Reach Versus Competition in Channels with Internet and Traditional Retailers

Barrie R. Nault
Haskayne School of Business, University of Calgary, Calgary, Alberta, Canada, nault@ucalgary.ca

Mohammad S. Rahman
Haskayne School of Business, University of Calgary, Calgary, Alberta, Canada, rahman@ucalgary.ca

We examine the strategic and welfare implications of competition between traditional (retail) and Internet channels for goods where characteristics such as trust in the seller, returns, after-sales support, and physical inspection are important. Terming these as fixed online disutility costs, we develop extensions to two paradigm models – the Salop (1979) "circle around the lake model" and the Balasubramanian (1998) "pure e-tailer in the center" model – to include traditional retailers selling through the Internet channel. In these extensions, we conceptualize and specify how the online disutility costs of purchasing can be mitigated if the purchase is from a dual-channel retailer, defining the extent of mitigation as a function of distance from the traditional physical outlet. We compare these four models in prices, profits, consumer, and social welfare. We find that the impact of competition from a pure e-tailer and reach from dual-channel retailers in the Internet channel improves consumer welfare while at the same time lowering social efficiency. This is because consumers incur greater online disutility costs than transportation costs in order to obtain lower prices that result from online competition. We also find that consumers do not receive the advantages of mitigation of online disutility costs when these costs are high as dual-channel retailer prices in both channels are greater than they are in the presence of a pure e-tailer. Yet the reverse occurs when online disutility costs are low as this increases the competition between the dual-channel retailers' Internet channels. However, it is only profitable for traditional retailers to extend into the Internet channel if the online disutility costs are high enough to forestall a pure e-tailer. Taken together, our results show how the extension of market reach when traditional retailers also sell through the Internet channel can confound the effects of competition.

Key words: market reach, channel competition, online disutility costs, consumer welfare, social welfare.
History:

---

1. Introduction & Background

Consumers generally believe that a wider selection of channels from which to purchase benefits them and society – a version of "more is better". This is especially true in the electronic retail, or e-tail, world of Internet commerce where in addition to greater reach, greater competition from, and in, Internet channels is commonly viewed as having advantages for consumers. However, this belief is based on the assumption that more channels do not negatively alter conditions consumers experience in existing channels. Despite the presence of dual-channel (traditional and Internet) retailers in most industries, there are few results that evaluate the benefit of having a traditional retail store linked to the Internet channel, for consumers and for society. For example, we do not find equivocal evidence about whether the presence of dual-channel retailers matters even in prices. In particular, Clay et al. (2002) found similar book prices through the Internet and traditional retail. In contrast, Chevalier and Goolsbee (2003) found Internet prices for books were sensitive to dual-channel retailer prices, and Goolsbee (2001) found that the decision to buy computers online is sensitive to the relative price of such in traditional retail stores. What is clear from prior research is that Internet prices are highly variable. Clay et al. (2002) found greater price dispersion for books through Internet channels, as did Baylis and Perloff (2002) for cameras and scanners, Tang and Xing (2001) for DVDs, Clemons et al. (2002) for airline tickets, and Baye et al. (2004) for other retail products. Most of this research studied homogeneous search goods such as CDs, books, and DVDs (Iyer and Pazgal 2003), a type of good for which there is limited impact of channel and seller characteristics. This suggests that, especially in cases where there is a significant effect of channel and seller attributes, it is not straightforward to characterize the strategic and welfare implications of competitive market structures that may involve only dual-channel retailers, or dual-channel retailers in addition to pure e-tailers those that sell through the Internet channel only.

We conceptualize and model a vital aspect of dual-channel retailing – mitigating the disutility of buying from the Internet – in studying goods for which channel and seller characteristics matter in the context of traditional and Internet channel competition. The extant literature suggests that key components that underlie the disutility of buying from the Internet include trust (Jarvenpaaa et al. 2000, Stewart 2003), challenges in returning the product (Forman et al. 2009), and the lack of "touch and feel" (Balasubramanian 1998). Because traditional retail stores have a physical location to interact with consumers, they dominate Internet retailers on service, after-sales support, and trust (Verhoef et al. 2007). Consequently, access to a traditional retail store of a dual-channel retailer helps mitigate the costs – reduce the disutility – of buying from the Internet channel, essentially increasing retailer reach.

Trust plays a significant role in consumer decision making when buying online (Hoffman et al. 1999). Not surprisingly, the traditional store of a dual-channel retailer enhances consumer trust when they are purchasing through the Internet. Using an experiential survey, Jarvenpaaa et al. (2000) found that for trust to exist a consumer must believe that the seller has both the ability and motivation to reliably deliver goods of the quality expected, and this trust is more difficult to engender for an Internet store than a traditional retail store. They speculate that "the presence of a physical store or the recognition of the merchant's name might have an effect on consumer trust in an Internet-based store" (Jarvenpaaa et al. 2000). This trust is sometimes referred to as institution-based trust, and is taken to be higher when an Internet retailer also does business in the traditional retail channel. Experimental results from Stewart (2003) have shown that a connection to a traditional retail store had a significant positive effect on intention to buy, suggesting that institutional factors are important to trusting intentions. Hence, dual-channel retailers are able to benefit from institution based trust because the trust transfers from a traditional retail store to the Internet channel (Stewart 2003). The evidence from this research indicates that the traditional store of a dual-channel retailer provides a distinct advantage to the retailer's Internet counterpart by mitigating the online disutility cost, while a pure e-tailer does not enjoy this benefit. For example, most consumers would recognize and relate Bestbuy.com with a Bestbuy traditional store and, consequently, would more comfortably trust and transact with Bestbuy.com. However, a consumer may not place the same level of trust in Buy.com, a site that is not associated with traditional stores.

A critical disadvantage of buying from the Internet channel is problems related to returning a product (Forman et al. 2009). The problems online consumers face in making returns can be substantially reduced by visiting a traditional store of the dual-channel retailer. If there are post-purchase issues consumers can drive to the store and return the product or get satisfactory service. Indeed, even the option of going to the nearest traditional retail store gives a consumer peace of mind and reduces the online disutility cost. Based on a survey of transaction costs, Liang and Huang (1998) found that some products are more suitable for selling through the Internet channel than others, and this depends on the need for characteristics such as post-purchase service. For example, Bestbuy.com consumers have the option of going to a nearby traditional Bestbuy store to talk to someone in person if there are issues to resolve.

Not being able to touch and feel the product often makes online consumers uncertain about the fit with their needs and this induces disutility cost (Balasubramanian 1998). The literature on consumer behavior suggests that the consumer decision process can be divided into five stages: need recognition, information search, alternative evaluation, purchase, and outcome (Engel et al. 1990, Kotler 2002). Accordingly, a traditional store of the dual-channel retailer may help in the information search stage as a consumer may go to the store for other reasons and inspect available products. Later, if the consumer decides to buy a product online that s/he already saw at the store or a product close to the one s/he saw at the store, there is less uncertainty about the product or the retailer that sells it. Thus, the traditional store of the dual-channel retailer can provide the "touch and feel" for its Internet consumers. In this case, the consumer does not incur any traveling cost in purchasing the focal product because the inspection was done in a trip that was not made for this product.

In sum, the presence of a traditional retail store nearby mitigates the online disutility cost of buying from the Internet channel of the dual-channel retailer, and this mitigation depends on how far a consumer is from a traditional store. Brynjolfsson et al. (2009) empirically demonstrate that having traditional stores nearby reduces the Internet demand for popular products, which are likely to be available locally. Similarly, using data on bookselling Forman et al. (2009) show that when a local traditional retail store opens, consumers substitute away from the Internet channel, which implies that the comparison between online disutility costs and transportation costs matters even for books. Consequently, it is critical to understand the strategic and welfare implications of mitigating the online disutility costs. The novelty in our work is to formally articulate and model the mitigation of online disutility costs and derive insights by comparing consumer welfare and social welfare between commonly observed market structures. Accordingly, in examining the impact of mitigating online disutility costs, we conceptualize an important component of e-commerce, which plays a significant role in the competition within the Internet channel and between Internet and traditional channels.

We study this channel competition when selling the types of goods for which a seller's physical presence is valuable to consumers when they make purchases online. We specify a model of how dual-channel retailers, when they also sell through the Internet channel, mitigate online disutility costs to consumers based on the consumers' distance from the traditional retail store. Using this specification we then formulate and solve an extension to each of the paradigm analytical models – Salop's (1979) "circle around the lake" and Balasubramanian's (1998) "pure e-tailer in the center" – whereby traditional retailers also sell through the Internet channel so that, first, the retailers are dual-channel retailers (Salop model with an Internet channel) and, second, there is competition in the Internet channel (Balasubramanian model with retailers in the Internet channel). For example, considering the market for home improvement products, Home Depot and Lowe's are the main competitors both in traditional retail stores and through the Internet, which matches our first extension (Salop model with an Internet channel). The market for running shoes, on the other hand, has dual-channel retailers (e.g., Finish Line and Foot Locker) as well as pure e-tailers (e.g., Zappos.com) (Balasubramanian model with traditional retailers in the Internet channel).

