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Versioning and Competition

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Abstract

We study a duopoly of information goods with and without free disposal (no-free-disposal – NFD) where we endogenize the firms' production decision regarding the development of the highest version. Competing firms incur research and development (R&D) costs without any additional versioning or marginal costs. Our findings show that competition degenerates into a Bertrand model of zero profits if both firms choose to produce the same highest quality, for both types of goods. However, for NFD goods, when both firms have relatively low costs but are still sufficiently differentiated in their R&D capabilities it is optimal for both firms to pursue versioning. Interestingly, while the high capability firm offers a monopolist's menu (albeit only for the high-type consumers in the market), the low capability firm offers socially efficient versions to the low-type consumers. We also show that relative differences in R&D capabilities can lead to equilibrium results where only one firm offers a menu while the other offers a single version. Finally, we are able to show that in a market for goods with free disposal such as those often characterized by extant research, where consumers enjoy non-decreasing multiplicative utilities, it is never optimal for either firm to pursue versioning strategy independent of R&D costs. This result extends a recent finding in monopoly markets that suggests that marginal/usage costs are the sole reason for versioning outcomes. Our work clearly identifies the individual impact of no free disposal property, marginal costs and capital production costs thus allowing for the reconciliation of extant work on multi-product competition.

Keywords: Versioning, competition, information goods, pricing, free disposal

1. Introduction

While firms' versioning strategy itself has received much attention in the IS literature, there is a limited understanding on the impact of competition on the versioning decisions of firms. Although a recent paper in management science (Jones and Mendelson 2011) examines information goods in a competitive context, both firms appear to be prejudiced towards a single product strategy. In other words, the firms do not engage in versioning competition. Similarly, a recent working paper (Wei & Nault, 2011) considers an information goods duopoly albeit where again either firm do not pursue versioning as its optimal strategy in competition.

We develop a more general model of competition and versioning and importantly consider information goods with and without free disposal. This is critical since prior research on monopolistic screening for information goods suggests that versioning as a strategy requires some form of marginal cost to be present (Chellappa & Mehra, 2011).

2. Model

We consider a market where consumers are heterogeneous in their marginal valuation for quality or features, indexed by a parameter θ which is distributed on the support $\left[\underline{\theta},\overline{\theta}\right]$, with the probability density function given by $f\left(\theta\right)$ and where $F\left(\theta\right)$ is the associated cumulative distribution function.

ASSUMPTION 1. *(Market Distribution) The density function* $f\left(\theta\right)$ *is uni-modal and satisfies* *the monotone hazard rate property.*[1]

Let $q : q \in \mathbb{R}^{+}$ be the number of features of the information good such that higher q implies a good of higher quality (greater number of features). Consumer surplus is represented by $U\left(q,\theta\right) - p$, where $p : p \in \mathbb{R}^{+}$ is the price of the good for quality q.

ASSUMPTION 2. *(No Free Disposal).*

$$U\left(q,\theta\right) = \theta q - \frac{1}{2}\lambda q^2; \quad \lambda > 0 \tag{1}$$

The no free disposal property (Mas-Colell 1992) is captured through a non-monotonic utility function where the quadratic term represents an intrinsic disutility associated with consumption of many information goods and services. We denote the magnitude of this usage-related cost by a parameter λ, and consider a market where consumers are homogeneous in this cost. Note that $\lambda = 0$ represents free disposal and the corresponding utility function $U\left(q,\theta\right) = \theta q$ then reduces to the standard increasing multiplicative utility considered in extant research on versioning and vertical segmentation (Mussa and Rosen 1978). The firm has to decide on the maximum quality $\left(q^H\right)$ it must create along with any versioning and pricing decisions. In order to endogenize this decision, we incorporate a quality-dependent cost of creating the maximum quality. It is a one-time investment in creating the total number of features.

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[1] A more formal assumption is $\frac{d}{d\theta}\left[\frac{1-F\left(\theta\right)}{f\left(\theta\right)}\right] \le 0 \le \frac{d}{d\theta}\left[\frac{F\left(\theta\right)}{f\left(\theta\right)}\right]$, which is standard is standard according to Jullien (2000).

After the maximum quality is created, the firm may degrade it to produce lower quality versions. For example, lower quality versions of software can be produced by removing, disabling or recombining functions. It is well-recognized that additional costs to generate low-quality versions are negligible compared to development costs (Bhargava & Choudhary, 2008).

Assumption 3. *(Development Costs) Firms incur research & development cost of producing the highest quality or version, given by*

$$C\left(q^H\right) = \frac{1}{2}c\left[q^H\right]^2$$

There are no versioning costs or marginal cost of reproduction and distribution.

Notations are summarized in Appendix A for readers' convenience.

2.1 Production stages

Consider a duopoly with two information goods firms each of whom incur R&D costs c_1 and c_2. This duopolistic game is characterized by three stages:

1. *Development stage:* Firms 1 and 2 simultaneously choose the highest qualities they will produce, q_1^H and q_2^H and incur the corresponding R&D costs $\frac{1}{2}c_1\left[q_1^H\right]^2$ and $\frac{1}{2}c_2\left[q_2^H\right]^2$.
2. *Versioning stage*: Firms 1 and 2 decide on the quality range or the number of versions they will offer in the market given by $\left[\underline{q}_1, \overline{q}_1\right] \subseteq \left[0, q_1^H\right]$ and $\left[\underline{q}_2, \overline{q}_2\right] \subseteq \left[0, q_2^H\right]$ respectively.
3. *Pricing stage*: Firm 1 and Firm 2 configure their incentive compatible menus, i.e., $\{q_1, p_1(q_1)\}_{q_1 \in [\underline{q}_1, \overline{q}_1]}$ and $\{q_2, p_2(q_2)\}_{q_2 \in [\underline{q}_2, \overline{q}_2]}$ to serve their respective market segments.

The full-game equilibrium concept employed in this paper is subgame perfect Nash equilibrium (SPNE) and stages of the game are common to both free disposal and NFD information goods.

Proposition 1. *There is no SPNE in which both firms choose to create the same maximum-quality at the development stage* $\left(q_1^H = q_2^H\right)$. ■

(All proofs are delegated to Appendix B)

Proposition 1 holds for both types of information goods, ones with free disposal (increasing multiplicative utility) and for those with no free disposal (non-monotonic utilities). Indeed this result holds independent of whether versioning or single version strategy is optimal. We can see that a number of earlier works on both physical and information goods where single version strategy is pursued observe similar outcomes (Moorthy 1988, Wei and Nault 2008). As Moorthy (1988) observes, while a monopolist's quality allocation is based on the firm's own discrimination strategy, the duopolistic competition requires that one firm differentiate its quality from the other firm. The same is true for a physical goods market where versioning is pursued although this work (Champsaur and Rochet 1989) does not consider the highest-quality development costs as in our model. We can confirm that this result is also true for NFD goods where capital development costs are involved.

As a result, for the following sections, we restrict our attention to situations with asymmetric highest qualities $\left(q_2^H > q_1^H\right)$. Moreover, by assuming that Firm 2 has a development cost smaller than its rival $\left(c_2 \le c_1\right)$, without loss of generality, we are able to focus on analyzing more efficient market equilibria where the lower cost firm ends up offering a higher quality. Consequently, we label Firm 2 the "high R&D capability" firm, and Firm 1 the "low R&D capability" firm. Similarly, suppose that at versioning stage, each firm serves its market segment with a single quality or version, then this assumption ensures that this version is a single quality line rather than multiple disjoint qualities, lines or any combination of them.

For arriving at the equilibrium solution, we shall first solve for stage 2 and then through backward induction we shall solve for the development stage. In §3 we first develop the versioning subgame for NFD goods and subsequently in §4, we shall consider the full game to identify the production of the highest quality and possible duopolistic equilibria. Note that we conduct most of our analysis for NFD goods whose utility is given in equation (1) not only because versioning strategies have not been studied for them, but also because we can derive the equilibrium conditions for the traditional form of utility from this more general analyses.

3. Versioning subgame for NFD goods

Let each firm offer a menu of competing versions and prices in the subgame. We first characterize this equilibrium allocation of each menu by its lower bound and upper bound, denoted by $[\underline{q}_1, \overline{q}_1] \subseteq [0, q_1^H]$ for Firm 1 and $[\underline{q}_2, \overline{q}_2] \subseteq [0, q_2^H]$ for Firm 2. In other words $\underline{q}_1 (\overline{q}_1)$ is the lowest (highest) version that Firm 1 considers offering during the versioning subgame and correspondingly $\underline{q}_2 (\overline{q}_2)$ is that considered by Firm 2. Mathematically, if $\underline{q}_1 = \overline{q}_1$ it simply implies that Firm 1 offers only one version and correspondingly for Firm 2. We use notation $U_1(\theta)$ and $U_2(\theta)$ to denote the surplus of type θ consumers under the two incentive compatible menus. We follow a solution technique similar to that of Champsaur and Rochet (1989) for physical goods segmentation where we first consider a marginal consumer type $\hat{\theta} \in \left[\underline{\theta}, \overline{\theta}\right]$ who is indifferent between the two firms. We first exogenously assume the existence of such a type and later endogenously identify $\hat{\theta}$ where the market (if) separates between the two firms.

Lemma 1. *In the versioning subgame, if firms offer non-overlapping quality menus* $(\underline{q}_2 > \overline{q}_1)$ *then there exists a unique consumer type given by* $\hat{\theta}$ *who is indifferent between the two firms such that* $U_1(\theta)=U_2(\theta)$; $U_1(\theta)>U_2(\theta)\quad \forall\theta\in\left[\underline{\theta},\hat{\theta}\right)$ *and* $U_2(\theta)>U_1(\theta)\quad \forall\theta\in\left(\hat{\theta},\overline{\theta}\right]$. ■

Lemma 1 is an intermediate result that tells that once the levels of $\underline{q}_2$ and $\overline{q}_1$ are determined, incentive compatibility of each menu ensures the existence of a unique market partition point. As a result of this lemma, we can specify each firm's market segment, and frame the two firms' objectives. For Firm 2, this gives us

$$\max_{q_2'(\cdot),\underline{p}_2}\int_{\hat{\theta}}^{\overline{\theta}}\left[\theta q_2(\theta)-\frac{1}{2}\lambda q_2(\theta)^2-U_2(\theta)\right]f(\theta)d\theta$$
$$\text{subject to}\quad \begin{array}{c} q_2'(\theta)\geq 0;\ U_2'(\theta)=q_2(\theta)\\ q_2(\theta)\geq \underline{q}_2;\ U_2(\hat{\theta})=U_1(\hat{\theta})\end{array} \tag{2}$$

Lemma 1 informs that the IR conditions for $\theta>\hat{\theta}$ are all slack, thus being excluded from Firm 2's objective. The IR condition for the marginal type of consumers $\left(U_2(\hat{\theta})=U_1(\hat{\theta})\right)$ is variable in the sense that the outside option is not exogenously given but endogenously determined by competition between the two firms. Similarly, for Firm 1 we have

$$\max_{q_1'(\cdot),\overline{p}_1}\int_{\underline{\theta}}^{\hat{\theta}}\left[\theta q_1(\theta)-\frac{1}{2}\lambda q_1(\theta)^2-U_1(\theta)\right]f(\theta)d\theta$$
$$\text{subject to}\quad \begin{array}{c} q_1'(\theta)\geq 0;\ U_1'(\theta)=q_1(\theta)\\ q_1(\theta)\leq \overline{q}_1;\ U_1(\theta)\geq 0\\ U_1(\hat{\theta})=U_2(\hat{\theta})\end{array} \tag{3}$$

Essentially, the presence of the competitor affects the focal firm's objective by determining the attractiveness of the outside option to its customers.

