ of consumers are willing to pay up to $v_H - c$ for the software, and a measure $x_{H,n}(1) + x_{L,n}(1)$ of consumers are willing to pay up to $v_L - c$ for the software. Thus, the monopolist's optimal strategy is to charge $p_1 = v_H - c$ if

$$x_{H,n}(1)(v_H - c) \geqslant (x_{H,n}(1) + x_{L,n}(1))(v_L - c), \tag{1}$$

and $p_1 = v_L - c$ otherwise. In period 0, the monopolist specifies $p_{s,0}$ to maximize his overall profit given consumers' decisions. By comparing the monopolist's profit levels given different initial selling prices ($p_{s,0}$), one can show that when $c \leqslant v_L < \beta v_H$, the unique SPNE of the game is characterized by the following strategy profile:

The monopolist's strategy: in period 0, $p_{s,0} = v_H + \delta v_L - c$; in period 1, $p_1 = v_H - c$ if (1) holds, and $p_1 = v_L - c$ otherwise.

A consumer's strategy: in period 0, buy if $p_{s,0} \leqslant v - c + \delta v_L$; otherwise, in period 1, buy if $p_1 \leqslant v - c$.

In equilibrium, the high-type consumers buy the software in period 0, and the low-type consumers buy in period 1. The monopolist's optimal profit is $\beta(v_H + \delta v_L - c) + \delta(1-\beta)(v_L - c)$. This result confirms that of the durable-goods theories (i.e., Coase 1972; Bulow 1982): The monopolist is tempted to cut the price and generate additional profit after any initial sales. Rational consumers expect the future price to drop and adjust their willingness to pay for the software upfront. In equilibrium, this leads to a declining pricing path and a lower profit for the monopolist.

When the software is only for lease, a consumer with valuation v who does not lease the software in period 0 leases the software in period 1 if the benefit of leasing is higher than the cost, or $v - c \geqslant p_1$. A returning lessee renews her lease in period 1 if the benefit of a renewal is higher than the lease renewal price, or $v \geqslant r_1$. A consumer with valuation v leases the software in period 0 if the payoff of leasing in period 0 is no less than that of leasing for the first time in period 1 or of not leasing the software at all. That is,

$$v - c - p_{l,0} + \delta \max(0, v - Er_1) \geqslant \max\{\delta (v - c - Ep_1), 0\},$$

where Er_1 denotes the expected lease renewal price in period 1.

The monopolist's optimal pricing strategy can be solved through backward induction. In period 1, a measure $x_{H,l}(1)+x_{L,l}(1)$ of consumers have leased the software in period 0, of which a measure $x_{H,l}(1)$ are willing to pay up to v_H for renewing their leases, and a measure $x_{H,l}(1)+x_{L,l}(1)$ are willing to pay up to v_L for renewing their leases. Thus, the optimal lease renewal price $r_1 = v_H$ if

$$x_{H,l}(1)\, v_H \geqslant (x_{H,l}(1) + x_{L,l}(1))\, v_L, \tag{2}$$

and $r_1 = v_L$ otherwise. Applying a similar argument, the new lease price in period 1 $p_1 = v_H - c$ if (1) holds, and $p_1 = v_L - c$ otherwise. In period 0, the monopolist specifies $p_{l,0}$ to maximize his overall profit given consumers' decisions. By comparing the monopolist's profit levels given different initial leasing prices ($p_{l,0}$), one can show that when $c \leqslant v_L < \beta v_H$, the unique SPNE of the game is characterized by the following strategy profile:

The monopolist's strategy: in period 0, $p_{l,0} = (1-\delta)\, v_H - c + \delta v_L$; in period 1, $p_1 = v_H - c$ if (1) holds, and $p_1 = v_L - c$ otherwise; $r_1 = v_H$ if (2) holds, and $r_1 = v_L$ otherwise.

A consumer's strategy: in period 0, lease if $p_{l,0} \leqslant v - c + \delta\,(v_L - v)$; in period 1, having leased in period 0, renew if $v \geqslant r_1$; having not leased previously, lease if $v - c \geqslant p_1$.

In equilibrium, the high-type consumers lease the software in period 0 and renew their leases in period 1; the low-type consumers lease the software in period 2. The monopolist's optimal profit is $\beta\,(v_H + \delta v_L - c) + \delta\,(1-\beta)\,(v_L - c)$.

Evidently, selling is as profitable as leasing in the absence of consumers' valuation uncertainty. Can the monopolist improve his profit by concurrently offering selling and leasing? Proposition 1 shows that combining selling and leasing cannot improve the monopolist's profit in the absence of consumers' valuation uncertainty. To see why, let us assume that in an SPNE, the monopolist concurrently offers selling and leasing with prices $(p_{s,0}, p_{l,0}, r_1, p_1)$; given these prices, some consumers buy the software in period 0, while others lease the software in period 0, some of whom renew their leases in period 1. Since consumers know their valuation for the software, a consumer buys the software in period 0 only if $p_{s,0} \leqslant p_{l,0} + \delta E r_1$. A consumer leases in period 0 and renews her contract in period 1 only if $p_{s,0} \geqslant p_{l,0} + \delta E r_1$. In an SPNE, $r_1 = E r_1$. Therefore, $p_{s,0} = p_{l,0} + \delta r_1$, in which case concurrent selling and leasing with prices $(p_{s,0}, p_{l,0}, r_1, p_1)$ is as profitable as pure leasing with prices $(p_{l,0}, r_1, p_1)$. In summary, we have the following conclusion:

Proposition 1 *In the absence of consumers' valuation uncertainty, selling is as profitable as leasing. Moreover, the monopolist cannot improve his profit by concurrently offering selling and leasing.*

3.2 A Two-Period Model with Valuation Uncertainty

This section considers a two-period model in which consumers do not know their true valuation for the software prior to adoption, but receive a signal about it when they enter the market. They learn their valuation after using the software and incurring the one-time implementation cost c. Again, I first solve for the optimal pure selling and pure leasing prices separately and then compare their profitability and discuss the option of concurrently selling and leasing the software.

3.2.1 Pure selling

When the software is only for sale, a consumer who receives signal y and does not buy the software in period 0 buys the software in period 1 if the expected benefit of adoption is higher than the cost, or $E\left[v|y\right] - c \geqslant p_1$. In period 0, a consumer buys the software if the expected payoff of buying in period 0 is no less than that of buying for the first time in period 1 or of not buying at all, or

$$(1+\delta)\,E\left[v|y\right] - c - p_{s,0} \geqslant \max\left\{\delta\left(E\left[v|y\right] - c - Ep_1\right), 0\right\}.$$

The monopolist's optimal pricing strategy given consumers' decision can be solved in the same fashion as in the previous pricing problem without consumer valuation uncertainty. Therefore the detailed discussion is omitted for brevity. One can show that two alternative equilibrium outcomes are possible depending on the range of parameter values. In particular, define the following two pricing strategies:

The PS1 strategy: in period 0, $p_{s,0} = E\left[v|H\right] + \delta E\left[v|L\right] - c$; in period 1, $p_1 = E\left[v|H\right] - c$ if

$$x_{H,n}(1)\left(E\left[v|H\right] - c\right) \geqslant \left(x_{H,n}(1) + x_{L,n}(1)\right)\left(E\left[v|L\right] - c\right), \tag{3}$$

and $p_1 = E\left[v|L\right] - c$ otherwise.

The PS2 strategy: in period 0, $p_{s,0} = (1+\delta)\,E\left[v|L\right] - c$; in period 1, $p_1 = E\left[v|H\right] - c$ if (3) holds, and $p_1 = E\left[v|L\right] - c$ otherwise.

Let $c_1 = E\left[v|L\right] - \psi^H E\left[v|H\right]$. One can show that c_1 decreases with α. When the signal is relatively accurate (α approaches 1), $c_1 \leqslant 0$, and the unique equilibrium outcome resembles that in the absence of consumers' valuation uncertainty: The optimal pricing strategy in an SPNE is the PS1 strategy, which features a declining pricing path in equilibrium. Given these prices, consumers who receive the high-type signal buy the software in period 0, and consumers who receive the low-type signal buy the software in period 1. The monopolist's overall profit in equilibrium is

$\psi^H (E[v|H] - c) + \delta E[v|L] - \delta\psi^L c$. The same equilibrium outcome holds if the signal is noisy (and, hence, $c_1 > 0$), but the implementation cost is not too low ($c_1 / (1-\delta)\psi^L \leqslant c \leqslant v_L$).