In our models, the tension between a dual-channel retailer's reach via two channels versus competition in the Internet channel yields surprising results for consumer and social welfare. We find that the impact of competition from a pure e-tailer and reach from dual-channel retailers in the Internet channel improves consumer welfare while at the same time lowering social efficiency. This is because consumers incur greater online disutility costs than transportation costs in order to obtain lower prices that result from online competition, and this reduces social welfare – noting that prices are a transfer and only included in consumer welfare. We also find that when traditional retailers enter the Internet channel, consumers do not receive the advantages of mitigation of online disutility costs when these costs are high because dual-channel retailer prices in both channels are greater than they are in the presence of a pure e-tailer. The reverse occurs when online disutility costs are low as this increases the competition between the dual-channel retailers' Internet channels, reducing prices to the extent that even with less mitigation the mitigation effect is stronger. However, our profitability results suggest that it is only profitable for traditional retailers to extend into the Internet channel if the online disutility costs are high enough to forestall a pure e-tailer. Because competition between dual-channel retailers and competition from a pure e-tailer are the two most commonly observed market structures, these surprising results are particularly relevant for empirical studies, and for the formulation of future analytical models. Finally, in our analyses, like Balasubramanian (1998), we exclusively focus on retail-level competition and abstract from search, from segmentation apart from distance in the Salop (1979) model, and from supply chain effects such as vertical integration and double marginalization.

Our analysis proceeds as follow. First, we briefly review the Salop (1979) and Balasubramanian (1998) models, and explain our specification of how a traditional store can mitigate online disutility costs faced by consumers when purchasing from a retailer's Internet channel. Using a consistent framework, we show the solutions to the Salop and Balasubramanian models, and then obtain the similar solutions for the extensions to each of these models when traditional retailers are also in the Internet channel. Subsequently, we develop our main results in the form of propositions that compare the solutions of the different models in terms of price, retailer profits, consumer welfare, and social welfare. We conclude with a discussion of our findings, our contributions, and implications for future research.

2. Our Model

Salop Model Our model and its variants are based on the well-known Salop model (Salop 1979) of a circle around the lake creating a circular spatial market, which itself is an extension of the Hotelling (1929) model of horizontal differentiation along a line. Salop's model has a continuum of consumers, $x \in [0,1]$, spread uniformly around a circle of unit circumference. Each consumer is in the market for one unit of the good, consumption of which yields utility $U \in \mathcal{R}^+$, which we assume is large enough so that demand is inelastic and retailers compete for the business. All transportation occurs along the circle and is subject to a unit cost of $t \in \mathcal{R}^+$. All customers have access to information regarding prices. The consumers' objective is to maximize their utility by purchasing from one of the (traditional) retailers, which, with inelastic demand, is equivalent to purchasing from the retailer that minimizes the sum of the transportation cost incurred, t times distance from the retailer, plus the price paid for the good.

Retailers operate traditional stores selling identical goods with a marginal cost normalized to zero. Each retailer is aware of the other's offering price, and faces a fixed entry cost $f \in \mathcal{R}^+$. This fixed entry cost together with the unit transportation cost determines the number of retailers in the market. To make our analysis more insightful and tractable, we assume that $4 \le t/f < 9$ which in the original Salop model results in an equilibrium with two retailers (Tirole 1988). This particular inequality is scaled by the size of the circumference, which in turn scales t. We index these retailers by $r \in \{A, B\}$. In this circular setting, each retailer gains by locating as far as possible from competitors (de Frutos et al. 1999), hence our location of the two retailers at opposite sides of the circle. Although we obtain similar qualitative results with n retailers, expressing the results is more tedious and less insightful, so we use the two-retailer formulation in our analyses.

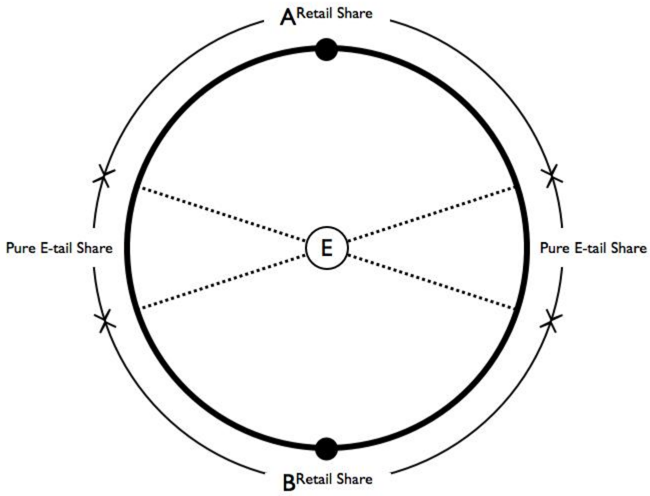


Figure 1 Balasubramanian Model — Two Traditional Retailers (A & B) and One Pure E-tailer.

Balasubramanian Model Balasubramanian's (1998) model extends the Salop model to include a pure e-tailer which offers a good identical to that of the traditional retailers. In Balasubramanian's model, the pure e-tailer has equal access to any point on the circumference. In Figure 1, the pure e-tailer is located at the center of the circle, with a radius distance to each point on the circumference. As an alternative to purchasing from one of the traditional stores as in the Salop model, consumers can purchase from the e-tailer and incur a fixed online disutility cost plus the price paid for the good. In line with Balasubramanian (1998), the fixed online disutility cost, which we denote as $\mu \in \mathcal{R}^+$, may include shipping and handling costs as well as disutility costs of purchasing electronically. These disutility costs may come from the privacy and security risks of purchasing online, the lack of trust in an e-tailer's ability and motivation to reliably deliver the quality expected, greater difficulties in getting help with or returning the good should there be problems post-purchase, and the lack of "touch and feel". In this model, the e-tailer has no entry costs.

Our Formulation Our formulation is based on traditional retailers also selling through the Internet channel, offering identical goods through both channels. To be consistent with Balasubramanian's model, we take retailers as having no online entry costs. When a dual-channel retailer sells through the Internet channel, consumers that purchase through the Internet channel incur the fixed online disutility cost, plus the price of the good. However, because of the existence of traditional retail stores, the disutility in the fixed online disutility cost of the Internet channel when purchasing from such retailers can be in part mitigated – along the three dimensions mentioned before: trust, after-sales support, and the lack of "touch and feel". The extent of the mitigation depends on the distance from the traditional store. For example, when purchasing from Bestbuy.com, a consumer who is 30 miles away from a Bestbuy store has a higher disutility mitigation compared to a consumer who is 60 miles away. In addition, in our equilibrium, retailers compete for those consumers that are closest to them: given prices are symmetric in equilibrium, then with a distance cost (through the traditional channel) or greater mitigation with proximity (through the Internet channel of dual-channel retailer), consumers cannot be better off choosing the retailer that is farther away.

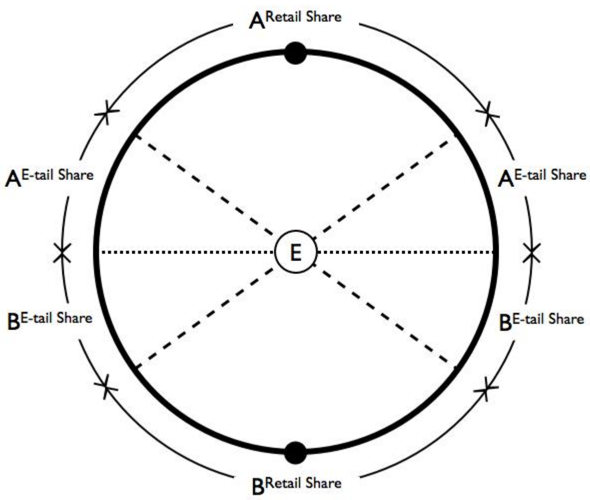


Figure 2 Salop Model with an Internet Channel — Two Dual-Channel Retailers.

Our formulation has two separate cases. The first is two retailers that compete across the traditional retail and Internet channels, which is essentially the addition of an Internet channel to the original Salop model with two retailers (see Figure 2). The second is two retailers that compete across the retail and Internet channels, and a pure e-tailer, which is essentially the addition of traditional retailers in the Internet channel to the original Balasubramanian model (see Figure 3). We set up our models as a simultaneous game of price setting, following the classic Salop and Balasubramanian articles. As such, in the two cases we develop, we implicitly assume that both traditional retailers enter the Internet market. Given retailers act symmetrically, there is no need to formally develop the step of whether traditional retailers enter the Internet market because it is relatively straightforward. Considering that there are no costs of entering the Internet channel (noting that is the case in Balasubramanian's model), and that our model is not about sequential entry, we also effectively model retailers deciding to enter the Internet channel simultaneously. Due to the symmetry in our model, there is no pure strategy equilibrium in which only one traditional retailer opens the Internet channel. Nonetheless, we provide conditions that describe when traditional retailer entry can be profitable in the Internet channel.

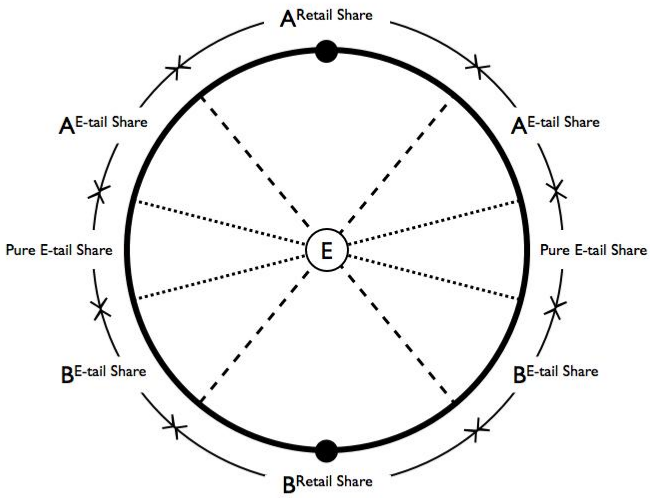


Figure 3 Balasubramanian Model with Retailers in the Internet Channel — Two Dual-Channel Retailers (A & B) and One Pure E-tailer.