3.1 Quality allocations for an exogenously given marginal consumer type $\hat{\theta}$

To facilitate the characterization of this complex interaction, we construct auxiliary problems for both firms with the value of outside options u and the market share $\hat{\theta}$ given. Specifically, for Firm 2 we have

$$\max_{q_2'(\theta)} \int_{\hat{\theta}}^{\bar{\theta}} \left[\theta q_2(\theta) - \frac{1}{2}\lambda q_2(\theta)^2 - U_2(\hat{\theta}) - \int_{\hat{\theta}}^{\theta} q_2(t)\,dt \right] f(\theta)\,d\theta$$

$$\text{subject to} \quad \begin{array}{c} q_2'(\theta) \geq 0 \\ q_2(\theta) \geq \underline{q}_2;\ U_2(\hat{\theta}) = u \end{array} \tag{4}$$

Solving this auxiliary problem independently for quality allocation ends up with the following lemma.

Lemma 2a. *In the versioning subgame, if the high R&D capability firm (Firm 2) pursues the versioning strategy, it will create a quality menu non-linear in consumer type* θ *such that*

$$q_{2\,proj}(\theta) = \begin{cases} \underline{q}_2 & \text{for } \theta \in \left[\hat{\theta}, q_2^{-1}(\underline{q}_2)\right) \\ \min\{\bar{q}_2, q_2(\theta)\} & \text{for } \theta \in \left[q_2^{-1}(\underline{q}_2), \bar{\theta}\right] \end{cases}$$

where $q_2(\theta) = \frac{1}{\lambda}\left(\theta - \frac{1-F(\theta)}{f(\theta)}\right)$ *and* $\underline{q}_2 < \bar{q}_2 \leq q_2(\bar{\theta})$. ■

This lemma characterizes optimal quality allocation in Firm 2's versioning menu. It also indicates that for $\underline{q}_2 \geq q_2(\overline{\theta})$, Firm 2 offers a single quality for all consumers with types $\forall \theta \in \left[\hat{\theta}, \overline{\theta}\right]$.

It is noticed that neither $\hat{\theta}$ nor u appears in the formula depicting quality allocation. In another word, the optimal quality allocation in the menu is independent of both the value of the outside option and the location of the market partition point. The reason underlying the independence of quality allocation is because the value of outside option is a lump-sum transfer to all served consumers (see in $\int_{\hat{\theta}}^{\overline{\theta}}\left[\theta q_2(\theta) - \frac{1}{2}\lambda q_2(\theta)^2 - \frac{1-F(\theta)}{f(\theta)} q_2(\theta) - u\right] f(\theta) d\theta$).

Different from the Firm 2's case, the price competition would affect quality allocation in Firm 1's menu, because participation from the lower end of the market is an issue to Firm 1. As Moorthy (1988) identified in single-quality competition, there may be some consumers locating at the lower end of the market left uncovered. This possibility is captured by the IR condition $U_1(\theta) \geq 0$. As a result, IR conditions could be binding at the two ends of its served segment for Firm 1. Specifically, for Firm 1 we have

$$\max_{q_1'(\theta)} \int_{\underline{\theta}}^{\hat{\theta}} \left[\theta q_1(\theta) - \frac{1}{2}\lambda q_1(\theta)^2 - U_1\left(\hat{\theta}\right) + \int_{\theta}^{\hat{\theta}} q_1(t)\,dt\right] f(\theta)\,d\theta$$

$$\text{subject to} \quad \begin{array}{c} q_1'(\theta) \geq 0, \\ q_1(\theta) \leq \overline{q}_1,\ U_1\left(\hat{\theta}\right) = u, \text{ and } U_1(\theta) \geq 0 \end{array} \tag{5}$$

LEMMA 2B. *In the versioning subgame, if the low R&D capability firm (Firm 1) pursues the versioning strategy, it will create a quality menu non-linear in consumer type* θ *such that*

$$q_{\gamma\, proj}(\theta) = \begin{cases} q_\gamma(\theta) & \text{for } \theta \in \left[\underline{\theta}, q_\gamma^{-1}(\overline{q}_1)\right) \\ \overline{q}_1 & \text{for } \theta \in \left[q_\gamma^{-1}(\overline{q}_1), \hat{\theta}\right] \end{cases}$$

where $q_\gamma(\theta)=\frac{1}{\lambda}\left(\theta+\frac{F(\theta)-\gamma}{f(\theta)}\right)$ *and* $\overline{q}_1>\underline{q}_1\geq q_\gamma(\underline{\theta})$*;* $\gamma=0$ *when the lowest type* $\underline{\theta}$ *is left a positive utility.* ■

This lemma implies that the market will be fully covered when Firm 1 pursues versioning. γ as well as $q_\gamma^{-1}(\overline{q}_1)$ is decreasing in u, which suggests that quality profile is increasing while the separating region is shrinking as the marginal type $\hat{\theta}$ is left more surplus. For some small u, γ is large enough to make $q_\gamma^{-1}(\overline{q}_1)>\hat{\theta}$, which means that Firm 1 will offer a fully-separating menu to serve consumers on $\left[\underline{\theta},\hat{\theta}\right]$. However, this could not be the equilibrium of the versioning subgame after we endogenize the choices of u and $\hat{\theta}$ as we will show.

3.2 Endogenizing the marginal type $\hat{\theta}$

We've characterized optimal quality allocation for given u and $\hat{\theta}$. In equilibrium, however, u and $\hat{\theta}$ are also the results of strategic responses. By definition, $\hat{\theta}$ represents the marginal type of consumers who are indifferent between offerings of Firm 1 and Firm 2 (i.e., $\hat{\theta}\underline{q}_2-\frac{1}{2}\lambda\underline{q}_2{}^2-\underline{p}_2=\hat{\theta}\overline{q}_1-\frac{1}{2}\lambda\overline{q}_1{}^2-\overline{p}_1$), and u is the utility left to them. Thereby, $\hat{\theta}$ is the function of both $\underline{p}_2$ and $\overline{p}_1$, namely

$$\hat{\theta}=\frac{\frac{1}{2}\lambda\left(\underline{q}_2-\overline{q}_1\right)\left(\underline{q}_1+\overline{q}_2\right)+\underline{p}_2-\overline{p}_1}{\left(\underline{q}_2-\overline{q}_1\right)} \tag{6}$$

The following lemma characterizes the equilibrium prices when both firms pursue versioning in the subgame.

LEMMA 3. *In the versioning subgame, the equilibrium price charged by the low R&D firm (Firm 1) for the highest quality it offers* $\left(\overline{q}_1\right)$ *is*

$$\overline{p}_1\left(\overline{q}_1,\underline{q}_2\right)=\frac{F\left(\hat{\theta}\right)}{f\left(\hat{\theta}\right)}\left(\underline{q}_2-\overline{q}_1\right)$$

The equilibrium price charged by the high R&D firm (Firm 2) for the lowest quality it offers $\left(\underline{q}_2\right)$ *is*

$$\underline{p}_2\left(\overline{q}_1,\underline{q}_2\right)=\frac{1-F\left(\hat{\theta}\right)}{f\left(\hat{\theta}\right)}\left(\underline{q}_2-\overline{q}_1\right)$$ *where* $\hat{\theta}\left(\overline{q}_1,\underline{q}_2\right)$ *is uniquely determined by solving*

$$\hat{\theta}+\frac{2F\left(\hat{\theta}\right)-1}{f\left(\hat{\theta}\right)}=\frac{1}{2}\lambda\left(\overline{q}_1+\underline{q}_2\right).\ \blacksquare$$

It can be observed that this pair of prices is increasing in difference between $\underline{q}_2$ and $\overline{q}_1$; as the difference vanishes, the two prices approach to 0, implying that a smaller differentiation in quality is detrimental to both firms' profits.

Once $\overline{p}_1$ and $\underline{p}_2$ are fixed, prices for other qualities in the two menus are also fixed due to incentive compatibility. It can be verified that IR condition $U_1\left(\theta\right)\geq 0$ for the lowest type $\underline{\theta}$ is slack, implying that Firm 1 would charge the price low enough to leave a positive surplus to the lowest type $\underline{\theta}$ and thus induce the full participation in the market. This is a consequence of intense competition for those intermediate consumers $\left(\hat{\theta}\right)$; both firms in equilibrium leave sufficiently high surpluses to marginal consumers, which dictate high surpluses to the lowest type in the market due to incentive compatibility. Accordingly, we establish the following lemma about quality allocation.

Lemma 4. *If Firm 2 pursues versioning in the full-game equilibrium with* $\underline{q}_2<\frac{\overline{\theta}}{\lambda}$, *Firm 2 will offer a quality menu defined by*

$$q_{2proj}(\theta)=\begin{cases}\underline{q}_2 & \text{for } \theta\in\left[\hat{\theta}\left(\overline{q}_1,\underline{q}_2\right),q_2^{-1}\left(\underline{q}_2\right)\right) \\ \min\left\{\overline{q}_2,q_2(\theta)\right\} & \text{for } \theta\in\left[q_2^{-1}\left(\underline{q}_2\right),\overline{\theta}\right]\end{cases}$$

If Firm 1 pursues versioning in the full-game equilibrium with $\overline{q}_1>\frac{\underline{\theta}}{\lambda}$, *Firm 1 will offer a quality menu defined by*

$$q_{1proj}(\theta)=\begin{cases}q_1(\theta) & \text{for } \theta\in\left[\underline{\theta},q_1^{-1}(\overline{q}_1)\right) \\ \overline{q}_1 & \text{for } \theta\in\left[q_1^{-1}(\overline{q}_1),\hat{\theta}\left(\overline{q}_1,\underline{q}_2\right)\right]\end{cases}\quad \textit{where}\ \ q_1(\theta)=\frac{1}{\lambda}\left(\theta+\frac{F(\theta)}{f(\theta)}\right)\ \blacksquare$$

The corresponding menu is depicted in Figure 1. If the high R&D capability firm pursues versioning, the market segment served by the separating portion of the menu consumes the exactly same quality as it would when served by a monopolist's menu (i.e., $q_2(\theta)$); there must be a bunching segment (starting from $\hat{\theta}$) preceding the separating portion of the menu, consumers from which are served with a quality higher than that they would be under a monopolist's menu.