However, when the signal is noisy (and, hence, $c_1 > 0$), and the implementation cost is low (i.e., $0 \leqslant c < c_1/(1-\delta)\psi^L$), a new equilibrium outcome emerges: In equilibrium, the monopolist's optimal pricing strategy is the PS2 strategy. Given these prices, all consumers buy the software as soon as they enter the market. The monopolist's overall profit in equilibrium is $(1+\delta) E[v|L] - c$. The implications of this result are discussed at the end of this section.

3.2.2 Pure leasing

When the software is only for lease, in period 1, a consumer who receives signal y and does not lease the software in period 0 leases the software for the first time if the expected benefit of adoption is higher than the cost, or $E[v|y] - c \geqslant p_1$. A returning lessee, after learning her valuation for the software v, renews her lease in period 1 if the benefit of a renewal is higher than the lease renewal price, or $v \geqslant r_1$. In period 0, a consumer who receives signal y leases the software if the expected payoff from leasing in period 0 is no less than that from delaying adoption until period 1 or from no adoption at all, or

$$\begin{aligned} & E[v|y] - c - p_{l,0} + \delta \max\left(0, \psi_{Hy}(v_H - Er_1), E[v|y] - Er_1\right) \\ \geqslant\ & \max\{0, \delta(E[v|y] - c - Ep_1)\}. \end{aligned}$$

The monopolist's optimal pricing strategy given consumers' decision can be solved in the same fashion as in the previous pricing problem without consumer valuation uncertainty. Therefore, the detailed discussion is omitted for brevity. One can show that two alternative equilibrium outcomes are possible in an SPNE depending on the range of parameter values. In particular, define:

The PL1 strategy: in period 0, $p_{l,0} = (1-\delta) E[v|H] + \delta E[v|L] - c$; in period 1, $p_1 = E[v|H] - c$ if (3) holds; otherwise, $p_1 = E[v|L] - c$; $r_1 = v_H$ if

$$(x_{H,l}(1)\psi_{HH} + x_{L,l}(1)\psi_{HL}) v_H \geqslant (x_{H,l}(1) + x_{L,l}(1)) v_L; \tag{4}$$

otherwise, $r_1 = v_L$.

The PL2 strategy: in period 0, $p_{l,0} = E[v|L] - c$; in period 1, $p_1 = E[v|H] - c$ if (3) holds; otherwise, $p_1 = E[v|L] - c$; $r_1 = v_H$ if (4) holds; otherwise, $r_1 = v_L$.

Let $c_2 = (1-\delta) c_1 + \delta\beta(1-\alpha) v_H$. One can show that c_2 also decreases with α. When the signal is relatively accurate (α approaches 1), $c_2 \leqslant 0$, and the unique equilibrium outcome resembles that in the absence of consumers' valuation uncertainty: The optimal pricing strategy is the PL1 strategy, which features a declining pricing path for new leases in equilibrium. Given these prices, consumers receiving the high-type signal lease the software in period 0, and consumers receiving the low-type signal lease the software for the first time in period 1. Nonetheless, different from the benchmark model, not all consumers who lease the software in period 0 renew their leases in period 1. When $\beta v_H > v_L$, the optimal lease renewal price is such that only the high-type consumers renew their leases, and the monopolist loses the demand from the low-type consumers because of this discriminative pricing. His overall profit in equilibrium is

$$\psi^H \left(E\left[v|H\right] + \delta E\left[v|L\right] - c - \delta \psi_{LH} v_L \right) + \delta \psi^L \left(E\left[v|L\right] - c \right).$$

The same equilibrium outcome holds if the signal is noisy (and, hence, $c_2 > 0$), but the implementation cost is not too low ($c_2 / \left(1 - \delta\right) \psi^L \leqslant c \leqslant v_L$).

However, when the signal is noisy (hence $c_2 > 0$), and the implementation cost is low (i.e., $0 \leqslant c < c_2 / \left(1 - \delta\right) \psi^L$), a new equilibrium outcome emerges: In equilibrium, the monopolist's optimal pricing strategy is the PL2 strategy. Given these prices, all consumers lease the software as soon as they enter the market, but they renew their leases in period 1 only if their true types are the high type. The monopolist's overall profit in equilibrium is $E\left[v|L\right] - c + \delta \beta v_H$.

3.2.3 Concurrent selling and leasing

Now consider the option to offer selling and leasing concurrently in period 0. First, consider consumers' decision: In period 0, a consumer's expected benefit from buying the software is $\left(1 + \delta\right) E[v|y] - c_s - p_{s,0}$; that from leasing the software is

$$E[v|y] - c_l - p_{l,0} + \max \delta \left(0, \psi_{Hy} \left(v_H - Er_1\right), E[v|y] - Er_1\right);$$

that from delaying her adoption until period 1 is $\delta \left(E[v|y] - Ep_1\right)$. When both selling and leasing are offered in period 0, a consumer chooses the option that maximizes her expected payoff.

Note that in period 0, buying is more desirable to leasing for consumers who receive the high-type signal than for those who receive the low-type signal. This is because the difference in payoff between buying and leasing, or

$$\begin{aligned} &\left(1 + \delta\right) E[v|y] - c - p_{s,0} \\ &- \left[E[v|y] - c - p_{l,0} + \max \delta \left(0, \psi_{Hy} \left(v_H - Er_1\right), E[v|y] - Er_1\right)\right], \end{aligned}$$

is nondecreasing in y. Intuitively, when offered both options, consumers who receive the high-type signal are more motivated to buy the software outright since their true types are likely to be the high type, and buying allows them to avoid a higher lease renewal price in the future. Consumers who receive the low-type signal, though, value leasing more, as it allows them to revise their adoption decisions once they have more information on their true types from the initial use. Thus, the monopolist may want to offer a menu of options so that consumers can choose according to their preferences.

The monopolist's optimal pricing strategy given consumers' decision can be solved by backward induction. In period 1, given the demand state $(x_{H,n}(1), x_{H,l}(1), x_{L,n}(1), x_{L,l}(1))$, the monopolist's optimal pricing strategy is relatively straightforward: $p_1 = E[v|H] - c$ if (3) holds; otherwise, $p_1 = E[v|L] - c$; $r_1 = v_H$ if (4) holds; otherwise, $r_1 = v_L$. In period 0, the monopolist's pricing problem is to choose $(p_{s,0}, p_{l,0})$ such that his overall profit is optimal given consumers' decisions. By comparing the monopolist's profit levels given different initial selling and leasing prices, one can show that one set of concurrent selling and leasing prices can be optimal. Define:

The CSL strategy: in period 0, $p_{s,0} = \delta E[v|H] + E[v|L] - c$, $p_{l,0} = E[v|L] - c$; in period 1, $p_1 = E[v|H] - c$ if (3) holds; otherwise, $p_1 = E[v|L] - c$; $r_1 = v_H$ if (4) holds; otherwise, $r_1 = v_L$.

Proposition 2 summarizes the monopolist's optimal pricing strategy when consumers do not know their true valuation for the software prior to adoption. The proof compares the profitability of the optimal selling, optimal leasing and optimal concurrent selling and leasing prices, and is omitted for brevity. Let $b_1 = (\beta - \psi_{HL}) / (\psi_{LL} - (1-\alpha)(1-\beta))$.

Proposition 2 *When consumers do not know their true valuation for the software prior to adoption, selling and leasing are not equally profitable. Define* α_1 *to be the solution of*

$$\beta - \psi_{HL} = \psi_{HL}\psi_{LL}.$$

Case 1 $1/2 < \alpha \leqslant \alpha_1$,

- *The CSL strategy is optimal if* $0 < v_L/v_H < \min(b_1, \psi_{HL})$ *and* $0 \leqslant c < c_2/(1-\delta)\psi^L$. *In equilibrium, consumers who receive the high-type signal buy the software in period 0; consumers who receive the low-type signal lease in period 0 and renew their leases in period 1 only if their true types are the high type. The monopolist's overall profit is* $\delta\psi^H E[v|H] + E[v|L] - c + \delta\psi^L\psi_{HL}v_H$.