To incorporate the mitigation of the fixed online disutility costs that the existence of a traditional retail store brings to consumers, we define two additional parameters. First is the marginal drop in mitigation of online disutility costs with distance, $a \in \mathcal{R}^+$, and the second is the maximum amount of mitigation of online disutility costs, ac where $c \in \mathcal{R}^+$. Thus, if a consumer at distance x from a traditional retailer purchases from that retailer's Internet channel, her costs of using the Internet channel is

$$\mu - a(c - x) = \mu - ac + ax. \tag{1}$$

A consumer adjacent to the traditional store faces a fixed online disutility cost less mitigation of $\mu - ac$, and that cost rises with x by a. Given this cost of distance cannot be higher than the transportation cost, we have $a < t$, and for the mitigation to be positive we have $c > x$. Finally, the fixed online disutility cost can never be completely mitigated, $\mu > ac$. For easier reference, we include these three inequalities below:

$$(i)\ a < t,\ (ii)\ c > x,\ (iii)\ \mu > ac. \tag{2}$$

This linear form in (1) is the simplest form we can use that is compatible with the Salop model.

We denote the traditional retail prices as p_r, the retailer prices through the Internet channel as p_{re}, and the pure e-tailer price as p_e. We use superscripts to denote the models, so that superscript s is the Salop model, b is the Balasubramanian model, se is our formulation of the Salop model with traditional retailers in the Internet channel, and br is our formulation of the Balasubramanian model with traditional retailers in the Internet channel.

2.1. Model Solutions

*Salop Model Solutions (*s*)* Two retailers offer identical goods. A consumer at the distance $x \in [0, 1/2]$ from retailer r is indifferent between purchasing from either retailer if $p_A + tx = p_B + t[1/2 - x]$. Based on this indifference equation we can determine retailer A's market share as $m_A = 2x = [p_B - p_A]/t + 1/2$. Retailer A's profit maximization problem is

$$\max_{p_A} \pi_A = \max_{p_A} \left\{ p_A \left[\frac{p_B - p_A}{t} + \frac{1}{2} \right] \right\}.$$

Retailer B has an identical market share and profit maximization problem. The resulting symmetric Nash equilibrium price is

$$p_r^s = t/2, \tag{3}$$

and there is no asymmetric equilibrium. Each retailer's market share is 1/2 and profits are $\pi_r^s = t/4$. The maximum distance for any consumer to a retailer is $x = 1/4$ and the minimum distance is 0, giving an average distance of 1/8 so that the total transportation cost incurred is $t/8$. The total cost to consumers is the sum of the retailer profits plus the transportation costs:

$$\omega^s = 2\pi_r^s + t/8 = 5t/8. \tag{4}$$

The social cost, which accounts for transfers between consumers and firms, is simply equal to the transportation cost in the Salop model:

$$\gamma^s = t/8. \tag{5}$$

*Balasubramanian Model Solutions (*b*)* The pure e-tailer offers the identical good at an effective price of $p_e + \mu$. The location of a consumer that is indifferent between purchasing from the e-tailer or a traditional retailer is determined by the indifference equation $p_e + \mu = p_r + tx$, giving the indifferent consumer's distance away from a retailer as $x = [p_e - p_r + \mu]/t$. Consumers closer to a given retailer than x purchase from that retailer, those that are farther than x purchase from the e-tailer.

The e-tailer's market share is $1 - 4x$, and each retailers' market share is $2x$. Each retailer's profit maximization problem is

$$\max_{p_r} \pi_r = \max_{p_r} \left\{ p_r \left[2\frac{p_e - p_r + \mu}{t} \right] \right\},$$

and the e-tailer's profit maximization problem is

$$\max_{p_e} \pi_e = \max_{p_e} \left\{ p_e \left[1 - 4\frac{p_e - p_r + \mu}{t} \right] \right\}.$$

The resulting Nash equilibrium prices are

$$p_e^b = t/6 - \mu/3 \text{ and } p_r^b = t/12 + \mu/3, \tag{6}$$

and there is no equilibrium where retailer prices are asymmetric. The e-tail market share is positive if $x < 1/4$, and consequently the pure e-tail price and market share are positive only if

$$\mu/t < 1/2, \tag{7}$$

and we restrict our attention to where (7) holds. It is worth noting that the magnitudes in this relation may appear unnatural until we recall that the circle is of unit circumference and thus the magnitude of distance, x, is small. Profits are

$$\pi_r^b = [t + 4\mu]^2/72t \text{ and } \pi_e^b = [t - 2\mu]^2/9t. \tag{8}$$

The total cost to consumers is the sum of the fixed online disutility costs, the transportation costs, and retailer and e-tailer profits:

$$\omega^b = \mu[1 - 4x] + 2t[x/2][2x] + 2\pi_r^b + \pi_e^b = \frac{11t^2 + 40\mu t - 16\mu^2}{72t}. \tag{9}$$

The social cost is the sum of fixed online disutility and transportation costs:

$$\gamma^b = \mu[1 - 4x] + 2t[x/2][2x] = \frac{t^2 + 56\mu t - 80\mu^2}{72t}. \tag{10}$$

Salop Model with an Internet Channel (se*)* The two dual-channel retailers compete across traditional retail and Internet channels. In the Internet channel, each retailer offers the identical good at $p_{re} + \mu - ac + ax$. We define two indifferent consumers. The first, x_1, is indifferent between the same retailer's traditional store and Internet channel. This indifferent consumer is defined by $p_r + tx_1 = p_{re} + \mu - ac + ax_1$, giving the distance away from the traditional retail store as $x_1 = [ac + p_r - p_{re} - \mu]/[a - t]$. The second, x_2, is indifferent between the two retailers' Internet channels. This indifferent consumer is defined by $p_{Ae} + \mu - ac + ax_2 = p_{Be} + \mu - ac + a[1/2 - x_2]$.

From our model formulation, we can show that because the retailers are identical, $x_2 = 1/4$. Each retailer's market share from its retail channel is $2x_1$, and from its Internet channel is $2[x_2 - x_1] = 1/2 - 2x_1$. Their profit maximization problems are

$$\max_{p_r, p_{re}} \pi_r = \max_{p_r, p_{re}} \left\{ p_r \left[2\frac{ac + p_r - p_{re} - \mu}{a - t} \right] + p_{re} \left[\frac{1}{2} - 2\frac{ac + p_r - p_{re} - \mu}{a - t} \right] \right\}.$$

The two retailers have identical best response functions, and the resulting Nash equilibrium prices are

$$p_r^{se} = [a - ac + \mu]/2 \text{ and } p_{re}^{se} = a/2, \tag{11}$$

both of which are positive, the first from (2)*(i)*. Both channels have positive market shares if $0 < x_1 = [\mu - ac]/2[t - a] < 1/4$. The first inequality is true from (2)*(i)* and *(iii)*, and the second is true if

$$t - a > 2[\mu - ac]. \tag{12}$$

For there to be positive mitigation, (2)*(ii)* requires $c > x_2$, or

$$c > 1/4. \tag{13}$$

Retailer profits are

$$\pi_r^{se} = \frac{a^2 - 2a^2c^2 - at + 4ac\mu - 2\mu^2}{4[a - t]}. \tag{14}$$

The total cost to consumers is the sum of fixed online disutility costs less mitigation, transportation costs, and retailer profits:

$$\begin{aligned} \omega^{se} &= [\mu - ac + a[1/4 + x_1]/2][1 - 4x_1] + 2t[2x_1][x_1/2] + 2\pi_r^{se} \\ &= \frac{5a^2 + 4a^2c^2 - 8a^2c + 8act - 5at - 8ac\mu + 8a\mu - 8\mu t + 4\mu^2}{8[a - t]}. \end{aligned} \tag{15}$$

The social cost is simply the sum of fixed online disutility costs less mitigation, plus transportation costs:

$$\begin{aligned} \gamma^{se} &= [\mu - ac + a[1/4 + x_1]/2][1 - 4x_1] + 2t[2x_1][x_1/2] \\ &= \frac{a^2 + 12a^2c^2 - 8a^2c - at + 8act + 8a\mu - 24ac\mu - 8\mu t + 12\mu^2}{8[a - t]}. \end{aligned} \tag{16}$$

Balasubramanian Model with Retailers in the Internet Channel (br*)* Two dual-channel retailers compete across traditional retail and Internet channels, and with a pure e-tailer in the Internet channel. The addition to the Balasubramnian model is the traditional retailer in the Internet channel. In the Internet channel, the retailer offers the identical goods at $p_{re}+\mu-ac+ax$. We define two indifferent consumers. The first, x_1, is indifferent between the same retailer's traditional store and Internet channel, and is defined as in the Salop model with an Internet channel above. The second, x_2, is indifferent between a dual-channel retailer's Internet channel and the pure e-tailer. This indifferent consumer is defined as $p_{re}+\mu-ac+ax_2=p_e+\mu$, giving $x_2=[ac+p_e-p_{re}]/a$.