If the low R&D capability firm pursues versioning, its menu always consists of a separating portion followed by a bunching portion. It is worthy of noting that a consumer on the separating region served by Firm 1 consumes a level of quality higher than the socially-efficient level. This is also a result of fierce price competition occurring to intermediate consumers. Given the sufficiently low price for $\overline{q}_1$, $\{\overline{q}_1,\overline{p}_1\}$ then becomes very attractive to the lower types of consumers served by Firm 1, creating "countervailing incentives" (i.e. low types have incentives to overstate their types). Simply lowering prices for qualities smaller than $\overline{q}_1$ to accommodate this incentive problem is suboptimal because this adjustment fails to take advantage of the fact that consuming a higher quality is more costly for those with a lower valuation for quality. Instead, Firm 1 can prevent consumers from overstating their types by increasing $q_1(\theta)$, which allows it to charge the lower segment of consumers with a relatively high price. In other words, Firm 1 will focus on extracting consumer surplus from the lower segment of the market

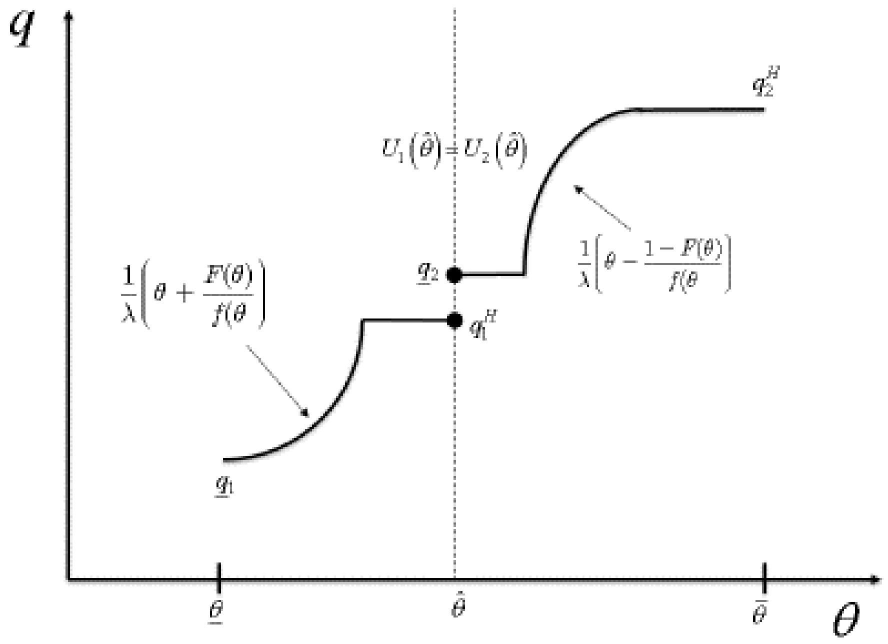

Figure 1: Quality allocation in the subgame when firms pursue versioning

First, note that the quality allocation menu in Figure 1 closely resembles the allocation in physical goods markets as shown by Champsaur and Rochet (1989). However, many key differences emerge from this earlier work. First, we observe bunching for the highest consumer types in market, i.e., quality distortion for the highest types consistent with monopoly offering. We know from recent research (Chellappa and Mehra 2011) that this is due to the capital R&D costs of production of the highest quality. Second, observe the price schedule dictated by the low R&D capability firm, Firm 1

$$p_1(\theta) = \theta q_1(\theta) - \frac{1}{2}\lambda q_1(\theta)^2 + \int_{\theta}^{\hat{\theta}} q_1(t)\,dt - u \tag{7}$$

We can see from equation (7) that this price schedule is non-increasing in θ as $\left[\theta - \lambda q_1(\theta)\right] q_1'(\theta) \leq 0$. The quality allocation in our paper is similar to Champsaur and Rochet (1989) but in the latter, consistent with common understanding, the price schedule is also increasing in θ. Simply put, consumers with higher marginal value for quality receive higher quality (or at least the same) and will pay a higher price (or at least the same). Our incentive compatible menu developed along standard methods however suggests that in the portion of the market served by Firm 1, consumers with higher marginal willingness to pay will pay a *lower* price (and that the firm would charge a lower price for a version of higher quality). While the mechanism design itself is individually rational and incentive compatible, this result is clearly inconsistent with industrial practice and conventional understanding of nonlinear pricing.

As a result, we impose an exogenous constraint on the monotonicity of the price schedule to meet the market expectation on the non-negative correlation between quality and price, and re-derive the menu $\left(\{q_1, p_1(q_1)\}_{q_1 \in [\underline{q}_1, \bar{q}_1]}\right)$ for Firm 1. Formally, we introduce the constraint

$$\left[\theta - \lambda q_1(\theta)\right] q_1'(\theta) \geq 0 \tag{8}$$

into the objective function of Firm 1 and get the following lemma:

Lemma 5. *Given the constraint* (8) *imposed on the price schedule, Firm 1 will offer a quality menu defined by* $q_{1proj}(\theta)=\begin{cases}\dfrac{\theta}{\lambda} & \text{for } \theta\in\left[\underline{\theta},q_1^{-1}(\overline{q}_1)\right) \\ \overline{q}_1 & \text{for } \theta\in\left[q_1^{-1}(\overline{q}_1),\hat{\theta}(\overline{q}_1,\underline{q}_2)\right]\end{cases}$ *if Firm 1 pursues versioning in the full-game equilibrium with* $\overline{q}_1>\dfrac{\theta}{\lambda}$. ■

After adopting non-decreasing pricing strategy, it is optimal for Firm 1 to allow all consumers with $\theta\in\left[\underline{\theta},q_1^{-1}(\overline{q}_1)\right)$ to consume their socially-efficient levels of quality. Consequently, a uniform price is charged by Firm 1 for all qualities it provides. Note the unique quality allocation menu given by Lemma 5; we indeed do not if this allocation is SPNE proof but if it is so then clearly the NFD property imposes certain constraints on firm that allows the lowest consumer types to enjoy their surplus maximizing quality. The economic intuition behind this result stems from the non-monotonic shape of the utility function. The incentive compatible menu in Lemma 4 results in overprovision of quality for these consumer types; while the overprovisioning impact is strictly suffered by the firm in free disposal goods markets (increasing utility functions) as in Champsaur and Rochet (1989), overprovisioning in non-monotonic utilities implies that a consumer is using a quality greater than his surplus maximizing quality, i.e., $q>\arg\max_{\{q\}}\left\{U(q,\theta)\right\}$. This implies that the provisioned quality for a given θ type is in the decreasing part (due to usage costs) of his utility curve. If the prices were not decreasing in type θ, this consumer will simply misrepresent himself as the type whose provisioned quality would be the type θ's surplus maximizing quality *and* ends up paying a lower price. Our result from Lemma 5 suggests therefore that the only available solution to Firm 1 is to offer these consumers their surplus maximizing quality. Note however that this does not mean that these types enjoy an unusually high surplus for low type consumers; the firm will extract this surplus through its price schedule.

3.3 Determining the highest versions of the low R&D capability firm $\overline{q}_1$ and the lowest version of the high R&D capability firm $\underline{q}_2$

Following the development of the menu, we need characterize the bounds of the offerings in the subgame. We shall first focus on the bounds that will affect the intermediate consumer types

Since both $\overline{p}_1$ and $\underline{p}_2$ are functions of the highest versions of the low R&D capability firm $\overline{q}_1$ and the lowest version of the high R&D capability firm $\underline{q}_2$ that will be offered in the market, the two firms' market shares and profits also depend on them. We formulate the two firms' objectives w.r.t. $\overline{q}_1$ and $\underline{q}_2$:

For Firm 1, if $\overline{q}_1 > q_1(\underline{\theta})$, it pursues versioning, hence

$$\begin{aligned} R_1(\overline{q}_1) &= \int_{\underline{\theta}}^{\hat{\theta}(\overline{q}_1,\underline{q}_2)} \left[\theta q_{1proj}(\theta) - \frac{1}{2}\lambda q_{1proj}(\theta)^2 + \frac{F(\theta)}{f(\theta)} q_{1proj}(\theta)\right] f(\theta)d\theta \\ &\quad - \left[\hat{\theta}(\overline{q}_1,\underline{q}_2)\overline{q}_1 - \frac{1}{2}\lambda \overline{q}_1^2 - \overline{p}_1(\overline{q}_1,\underline{q}_2)\right] F(\hat{\theta}) \\ &= \int_{\underline{\theta}}^{q_1^{-1}(\overline{q}_1)} \left[\theta q_1(\theta) - \frac{1}{2}\lambda q_1(\theta)^2 + \frac{F(\theta)}{f(\theta)} q_1(\theta)\right] f(\theta)d\theta \\ &\quad - \int_{\underline{\theta}}^{q_1^{-1}(\overline{q}_1)} \left[\theta \overline{q}_1 - \frac{1}{2}\lambda \overline{q}_1^2 + \frac{F(\theta)}{f(\theta)} \overline{q}_1\right] f(\theta)d\theta + \frac{F(\hat{\theta})^2}{f(\hat{\theta})}(\underline{q}_2 - \overline{q}_1) \end{aligned} \tag{9}$$

Otherwise, $R_1(\overline{q}_1) = \frac{F(\hat{\theta})^2}{f(\hat{\theta})}(\underline{q}_2 - \overline{q}_1)$ for single-quality strategy.

If the equilibrium strategy of the low R&D capability firm is to pursue versioning, it always aligns the highest quality $\overline{q}_1$ in the versioning subgame with its maximum quality, i.e., $\bar{q}_1 = q_1^H$. Otherwise, if the versioning stage ends up with $\bar{q}_1 < q_1^H$, Firm 1 anticipates this and will lower q_1^H in the R&D stage. This deviation will not affect the competition outcome and hence Firm 1's revenue in the second stage. However, it helps reduce Firm 1's R&D cost. Therefore, $\bar{q}_1$ is determined at the R&D stage, and is not adjustable in the versioning stage.

Different from Firm 1's case, if versioning is an equilibrium strategy for Firm 2, its highest produced quality $\underline{q}_2$ is determined at the versioning stage.