- *The PS1 strategy is optimal if* $\psi_{HL} \leqslant v_L/v_H < \beta$ *and* $c_1/(1-\delta)\psi^L \leqslant c \leqslant v_L$*; or* $0 < v_L/v_H < \psi_{HL}$ *and* $\max(c_1, c_2)/(1-\delta)\psi^L \leqslant c \leqslant v_L$*.*

- *The PS2 strategy is optimal if* $\min(b_1, \psi_{HL}) \leqslant v_L/v_H < \beta$ *and* $0 \leqslant c < c_1/(1-\delta)\psi^L$*.*

Case 2 $\alpha_1 < \alpha < 1$*,*

- *The CSL strategy is optimal if* $0 < v_L/v_H < \psi_{HL}$ *and* $0 \leqslant c < \max(c_2, 0)/(1-\delta)\psi^L$*.*

- *The PS1 strategy is optimal if* $0 < v_L/v_H < \psi_{HL}$ *and* $\max(0, c_2)/(1-\delta)\psi^L \leqslant c \leqslant v_L$*; or* $\psi_{HL} \leqslant v_L/v_H < \beta$ *and* $\max(c_1, c_1 + \delta(\beta v_H - E[v|L]))/(1-\delta)\psi^L \leqslant c \leqslant v_L$*.*

- *The PS2 strategy is optimal if* $(\beta - \psi_{HL})/\psi_{LL} \leqslant v_L/v_H < \beta$ *and* $0 \leqslant c < \max(c_1, 0)/(1-\delta)\psi^L$*.*

- *The PL2 is optimal if* $\psi_{HL} \leqslant v_L/v_H < (\beta - \psi_{HL})/\psi_{LL}$ *and*

$$0 \leqslant c < (c_1 + \delta(\beta v_H - E[v|L]))/(1-\delta)\psi^L.$$

The above result is illustrated in Figure 1. The relevant areas are those above the 45-degree line (i.e., $0 \leqslant c \leqslant v_L$). When the high- and low-type consumers are very different, and the implementation cost is not very high (Region I), concurrent selling and leasing (the CSL strategy) is optimal. When the high- and low-type consumers are not very different, and the implementation cost is relatively low (Region II), the optimal pricing policy is pure selling, and the prices are such that all consumers buy the software as soon as they enter the market (the PS2 strategy). When the implementation cost is not very low (Region III), the optimal pricing policy is pure selling, and the prices are such that consumers receiving the high-type signal buy the software in period 0, and consumers receiving the low-type signal buy the software in period 1 (the PS1 strategy). Pure leasing is optimal only if the signal is relatively accurate, the high- and low-type consumers are moderately different, and the implementation cost is relatively low (Region IV). The prices are such that all consumers lease the software in period 0 and renew their leases in period 1 only if their true types are the high type (the PL2 strategy).

This result has three implications: First, selling can be more profitable than leasing when consumers' valuation uncertainty is present. Leasing allows consumers to make sequential adoption decisions and, hence, reduce their risk of paying upfront for software that later turns out to be a misfit. This option may not benefit the monopolist, however. After the initial lease, some consumers are willing to pay more, while others are willing to pay less to renew their leases. The

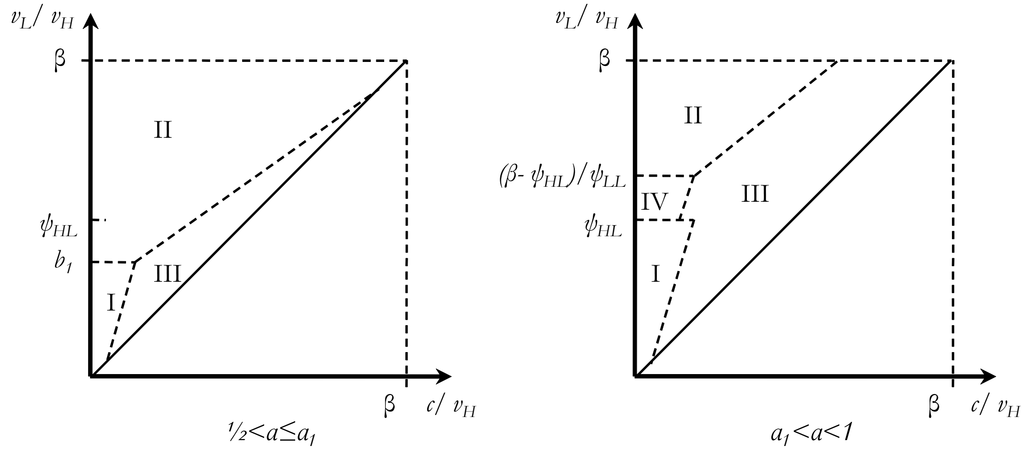

Figure 1: SPNE in a Two-period Game with Valuation Uncertainty

monopolist may be tempted to raise the lease renewal price if the high-end demand is sufficiently large. Nonetheless, when the lease renewal price is discriminative, the monopolist loses the demand from the low-type consumers in period 1. These consumers are willing to pay more for the software before learning their true types in period 0, and selling allows the monopolist to benefit somewhat from such a perception. The tradeoff between selling and leasing depends on the gain from a higher lease renewal price as compared to the loss from a lower demand in period 1.

Second, the implementation cost c acts as another factor that impacts the initial demand distribution. Although it does not have an impact on the equilibrium outcome in the benchmark model when $\beta v_H > v_L$, this is no longer true when consumers' valuation uncertainty changes their willingness to pay for the software prior to adoption (i.e., $v_L < E[v|L] < E[v|H] < v_H$). A higher implementation cost makes the low-end market–consumers receiving the low-type signal–less appealing to the monopolist, as it represents a larger fraction of these consumers' expected valuation for the software. More importantly, it makes leasing more costly to the monopolist because he has to subsidize more heavily in the initial lease, while only a fraction of the initial adopters are willing to renew their leases at a higher price later on. As Figure 1 shows, when c is relatively large, offering leasing is not an optimal pricing option.

Third, while concurrent selling and leasing is not profit-improving in the absence of consumer valuation uncertainty, this is no longer true when such uncertainty exists. Buying outright may be preferable to some consumers who perceive the software favorably and want to avoid paying high prices for the software later on. Leasing may be preferable to others who value the option to revise their adoption decision once they obtain more information on their true valuation for the software. Concurrent selling and leasing allows a menu of options so that consumers can choose according to their private information on their true types. In contrast, when consumers know their true valuation upfront, selling and leasing differ only in payment schedule. Consumers with rational expectations do not need to revise their adoption decisions and simply choose the option that gives them the highest payoff. Therefore, in the absence of consumer valuation uncertainty, having more pricing options does not improve the monopolist's profit.

4 An Infinite-Period Model

In this section, I consider the full infinite-period model with entry of new consumers. Recall that a unit mass of identical new consumers enters the market at the beginning of each period and lives for two periods. The monopolist sells and/or leases a software in each period and lives forever. There are no age-2 consumers in the initial period (i.e., $x_{i,j}(0) = 0$, $i = H$ or L, $j = n$ or l).

As in many dynamic games, there are multiple equilibria in this game even if we constrain ourselves to SPNE. Since the monopolist announces his pricing policy first and acts like a Stackelberg leader, he is motivated to choose the equilibrium in which he achieves the highest overall profit, and signals his choice to consumers. Thus, below, I focus on the subset of SPNE in which, among all SPNE of the game, the monopolist achieves the highest level of overall profit. Let us call this subset of equilibria the *profit-maximizing SPNE*.

4.1 Profit-Maximizing SPNE

One main finding of this paper is that when the implementation cost is not too low, all profit-maximizing SPNE of this game have the same properties that on the equilibrium path, only selling is offered at a stationary price, and the selling price is such that only consumers who receive the high-type signal buy the software as soon as they enter the market. Consumers who receive the low-type signal never adopt the software. The monopolist achieves the optimal overall profit

$$\psi^H \left((1 + \delta) E[v|H] - c \right) / (1 - \delta) . \tag{5}$$

This set of profit-maximizing SPNE may differ in players' strategies off the equilibrium path, but on the equilibrium path, they all share the same observed pricing and adoption behavior. The following proposition formalizes this result. Next, I investigate the impact of the new SaaS model on the equilibrium pricing behavior.