The retailer's market shares are as defined in the Salop model with an Internet channel, except that in this case x_2 is not equal to 1/4. The pure e-tailer market share is $1-4x_2$. Each retailer's profit maximization problem is

$$\max_{p_r,p_{re}} \pi_r = \max_{p_r,p_{re}} \left\{ p_r \left[2\frac{ac+p_r-p_{re}-\mu}{a-t} \right] + p_{re}2 \left[\frac{ac+p_e-p_{re}}{a} - \frac{ac+p_r-p_{re}-\mu}{a-t} \right] \right\}.$$

The e-tailer's profit maximization problem is

$$\max_{p_e} \pi_e = \max_{p_e} \left\{ p_e \left[1-4\frac{ac+p_e-p_{re}}{a} \right] \right\}. \tag{17}$$

Again, as the dual-channel retailer best response functions are symmetric, the only equilibrium is where retailer prices are symmetric, and the resulting Nash equilibrium prices are

$$p_r^{br}=[a-2ac+6\mu]/12,\ \ p_{re}^{br}=[a+4ac]/12,\ \text{and}\ p_e^{br}=[a-2ac]/6. \tag{18}$$

Prices are positive from (2)*(i)* and when $c<1/2$. The market shares depend on x_1 and x_2 which are

$$x_1=[\mu-ac]/2[t-a]\ \text{and}\ x_2=c/3+1/12.$$

x_1 is positive from (2)*(i)* and *(iii)*, the same as in the Salop model with an Internet channel. For the pure e-tailer to have positive market share requires $x_2<1/4$, or $c<1/2$, which is the same as the positive price condition. For the retailer to have a positive Internet market share requires $x_2-x_1>0$, or

$$t-a>6\mu-2ac-4ct. \tag{19}$$

For there to be positive mitigation, (2)*(ii)* requires $c>x_2$, or $c>1/8$. Stating the bounds on c:

$$1/2>c>1/8. \tag{20}$$

Retailer profits are

$$\pi_r^{br}=\frac{a^2-20a^2c^2+8a^2c-a[1+4c]^2t+72ac\mu-36\mu^2}{72[a-t]}, \tag{21}$$

and e-tailer profits are

$$\pi_e^{br} = a[1-2c]^2/9. \tag{22}$$

The condition in (19) is sufficient for the dual-channel retailers Internet channel to have positive market share, and (22) being positive is sufficient for the pure e-tailer to be profitable – thus the Balasubramanian model with retailers in the Internet channel obtains when (19) is true. The total cost to consumers is the sum of fixed online disutility costs on purchases from the e-tailer, the sum of fixed online disutility costs less mitigation on Internet purchases from dual-channel retailers, transportation costs, and retailer and e-tailer profits:

$$\begin{aligned}\omega^{br} &= \mu[1-4x_2] + [\mu - ac + a[x_2+x_1]/2][4[x_2-x_1]] + 2t[x_1/2][2x_1] + 2\pi_r^{br} + \pi_e^{br} \\ &= \frac{11a^2 + 20a^2c^2 - 32a^2c - 11at + 32act + 16ac^2t - 72ac\mu + 72a\mu - 72\mu t + 36\mu^2}{72[a-t]}.\end{aligned} \tag{23}$$

The social cost is sum of fixed online disutility costs on purchases from the e-tailer, the sum of online disutility costs less mitigation on Internet purchases from dual-channel retailers, plus transportation costs:

$$\begin{aligned}\gamma^{br} &= \mu[1-4x_2] + [\mu - ac + a[x_2+x_1]/2][4[x_2-x_1]] + 2t[x_1/2][2x_1] \\ &= \frac{a^2 + 28a^2c^2 - 16a^2c - at + 16act + 80ac^2t - 216ac\mu + 72a\mu - 72\mu t + 108\mu^2}{72[a-t]}.\end{aligned} \tag{24}$$

3. Main Results

In some of propositions that follow rather than state the proof directly in the text we use constraint plots to more clearly demonstrate our results. The proofs that underlie these propositions are provided in the Appendix.

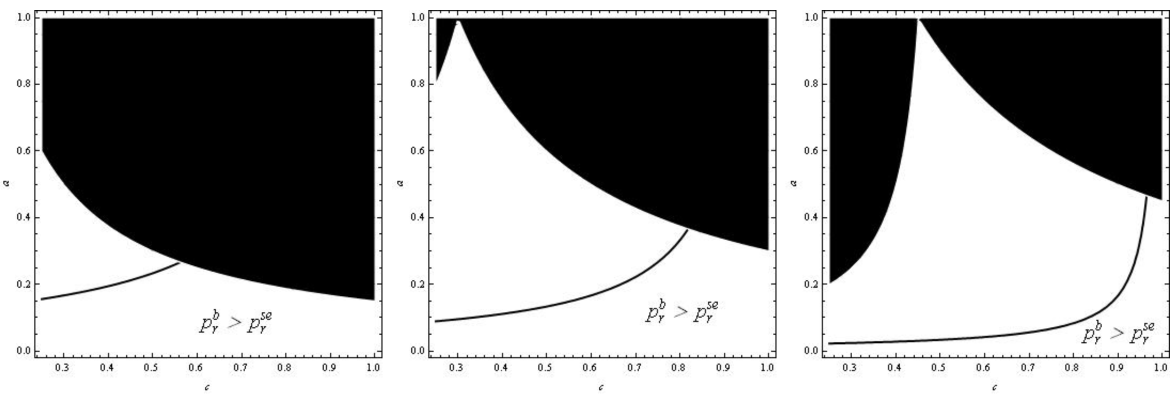
3.1. Prices and Profits

Prices To begin, the presence of an Internet channel in the Salop model (*se*) reduces traditional retail prices as the Internet channel increases reach by separating the market into those consumers that incur transportation costs and those that incur online disutility costs. To understand why this occurs, in the Salop model (*s*), each consumer except the indifferent consumer has a lower transportation cost with one or the other retailer. However, with the Internet channel – even with mitigation – for a given consumer the difference in the online disutility costs is smaller than the difference in transportation costs. As a consequence, the price competition between the retailers' Internet channels affects Internet prices and causes the traditional retail channel prices to be lower. Thus, the additional reach engenders increased competition.

Next, comparing the Balasubramanian model (*b*) to traditional retailers in the Internet channel in the Salop model (*se*), competition from a pure e-tailer, surprisingly, does not always extert downward pressure on prices so long as the fixed online disutility cost is not too high. In other words, if the online disutility cost is not too high, competition from a pure e-tailer can support higher traditional retail and Internet prices than can additional reach from dual-channel retailers. Otherwise, retailers in the Internet channel supports higher traditional retail and Internet prices. The following proposition provides the formal statement.

Proposition 1. **The Effect of Reach versus Competition on Prices**
If fixed online disutility costs are high (low), then dual-channel retailers charge higher (lower) traditional retail and Internet prices.

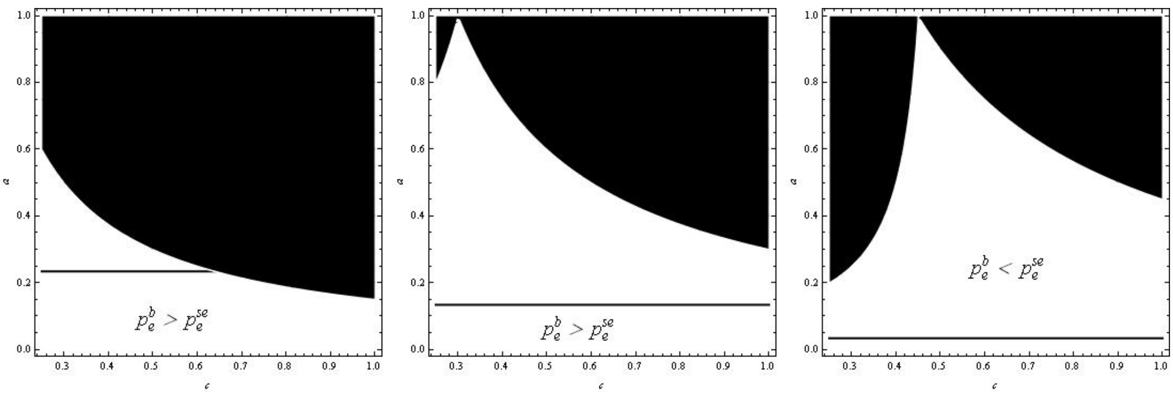
We show the result numerically through the constraint plots in Figures 4 and 5. Without loss of generality, we set the unit transportation cost to unity so that $t = 1$. By construction $t > a$, so that the marginal drop in mitigation with distance is $a < 1$. We increase the online disutility costs, μ, successively moving from Figure 4(a) to 4(c). The shaded areas reflect our constraints: the market share condition from (12) as well as the constraints on the mitigation parameters in (2). As μ increases, an increasing proportion of the parameter space supports $p_r^{se} > p_r^b$. A similar pattern is true for Internet prices in Figure 5, i.e., $p_e^{se} > p_e^b$.



(a) Price: $\mu = 0.15$, $t = 1$ (b) Price: $\mu = 0.30$, $t = 1$ (c) Price: $\mu = 0.45$, $t = 1$

Figure 4 Comparing traditional retail prices in the Balasubramanian model (b) to retailers in the Internet channel in the Salop model (se).

Proposition 1 is important and surprising because it shows that the effect of reach – even with retailers as dual-channel monopolists – does not necessarily lead to higher prices as compared to competition from a pure e-tailer, the Balasubramanian model. The conditions under which this occurs is low online disutility costs and a lower marginal drop in mitigation with distance. The latter means that the mitigation applies to a greater range of consumers, which in turn, intensifies



(a) Price: $\mu = 0.15$, $t = 1$ (b) Price: $\mu = 0.30$, $t = 1$ (c) Price: $\mu = 0.45$, $t = 1$

Figure 5 Comparing Internet prices in the Balasubramanian model (b) to retailers in the Internet channel in the Salop model (se).

the competition between the Internet channels of the dual-channel retailers. This intensified online competition creates pressure on traditional retail prices, increasing the rivalry between the two retailers. In contrast, the pure e-tailer in the Balasubramanian model competes directly with each traditional retailer and does not increase the rivalry between the retailers (see Figure 1). With higher online disutility costs or as the marginal drop in mitigation increases and mitigation becomes less substantial over distance, the rivalry between the Internet channels of the two dual-channel retailers (se) is reduced, and higher Internet and traditional retail prices can be sustained.

For Internet prices, the proposition shows that when μ is low, both dual-channel retailer and pure e-tail prices are higher in a competitive market as opposed to dual-channel monopolies for a larger parameter space. A lower μ facilitates this because the pure e-tailer is less disadvantaged, whereas the Internet channel of the dual-channel retailer faces greater competition from its rival. However, when online disutility costs are high, then mitigation from greater reach matters more, and dual-channel retailers can sustain higher Internet prices.