Then Firm 2's objective is formulated as

$$R_2\left(\underline{q}_2,\bar{q}_2\right)=\int_{\hat{\theta}\left(q_1^H,\underline{q}_2\right)}^{\bar{\theta}}\left[\theta q_{2proj}\left(\theta\right)-\frac{1}{2}\lambda\left(q_{2proj}\left(\theta\right)\right)^2+\frac{F\left(\theta\right)-1}{f\left(\theta\right)}q_{2proj}\left(\theta\right)\right]f\left(\theta\right)d\theta \\ -\left[\hat{\theta}\left(q_1^H,\underline{q}_2\right)\underline{q}_2-\frac{1}{2}\lambda\underline{q}_2^{\,2}-\underline{p}_2\left(q_1^H,\underline{q}_2\right)\right]\left(1-F\left(\hat{\theta}\right)\right) \tag{10}$$

when $\underline{q}_2 < \dfrac{\bar{\theta}}{\lambda}$. When $\underline{q}_2 \geq \dfrac{\bar{\theta}}{\lambda}$, $R_2\left(\underline{q}_2\right)=\dfrac{\left(1-F\left(\hat{\theta}\right)\right)^2}{f\left(\hat{\theta}\right)}\left(\underline{q}_2-\bar{q}_1\right)$ for single-quality strategy.

Lemma 6. *If the equilibrium strategy of the high R&D capability firm (Firm 2) is to pursue versioning, the lowest quality* $\underline{q}_2$ *it offers in the pricing subgame is a solution to*

$$\frac{\left(1-F\left(\hat{\theta}\right)\right)^2}{f\left(\hat{\theta}\right)}-\frac{\left(1-F\left(q_2^{-1}\left(\underline{q}_2\right)\right)\right)^2}{f\left(q_2^{-1}\left(\underline{q}_2\right)\right)}+\left(\underline{q}_2-q_1^H\right)\frac{\partial}{\partial\theta}\left[\frac{\left(1-F\left(\hat{\theta}\right)\right)^2}{f\left(\hat{\theta}\right)}\right]\frac{\partial\hat{\theta}\left(q_1^H,\underline{q}_2\right)}{\partial\underline{q}_2}=0 \tag{11}$$

such that $\hat{\theta}=\hat{\theta}\left(q_1^H,\underline{q}_2\right)$ *is defined in Lemma 3; Firm 2 always aligns the highest quality with its maximum attainable quality, i.e.,* $\bar{q}_2 = q_2^H$. ■

The first term in equation (11) indicates the loss in the versioning benefit due to the increase of the lower bound of versioning (i.e., $\underline{q}_2$). The latter two capture the implication of the competition force on Firm 2's profit, and indicate that differentiating its quality line (i.e., the increase of $\underline{q}_2$) with its competitors moderates the price competition.

It is always a profit improving strategy for Firm 2 to align the highest quality it offers to the market $\overline{q}_2$ with the maximum attainable quality $q_2^H \leq \frac{\overline{\theta}}{\lambda}$.

Till now, we are exploring the subgame equilibrium at versioning stage in the domain of non-overlapping quality lines; that is, the equilibrium is derived by presuming that $\underline{q}_2 > \overline{q}_1$. In the next lemma, we relax this restriction, and show that avoiding direct quality competition is indeed the equilibrium response.

LEMMA 7. *If the equilibrium strategy of the high R&D capability firm (Firm 2) is to pursue versioning in the subgame, it is necessary to have* $\underline{q}_2\left(q_1^H\right) > q_1^H$. ■

PROPOSITION 2. *SPNE requires the firms to provide non-overlapping qualities/quality-menus in a competitive market.* ■

Together with the well-known result with the single quality competition from Moorthy (1988), it ends up by the following proposition.

4. SPNE – Highest quality development for NFD goods

Now having characterized the incentive compatible menus that the firms will develop in the versioning subgame, we need to determine the equilibrium highest quality that will be developed by two competing firms anticipating the subgame. We do so by backward inducting the later stage into the first stage decision.

4.1 Versioning strategy by the high R&D capability firm

If pursuing versioning is SPNE for the high R&D capability firm (Firm 2) then it defers its decision regarding the production of its highest quality $\left(\underline{q}_2\right)$ to the versioning stage.

Lemma 8. *When the high R&D capability firm (Firm 2) pursues versioning, the SPNE maximum quality it will produce* $\left(q_2^{H*}\right)$ *is obtained by solving*

$$\frac{\left(1-F\left(q_2^{-1}\left(q_2^H\right)\right)\right)^2}{f\left(q_2^{-1}\left(q_2^H\right)\right)}=c_2 q_2^H \tag{12}$$

In this case the low R&D firm (Firm 1) will produce a maximum quality $\left(q_1^{H*}\right)$ *which is a solution to*

$$\begin{aligned}&\left(\underline{q}_2\left(q_1^H\right)-q_1^H\right)\frac{\partial}{\partial\theta}\left[\frac{F\left(\hat{\theta}\right)^2}{f\left(\hat{\theta}\right)}\right]\left[\frac{\partial\hat{\theta}\left(q_1^H,\underline{q}_2\left(q_1^H\right)\right)}{\partial q_1^H}+\frac{\partial\hat{\theta}\left(q_1^H,\underline{q}_2\left(q_1^H\right)\right)}{\partial\underline{q}_2}\frac{\partial\underline{q}_2\left(q_1^H\right)}{\partial q_1^H}\right]\\&+\frac{F\left(\hat{\theta}\right)^2}{f\left(\hat{\theta}\right)}\left[\frac{\partial\underline{q}_2\left(q_1^H\right)}{\partial q_1^H}-1\right]=c_1 q_1^H\end{aligned} \tag{13}$$

such that $\underline{q}_2\left(q_1^H\right)$ *is given in equation* (11) *and* $\hat{\theta}\left(q_1^H,\underline{q}_2\right)$ *is given in Lemma 3.* ■

For a given distribution function $F(\theta)$, the solution of q_1^H to equation (13) is solely dependent on c_1. If $q_1^H(c_1)$ is larger than $q_1(\underline{\theta})=\frac{\theta}{\lambda}$, Firm 1 pursuing versioning is then the full-game equilibrium; otherwise, Firm 1 will pursue the single-quality strategy. $q_1^H(c_1)>\frac{\theta}{\lambda}$ gives the threshold of c_1 that make Firm 1 pursuing versioning the full-game equilibrium.

The condition that is necessary for solutions to equations in this lemma to be an equilibrium is that c_2 satisfies $\frac{\left(1-F\left(q_2^{-1}\left(\underline{q}_2\right)\right)\right)^2}{f\left(q_2^{-1}\left(\underline{q}_2\right)\right)} > c_2\underline{q}_2$. Otherwise, the maximum quality q_2^H would be smaller than the lowest version of the high R&D capability firm $\underline{q}_2\left(q_1^H\right)$, thus contradicting the definition of versioning. As $\underline{q}_2$ is a function of $q_1^H\left(c_1\right)$, we can characterize the relationship between c_1 and c_2 through $\frac{\left(1-F\left(q_2^{-1}\left(\underline{q}_2\right)\right)\right)^2}{f\left(q_2^{-1}\left(\underline{q}_2\right)\right)} > c_2\underline{q}_2$ that make both firms pursuing versioning the full-game equilibrium.

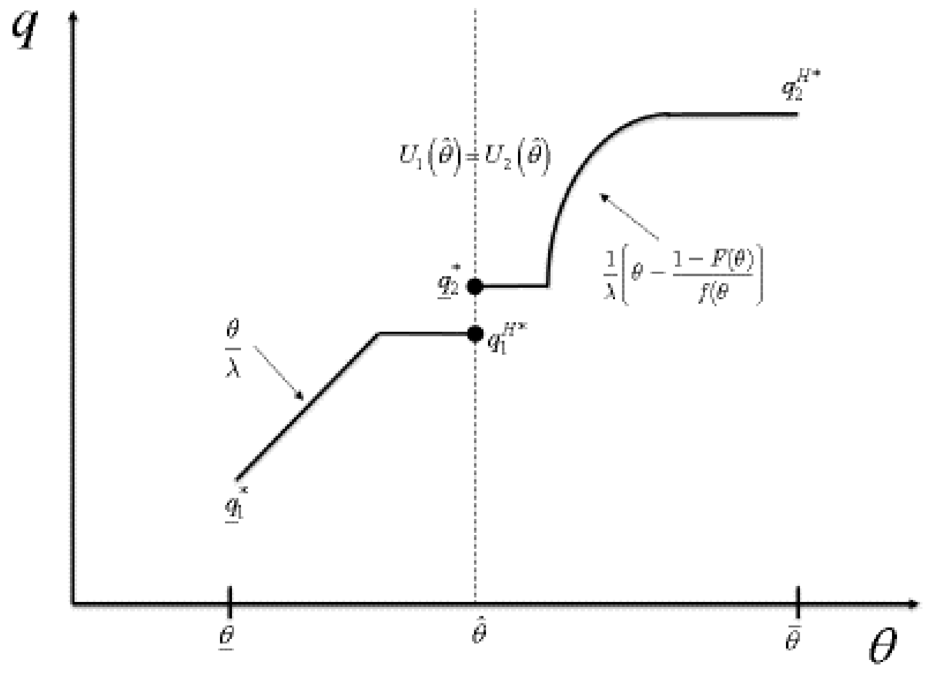

Figure 2: Both firms pursuing versioning in equilibrium

Proposition 3. *When the two firms' R&D costs are sufficiently different and when the low-type firm's costs are lower than a threshold then both firms will adopt a versioning strategy.*

■

With a relatively large c_1, the solution of q_1^H to equation (13) is smaller than or equal to $q_1(\underline{\theta}) = \frac{\underline{\theta}}{\lambda}$, Firm 1 pursuing a single-quality strategy is then the full-game equilibrium.