Let $c_3 = \left(\left(c_2 + \delta c_1\right)/\psi^L + E\left[v|L\right]\right)/2$, and $c_4 = \left(\left(1+\delta\right)c_1/\psi^L + E\left[v|L\right]\right)/2$.

Proposition 3 *When the implementation cost is not too low, such that*

$$\max\left(c_3, c_4, 0\right) < c \leqslant v_L, \tag{6}$$

all profit-maximizing SPNE of the game have the same properties that on the equilibrium path, only selling is offered at a stationary price, $p_{s,t} = \left(1+\delta\right)E[v|H] - c,\ t = 0, 1, 2\ldots$ *The monopolist achieves the optimal profit defined in (5).*

A sketch of the proof: The proof utilizes a preliminary result (Lemma 1 in the Appendix), which characterizes the monopolist's optimal pricing strategy if he is able to commit to a price trajectory for an infinite time horizon in which selling and leasing prices may vary over time. Since the monopolist can commit to any price trajectory, including those that appear on the equilibrium path of an SPNE, this exercise allows one to obtain an upper bound of the monopolist's overall profit in a profit-maximizing SPNE. In particular, it shows that given (6), the monopolist's optimal profit is no greater than (5). The result also characterizes the set of possible price trajectories that would allow the monopolist to achieve this optimal profit level if they can be enforced in an SPNE.

The proof then establishes that given (6), there exists an SPNE of the game in which, on the equilibrium path, only selling is offered at a stationary price, and the monopolist achieves the maximum profit level as defined in (5). Next, this result is generalized. Recall that the preliminary result (i.e., Lemma 1) characterizes the set of possible price trajectories that would allow the monopolist to achieve the optimal profit level (5). The rest of the proof discusses the monopolist's ability to enforce these price trajectories in an SPNE. It is shown that given (6), the only price trajectory that can be on the equilibrium path of an SPNE is the one in which the monopolist offers pure selling at a stationary price, $p_{s,t} = \left(1+\delta\right)E[v|H] - c$ for $\forall t$. This concludes the proof. All proofs not included in the paper are in the Appendix.

This finding extends that of the two-period model and offers a potential explanation for the prevalence of perpetual licensing in the enterprise software market before the lease-based SaaS model became available. Selling is more profitable than leasing, as it allows the monopolist to charge upfront and shift the risk of valuation uncertainty to consumers. In addition, when the implementation cost is not low, leasing becomes costly to the monopolist as he has to subsidize heavily for the initial lease, while only a fraction of early adopters are willing to pay a higher price for lease renewal. Entry of new consumers brings in fresh high-end demand (i.e., consumers receiving the high-type signal), and thus, the monopolist is less motivated to lower his prices overtime. As a result, the optimal pricing strategy is characterized by pure selling at a stationary price overtime. The adoption pattern suggested in this finding is consistent with the observation that many enterprise software vendors' pricing policy in the past mainly targeted the high-end market (e.g., large companies) (Cusumano 2007).

Advances in web-based technologies enable the new lease-based SaaS model, which requires a lower upfront implementation cost. How would this exogenous technological development impact the long-run market equilibrium? Assume that the in-house implementation cost c_s remains the same. The following proposition shows that when firms do not heavily discount future benefits, and the difference between c_s and c_l is relatively small, all profit-maximizing SPNE of the game still feature the same pricing policy and adoption behavior on the equilibrium path: Only selling is offered at a stationary price, and the prices are such that only consumers receiving the high-type signal buy the software as soon as they enter the market. Let

$$c_5 = \min\left\{(1-\delta)\left(\psi_{HH}v_H - v_L\right), \delta\psi_{LH}v_L, c_s - c_3, 2\left(c_s - c_4\right), \psi^L c_s - (1+\delta)\, c_1\right\}. \tag{7}$$

Proposition 4 *Given* $\max\left(c_3, c_4, 0\right) < c_s \leqslant v_L$*, when the discount factor is not too low so that* $(1-\beta)\, v_L/\beta v_H \leqslant \delta < 1$*, and the difference between* c_s *and* c_l *is relatively small such that* $0 \leqslant c_s - c_l < c_5$*, all profit-maximizing SPNE of the game have the properties that on the equilibrium path, only selling is offered at a stationary price* $p_{s,t} = (1+\delta)\, E[v|H] - c_s$*,* $t = 0, 1, 2...$*, and the monopolist gains the optimal profit of* $\psi^H \left((1+\delta)\, E[v|H] - c_s\right)/\left(1-\delta\right)$*.*

When the cost reduction (i.e., $c_s - c_l$) is significant, however, I am interested in characterizing the monopolist's optimal pricing policy in a profit-maximizing SPNE given different ranges of parameter values that may represent features of different software-applications markets. I discuss two such results. In particular, the following proposition shows that when the software market is characterized by 1) the high- and low-type consumers having relatively different valuations for the software, and 2) the signal being relatively inaccurate, and if SaaS lowers a significant amount of the implementation cost, then the monopolist offers only the lease-based SaaS model in a profit-maximizing SPNE. Let

$$c_6 = \max\left\{\delta\psi_{LH}v_L, c_s - \left[c_2 + \delta c_1 - \delta\,(1-\alpha)\,(1-\beta)\, v_L\right]/\psi^L\right\}.$$

Proposition 5 *When the high- and low-type consumers' valuations are relatively different or* $0 < v_L/v_H \leqslant \beta\left(2 - 1/\alpha\right)$*, and the difference between* c_s *and* c_l *is such that* $c_6 \leqslant c_s - c_l < v_L$*, all* *profit-maximizing SPNE of the game have the property that, on the equilibrium path, only the lease-based SaaS model is offered, and the prices are* $p_{l,t} = E[v|L] - c_l$, $r_{t+1} = v_H$, *for* $t = 0, 1, 2....$ *In equilibrium, all consumers lease the software as soon as they enter the market and renew their leases only if their true types turn out to be the high type. The monopolist's overall profit is* $(E[v|L] - c_l + \delta\beta v_H) / (1 - \delta)$.

Evidently, $c_5 \leqslant c_6$. Thus, the range of parameter values in Proposition 5 does not overlap with that in Proposition 4. Moreover, a necessary condition for this proposition to hold is

$$c_s - [c_2 + \delta c_1 - \delta (1 - \alpha)(1 - \beta) v_L] / \psi^L \leqslant c_6 < c_s,$$

or

$$c_2 + \delta c_1 - \delta (1 - \alpha)(1 - \beta) v_L > 0. \tag{8}$$

One can show that the left-hand side of (8) is a decreasing function of α and is negative at $\alpha = 1$. Thus, there exists an $\alpha^* \in (1/2, 1)$, so that the above proposition holds only if the accuracy of the signal is low, or $1/2 < \alpha < \alpha^*$.

The intuition behind this result is as follows. A significantly lower implementation cost makes leasing through SaaS an attractive pricing option for the monopolist. Leasing through SaaS may be more appealing in software markets in which the high- and low-type consumers value the software relatively differently, and the signal is very noisy. This is because in these markets, consumers who receive the low-type signal are willing to pay more to try out the software for the first time, as there is a better chance that their true types may be the high type. Moreover, the monopolist is more motivated to help consumers learn their true types, as a larger fraction of high-type consumers may have received the wrong signal, and once they have learned their true types, their willingness to pay for the software is much improved. As a result, in these software markets, the monopolist moves entirely to offering only the lease-based SaaS solution. The prices of their SaaS solutions feature a low introductory offer for new customers and a higher price for lease renewals.