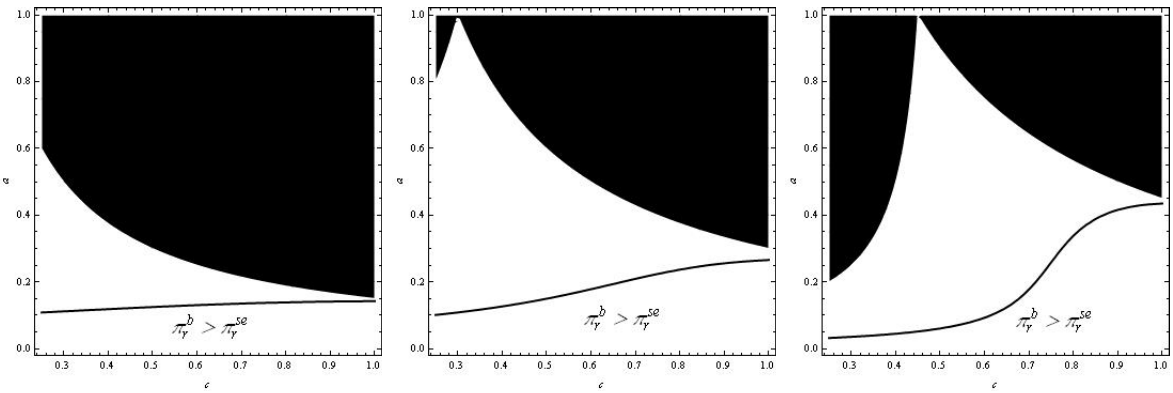
Finally, traditional retail and Internet prices are lower when there is competition from a pure e-tailer in the Internet channel (br) versus dual-channel retailers (se) as the competition from a pure e-tailer reduces Internet prices offered by the dual-channel retailer, and this in turn further lowers traditional retail prices. Hence, competition is more powerful than reach in determining prices. Using our results, the relationship between prices is

$$p_r^s > p_r^{se} > p_r^b > p_r^{br} \text{ and } p_{re}^{se} > p_e^b > p_e^{br},\, p_{re}^{se} > p_{re}^{br}, \tag{25}$$

where the comparison between retailers in the Internet channel (se) and the Balasubramanian model (b) are from Proposition 1 when online disutility costs are high.

Profits In comparisons between the original Salop model (s) and the Salop model with retailers in the Internet channel (se), and between the original Balasubramanian model (b) and the Balasubramanian model with retailers in the Internet channel (br), dual-channel retailer profits from one model versus the other can be higher or lower depending on the value of the additional channel. Generally, we expect that a lower fixed online disutility cost and a higher maximum mitigation favors retailer profits from the e-tail channel, and vice versa.

Not surprisingly, dual-channel retailer profits are higher when there is no pure e-tailer since the pure e-tailer adds direct competition to the Internet channel, and indirect competition to the traditional retail channel when the retailers sell through both channels. Surprisingly, retail profits are higher in the Salop model as compared to the Salop model with retailers in the Internet channel because the additional channel increases competition between the retailers. Consequently, $\pi^s \geq \pi^{se}, \pi_r^b, \pi_r^{br}$ and $\pi^{se} > \pi_r^{br}$. Corollary 1 compares profits in the Balasubramanian model (b) to retailers in the Internet channel in the Salop model (se). The profit relationship follows the general trend in prices presented in Proposition 1: when online disutility costs are high, then the effects of reach with mitigation dominate those of increased competition between dual-channel retailers in the Internet channel, and vice versa.



(a) Price: $\mu = 0.15$, $t = 1$ (b) Price: $\mu = 0.30$, $t = 1$ (c) Price: $\mu = 0.45$, $t = 1$

Figure 6 Retail profits in the Balasubramanian model (b) compared to the dual-channel retailers (se).

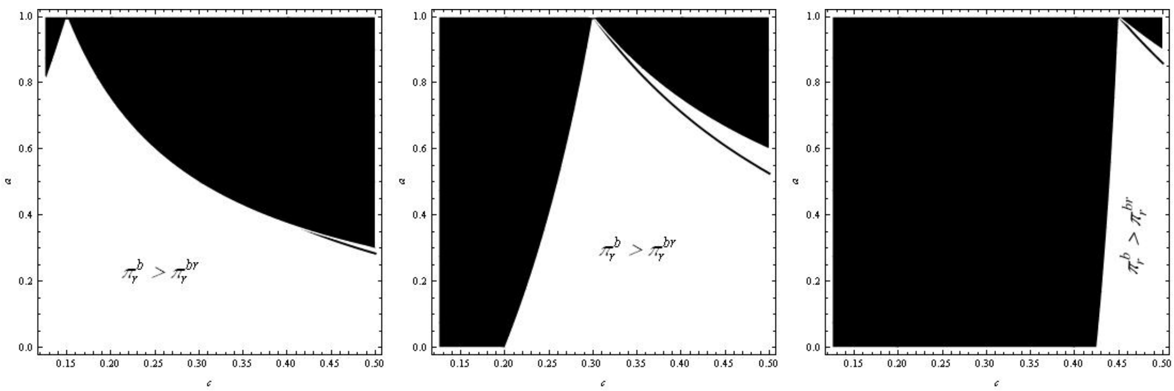
Corollary 1. *If fixed online disutility costs are high (low), then dual-channel retailers are more (less) profitable than traditional retailers with a pure e-tailer in the Internet channel.*

We show the result numerically through the constraint plots in Figure 6. Without loss of generality, we set the unit transportation cost to unity so that $t = 1$. By construction $t > a$, so that the marginal drop in mitigation with distance is $a < 1$. We increase μ successively moving from Figures 6(a) to 6(c). The shaded areas reflect our constraints: the market share condition from (12) as well as the constraints on the mitigation parameters in (2). As μ increases, an increasing proportion of the parameter space supports $\pi_r^b > \pi_r^{se}$.

Our next proposition compares profits in the Balasubramanian model (b) to those from the Balasubramanian model with retailers in the Internet channel (br).

PROPOSITION 2. **The Effect of Reach on E-tail Competition**
Entry into the Internet channel and consequent reach is rarely profitable for a traditional retailer.

We show the result numerically through the constraint plots in Figure 7. Without loss of generality, we set the unit transportation cost to unity so that $t = 1$. By construction $t > a$, so that the marginal drop in mitigation with distance is $a < 1$. We increase μ successively moving from Figure 7(a) to 7(c). The shaded areas reflect our constraints: the market share condition from (19) as well as the constraints on the mitigation parameters in (2). $\pi_r^b > \pi_r^{br}$ across most of the range of c and a. In the small feasible areas above the line, the opposite is true.



(a) Price: $\mu = 0.15$, $t = 1$ (b) Price: $\mu = 0.30$, $t = 1$ (c) Price: $\mu = 0.45$, $t = 1$

Figure 7 **Retail profits in the Balasubramanian model (b) compared to the extended the Balasubramanian model (br).**

Another way to show Proposition 2 is through Corollary 1 and $\pi^{se} > \pi_r^{br}$. As Figure 7 shows, Proposition 2 holds over a substantial range of the parameter space (more than Corollary 1), and the converse holds only over a small part of the parameter space. As in some of our pricing results, the additional reach from traditional retailers in the Internet channel increases competition in that channel, and this increased competition carries through to the traditional retail channel. Using our results from Corollary 1 and Proposition 2, we have the following profit relationships for the retailers:

$$\pi^s \geq \pi^{se} \geq \pi_r^b \geq \pi_r^{br}. \tag{26}$$

The change in pure e-tailer profits when there is additional competition from traditional retailers selling through the Internet channel is straightforward as the additional competition dissipates profits:

$$\pi_e^b > \pi_e^{br}. \tag{27}$$

3.2. Consumer Welfare

We now examine the impact of reach as compared to competition on consumer welfare. In consumer welfare we account for prices, the online disutility costs less mitigation and the transportation costs. Thus, our analysis is based on total costs to consumers, recognizing that with our implicit assumption that the market is covered, each consumer derives the same value from consumption across models and, consequently, they only differ in their costs. Nonetheless, we describe our results in terms of consumer welfare because it is a more common and natural description.

Our first result is that starting from the Salop model, consumer welfare is increased with a pure e-tailer in the market. This result, that we choose not to state formally in a proposition, is that the total cost to consumers is lower in the Balasubramanian model than in the Salop model:

$$\omega^s > \omega^b. \tag{28}$$

This is straightforward from (4) and (9), recognizing that the constraint for the relationship between the fixed online disutility cost and the transportation cost in (7) must hold.

The following proposition, stated in two parts, shows that both reach and competition contribute to consumer welfare. The first part of the proposition compares the Salop model (s) and the Salop model with retailers in the Internet channel (se). The second part compares the Salop model with retailers in the Internet channel (se) and the Balasubramanian model with retailers in the Internet channel (br).

Proposition 3. **The Effect of Reach and Then Competition**

(i) *The additional reach of traditional retailers in the Internet channel increases consumer welfare.*

(ii) *The additional competition from a pure e-tailer in a market with dual-channel retailers increases consumer welfare.*

Proof: (i) From the total costs to consumers in (4) and (15),

$$\frac{\partial[\omega^s - \omega^{se}]}{\partial c} = \frac{a(a - ac - t + \mu)}{a - t} \text{ and } \frac{\partial^2[\omega^s - \omega^{se}]}{\partial c^2} = \frac{-a^2}{a - t} > 0$$

so that $[\omega^s - \omega^{se}]$ is convex. In addition, $[\omega^s - \omega^{se}]$ has no real roots, but only complex roots. Hence, $\omega^s > \omega^{se}$.