Proposition 4. *When the two firms' R&D costs are sufficiently different and when the low-type firm's costs are higher than a threshold, the equilibrium strategy dictates that the high capability firm pursues versioning while the low capability firm offers one version to the market.*

■

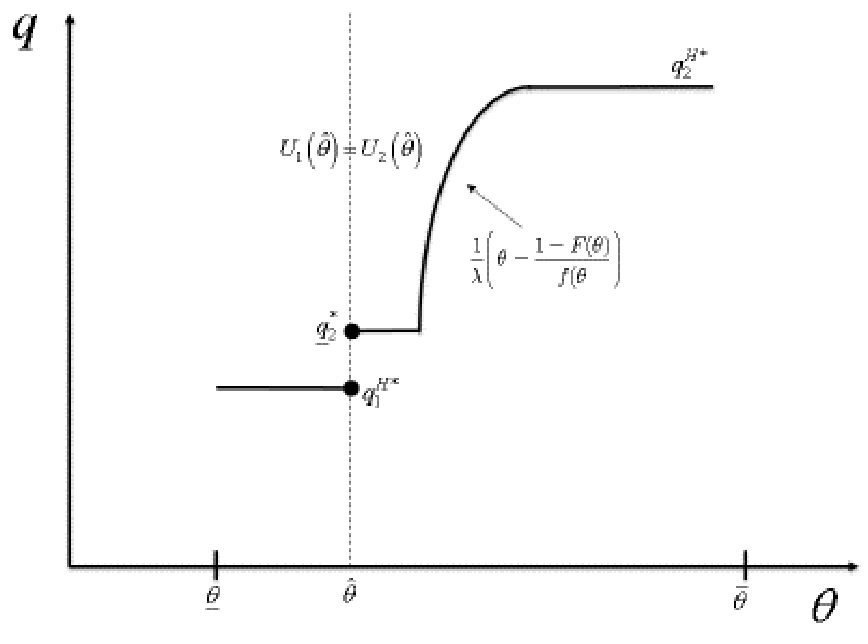

Figure 3: Asymmetrical versioning strategies when firms are very different in their R&D capabilities

Figure 2 depicts quality allocation and market coverage in the full game if Firm 2 pursues versioning, while Firm 1 pursues the single-quality strategy.

The solution to equation (13) may lead to a situation that q_1^H is sufficiently low so that there exist some levels of $\underline{q}_2$ such that $\underline{q}_2\left(q_1^H\right) > \underline{q}_2 \geq q_2\left(\underline{\theta}\right)$ and that the inequality $\frac{1}{2}\lambda\left(\underline{q}_2 + q_1^H\right) > \underline{\theta} - \frac{1-F\left(\underline{\theta}\right)}{f\left(\underline{\theta}\right)} + \frac{F\left(\underline{\theta}\right)}{f\left(\underline{\theta}\right)}$ is violated (see Lemma 3), implying that Firm 2 can attract all consumers. Then it could be profitable for Firm 2 to set the lowest quality at $q_2\left(\underline{\theta}\right)$ to deter its inferior competitor (by setting the lowest quality at $q_2\left(\underline{\theta}\right)$). For any q_1^H that leads to aggressive versioning on Firm 2's side, the duopoly competition is no longer an equilibrium, as Firm 1 cannot obtain a positive market share from entry. This informs the lower bound of q_1^H and correspondingly the upper bound of c_1 that sustains a competitive equilibrium. Upon observing c_1 that produces those levels of q_1^H, Firm 1 will detain from the market.

4.2 When Pursuing Single-quality Strategy is SPNE for the High R&D Capability Firm

The high R&D capability firm pursuing single-quality strategy gives up quality adjustment at the versioning stage. Hence its quality (q_2^H in this case) is determined at the R&D stage. Then the two firms determine their maximum qualities in a manner of Nash moves.

LEMMA 9. *When the high R&D capability firm (Firm 2) pursues single-quality strategy, the SPNE maximum quality it will produce* $\left(q_2^{H*}\right)$ *and the maximum quality produced by Firm 1* $\left(q_1^{H*}\right)$ *is obtained by simultaneously solving the following two equations*

$$-\frac{F\left(\hat{\theta}\right)^2}{f\left(\hat{\theta}\right)}+\left(q_2^H-q_1^H\right)\frac{\partial}{\partial\theta}\left[\frac{F\left(\hat{\theta}\right)^2}{f\left(\hat{\theta}\right)}\right]\frac{\partial\hat{\theta}\left(q_1^H,q_2^H\right)}{\partial q_1^H}=c_1q_1^H \tag{14}$$

$$\frac{\left[1-F\left(\hat{\theta}\right)\right]^2}{f\left(\hat{\theta}\right)}+\left(q_2^H-q_1^H\right)\frac{\partial}{\partial\theta}\left[\frac{\left(1-F\left(\hat{\theta}\right)\right)^2}{f\left(\hat{\theta}\right)}\right]\frac{\partial\hat{\theta}\left(q_1^H,q_2^H\right)}{\partial q_2^H}=c_2q_2^H \tag{15}$$

■

In this case, both $q_1^H\left(c_1,c_2\right)$ and $q_2^H\left(c_1,c_2\right)$ are functions of c_1 and c_2. If $q_1^H\left(c_1,c_2\right)$ is larger than $q_1\left(\underline{\theta}\right)$, Firm 1 pursuing versioning is then the full-game equilibrium.

Similarly to the situation in the last section, solutions to equations in this lemma are not necessarily equilibriums due to a possible deviation of Firm 2 to the versioning strategy at the second stage. For an illustrative purpose, consider a cost structure in which the lowest version of the high R&D capability firm $\underline{q}_2$ from equation (11) coincides with q_2^H from equation (12) when Firm 2 pursues versioning (i.e., $\frac{\left(1-F\left(q_2^{-1}\left(\underline{q}_2\right)\right)\right)^2}{f\left(q_2^{-1}\left(\underline{q}_2\right)\right)}=c_2\underline{q}_2$). If Firm 2 anticipates this degeneration of versioning strategy, it could ex ante pursue single-quality strategy. However, pursing single-quality strategy returns smaller $q_1^H{}_{Single-quality}$ and $q_2^H{}_{Single-quality}$ than $q_1^H{}_{Versioning}$ and $\underline{q}_{2\ Versioning}$ under the same cost structure. Given $q_1^H{}_{Single-quality}$ and $q_2^H{}_{Single-quality}$, it is profitable for Firm 2 to find a new $\underline{q}_2<q_2^H{}_{Single-quality}$, and it turns out that there's no SPNE in the full game under this cost structure.

In order to ensure that it is not profitable for Firm 2 to deviate to the versioning strategy, we need a necessary condition that requires the solutions to equations in Lemma 9 to satisfy

$$\frac{\left(1-F\left(\hat{\theta}\right)\right)^2}{f\left(\hat{\theta}\right)}-\frac{\left(1-F\left(q_2^{-1}\left(q_2^H\right)\right)\right)^2}{f\left(q_2^{-1}\left(q_2^H\right)\right)}+\left(q_2^H-q_1^H\right)\frac{\partial}{\partial\theta}\left[\frac{\left(1-F\left(\hat{\theta}\right)\right)^2}{f\left(\hat{\theta}\right)}\right]\frac{\partial\hat{\theta}\left(q_1^H,q_2^H\right)}{\partial\underline{q}_2}\geq 0$$

This ensures that Firm 2's commitment to single-quality strategy is credible. Otherwise, Nash moves at the R&D stage will make Firm 1 underinvest on its maximum q_1^H so as to leave Firm 2 some room to lower its lowest version at the versioning stage. The two inequalities

$$q_1^H(c_1,c_2)>\frac{\underline{\theta}}{\lambda}$$

$$\frac{\left(1-F\left(\hat{\theta}\right)\right)^2}{f\left(\hat{\theta}\right)}-\frac{\left(1-F\left(q_2^{-1}\left(q_2^H\right)\right)\right)^2}{f\left(q_2^{-1}\left(q_2^H\right)\right)}+\left(q_2^H-q_1^H\right)\frac{\partial}{\partial\theta}\left[\frac{\left(1-F\left(\hat{\theta}\right)\right)^2}{f\left(\hat{\theta}\right)}\right]\frac{\partial\hat{\theta}\left(q_1^H,q_2^H\right)}{\partial\underline{q}_2}\geq 0$$

dictate the cost structure that makes that the low R&D capability firm pursues versioning and the high R&D capability firm pursues a single-quality strategy the full-game equilibrium. This is summarized by the following proposition:

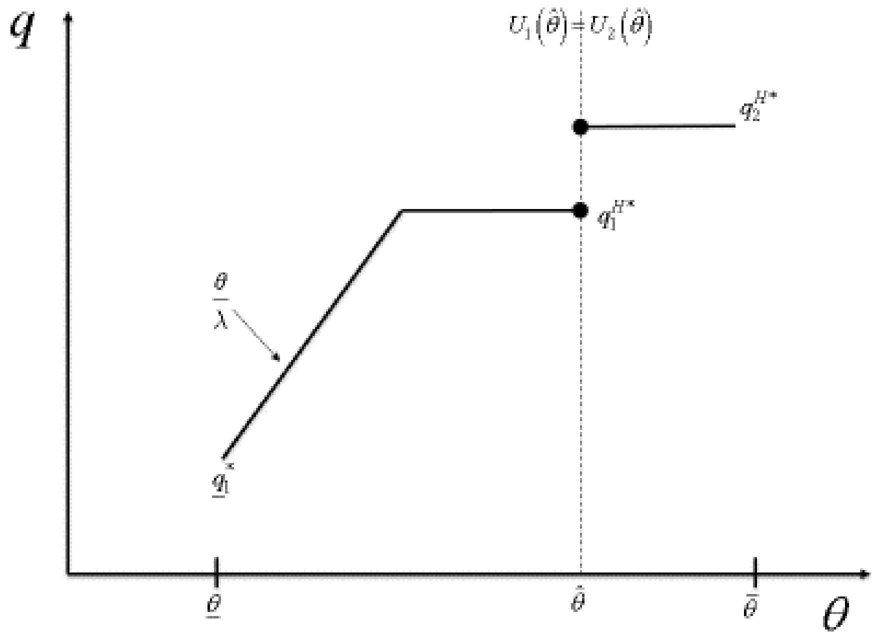

Figure 4: Asymmetrical versioning strategies when the low R&D firm is closer in costs to its competitor

Proposition 5. *When the R&D costs of the two firms are NOT sufficiently different and the low R&D capability firm's costs are low enough then there exists an equilibrium strategy where the high capability firm offers one version to the market while the low capability firm pursues versioning.* ■

Figure 3 depicts quality allocation and market coverage in the full game if Firm 1 pursues versioning, while Firm 2 pursues the single-quality strategy.

5. Competitive strategies for goods with free disposal

Our result that versioning could be optimal in competition of simultaneous-move settings under some cost structures seems to be inconsistent with findings of recent studies by Jones and Mendelson (2011) and Wei and Nault (2011). The following proposition indicates why versioning could not be optimal in their models, and highlights the role of the non-free-disposal property in forming the versioning strategy of information goods.