When the difference in implementation cost between the two pricing options is intermediary, the monopolist may want to offer both so that consumers can choose according to their preferences. Recall that the benefit of buying over leasing is higher for new consumers receiving the high-type signal than for those receiving the low-type signal. Offering leasing at a relatively low price may cannibalize the monopolist's revenue from selling that targets the high-end market, however. The following proposition shows that when the software market is characterized by a small fraction of high-type consumers who have very different valuations for the software than the rest of consumers, and the difference in implementations costs between buying and leasing the software via SaaS is intermediary, the monopolist's optimal pricing strategy in a profit-maximizing SPNE involves concurrently offering selling and the lease-based SaaS model. In equilibrium, consumers receiving the high-type signal buy the software as soon as they enter the market, and consumers receiving the low-type signal lease the software when they enter the market and renew their leases only if their true types turn out to be the high type. The monopolist offers the lease-based option at an introductory price to expand his market coverage. His long-run pricing strategy still targets the high-type consumers. This result is formally stated in the following proposition. Let

$$
\begin{aligned}
c_7 &= \delta\psi_{LH}v_L + \delta\min\left\{0, \left(\beta v_H - E\left[v|L\right]\right)/\psi^H\right\}, \\
c_8 &= \frac{\psi^L c_s - \left(E[v|L] - \psi^H E\left[v|H\right] + \delta\beta\left(1-\alpha\right)v_H\right)}{\psi^L - \psi^H}.
\end{aligned}
$$

Proposition 6 *When $0 < \beta \leqslant 1/2$, the high- and low-type consumers' valuations are very different, so that $0 < v_L/v_H \leqslant \psi_{HL}$, and the implementation costs are such that $c_8 < c_s - c_l \leqslant c_7$, all profit-maximizing SPNE of this game have the property that, on the equilibrium path, both selling and the lease-based SaaS model are offered, and the prices are stationary, characterized by $p_{s,t} = E[v|L] - c_s + \delta E[v|H]$, $p_{l,t} = E[v|L] - c_l$ and $r_{t+1} = v_H$, $t = 0, 1, 2....$ The monopolist's overall profit is $\left[E[v|L] + \delta\psi^H E[v|H] - \left(\psi^H c_s + \psi^L c_l\right) + \delta\beta\left(1-\alpha\right)v_H\right]/\left(1-\delta\right)$.*

5 Discussion

This section discusses the economic implications of the results.

5.1 SaaS or not-SaaS

According to Proposition 4, a necessary condition under which an established software vendor would find it optimal to start offering the lease-based SaaS model is $c_s - c_l > c_5$, where c_5 is defined in (7). As the signal becomes more accurate (or α approaches 1), c_5 approaches zero, in which case a small difference in implementation cost (i.e., $c_s - c_l$) could make offering SaaS a profitable option. One important implication of this finding is that offering SaaS is likely to be optimal for software applications that feature a low level of customer valuation uncertainty–e.g., for small-scale applications that support relatively simple functions, such as online office suites. Perpetual licensing was more profitable than lease-based licensing because it allowed the monopolist to shift the risk of valuation uncertainty to customers. In these application markets, however, the threat of valuation uncertainty is low. Therefore, a small change in implementation cost that favors the lease-based SaaS model is likely to make offering SaaS a profitable option.

Moreover, c_5 is a non-decreasing function of c_s. Unless SaaS reduces a significant fraction of the in-house implementation cost (c_s), the lease-based SaaS model will not become a profitable option. Software implementation costs have several components. SaaS reduces implementation costs mainly in terms of infrastructure costs and technical deployment costs. Clients still incur the costs associated with customization, process transformation, user training, etc. (Xin and Levina 2008). Another implication of the result is that when the latter costs represent a large fraction of the overall implementation cost, the software vendor is less likely to offer the software via the lease-based SaaS model. SaaS is likely to be popular for software applications that, for instance, support common business processes that require fewer customization efforts, such as payroll processes.

These findings are consistent with the empirical observations and offer insights on the variation in SaaS adoption across software markets. Market research finds that payment flexibility and the ability to avoid misspent IT investments are the main drivers for SaaS adoption (Mertz et al. 2008). Moreover, SaaS adoption is slow for large or complex software applications, such as the ERP software, which supports complex business processes and, hence, is hard to evaluate outright. Since these business processes are also likely to vary significantly across organizations, extensive customization efforts may be required in order to fit businesses' specific needs (Hitt et al. 2002). SaaS adoption, though, is more active for applications that support common business processes, such as sales force automation, and simple applications such as online office suites are the fastest-growing markets for SaaS (Mertz et al. 2008).

5.2 Vendor's profit and social welfare

The set of SPNE described in Proposition 3 resembles the market outcome prior to SaaS. The monopolist's discounted sum of profit on the equilibrium path is $\psi^H \left[(1+\delta)\, E\left[v|H\right] - c_s\right] / \left(1-\delta\right)$, which increases with the accuracy of the signal since

$$\frac{\partial \left\{\psi^H \left[(1+\delta)\, E\left[v|H\right] - c_s\right] / \left(1-\delta\right)\right\}}{\partial \alpha} > 0$$

when $c_s \leqslant v_L < \beta v_H$. The more accurate the signal (i.e., α approaches 1), the more likely it is that a customer who receives the high-type signal is also the high type and, hence, the more the customer is willing to pay to buy the software outright. Although the fraction of clients who receive the high-type signal may decrease with α, this effect is more than offset by the increase in adopters' willingness to pay. Therefore, the vendor is motivated to invest in improving the quality of the signal by, for example, investigating the performance impact of his software on different kinds of clients (e.g., the establishment of the Software Economics Council), or by educating customers on his product (e.g., through trade shows), which are common practices in the enterprise software industry.

Since I assume that the marginal cost of software production is zero and $c_l < c_s \leqslant v_L < v_H$, social welfare increases with the adoption of the software and is maximized when the whole market is fully covered in each period. Prior to SaaS, perpetual licensing was the prevailing pricing strategy. According to this paper, one consequence of this pricing practice was that only customers whose perceived valuation for the software was high adopted the software. Although the remaining customers may have received the wrong signal, the high implementation costs made it simply too costly for the monopolist to adjust his prices so that these customers could afford to try out the software and learn their true types.

SaaS requires lower upfront costs and, therefore, presents an opportunity for an expanded adoption basis and improved social welfare in some markets. As I pointed out earlier, this is more likely to happen for software applications that support relatively simple business processes and tend to require low levels of client-specific implementation. Advances in cloud computing technologies are likely to bring down the cost of delivering software services further. This will make SaaS an attractive option in more software markets and potentially increase the total social surplus.

6 Conclusion

This paper studies a software vendor's decision to offer perpetual or lease-based licensing when market demand is characterized by two features: 1) customers are uncertain about their valuation of the software prior to adoption, although they receive an informative signal about it and can learn it through use; and 2) software adoption requires a fixed upfront implementation cost. The pricing problem is investigated in a standard overlapping-generations model with an infinitely-lived monopoly software vendor and overlapping generations of customers. I focus on the Subgame Perfect Nash Equilibrium (SPNE) of the game.

One of the main findings is that perpetual licensing can be more profitable than lease-based licensing when the demand is characterized by these two features. This result sheds light on the prevalence of perpetual licensing in the enterprise software market and contrasts with the well-known results of durable-goods pricing (Coase 1972; Bulow 1982), which suggest that leasing is at least as profitable as, if not more profitable than, selling.

Intuitively, when customers do not know their true valuation for the software prior to use, leasing allows them to make sequential adoption decisions and, thus, reduces their risk of paying upfront for software that later turns out to be a misfit. This option may not benefit the monopolist, however. Some customers are willing to pay more, while others are willing to pay less for the software after learning their true valuation through the initial lease. The monopolist may be tempted to charge a higher price for lease renewal. When the lease renewal price is discriminative, the monopolist loses the demand from those returning lessees whose true valuation for the software turns out to be lower than their initial perception. These customers are willing to pay more for the software before learning their valuation, and perpetual licensing allows the monopolist to benefit somewhat from this perception. The implementation cost acts as another factor that impacts new customers' demand distribution, and a high implementation cost makes leasing more costly to the monopolist because he has to subsidize more heavily in the initial lease, while only a fraction of the initial adopters are willing to renew their leases at a higher price.

Advances in web-based technologies enable the lease-based SaaS model, which requires a lower implementation cost compared to perpetual licensing. The paper then analyzes the new long-run market equilibrium after SaaS becomes available, treating SaaS as a type of leasing. Proposition 4 establishes that offering SaaS would not be optimal unless the implementation cost saving is large enough; the threshold depends on customers' valuation uncertainty for the software.

One implication of this finding is that SaaS is likely to be popular for software applications that feature a low level of customer valuation uncertainty, such as small-scale applications that support relatively simple functions (e.g., online office suites), which are easier to evaluate upfront. Moreover, SaaS offers cost savings through shared IT infrastructure and operational services, but not customer-specific solutions. Another implication of this finding is that SaaS is likely to be popular for applications for which the latter costs are minimum; for example, applications that support common business processes and require fewer customization efforts (e.g., payroll processes). These implications are consistent with published data on SaaS adoption, which shows that SaaS is less popular for software applications that support complex business processes and are hard to evaluate outright or for those that require substantial customization efforts, such as ERP applications (Mertz et al. 2007).