(ii) From the total costs to consumers in (15) and (23), and using the constraints in (13) for the Salop model with retailers in the Internet channel together with those in (20) for the Balasubramanian model with retailers in the Internet channel:

$$\omega^{se} - \omega^{br} = 20c - 8c^2 - 17 > 0 \ \forall \ c \ni \ 1/2 \geq c \geq 1/4.$$

Q.E.D.

Traditional retailers in the Internet channel are effectively dual-channel monopolists. However, the reduction in online disutility costs from a sufficiently low marginal drop in mitigation with distance for consumers that purchase through the Internet more than offsets the potentially higher retail prices for those customers that purchase through the traditional retail channel. Consequently, from Proposition 3*(i)*, an average consumer is better off with traditional retailers in the Internet channel – greater reach – when the market involves dual-channel retailers only.

Proposition 3*(ii)* is important because it shows that for the average consumer, the possible increases in online disutility costs that occurs from competition in the Internet channel whereby the pure e-tailer as well as the dual-channel retailers in both channels have a positive market share are dominated by the decreases in prices that comes from the same competition. The possible increases in online disutility costs is due to the lack of mitigation of online disutility costs offered by dual-channel retailers in the Internet channel when the pure e-tailer has a positive market share. As described earlier, competition in the Internet channel reduces Internet prices, which in turn puts downward pressure on traditional retail prices. Consequently, competition compounds the effects of reach on consumer welfare.

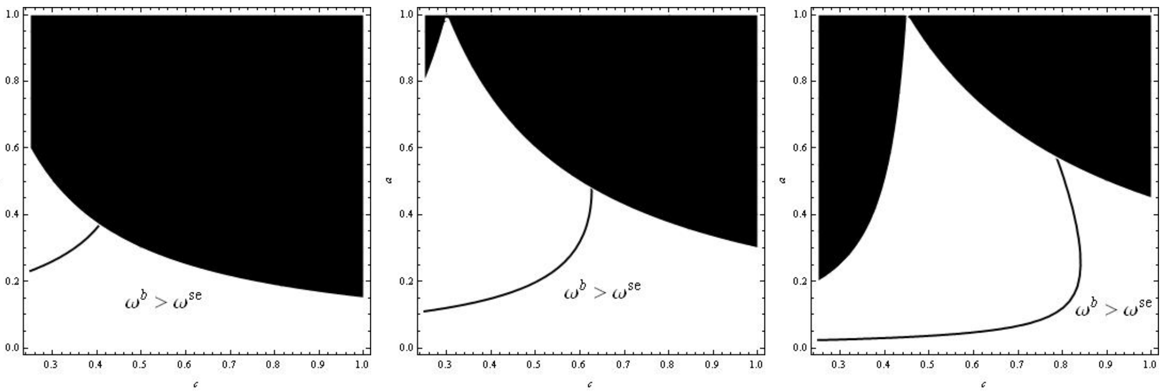
Overall, Proposition 3 establishes that traditional retailers selling through the Internet channel increase consumer welfare as compared to the Salop model, and that competition in the Internet channel from a pure e-tailer further increases consumer welfare. In terms of total costs to consumers, we have

$$\omega^s > \omega^{se} > \omega^{br}. \tag{29}$$

It remains to determine whether consumers are better off as a result of increased traditional retailer reach – the Salop model with retailers in the Internet channel (se) – or increased competition – only a pure e-tailer in the e-tail channel, the Balasubramanian model (b).

PROPOSITION 4. **The Effect of Reach versus Competition on Consumer Welfare**
If fixed online disutility costs are high (low), then a pure e-tailer increases (decreases) consumer welfare relative to dual-channel retailers.

We show the result numerically through the constraint plots in Figure 8. Without loss of generality, we set the unit transportation cost to unity so that $t = 1$. By construction $t > a$, so that the marginal drop in mitigation with distance is $a < 1$. We increase μ successively moving from Figure 8(a) to 8(c). The shaded areas reflect our constraints: the market share condition from (12) as well as the constraints on the mitigation parameters in (2). When μ increases, and increasing proportion of the parameter space supports $\omega^{se} > \omega^b$.



(a) Price: $\mu = 0.15$, $t = 1$ (b) Price: $\mu = 0.30$, $t = 1$ (c) Price: $\mu = 0.45$, $t = 1$

Figure 8 Consumer welfare in the Balasubramanian model (b) relative to that with the Internet channel in the Salop model (se).

Proposition 4 is important and surprising because it shows that the consumer welfare-increasing effect of mitigating online disutility costs overcomes the consumer welfare-increasing effects of competition on prices in the Balasubramanian model mostly at lower levels of the online disutility costs. As online disutility costs increase, the effects of competition are greater than those of mitigation over an increasingly greater range of the parameter space. Consequently, mitigation is only consumer welfare-increasing if the marginal drop in mitigation (a) is low and the maximum mitigation is high (ac).

It is also possible to show that adding retailers in the Internet channel to the Balasubramanian model increases consumer welfare because both competition and reach work to lower prices in both channels (see (25)). Putting the relations together over the different market configurations, we have

$$\omega^s > \omega^{se} > \omega^b > \omega^{br}, \tag{30}$$

where the comparison between retailers in the Internet channel (se) and the Balasubramanian model (b) are from Proposition 4 when online disutility costs are high.

3.3. Social Welfare

Accounting for the fact that prices, and thus profits, are a transfer between sellers and consumers, social costs are a subset of total costs to consumers that only include online disutility costs less mitigation and transportation costs. Consequently, the relative effects of different channel configurations on social welfare may differ from the relative effects of different channel configurations on consumer welfare. As with consumer welfare, with the market covered, each consumer derives the same value from consumption across models. Although our analysis of social welfare is done based on social costs, we describe our results in terms of social welfare as it is more common and natural.

3.3.1. The Effects of an Internet Channel We begin by establishing the condition that determines if social welfare is increased by the addition of a pure e-tailer to a market that only contains traditional retailers, that is, when the Balasubramanian model (b) increases social welfare over the Salop model (s). The following proposition provides the condition.

PROPOSITION 5. **The Effect of a Pure E-tailer on Social Welfare from the Salop Model** *If fixed online disutility costs are more (less) than 20% of unit transportation costs, then the addition of a pure e-tailer to the Salop model – the Balasubramanian model – decreases (increases) social welfare.*

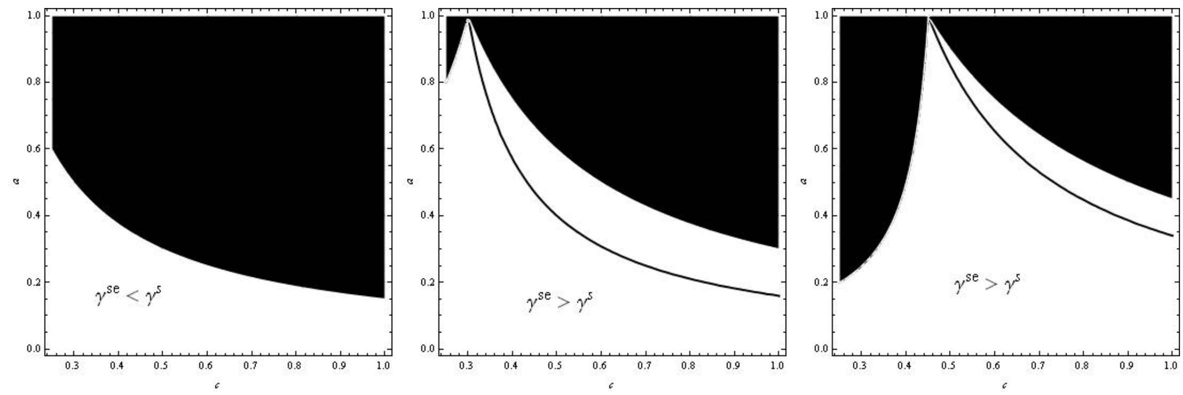
Proof: Directly from (5) and (10), if $t < 5\mu$ then $\gamma^b > \gamma^s$. Q.E.D.

The impact in Proposition 5 comes from the combined effect of some consumers substituting online disutility costs for transportation costs and of additional price competition from the pure e-tailer determining how many consumers make that substitution. Although it is perhaps surprising, recalling the constraint from the Balasubramanian model in (7), $2\mu < t$, and the condition in the Proposition, there is a substantial range in the relationship between online disutility costs and transportation costs whereby either a traditional retail only channel market or competition between channels can have greater social welfare.

Our next social welfare proposition compares the social costs in the original Salop model (s) to the Salop model with retailers in the Internet channel (se).

PROPOSITION 6. **The Effect of Retailer Reach on Social Welfare** *If fixed online disutility costs are high, then social welfare is lower with traditional retailers in the Internet channel.*

We show the result numerically through the constraint plots in Figure 9. Without loss of generality, we set the unit transportation cost to unity so that $t = 1$. By construction $t > a$, so that the marginal drop in mitigation with distance is $a < 1$. We increase μ successively moving from Figure 9(a) to 9(c). The shaded areas reflect our constraints: the market share condition from (12) and the constraints on the mitigation parameters in (2). To be consistent across models, we also impose $\mu \leq 1/2$ from (7). When μ is moderate to high in Figures 9(b) and 9(c), then $\gamma^{se} > \gamma^{s}$ over most of the parameter space. When μ is low then from Figure 9(a), $\gamma^{s} > \gamma^{se}$.