Proposition 6. *Even in a competitive market, for information goods (zero marginal and versioning costs) with free disposal, there is never an equilibrium strategy in versioning. This result is independent of the presence of R&D costs.* ■

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Appendix A: Notations

Symbol	Definition
q^H	The maximum quality created at development stage
c	Cost incurred by creating one unit of quality at development stage
$C\left(q^H\right)$	The one-short, quality-dependent cost of creating the maximum quality at development stage.
$\overline{q}$	The highest quality of the quality line a firm offers to the market
$\underline{q}$	The lowest quality of the quality line a firm offers to the market
p	Price of the information good
λ	Usage-related cost. $\lambda > 0$ for information goods with no-free-disposal property, or $\lambda = 0$ for goods with free disposal
$\theta \in \left[\underline{\theta}, \overline{\theta}\right]$	Consumer marginal valuation for quality of information goods—distributed with pdf $f\left(\cdot\right)$ and cdf $F\left(\cdot\right)$
$U\left(q, \theta\right)$	Surplus that consumer θ obtains from consuming information goods of quality q.
$\left\{q, p(q)\right\}$	General pricing menu
$\left\{q\left(\theta\right), p(\theta)\right\}$	Incentive compatible menu
$U(\theta)$	Surplus left to consumer under an incentive compatible menu
$q_{proj}\left(\theta\right)$	Quality allocation subject to a predetermined quality line
$\hat{\theta}$	The marginal consumer type that is indifferent between two competing firms, representing the market partition point.
$R\left(\cdot\right)$	Revenues generated in the versioning subgame
$\Pi\left(\cdot\right)$	Profits earned by entering the market and developing a maximum quality

The number in subscript is used to label firms.

* in superscript is used to denote SPNE.

Appendix B: Proof of Lemmas & Propositions

Proof of Proposition 1

First, at a competitive equilibrium at the pricing stage, both firms charge price 0 for those qualities they both offer due to the Bertrand effect. Therefore, in order to avoid a zero-profit result and justify the R&D costs incurred by market entry, each firm must provide some qualities non-overlapping with its rival's offerings in equilibrium.

Second, if an equilibrium exists where Firm i chooses its highest version of a quality $\overline{q}_i$ lower than the maximum quality and surrender the quality range $\left(\overline{q}_i, q_i^H\right]$ to its rival at the versioning stage, anticipating this, this firm could deviate to create a lower maximum quality and save the R&D cost accordingly at the R&D stage.

Third, if both firms provide a quality interval with the upper boundary at $q_i^H = q_j^H$. Denote the lower boundaries of the two intervals by $\underline{q}_i^1$ and $\underline{q}_j^1$ in which superscript 1 indicates the first quality interval from above for each firm. Without loss of generalizability, suppose $\underline{q}_i^1 \geq \underline{q}_j^1$. According to our discussion in the first paragraph, Firm i must also provide some qualities strictly lower than $\underline{q}_j^1$ which does not overlap Firm j's offerings. Denote the supremum of those qualities by q_i^S. Eliminating $\left[\underline{q}_i^1, q_i^H\right]$ at the versioning stage will not harm Firm i's profit. Doing so also allows Firm i to save the R&D cost, since the maximum quality it must create then is q_i^S rather than q_i^H. ■

Proof of Lemma 1

Incentive compatibility requires that $U_1'(\theta) = q_1(\theta)$ and $U_2'(\theta) = q_2(\theta)$. $q_1(\theta) \le \overline{q}_1 < \underline{q}_2 \le q_2(\theta)$. For the marginal type $\hat{\theta}$ who are indifferent between the menu of Firm 1 and that of Firm 2, $U_1(\hat{\theta}) = U_2(\hat{\theta})$ and $\left(U_2(\hat{\theta}) - U_1(\hat{\theta})\right)' > 0$. ■

Proof of Lemma 2A

The proof of this lemma is standard in adverse selection models, thus being omitted here. Quality projection identified in this lemma is a result similar to Lemma 2 in Chellappa and Mehra (2011).

Proof of Lemma 2B

According to Lemma 2 in Jullien (2000) that the optimal contract induces full participation from the market segment $\left[\underline{\theta}, \hat{\theta}\right]$ if $u \le \hat{\theta}\underline{q}_2 - \frac{1}{2}\lambda \underline{q}_2^2$; and Proposition 2 in Jullien (2000) gives the definition of γ and the formulation of $q_\gamma(\theta)$. Specifically, γ is a solution to $\int_{\underline{\theta}}^{\min\left\{q_\gamma^{-1}(\overline{q}_1), \hat{\theta}\right\}} \frac{1}{\lambda}\left[\theta + \frac{F(\theta) - \gamma}{f(\theta)}\right] d\theta + \int_{\min\left\{q_\gamma^{-1}(\overline{q}_1), \hat{\theta}\right\}}^{\hat{\theta}} \overline{q}_1 d\theta = u$, which is derived by applying $U_1(\underline{\theta}) = 0$. By implicit function theorem, $\frac{\partial \gamma}{\partial u} = -\frac{1}{\int_{\underline{\theta}}^{q_\gamma^{-1}(\overline{q}_1)} \frac{1}{\lambda}\frac{1}{f(\theta)} d\theta} < 0$ ■

Proof of Lemma 3

Firm 1 sets the price for $\overline{q}_1$ with considering Firm 2's response, i.e., the price Firm 2 sets for $\underline{q}_2$. Thereby, if $q_\gamma^{-1}(\overline{q}_1) \le \hat{\theta}$, the Firm 1's objective is represented by

$$\max_{\overline{p}_1} \int_{\underline{\theta}}^{\hat{\theta}} \left[\theta q_{\gamma\,proj}(\theta) - \frac{1}{2}\lambda q_{\gamma\,proj}(\theta)^2 + \int_{\theta}^{\hat{\theta}} q_{\gamma\,proj}(t)\,dt - U_1(\hat{\theta})\right] f(\theta)\,d\theta \tag{A1}$$

According to Lemma 2B,

$$\begin{aligned} &= \int_{q_\gamma^{-1}(\overline{q}_1)}^{\hat{\theta}} \left[\theta \overline{q}_1 - \frac{1}{2}\lambda \overline{q}_1^2 + \int_{\theta}^{\hat{\theta}} \overline{q}_1 dt - \left(\hat{\theta}\overline{q}_1 - \frac{1}{2}\lambda \overline{q}_1^2 - \overline{p}_1\right)\right] f(\theta)\,d\theta \\ &+ \int_{\underline{\theta}}^{q_\gamma^{-1}(\overline{q}_1)} \left[\theta q_\gamma(\theta) - \frac{1}{2}\lambda q_\gamma(\theta)^2 + \left(\hat{\theta} - q_\gamma^{-1}(\overline{q}_1)\right)\overline{q}_1 + \int_{\theta}^{q_\gamma^{-1}(\overline{q}_1)} q_\gamma(t)\,dt - \left(\hat{\theta}\overline{q}_1 - \frac{1}{2}\lambda \overline{q}_1^2 - \overline{p}_1\right)\right] f(\theta)\,d\theta \end{aligned}.$$

Integration by parts yields

$$= \int_{q_\gamma^{-1}(\overline{q}_1)}^{\hat{\theta}} \overline{p}_1 f(\theta)\,d\theta$$
$$+\int_{\underline{\theta}}^{q_\gamma^{-1}(\overline{q}_1)} \left[\theta q_\gamma(\theta) - \frac{1}{2}\lambda q_\gamma(\theta)^2 + \frac{F(\theta)}{f(\theta)} q_\gamma(\theta) - \left(q_\gamma^{-1}(\overline{q}_1)\overline{q}_1 - \frac{1}{2}\lambda \overline{q}_1^{\,2} - \overline{p}_1\right)\right] f(\theta)\,d\theta\,.$$

Differentiating it w.r.t. $\overline{p}_1$ yields

$$\int_{\underline{\theta}}^{\hat{\theta}} f(\theta)\,d\theta + \left[\int_{\underline{\theta}}^{q_\gamma^{-1}(\overline{q}_1)} \left[\theta - \lambda q_\gamma(\theta) + \frac{F(\theta)}{f(\theta)}\right] f(\theta) \frac{-1}{f(\theta)}\,d\theta\right] \frac{1}{\lambda}\frac{\partial \gamma}{\partial \overline{p}_1} + \overline{p}_1 f(\hat{\theta}) \frac{\partial \hat{\theta}}{\partial \overline{p}_1}\,.$$

Substituting $\frac{\partial \hat{\theta}}{\partial \overline{p}_1} = -\frac{1}{(\underline{q}_2 - \overline{q}_1)}$ and $\frac{\partial \gamma}{\partial \overline{p}_1} = \frac{1}{\int_{\underline{\theta}}^{q_\gamma^{-1}(\overline{q}_1)} \frac{1}{\lambda}\frac{1}{f(\theta)}\,d\theta}$ into the expression above and

simplifying it yield

$$\left((\overline{q}_1 - \underline{q}_2)F(\hat{\theta}) + \overline{p}_1 f(\hat{\theta})\right)\frac{1}{(\overline{q}_1 - \underline{q}_2)} - \gamma = 0\,.$$

Hence, $\overline{p}_1 = \frac{F(\hat{\theta}) - \gamma}{f(\hat{\theta})}(\underline{q}_2 - \overline{q}_1)$ with $F(\hat{\theta}) \geq \gamma$ to ensure that it is well-defined.

Similarly, Firm 2's objective is represented by

$$\max_{\underline{p}_2} \int_{\hat{\theta}}^{\overline{\theta}} \left[\theta q_{2proj}(\theta) - \frac{1}{2}\lambda q_{2proj}(\theta)^2 - U_2(\hat{\theta}) - \int_{\hat{\theta}}^{\theta} q_{2proj}(t)\,dt\right] f(\theta)\,d\theta$$

Its f.o.c. is

$$\left[\underline{p}_2 - \frac{1 - F(\hat{\theta})}{f(\hat{\theta})}\underline{q}_2\right] f(\hat{\theta}) + \int_{\hat{\theta}}^{\overline{\theta}} \overline{q}_1 f(\theta)\,d\theta = 0\,.$$

Hence, $\underline{p}_2 = \frac{1 - F(\hat{\theta})}{f(\hat{\theta})}(\underline{q}_2 - \overline{q}_1)$. Therefore, given $\underline{q}_2$ and $\overline{q}_1$, two unknowns are determined by

two equations, i.e.,

$$\hat{\theta}\overline{q}_1 - \frac{1}{2}\lambda \overline{q}_1^{\,2} - \frac{F(\hat{\theta}) - \gamma}{f(\hat{\theta})}(\underline{q}_2 - \overline{q}_1) = \hat{\theta}\underline{q}_2 - \frac{1}{2}\lambda \underline{q}_2^{\,2} - \frac{1 - F(\hat{\theta})}{f(\hat{\theta})}(\underline{q}_2 - \overline{q}_1),$$ and

$$\hat{\theta}\overline{q}_1 - \frac{1}{2}\lambda\overline{q}_1^{\,2} - \frac{F(\hat{\theta}) - \gamma}{f(\hat{\theta})}(\underline{q}_2 - \overline{q}_1) = \int_{\underline{\theta}}^{q_\gamma^{-1}(\overline{q}_1)} q_\gamma(\theta)d\theta + \int_{q_\gamma^{-1}(\overline{q}_1)}^{\hat{\theta}} \overline{q}_1 d\theta .$$

Construct a function of γ

$$\begin{aligned}\omega(\gamma) &= \hat{\theta}\overline{q}_1 - \frac{1}{2}\lambda\overline{q}_1^{\,2} - \frac{F(\hat{\theta}) - \gamma}{f(\hat{\theta})}(\underline{q}_2 - \overline{q}_1) - \frac{1}{\lambda}\int_{\underline{\theta}}^{q_\gamma^{-1}(\overline{q}_1)}\left(\theta + \frac{F(\theta) - \gamma}{f(\theta)}\right)d\theta - \overline{q}_1\left(\hat{\theta} - q_\gamma^{-1}(\overline{q}_1)\right) \\ &= q_\gamma^{-1}(\overline{q}_1)\overline{q}_1 - \frac{1}{2}\lambda\overline{q}_1^{\,2} - \frac{F(\hat{\theta}) - \gamma}{f(\hat{\theta})}(\underline{q}_2 - \overline{q}_1) - \frac{1}{\lambda}\int_{\underline{\theta}}^{q_\gamma^{-1}(\overline{q}_1)}\left(\theta + \frac{F(\theta) - \gamma}{f(\theta)}\right)d\theta\end{aligned},$$

in which $\frac{2F(\hat{\theta}) - (1+\gamma)}{f(\hat{\theta})} + \hat{\theta} = \frac{1}{2}\lambda(\underline{q}_2 + \overline{q}_1)$.