Propositions 5 and 6 explore different SPNE outcomes that involve the offering of the lease-based SaaS model on the equilibrium path. Proposition 6 suggests that SaaS may be offered as a complement to perpetual licensing, so that customers whose perceived valuation for the software is relatively low can try out the software at a low entry cost and decide if they want to continue using the software after having more information on it. When the cost saving is substantial, the software vendor may choose to offer the lease-based SaaS model only. Compared to perpetual licensing, SaaS gives the vendor more flexibility in adjusting his prices over time. When the high- and low-type consumers are relatively different, according to Proposition 5, the SaaS offering may feature a low introductory price for new customers and a high lease renewal price for existing customers.

This study can be extended in a number of directions. First, I have assumed that the monopolist knows the demand state in each period. It is, nonetheless, feasible that some exogenous shocks might cause a positive measure of consumers to cooperatively deviate from their strategy. In this case, the monopolist cannot calculate the demand state precisely without knowing consumers' private signals, although he remembers the history of the game (i.e., prices and sales). Each history may correspond to a set of multiple possible demand states, which complicates the monopolist's pricing problem. A demand state is *consistent with the history* if there exists a sequence of adoption behaviors that produces the observed history of sales and leads to this demand state. (The sequence of adoption behaviors may or may not be optimal from consumers' perspective given the history of prices.)

One approach to generalize the current results could be to consider NE using undominated strategies (NEU). A strategy is *undominated* if there is no other strategy that is at least as good for some history and strictly better for others. The monopolist's strategy, therefore, can be thought of as having 2 components: an algorithm that predicts the current demand state (i.e., $x_{H,n}(t), x_{H,l}(t), x_{L,n}(t), x_{L,l}(t)$) given the history of the game; and a pricing function that specifies a set of selling and leasing prices (i.e., $p_{s,t}$, $p_{l,t}$, r_t) given the predicted demand state. A strategy profile is an NEU if: 1) the monopolist's algorithm makes the right prediction on the demand state on the equilibrium path; and his prediction off the equilibrium path is consistent with the history, although not necessarily correct; 2) the monopolist's pricing policy maximizes his profit given his prediction on the demand state; and 3) consumers' adoption decisions maximize their expected utility given their history of observations and the current prices.

Second, Proposition 3 characterizes the SPNE of the game that describes the observed pricing practice prior to SaaS. Propositions 4 through 6 explore the long-run market equilibrium after SaaS becomes available. It remains to be shown what the transitioning pricing path would be before the market reaches the new long-run equilibrium. Finally, I have focused on demand factors that drive the adoption of SaaS in this work. Another potential driver for established software vendors to offer SaaS is competition from the low-end market entrants whose pricing strategies are largely SaaS-based. One future step would be to extend the monopoly model to a duopoly model and study how competition may change a market incumbent's optimal pricing policy.

The growth of the SaaS market has highlighted the importance of term-based pricing. The unique demand features such as customers' valuation uncertainty present many fruitful opportunities for future research.

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Appendix

Proof of Proposition 3. Proof of Proposition 3 utilizes the following preliminary results.

Lemma 1 *In the infinite-period game, suppose that the monopolist is able to commit to a price trajectory from period* $\widehat{t}$ *on. If the initial demand state in period* $\widehat{t}$ *is such that* $x_{H,n}\left(\widehat{t}\right) = x_{L,n}\left(\widehat{t}\right) = 0$, *and* $x_{H,l}\left(\widehat{t}\right) = x_{L,l}\left(\widehat{t}\right) = 0$, *and the implementation cost is not too low such that*

$$\left(\max\left(c_1, c_2\right) + \delta c_1\right)/\psi^L < c \leqslant v_L, \tag{9}$$

the monopolist's optimal price trajectory from period $\widehat{t}$ *on is characterized by: for* $\forall t \geqslant \widehat{t}$, *either* $p_{s,t} = (1+\delta) E[v|H] - c$, *and* $p_{l,t} = \overline{p_l}$ *or* $p_{s,t} = \overline{p_s}$, $p_{l,t} = (1+\delta) E[v|H] - c - \delta r_{t+1}$, *and* $0 < r_{t+1} \leqslant v_L$. *His optimal continuation profit from period* $\widehat{t}$ *on is (5).*

Proof. *First, let us consider a different problem: In the 2-period model (without entry of new consumers in period 1) with consumer valuation uncertainty, if the monopolist is able to commit to a set of selling, leasing and lease renewal prices in periods 0 and 1 (i.e.,* $p_{s,0}, p_{l,0}, r_1, p_1$*), what is the monopolist's optimal pricing policy? With some relatively straightforward algebra, one can show that given (9), the monopolist's optimal (committed) pricing policy is either* $p_{s,0} = (1+\delta) E[v|H] - c$, $p_{l,0} = \overline{p_l}$, $p_1 \geqslant E[v|H] - c$, *or* $p_{s,0} = \overline{p_s}$, $p_{l,0} = (1+\delta) E[v|H] - c - \delta r_1$, $0 < r_1 \leqslant v_L$ *and* $p_1 \geqslant E[v|H] - c$. *The monopolist's optimal profit is* $\psi^H\left((1+\delta) E[v|H] - c\right)$.

Now consider the infinite-period model. Since $x_{i,j}\left(\widehat{t}\right) = 0$ *(*$i = H$ *or* L, $j = n$ *or* l*), or there is no residual demand from age-2 consumers in period* $\widehat{t}$, *the monopolist's overall profit from period* $\widehat{t}$ *on can be seen as the sum of his profit from each generation of consumers who enter the market in period* $t \geqslant \widehat{t}$, *which, according to the early discussion, is no greater than* $\psi^H\left((1+\delta) E[v|H] - c\right)$ *given (9). Thus, the monopolist's overall profit from period* $\widehat{t}$ *on is no greater than (5). One can show that the monopolist gains this optimal profit by committing to the price trajectories described in the lemma. Moreover, no other price trajectories can achieve the same profit. This is because to achieve this profit, the monopolist's profit from each generation of consumers needs to be* $\psi^H\left((1+\delta) E[v|H] - c\right)$. *From the 2-period model, we know this requires for* $\forall t \geqslant \widehat{t}$ *either* $p_{s,t} = (1+\delta) E[v|H] - c$, $p_{l,t} = \overline{p_l}$, $\min\left(p_{s,t+1}, p_{l,t+1}\right) \geqslant E[v|H] - c$, *or* $p_{s,t} = \overline{p_s}, p_{l,t} = (1+\delta) E[v|H] - c - \delta r_{t+1}$, $0 < r_{t+1} \leqslant v_L$, $\min\left(p_{s,t+1}, p_{l,t+1}\right) \geqslant E[v|H] - c$. *This concludes the proof.* ■

Lemma 2 *When price commitment is possible, and the initial demand state in period* $\widehat{t}$ *is such that* $x_{H,n}\left(\widehat{t}\right) = 0$, $x_{L,n}\left(\widehat{t}\right) = \psi^L$, *and* $x_{H,l}\left(\widehat{t}\right) = x_{L,l}\left(\widehat{t}\right) = 0$, *if the implementation cost is not too low such that (6) holds, the monopolist's optimal price trajectory from period* $\widehat{t}$ *on is characterized by: for* $\forall t \geqslant \widehat{t}$, *either* $p_{s,t} = (1+\delta) E[v|H] - c$, *and* $p_{l,t} = \overline{p_l}$ *or* $p_{s,t} = \overline{p_s}, p_{l,t} = (1+\delta) E[v|H] - c - \delta r_{t+1}$, *and* $0 < r_{t+1} \leqslant v_L$. *His optimal continuation profit is (5).*

Proof. *Clearly, (6) is sufficient for (9). Consider two types of price trajectories: one is such that* $\min\left(p_{s,\widehat{t}}, p_{l,\widehat{t}}\right) > E[v|L] - c$*; the other is such that* $\min\left(p_{s,\widehat{t}}, p_{l,\widehat{t}}\right) = E[v|L] - c$ *(Note that one can show that on any optimal price trajectory,* $\min(p_{s,t}, p_{l,t}) \geqslant E[v|L] - c$, $\forall t$*).*