(a) Price: $\mu = 0.15$, $t = 1$ (b) Price: $\mu = 0.30$, $t = 1$ (c) Price: $\mu = 0.45$, $t = 1$

Figure 9 **Social welfare in the Salop model (s) relative to that with the Internet channel in the Salop model (se).**

Proposition 6 is important because it shows that counter to intuition, consumers can incur greater costs – in other words, social welfare is reduced – with the addition of traditional retailers selling through the Internet channel. Hence, we have the surprising result that exclusive of prices, an extra channel can effectively increase costs to society. These social costs are higher when online disutility costs are sufficiently large and when the maximum mitigation (i.e., ac) is not sufficiently large to offset the high online disutility costs, in the context of a dual-channel retailers that price discriminate between channels such that some consumers incur the higher net online disutility costs in exchange for possibly lower prices (note $p_r^{se} \geq p_{re}^{se}$ from (25)). Proposition 6 also shows that the lower are the online disutility costs relative to transportation costs (having normalized $t = 1$), the greater is the social gain from consumers substituting the Internet channel for the traditional retail channel.

Combining Propostions 5 and 6 we find the surprising result that when online disutility costs are sufficiently high, then the addition of an Internet channel – whether from a pure e-tailer or from the traditional retailers in the Internet channel – decreases social welfare. In terms of social costs we have

$$\gamma^{se}, \gamma^{b} > \gamma^{s}.$$

3.3.2. The Effects of Competition in the Internet Channel We examine the effects of competition in the Internet channel, and compare these effects with those of reach. We begin with a conclusive theorem that shows competition in the Internet channel (br) decreases social welfare relative to reach from traditional retailers in the Internet channel (se).

PROPOSITION 7. **The Effect of Competition from a Pure E-tailer in the Internet Channel on Social Welfare**
Compared with traditional retailers in the Internet channel, additional competition in the Internet channel from a pure e-tailer decreases social welfare.

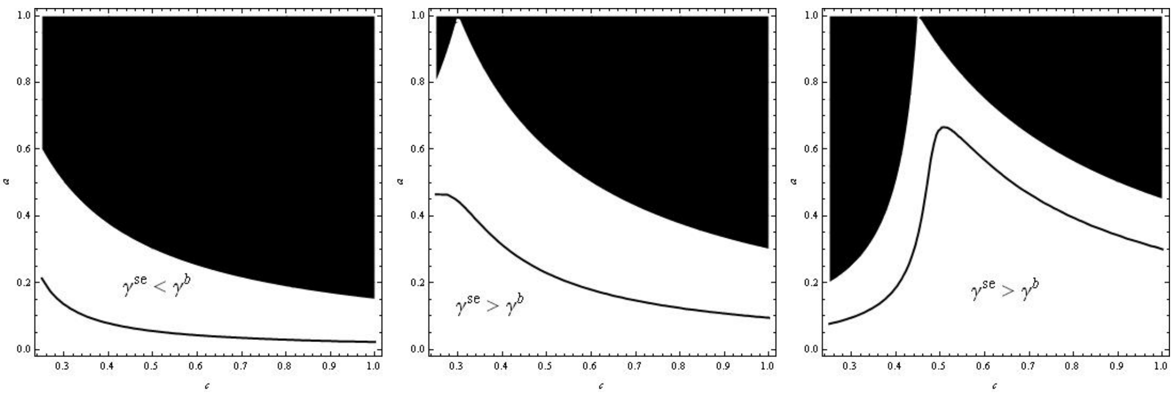
Proof: Using (16) and (24), we find $\gamma^{se} - \gamma^{br} = \frac{a}{9}(1 + c(10c - 7))$. Therefore, if $(7 - 10c)c > 1$, then $\gamma^{se} < \gamma^{br}$. Combining constraints from (13) and (20) to obtain the range of $1/4 < c < 1/2$, the inequality is true for all $c \in (1/4, 1/2)$, and social costs are equal if $c = 1/2$. Q.E.D.

This proposition is important and surprising because it shows that social welfare is reduced by competition in the Internet channel from a pure e-tailer. It is also a strong theorem in that it does not depend on conditions outside the two model's solutions. The reason welfare is reduced is because net of prices – which are traditional retail and Internet profits and do not enter into social costs – consumers located far from a dual-channel retailer purchase from the pure e-tailer and do not benefit from mitigation of their online disutility costs as they would if they purchased from a retailer's Internet channel. In other words, with only traditional retailers in the Internet channel, the online disutility costs are always mitigated for consumers that buy through the Internet channel. The reversal of the effects on costs, total versus social, from Proposition 3 is because prices fall with competition between dual-channel retailers and a pure e-tailer in the Internet channel (see (25)), and more than offset the differences in transportation and online disutility costs.

Next, we compare social welfare in the Balasubramanian model (b) with that from the Salop model with retailers in the Internet channel (se).

PROPOSITION 8. **The Effect of Dual-Channel Retailers versus a Pure E-tailer in the Internet Channel**
If fixed online disutility costs are high and the marginal drop in mitigation is low, then social welfare is higher with a pure e-tailer in the Internet channel than with traditional retailers in the Internet channel.

We show the result numerically through constraint plots in Figure 10. Without loss of generality, we normalize the unit transportation cost to unity, $t = 1$. By construction $t > a$, so that the marginal drop in mitigation with distance from (1) is $a < 1$. We increase μ successively moving from Figure 10(a) to 10(c). The shaded areas reflect our constraints: the market share condition from (12) as



(a) Price: $\mu = 0.15$, $t = 1$ (b) Price: $\mu = 0.30$, $t = 1$ (c) Price: $\mu = 0.45$, $t = 1$

Figure 10 Social welfare in the Balasubramanian model (b) relative to that with the Internet channel in the Salop model (se).

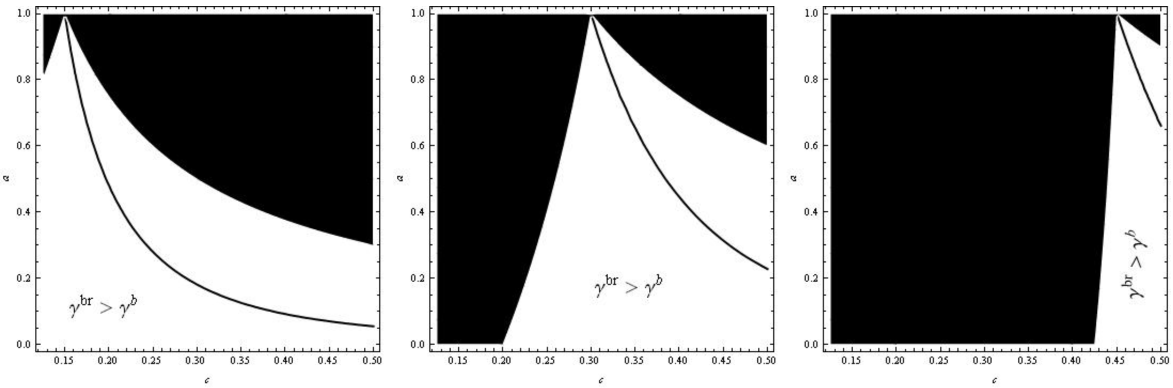
well as the constraints on the mitigation parameters in (2). As μ increases across Figures 10(a) through 10(c), when a is low, then $\gamma^{se} > \gamma^{b}$.

Proposition 8 is important and very surprising because it shows that competition between a pure e-tailer in the Internet channel and traditional retailers can increase social welfare relative to the increased reach and consequent mitigation of online disutility costs from traditional retailers in the Internet channel – dual-channel retailers. This occurs because when price is excluded, the calculation of social costs is based on transportation costs and online disutility costs. The online disutility costs are mitigated for consumers with dual-channel retailers selling through the Internet channel relative to the case when the pure e-tailer is alone in the Internet channel. Examining the premise of the theorem, a low marginal drop in mitigation of online disutility costs with distance favors a pure e-tailer relative to traditional retailers in the Internet channel in terms of lower social costs. Moreover, a low marginal drop in mitigation also reduces the maximum mitigation of online disutility costs, which impacts the net online disutility costs incurred by consumers.

Our last proposition compares social welfare in the Balasubramanian model (b) to the extended Balasubramanian model where a pure e-tailer competes with dual-channel retailers in the Internet channel (br).

Proposition 9. **The Effect of Competition in the Internet Channel**
If fixed online disutility costs are high or the maximum mitigation of fixed online disutility costs or the marginal drop in mitigation are low, then social welfare is lower with competition in the Internet channel.

We show the result numerically through constraint plots in Figure 11. Without loss of generality we normalize the unit transportation cost to unity, $t = 1$. By construction $t > a$, so that the marginal drop in mitigation with distance from (1) is $a < 1$. We increase μ successively moving from Figures 11(a) to 11(c). The shaded areas reflect our constraints: the market share condition from (19) as well as the constraints on the mitigation parameters in (2). As μ increases across Figures 11(a) through 11(c), when a is low, then $\gamma^{br} > \gamma^b$.



(a) Price: $\mu = 0.15$, $t = 1$ (b) Price: $\mu = 0.30$, $t = 1$ (c) Price: $\mu = 0.45$, $t = 1$

Figure 11 Social welfare in the Balasubramanian model (b) relative to that in the extended Balasubramanian model (br).

Reflecting the result from Proposition 7, the premise in Proposition 9 is weaker than that in Proposition 8. Proposition 9 is important in that exclusive of prices, consumers can incur greater social costs with the addition of traditional retailers selling through the Internet channel – this time beyond an Internet channel served by a pure e-tailer. Thus, we find the surprising result that the combination of competition in the Internet channel between a pure e-tailer and traditional retailers in the Internet channel together with reach from the dual-channel retailers can also reduce social welfare. Because of lower prices with greater competition, the Internet channel – both the pure e-tailer and traditional retailers in the Internet channel – has greater reach where higher online disutility costs are offset by lower prices. Thus, social costs are higher because more consumers incur greater online disutility costs.