Differentiating it w.r.t. γ yields

$$\begin{aligned}&(\overline{q}_1 - \overline{q}_1)\frac{\partial\left(q_\gamma^{-1}(\overline{q}_1)\right)}{\partial\gamma} - (\underline{q}_2 - \overline{q}_1)\frac{d}{d\theta}\left(\frac{F(\hat{\theta}) - \gamma}{f(\hat{\theta})}\right)\frac{\partial\hat{\theta}}{\partial\gamma} + \frac{1}{f(\hat{\theta})}(\underline{q}_2 - \overline{q}_1) + \frac{1}{\lambda}\int_{\underline{\theta}}^{q_\gamma^{-1}(\overline{q}_1)}\left(\frac{1}{f(\theta)}\right)d\theta \\ &= \frac{\left[1 - \frac{\left(F(\hat{\theta}) - 1\right)f'(\hat{\theta})}{f(\hat{\theta})^2}\right]}{\left[3 - \frac{\left(2F(\hat{\theta}) - (1+\gamma)\right)f'(\hat{\theta})}{f(\hat{\theta})^2}\right]}\frac{(\underline{q}_2 - \overline{q}_1)}{f(\hat{\theta})} + \frac{1}{\lambda}\int_{\underline{\theta}}^{q_\gamma^{-1}(\overline{q}_1)}\left(\frac{1}{f(\theta)}\right)d\theta > 0\end{aligned}.$$

The inequality follows from Assumption 1.

$\omega(\gamma)$ is an increasing function, contradicting the definition of γ (which requires $\omega(\gamma)$ to be zero always). Hence $\gamma = 0$ in $\overline{p}_1 = \frac{F(\hat{\theta}) - \gamma}{f(\hat{\theta})}(\underline{q}_2 - \overline{q}_1)$ .The uniqueness of $\hat{\theta}$ in

$\hat{\theta} = \frac{1}{2}\lambda(\overline{q}_1 + \underline{q}_2) - \frac{2F(\hat{\theta}) - 1}{f(\hat{\theta})}$ is ensured by Assumption 1. ■

Lemma 4 is a direct result of Lemma 2 and Lemma 3 and thus omitted here.

Proof of Lemma 5

$$\max_{q_1(\cdot),\overline{p}_1}\int_{\underline{\theta}}^{\hat{\theta}}\left[\theta q_1(\theta) - \frac{1}{2}\lambda q_1(\theta)^2 - U_1(\theta)\right]f(\theta)d\theta,$$

$$\text{subject to } \begin{array}{l} q_1'(\theta) = t(\theta) \geq 0,\ U_1'(\theta) = q_1(\theta) \\ (\theta - \lambda q_1(\theta))t(\theta) \geq 0 \\ q_1(\theta) \leq \overline{q}_1,\ U_1(\theta) \geq 0 \\ U_1(\hat{\theta}) = U_2(\hat{\theta}) \end{array} .$$

Suppose there exists a function $\alpha_1(\theta)$ and $\alpha_2(\theta)$ for all θ, designated as the multiplier of $q_1'(\theta) = t(\theta)$, and a function $\alpha_2(\theta)$ for all θ, designated as the multiplier of $U_1'(\theta) = q_1(\theta)$. then the Hamiltonian is defined as

$$H(q_1, U_1, t, \alpha, \theta) = \theta q_1(\theta) - \frac{1}{2}\lambda q_1(\theta)^2 - U_1(\theta) + \alpha_1(\theta)t(\theta) + \alpha_2(\theta)q_1(\theta) \tag{A2}$$

Suppose we can further find $\beta_1(\theta)$ and $\beta_2(\theta)$, and non-decreasing functions $\gamma_1(\theta)$ and $\gamma_2(\theta)$ to define the Lagrangian (Seierstad and Sydsæter (1987)'s theorem 5, p.372) as

$$\begin{aligned} &L(q_1, U_1, t, \alpha, \beta, \gamma, \theta) \\ &= H(q_1, U_1, t, \alpha, \theta) + \beta_1(\theta)(\theta - \lambda q_1(\theta))t(\theta) + \beta_2(\theta)t(\theta) + \gamma_1(\theta)t(\theta) - \gamma_2(\theta)q_1(\theta) \end{aligned} \tag{A3}$$

where $\alpha_1(\theta) - \gamma_1(\theta)$ and $\alpha_2(\theta) + \gamma_2(\theta)$ are continuous.

From the Pontryagin principle, we have

$$\begin{aligned} \alpha_1'(\theta) - \gamma_1'(\theta) &= -\frac{\partial L(q_1, U_1, t, \alpha, \beta, \gamma, \theta)}{\partial q_1} = -\left[\theta - \lambda q_1(\theta) + \alpha_2(\theta) - \lambda t(\theta)\beta_1(\theta) - \gamma_2(\theta)\right] \\ \alpha_2'(\theta) + \gamma_2'(\theta) &= -\frac{\partial L(q_1, U_1, t, \alpha, \beta, \gamma, \theta)}{\partial U_1} = 1 . \end{aligned}$$

Without loss of generality we set (see Note 3 on p.333 Seierstad and Sydsæter (1987)) $\gamma_1(\underline{\theta}) = 0$ and $\gamma_2(\underline{\theta}) = 0$. Optimality requires $\alpha_1(\underline{\theta}) = \alpha_2(\underline{\theta}) = 0$. It can be verified that $q_1(\theta)$ in Lemma 5 satisfies the necessary conditions required by theorem 5 of Seierstad and Sydsæter (1987). Because concavity conditions of theorem 6 are also satisfied, the solution satisfying the necessary conditions is indeed optimal solution. ■

Proof of Lemma 6

When pursuing versioning is the equilibrium, Lemma 2A informs that $\underline{q}_2 < \overline{q}_2 \le q_2(\overline{\theta}) = \overline{\theta}$.

According to Lemma 3, $\underline{p}_2(\overline{q}_1, \underline{q}_2) = \frac{1 - F(\hat{\theta})}{f(\hat{\theta})}(\underline{q}_2 - \overline{q}_1)$.

Applying integration by parts, equation (9) can be simplified as

$$\begin{aligned} R_2(\underline{q}_2, \overline{q}_2) = & \int_{q_2^{-1}(\underline{q}_2)}^{q_2^{-1}(\overline{q}_2)} \left[\theta q_2(\theta) - \frac{1}{2}\lambda q_2(\theta)^2 + \frac{F(\theta) - 1}{f(\theta)} q_2(\theta) \right] f(\theta) d\theta \\ & + \int_{q_2^{-1}(\overline{q}_2)}^{\overline{\theta}} \left[\theta \overline{q}_2 - \frac{1}{2}\lambda \overline{q}_2^{\,2} + \frac{F(\theta) - 1}{f(\theta)} \overline{q}_2 \right] f(\theta) d\theta \\ & - \int_{q_2^{-1}(\underline{q}_2)}^{\overline{\theta}} \left[\theta \underline{q}_2 - \frac{1}{2}\lambda \underline{q}_2^{\,2} + \frac{F(\theta) - 1}{f(\theta)} \underline{q}_2 \right] f(\theta) d\theta \\ & + \frac{(1 - F(\hat{\theta}))^2}{f(\hat{\theta})} (\underline{q}_2 - q_1^H) \end{aligned}.$$

$\frac{\partial R_2(\underline{q}_2, \overline{q}_2)}{\partial \overline{q}_2} > 0$, implying that it is always a profit improving strategy for Firm 2 to increase $\overline{q}_2$ until it reaches the maximum quality $q_2^H \le q_2(\overline{\theta}) = \frac{\overline{\theta}}{\lambda}$ (when pursuing versioning, Firm 2 has no incentive to provide $q_2^H > \frac{\overline{\theta}}{\lambda}$). Therefore, the vendor 2's objective is solely dependent on $\underline{q}_2$.

$$\begin{aligned} \max_{\underline{q}_2 < q_2^H} R_2(\underline{q}_2) = & \int_{q_2^{-1}(\underline{q}_2)}^{q_2^{-1}(q_2^H)} \left[\theta q_2(\theta) - \frac{1}{2}\lambda q_2(\theta)^2 + \frac{F(\theta) - 1}{f(\theta)} q_2(\theta) \right] f(\theta) d\theta \\ & + \int_{q_2^{-1}(q_2^H)}^{\overline{\theta}} \left[\theta q_2^H - \frac{1}{2}\lambda (q_2^H)^2 + \frac{F(\theta) - 1}{f(\theta)} q_2^H \right] f(\theta) d\theta \\ & - \int_{q_2^{-1}(\underline{q}_2)}^{\overline{\theta}} \left[\theta \underline{q}_2 - \frac{1}{2}\lambda \underline{q}_2^{\,2} + \frac{F(\theta) - 1}{f(\theta)} \underline{q}_2 \right] f(\theta) d\theta \\ & + \frac{(1 - F(\hat{\theta}))^2}{f(\hat{\theta})} (\underline{q}_2 - q_1^H) \end{aligned},$$

such that $\frac{1}{2}\lambda(\underline{q}_2 + q_1^H) > \underline{\theta} - \frac{1 - F(\underline{\theta})}{f(\underline{\theta})} + \frac{F(\underline{\theta})}{f(\underline{\theta})}$.

The partial derivative w.r.t. $\underline{q}_2$ gives the expression stated in the lemma.