When $\min\left(p_{s,\widehat{t}}, p_{l,\widehat{t}}\right) > E[v|L] - c$*, the age-2 consumers receiving the low-type signal (who have not adopted the software in period* $\widehat{t} - 1$*) do not adopt the software in period* $\widehat{t}$*. The monopolist's continuation profit is the sum of his profit from consumers who enter the market in period* $t \geqslant \widehat{t}$*. Applying Lemma 1, we know the optimal price trajectory is the proposed price trajectory.*

When $\min\left(p_{s,\widehat{t}}, p_{l,\widehat{t}}\right) = E[v|L] - c$*, the age-2 consumers receiving the low-type signal adopt the software in period* $\widehat{t}$*, and so do all age-1 consumers. So the demand state in period* $\widehat{t} + 1$ *is* $x_{H,n}\left(\widehat{t} + 1\right) = x_{L,n}\left(\widehat{t} + 1\right) = 0$*. Applying Lemma 1, we know the monopolist's optimal price trajectory from period* $\widehat{t} + 1$ *on is:* $r_{\widehat{t}+1} = v_H$ *if* $\left(x_{H,l}\left(\widehat{t}+1\right)\psi_{HH} + x_{L,l}\left(\widehat{t}+1\right)\psi_{HL}\right) v_H \geqslant \left(x_{H,l}\left(\widehat{t}+1\right) + x_{L,l}\left(\widehat{t}+1\right)\right) v_L$*, and* $r_{\widehat{t}+1} = v_L$ *otherwise; and* $\forall t > \widehat{t}$*, either* $p_{s,t} = (1+\delta) E[v|H] - c$*,* $p_{l,t} = \overline{p_l}$ *or* $p_{s,t} = \overline{p_s}$*,* $p_{l,t} = (1+\delta) E[v|H] - c - \delta r_{t+1}$*, and* $0 < r_{t+1} \leqslant v_L$*. Now the monopolist simply needs to choose* $\left(p_{s,\widehat{t}}, p_{l,\widehat{t}}, r_{\widehat{t}+1}\right)$ *so as to optimize his overall profit given* $\min\left(p_{s,\widehat{t}}, p_{l,\widehat{t}}\right) = E[v|L] - c$*. One can show that four possibilities can be optimal: 1)* $p_{s,\widehat{t}} = \overline{p_s}$*,* $p_{l,\widehat{t}} = E[v|L] - c$ *and* $r_{\widehat{t}+1} = v_H$*; 2)* $p_{s,\widehat{t}} = E[v|L] + \delta E[v|H] - c$*,* $p_{l,\widehat{t}} = E[v|L] - c$*, and* $r_{\widehat{t}+1} = v_H$*; 3)* $p_{s,\widehat{t}} = E[v|L] + \delta v_L - c$*,* $p_{l,\widehat{t}} = E[v|L] - c$*, and* $r_{\widehat{t}+1} = v_L$*; 4)* $p_{s,\widehat{t}} = (1+\delta) E[v|L] - c$*,* $p_{l,\widehat{t}} = E[v|L] - c$*, and* $r_{\widehat{t}+1} > v_H$*. Compare the profitability of these four alternative price trajectories with that of the proposed price trajectory. One can show that when (6) holds, it is suboptimal to commit to a price trajectory such that* $\min\left(p_{s,\widehat{t}}, p_{l,\widehat{t}}\right) = E[v|L] - c$*. This concludes the proof.* ■

Proof of Proposition 3. From Lemma 1, we know that when (9) holds, to achieve the optimal profit defined in (5), the monopolist's pricing policy on the equilibrium path is: either (pure selling) $p_{s,t} = (1+\delta) E[v|H] - c$, $p_{l,t} = \overline{p_l}$, or (pure leasing) $p_{s,t} = \overline{p_s}$, $p_{l,t} = (1+\delta) E[v|H] - c - \delta r_{t+1}$, and $0 < r_{t+1} \leqslant v_L$, $\forall t \geqslant 0$. Consider the following strategy profile; let's call it the PSS strategy:

The monopolist: If $x_{H,n}(t) = 0$, $p_{s,t} = (1+\delta) E[v|H] - c$, and $p_{l,t} > E[v|H] - c$; otherwise, $p_{s,t} = (1+\delta) E[v|H] - c$, $p_{l,t} = E[v|H] - c$; if $(x_{H,l}(t)\psi_{HH} + x_{L,l}(t)\psi_{HL}) v_H \geqslant (x_{H,l}(t) + x_{L,l}(t)) v_L$, $r_t = v_H$, otherwise $r_t = v_L$.

New consumers who receive signal y: Buy if $p_{s,t} \leqslant (1+\delta) E[v|y] - c$ and $p_{l,t} \geqslant p_{s,t} - \delta E[v|y]$; Lease if $p_{l,t} \leqslant E[v|y] - c$ and $p_{l,t} < p_{s,t} - \delta E[v|y]$; no adoption otherwise.

Consumers who have leased the software in period $t-1$: Renew lease only if $r_t \leqslant v$.

Age-2 consumers who did not adopt the software at age 1 and receive signal y: Buy if $p_{s,t} \leqslant E[v|y] - c$ and $p_{l,t} \geqslant p_{s,t}$; Lease if $p_{l,t} \leqslant E[v|y] - c$ and $p_{l,t} < p_{s,t}$; no adoption otherwise.

Given (6), the above strategy profile is a profit-maximizing SPNE: It is straightforward to check that given the monopolist's strategy, consumers do not gain from any deviation. Applying Lemmas 1 and 2, one can show that the monopolist gains the maximum profit by following the specified strategy in each period. Thus, the proposed strategy profile is a profit-maximizing SPNE. On the equilibrium path, only selling is offered at a stationary price $p_{s,t} = (1+\delta) E[v|H] - c$, $\forall t \geqslant 0$.

Next, I show that given (6), there are no profit-maximizing SPNE of the game in which leasing is offered on the equilibrium path. Assume that on the equilibrium path, pure leasing is offered in at least one period, say in period $\widetilde{t}$ with prices $p_{s,\widetilde{t}} = \overline{p_s}$, $p_{l,\widetilde{t}} = (1+\delta) E[v|H] - c - \delta Er_{\widetilde{t}+1}$, $0 < Er_{\widetilde{t}+1} \leqslant v_L$, where $Er_{\widetilde{t}+1}$ represents consumers' expectation about the next-period lease renewal price. I establish the result by showing that there exists a profitable deviation to the monopolist's equilibrium pricing policy and, hence, a contradiction.

Now consider an alternative price trajectory from period $\widetilde{t}+1$ on: $r_{\widetilde{t}+1} = v_H$, and the monopolist plays his PSS strategy for $t \geqslant \widetilde{t}+1$. On the demand side, the age-2 consumers who have leased the software in period $\widetilde{t}$ and whose true types are the high-type renew their leases in period $\widetilde{t}+1$. New consumers who receive the high-type signal and enter the market in period $t \geqslant \widetilde{t}+1$ buy the software at age 1 since they receive the same payoff as if they were on the equilibrium path, and there is no need to delay adoption, as the PSS strategy is indeed optimal for the monopolist for the remaining game when (6) holds. In fact, this subgame itself is an SPNE. The monopolist's overall profit from this alternative pricing policy is $\psi^H \psi_{HH} v_H + \psi^H \frac{(1+\delta)E[v|H]-c}{1-\delta}$, which is higher than his optimal profit if he remains on the equilibrium path, or $\psi^H v_L + \psi^H \frac{(1+\delta)E[v|H]-c}{1-\delta}$ given $\beta v_H > v_L$. Contradiction. This concludes the proof.

Proof of Proposition 4. Two preliminary results are necessary for proving Proposition 4.