This is the local extremum when $\frac{1}{2}\lambda(\underline{q}_2 + q_1^H) > \underline{\theta} - \frac{1 - F(\underline{\theta})}{f(\underline{\theta})} + \frac{F(\underline{\theta})}{f(\underline{\theta})}$.

For some small q_1^H, Firm 2 can continue to lower $\underline{q}_2$ to make $\frac{1}{2}\lambda\left(\underline{q}_2+q_1^H\right)\leq\underline{\theta}-\frac{1-F(\underline{\theta})}{f(\underline{\theta})}+\frac{F(\underline{\theta})}{f(\underline{\theta})}$, all consumers then will be served by Firm 2, and Firm 1 will respond to this aggressive versioning by setting its price to $p_1^H=0$. For all $\underline{q}_2$ such that $\frac{1}{2}\lambda\left(\underline{q}_2+q_1^H\right)\leq\underline{\theta}-\frac{1-F(\underline{\theta})}{f(\underline{\theta})}+\frac{F(\underline{\theta})}{f(\underline{\theta})}$, Firm 2's profit is represented by

$$\begin{aligned}\tilde{R}_2\left(\underline{q}_2\right)=&\int_{\underline{\theta}}^{q_2^{-1}(\underline{q}_2)}\left[\theta\underline{q}_2-\frac{1}{2}\lambda\underline{q}_2^{\,2}+\frac{F(\theta)-1}{f(\theta)}\underline{q}_2-\left(\underline{\theta}q_1^H-\frac{1}{2}\lambda\left(q_1^H\right)^2\right)\right]f(\theta)d\theta\\&+\int_{q_2^{-1}(\underline{q}_2)}^{q_2^{-1}(q_2^H)}\left[\theta q_2(\theta)-\frac{1}{2}\lambda q_2(\theta)^2+\frac{F(\theta)-1}{f(\theta)}q_2(\theta)-\left(\underline{\theta}q_1^H-\frac{1}{2}\lambda\left(q_1^H\right)^2\right)\right]f(\theta)d\theta\quad,\\&+\int_{q_2^{-1}(q_2^H)}^{\bar{\theta}}\left[\theta q_2^H-\frac{1}{2}\lambda\left(q_2^H\right)^2+\frac{F(\theta)-1}{f(\theta)}q_2^H-\left(\underline{\theta}q_1^H-\frac{1}{2}\lambda\left(q_1^H\right)^2\right)\right]f(\theta)d\theta\end{aligned}$$

in which $\frac{\partial\tilde{R}_2\left(\underline{q}_2\right)}{\partial\underline{q}_2}\Big|_{\underline{q}_2>q_2(\underline{\theta})}<0$ and $\frac{\partial\tilde{R}_2\left(\underline{q}_2\right)}{\partial\underline{q}_2}\Big|_{\underline{q}_2=q_2(\underline{\theta})}=0$.

Hence $\underline{q}_2=q_2\left(\underline{\theta}\right)$ maximizes $\tilde{R}_2\left(\underline{q}_2\right)$, and the corresponding profit is denoted by

$$\begin{aligned}\tilde{R}_2=\int_{\underline{\theta}}^{q_2^{-1}(q_2^H)}\frac{1}{2\lambda}\left[\theta+\frac{F(\theta)-1}{f(\theta)}\right]^2 f(\theta)d\theta+\int_{q_2^{-1}(q_2^H)}^{\bar{\theta}}\left[\theta q_2^H-\frac{1}{2}\lambda\left(q_2^H\right)^2+\frac{F(\theta)-1}{f(\theta)}q_2^H\right]f(\theta)d\theta\\-\int_{\underline{\theta}}^{\bar{\theta}}\left[\underline{\theta}q_1^H-\frac{1}{2}\lambda\left(q_1^H\right)^2\right]f(\theta)d\theta\end{aligned}.\blacksquare$$

Proof of Lemma 7

For $q_1^H\geq q_2\left(\underline{\theta}\right)$, in equation(10), as $\underline{q}_2$ approaches to q_1^H,

$$\begin{aligned}&\frac{\left(1-F\left(\hat{\theta}\right)\right)^2}{f\left(\hat{\theta}\right)}-\frac{\left(1-F\left(q_2^{-1}\left(\underline{q}_2\right)\right)\right)^2}{f\left(q_2^{-1}\left(\underline{q}_2\right)\right)}+\left(\underline{q}_2-q_1^H\right)\frac{\partial}{\partial\theta}\left[\frac{\left(1-F\left(\hat{\theta}\right)\right)^2}{f\left(\hat{\theta}\right)}\right]\frac{\partial\hat{\theta}\left(q_1^H,\underline{q}_2\right)}{\partial\underline{q}_2}\\&\rightarrow\frac{\left(1-F\left(\hat{\theta}\right)\right)^2}{f\left(\hat{\theta}\right)}-\frac{\left(1-F\left(q_2^{-1}\left(\underline{q}_2\right)\right)\right)^2}{f\left(q_2^{-1}\left(\underline{q}_2\right)\right)}>0\end{aligned}.$$

It implies that $\underline{q}_2$ is increasing in the right neighborhood of q_1^H. The remaining thing we need to check is that Firm 2 can do no better when it lowers $\underline{q}_2$ to make $\underline{q}_2\leq q_1^H$.

When $\underline{q}_2 = q_1^H = q$, $\hat{\theta}$ is not well defined. Bertrand effects then drive the two firms to charge customers lying between $\left[q_1^{-1}(q), q_2^{-1}(q)\right]$ zero price in equilibrium. Then both firms make zero profit from consumers of type $\theta \in \left[q_1^{-1}(q), q_2^{-1}(q)\right]$. In addition, due to the increased value of outside options for customers served by the separating portions of menus, firms also make less profits from their exclusively served portion of the market.

If the top firm further lower $\underline{q}_2$ to make $\underline{q}_2 < q_1^H$, the two product lines would overlap with each other. Bertrand effects still induce that firms charge 0 prices for the overlapped qualities. With q_1^H unchanged, lowering $\underline{q}_2$ renders its profit invariant. In all, Firm 2 prefers to differentiate the profile lines from that of its competitors.

If $q_1^H < q_2(\underline{\theta})$, even though pursuing aggressive versioning is Firm 2's equilibrium strategy, $\underline{q}_2 \geq q_2(\underline{\theta})$. Hence $\underline{q}_2 > q_1^H$. ■

Proof of Lemma 8

If pursuing versioning in competition is Firm 2's SPNE strategy, Firm 2 determines $\underline{q}_2$ according to equation (10). Then Firm 2's objective at development stage is represented by

$$\begin{aligned}\Pi_2\left(q_2^H\right) = &\int_{q_2^{-1}(\underline{q}_2)}^{q_2^{-1}(q_2^H)}\left[\theta q_2(\theta) - \frac{1}{2}\lambda q_2(\theta)^2 + \frac{F(\theta)-1}{f(\theta)}q_2(\theta)\right]f(\theta)d\theta \\ &+\int_{q_2^{-1}(q_2^H)}^{\bar{\theta}}\left[\theta q_2^H - \frac{1}{2}\lambda\left(q_2^H\right)^2 + \frac{F(\theta)-1}{f(\theta)}q_2^H\right]f(\theta)d\theta \\ &-\int_{q_2^{-1}(\underline{q}_2)}^{\bar{\theta}}\left[\theta \underline{q}_2 - \frac{1}{2}\lambda \underline{q}_2^2 + \frac{F(\theta)-1}{f(\theta)}\underline{q}_2\right]f(\theta)d\theta \\ &+\frac{\left(1-F(\hat{\theta})\right)^2}{f(\hat{\theta})}\left(\underline{q}_2 - q_1^H\right) - \frac{1}{2}c_2 q_2^{H2}\end{aligned},$$

in which $\hat{\theta}\left(q_1^H, \underline{q}_2\right)$ is irrelevant to q_2^H.

Its partial derivative w.r.t. q_2^H gives equation (11). And Firm 1's objective is either

$\Pi_1\left(q_1^H\right)=\frac{F\left(\hat{\theta}\right)}{f\left(\hat{\theta}\right)}\left(\underline{q}_2\left(q_1^H\right)-q_1^H\right)F\left(\hat{\theta}\right)-\frac{1}{2}c_1q_1^{H2}$ if it pursues single-quality strategy; or

$$\Pi_1\left(q_1^H\right)=\int_{\underline{\theta}}^{q_1^{-1}\left(q_1^H\right)}\left[\theta q_1(\theta)-\frac{1}{2}\lambda q_1(\theta)^2+\frac{F(\theta)}{f(\theta)}q_1(\theta)\right]f(\theta)d\theta$$
$$-\int_{\underline{\theta}}^{q_1^{-1}\left(q_1^H\right)}\left[\theta q_1^H-\frac{1}{2}\lambda\left(q_1^H\right)^2+\frac{F(\theta)}{f(\theta)}q_1^H\right]f(\theta)d\theta+\frac{F\left(\hat{\theta}\right)^2}{f\left(\hat{\theta}\right)}\left(\underline{q}_2\left(q_1^H\right)-q_1^H\right)-\frac{1}{2}c_1\left(q_1^H\right)^2,$$

in which $\hat{\theta}\left(q_1^H,\underline{q}_2\left(q_1^H\right)\right)$ is a function of q_1^H, if it pursues versioning.

In either case, the partial derivative w.r.t. q_1^H gives equation (12). ■

Proof of Lemma 9

If pursuing single-quality strategy is Firm 2's SPNE strategy, its objective at R&D stage is represented by

$$\Pi_2\left(q_2^H\right)=\frac{\left(1-F\left(\hat{\theta}\right)\right)^2}{f\left(\hat{\theta}\right)}\left(q_2^H-q_1^H\right)-\frac{1}{2}c_2q_2^{H2}.$$

Its partial derivative w.r.t. q_2^H gives equation (14).

And Firm 1's objective is either .. if it pursues single-quality strategy; or

$$\Pi_1\left(q_1^H\right)=\int_{\underline{\theta}}^{q_1^{-1}\left(q_1^H\right)}\left[\theta q_1(\theta)-\frac{1}{2}\lambda q_1(\theta)^2+\frac{F(\theta)}{f(\theta)}q_1(\theta)\right]f(\theta)d\theta$$
$$-\int_{\underline{\theta}}^{q_1^{-1}\left(q_1^H\right)}\left[\theta q_1^H-\frac{1}{2}\lambda\left(q_1^H\right)^2+\frac{F(\theta)}{f(\theta)}q_1^H\right]f(\theta)d\theta+\frac{F\left(\hat{\theta}\right)^2}{f\left(\hat{\theta}\right)}\left(q_2^H-q_1^H\right)-\frac{1}{2}c_1\left(q_1^H\right)^2,$$

if it pursues versioning. In either case, the partial derivative w.r.t. q_1^H gives equation (13). ■

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