Lemma 3 *When price commitment is possible, and the initial demand state in period $\widehat{t}$ is such that $x_{H,n}(\widehat{t}) = 0$, $x_{L,n}(\widehat{t}) \leqslant \psi^L$, and $x_{H,l}(\widehat{t}) = x_{L,l}(\widehat{t}) = 0$, if $0 < c_s - c_l < c_5$, the monopolist's optimal price trajectory from period $\widehat{t}$ on is: $p_{s,t} = \overline{p_s}$, $p_{l,t} = (1+\delta) E[v|H] - c_l - \delta r_{t+1}$, and $0 < r_{t+1} \leqslant v_L$, for $t \geqslant \widehat{t}$. His optimal continuation profit is $\psi^H ((1+\delta) E[v|H] - c_l) / (1-\delta)$.*

Lemma 4 *When price commitment is possible, and the initial demand state in period $\widehat{t}$ is such that $x_{H,n}(\widehat{t}) = 0$, $x_{L,n}(\widehat{t}) \leqslant \psi^L$ and $x_{H,l}(\widehat{t}) = x_{L,l}(\widehat{t}) = 0$, and the monopolist can only commit to a price trajectory that satisfies $\nexists t$, s.t. $p_{s,t} = \overline{p_s}$, $p_{l,t} = (1+\delta) E[v|H] - c_l - \delta r_{t+1}$, and $0 < r_{t+1} \leqslant v_L$ for $t \geqslant \widehat{t}$, his optimal price trajectory from period $\widehat{t}$ on is characterized by: $p_{s,t} = (1+\delta) E[v|H] - c_s$, $p_{l,t} = \overline{p_l}$, for $t \geqslant \widehat{t}$, if the discount factor is not too low so that $(1-\beta) v_L / \beta v_H < \delta < 1$ and the* *difference between* c_s *and* c_l *is relatively small such that* $0 < c_s - c_l < c_5$.

The proofs of the above 2 lemmas use the same technique as in the proofs of Lemmas 1 and 2. I first consider the monopolist's pricing problem in a 2-period game in which price commitment is possible, only in the case of Lemma 4, I consider a subset of pricing policies excluding the possibility: $p_{s,0} = \overline{p_s}$, $p_{l,0} = (1+\delta) E[v|H] - c_l - \delta r_1$, and $0 < r_1 \leqslant v_L$. I then extend the results to consider the optimal price trajectory in the infinite-period game. The price trajectory may or may not induce the age-2 consumers receiving the low-type signal to adopt the software in period $\widehat{t}$. After comparing the profitability of alternative price trajectories, one can obtain the results described in the lemmas. Details of the proofs are obmitted to avoid repetition.

Proof of Proposition 4. I first show that given the specified conditions, there are no SPNE of the game in which on the equilibrium path, consumers lease the software at a price $(1+\delta) E[v|H] - c_l - \delta v_L \leqslant p_{l,t} < (1+\delta) E[v|H] - c_l$. I establish this by constructing a contradiction. Assume this is true, and new consumers receiving the high-type signal lease the software at such a price in period $\widehat{t}$. From Lemma 3, we know that when $0 < c_s - c_l < c_5$, the monopolist's continuation profit from period $\widehat{t}+1$ on is no greater than $\psi^H r_{\widehat{t}+1} + \psi^H ((1+\delta) E[v|H] - c_l) / (1-\delta)$, where $0 < r_{\widehat{t}+1} \leqslant v_L$. I show that there exists a profit-maximizing SPNE in this sub-game in which the monopolist's strategy is described by $r_{\widehat{t}+1} = v_H$, and for $\forall t \geqslant \widehat{t}+1$, if $x_{H,n}(t) = 0$, $p_{s,t} = (1+\delta) E[v|H] - c_s$, $p_{l,t} = \overline{p_l}$; if $x_{H,n}(t) > 0$, $p_{s,t} = (1+\delta) E[v|H] - c_s$, $p_{l,t} = E[v|H] - c_l$, and his continuation profit in period $\widehat{t}+1$ is $\psi^H \psi_{HH} v_H + \psi^H ((1+\delta) E[v|H] - c_s) / (1-\delta)$, which is greater than $\psi^H v_L + \psi^H ((1+\delta) E[v|H] - c_l) / (1-\delta)$, when $c_s - c_l \leqslant (1-\delta)(\psi_{HH} v_H - v_L)$.

The monopolist's equilibrium price trajectory in the subgame given $r_{\widehat{t}+1} = v_H$ has 2 possibilities: 1) If $\nexists t \geqslant \widehat{t}+1$, such that $p_{s,t} = \overline{p_s}$, $p_{l,t} = (1+\delta) E[v|H] - c_l - \delta r_{t+1}$, $0 < r_{t+1} \leqslant v_L$, from Lemma 4, we know under the specified conditions, the monopolist achieves the maximum profit if he offers pure selling at $p_{s,t} = (1+\delta) E[v|H] - c_s$ for $t \geqslant \widehat{t}+1$ on the equilibrium path. 2) Otherwise, the equilibrium requires that new consumers are unwilling to lease the software at $(1+\delta) E[v|H] - c_l - \delta v_L \leqslant p_{l,t} < (1+\delta) E[v|H] - c_l$ again until a period of penalty offsets the monopolist's gain from his deviation in period $\widehat{t}+1$. This implies that the monopolist's continuation profit in period $\widehat{t}+1$ is no greater than $\psi^H v_L + \psi^H ((1+\delta) E[v|H] - c_l) / (1-\delta)$, which is less than $\psi^H \psi_{HH} v_H + \psi^H ((1+\delta) E[v|H] - c_s) / (1-\delta)$, when $c_s - c_l \leqslant (1-\delta)(\psi_{HH} v_H - v_L)$.

Given the monopolist's strategy, consumers who enter the market and lease the software in period $\widehat{t}$ renew their leases in period $\widehat{t}+1$ if their true types are the high type. Consumers who enter the market in period $t \geqslant \widehat{t}+1$ buy the software as soon as they enter the market if they receive the high-type signal and do not adopt the software otherwise. Given consumers' response, the monopolist does not have incentives to deviate given the discussion in the previous paragraph.

Second, given the above finding, from Lemma 4, we know that under the specified conditions the monopolist achieves the maximum profit in an SPNE if he offers pure selling only at a stationary price on the equilibrium path. This concludes the proof.

Proofs of Propositions 5 and 6: Proofs of Propositions 5 and 6 utilize the same technique as that of Proposition 3 and, hence, are omitted to avoid repetition. In the following, I provide a sketch of the proofs: I first consider a special case in which price commitment is possible and obtain the following Lemma 5. The purpose of this exercise is twofold: First, since the monopolist is unable to achieve a higher profit without the ability of price commitment, this lemma gives an upper bound to the optimal profit in the infinite-period game in which price commitment is impossible. Let us call this the maximum profit levels. Second, it also allows one to characterize the set of price trajectories that would allow the monopolist to achieve these maximum profit levels. Proof of this lemma uses the same technique as that of Lemma 1 and, hence, is omitted for brevity.

Next, I show that under the specified conditions, there exists an SPNE of the game in which the monopolist's pricing on the equilibrium path is described in the proposition, and the monopolist achieves the maximum profit level. I establish this by providing an example of such a strategy profile and proving it to be an SPNE. Finally, applying Lemma 5, one can show that under the specified conditions, if the monopolist achieves the maximum profit level in an SPNE, his equilibrium pricing behavior has to have certain properties that are consistent with the claims in the propositions. This concludes the proofs.

Lemma 5 *In the infinite-period model, when price commitment is possible, the monopolist's optimal price trajectory is:*

a) $p_{s,t} = E[v|L] - c_s + \delta E[v|H]$, $p_{l,t} = E[v|L] - c_l$ *and* $r_{t+1} = v_H$, *for* $t \geqslant 0$, *if* $\psi^L c_l < E[v|L] - (1+\delta)\psi^H E[v|H] + \delta\beta v_H$ *and* $\psi^H (c_s - c_l) < \delta(1-\alpha)(1-\beta) v_L + \delta \min\{0, \beta v_H - E[v|L]\}$. *The monopolist's optimal profit is* $\left(E[v|L] + \delta\psi^H E[v|H] - \left(\psi^H c_s + \psi^L c_l\right) + \delta\beta(1-\alpha) v_H\right)/(1-\delta)$.

b) $p_{s,t} = \overline{p_s}$, $p_{l,t} = E[v|L] - c_l$, $r_{t+1} = v_H$, *for* $t \geqslant 0$, *if* $\beta v_H > E[v|L]$, $\psi^H (c_s - c_l) > \delta(1-\alpha)(1-\beta) v_L$, *and* $\psi^L c_l < E[v|L] - (1+\delta)\psi^H E[v|H] + \delta\beta v_H$. *The monopolist's optimal profit is* $\left(E[v|L] - c_l + \delta\beta v_H\right)/(1-\delta)$.