Combining the results of Propositions 7, 8 and 9, if online disutility costs are sufficiently large then greater competition and reach in the Internet channel can reduce social welfare. Moreover, together with those from Propositions 5 and 6 we have

$$\gamma^{br} > \gamma^{se} > \gamma^b > \gamma^s. \tag{31}$$

This is a dramatic result in that under reasonable conditions maximum social welfare is the Salop model without an Internet channel. In other words, for types of goods where a seller's retail presence is valuable to consumers, an Internet channel *reduces* social efficiency. This is in direct contrast to our results on consumer welfare where reduced prices that are a consequence of an Internet channel benefitting consumers to the degree that they are better off incurring larger online disutility costs than the alternative transportation costs in order to pay a lower price. Even without the results from Propositions 5 and 6, the combination of Propositions 7, 8 and 9,

$$\gamma^{br} > \gamma^{se} > \gamma^{b},$$

shows how increasingly greater competition and reach causes consumers to pay greater online disutility costs in order to benefit from lower prices.

3.4. Entry Conditions

The relationships between our different market configurations for profits, when online disutility costs are high, is (26) summarized below:

$$\pi^{s} \geq \pi^{se} \geq \pi_{r}^{b} \geq \pi_{r}^{br} \text{ and } \pi_{e}^{b} > \pi_{e}^{br}.$$

Of course, the inclusion of fixed entry costs into the Internet channel could alter these inequalities simply by reducing the profits of online alternatives by a fixed amount. However, so long as these fixed entry costs remain below a certain threshold, our results continue to hold.

Although our model does not include the case when only one retailer is in the Internet channel, our profit results suggest that, when μ is high, it is more profitable for the traditional retailers to enter the Internet channel than to wait for a pure e-tailer to do so. However, if a pure e-tailer is established in a market, or if there is a threat from a pure e-tailer entering the market, especially when μ is low, the traditional retailers are better off ceding the Internet channel to the pure e-tailer in order to avoid direct competition in the Internet channel. In other words, competition is more damaging to dual-channel retailers than the benefits they can obtain with increased reach. The market for books can be considered as an example of such market where online disutility cost is arguably low and we do see dual-channel retailers struggling to compete with pure e-tailers.

Many of our results depend on high online disutility costs. Indeed, for the Internet channel to have positive market share requires the Balasubramanian model (b) to limit the ratio of online disutility costs to transportation costs (7). For retailers in the Internet channel, the condition required for the Internet channel to have positive market share is (12), which is less restrictive than (7) by way of allowing for a higher fixed online disutility cost. Consequently, for sufficiently high fixed online disutility cost, the market may support traditional retailers in the Internet channel and not support a pure e-tailer exactly because of the mitigation of online disutility costs. Perhaps, home improvement products market relate to a structure where online disutility cost is arguably high and, therefore, we primarily see dual-channel retailers (e.g.,Home Depot and Lowes) serving the market.

4. Discussion

Our goal has been to examine competition between traditional retail and Internet channels when having a traditional retail store can mitigate the online disutility costs that consumers incur purchasing through the Internet channel. The mitigation of these online disutility costs is based on a traditional retail store engendering greater trust, opportunities for inspection, returns and support when consumers purchase through its Internet channel. To examine this competition, we extended two paradigm models – the Salop (1979) "circle around the lake" model and the Balasubramanian (1998) "pure e-tailer in the center" model – to include traditional retailers extending their reach by selling through the Internet channel, specifying how the online disutility costs of consumers could be mitigated if they purchased online from these dual-channel retailers. We compared the four models from the perspective of prices, profits, consumer welfare, and social welfare. In this, we juxtaposed the effects of competition both from a pure e-tailer and an additional channel with those of reach from traditional retailers entering the Internet channel.

Our first important result is the contrast between what maximizes consumer welfare and what is socially efficient – the greatest social welfare. The combination of competition from a pure e-tailer together with traditional retailers in the Internet channel increasing dual-channel retailer reach yields the highest consumer welfare: competition in the Internet channel puts downward pressure on Internet prices, and this downward pressure extends to traditional retail prices. In stark contrast, a pure e-tailer with traditional retailers in the Internet channel is the *least* socially efficient market configuration. This is because social welfare only considers online disutility costs, with some mitigation, and transportation costs – prices are not included because they are a transfer. Consequently, the lower prices from both competition and increased reach cause some consumers to incur greater online disutility costs than their alternative transportation cost by purchasing through the Internet channel in order to take advantage of prices. Indeed, the most socially efficient market configuration is the Salop model with only a traditional retail channel, which implies that the lower prices that obtain in the market configurations that include an Internet channel always induce some consumers to incur higher online disutility costs than their alternative transportation costs.

Examining consumer welfare more closely, we find that consumers do not benefit from the additional reach engendered by traditional retailers in the Internet channel in spite of the mitigation of online disutility costs. This is because, relative to a pure e-tailer, dual-channel retailers maintain some monopoly power over consumers that are closer to them than to other retailers, and this translates into higher prices that outweigh the benefits of mitigation. Thus, competition from a pure e-tailer more than offsets the advantages of mitigation that comes with increased retailer reach. Interestingly, if the online disutility costs are relatively low, then mitigation of these costs is less important, and competition between dual-channel retailers in the Internet channel can have a similar effect to that of a pure e-tailer. In this circumstance, consumer welfare can be higher from the additional reach because for consumers it combines the benefits of competition and mitigation.

Another significant result concerns entry. Although we do not model entry decisions explicitly, we can anticipate some conditions from our profit comparisons. In our models, in line with previous related studies, we take the market as covered, so the Salop model is the most profitable for retailers because competition between them is limited. However, when faced with a (potentially) profitable pure e-tailer, retailers are more profitable remaining out of the Internet channel. This is similar to the results of Judd (1985) and Ghemawat (1991) in the context of duopolies with differentiated goods, and Nault (1997) in the context of goods supported and not supported by interorganizational systems, whereby competition within the Internet channel dissipates profits from the traditional channel making retailers worse off. Nonetheless, a pure e-tailer may not be profitable if the online disutility costs are high (see (7) from the Balasubramanian model) where an Internet channel of a traditional retailer can be because of the mitigation of those same online disutility costs.

The key element of our formulation, and the element responsible for our results, is the partial mitigation of online disutility costs when consumers purchase through the Internet from a seller that also has a traditional retail store. Others have obtained some pricing results similar to ours, however these results usually are a consequence of including consumer search to resolve price uncertainty as part of the model formulation (see Lal and Sarvary (1999)), and higher prices can be maintained because of the cost of additional search. Similarly, higher prices and consumer welfare can be maintained from vertical strategic interactions – through integration and double marginalization – between an upstream manufacturer and downstream retailers within a mixed channel system (see Yoo and Lee (2011)). Jeffers and Nault (2011) consider pure e-tail entry into the traditional retail channel without accounting for the mitigation obtained from the store presence, which makes traditional retailer entry into the Internet channel infeasible because of zero profit. Thus, explicitly modeling the partial mitigation of online disutility costs based on distance from a traditional store is heart of the technical contribution of our work, and at a more general level, the mitigation is responsible for the augmentation of traditional retailer reach.

The limitations of our analysis resemble previous related studies. For example, we do not explicitly model the sequence of entry decisions, and we assume at the start that transportation costs and fixed entry costs are such that the Salop model is in equilibrium with two retailers. Moreover, we assume that the market is covered, which depending on other assumptions can have a substantial impact on the results of the Salop model and the Salop model with retailers in the Internet channel as dual-channel retailers may not find it profitable to reduce price sufficiently to serve consumers at a distance. This does not happen in the Balasubramanian model or its extension to traditional retailers in the Internet channel because all consumers incur the same online disutility costs from a pure e-tailer. In addition, consumers in our model do not differ in their online disutility costs, and a significantly more complex model would result if consumers differed in their online disutility costs – in effect another dimension of differentiation beyond the spatial one in the Salop model.

Dual-channel retailers commonly charge different prices for the same product across channels (e.g., web-only prices or in-store specials). Accordingly, in our formulation we allowed the dual-channel retailers to choose channel-specific prices. However, in some cases retailers are limited to charging the same prices in both channels. Adding an additional constraint to require identical traditional retail and Internet prices would change the focus and structure of our models. Although some of the results are similar to the current study even after adding such a constraint, future research may examine the incentives of retailers to impose such a constraint as well as the implications for consumer welfare and for social welfare.

Acknowledgments

We thank Jian (Ray) Zhang for research assistance. We also thank the Natural Science and Engineering Research, the Social Science and Humanities Research Councils of Canada, the Robson Professorship Endowment, the Informatics Research Center and the Center for the Digital Economy for support.

Appendix.

A. Proofs

Without loss of generality, we assume $t = 1$ in our proofs. Note that the value of t only scales the problem. Also, for exposition, we define the following:

Definition 1. $L = ac$, $U = 1/2 - a/2 + ac$ and $U' = [1 - a + 2ac + 4c]/6$.

Using Definition 1 and given conditions on μ defined in Section 2.1, we have

- In b model, $\mu \in (L, 1/2)$.
- In se model, $\mu \in (L, U)$.
- In br model, $\mu \in (L, U')$.

A.1. Proof of Proposition 1

Part 1 (Traditional retail prices)

In comparing p_r^{se} and p_r^b, we have

$$p_r^{se} - p_r^b = \frac{1}{12}[6a - 6ac - 1 + 2\mu],$$

which is an increasing linear function of μ.

From Definition 1 we have that when b and se models are considered together, $\mu \in (L, \min\{\frac{1}{2}, U\})$. Note when $c \leq 1/2$, $U \leq 1/2$; otherwise, if $c > 1/2$, $U > 1/2$.

Case 1: $c \leq 1/2$. If $c \leq 1/2$, we have $\mu \in \{L, U\}$. Because $p_r^{se} - p_r^b$ is continuous in μ, we have

$$[p_r^{se} - p_r^b]\Big|_{\mu \to U} = [p_r^{se} - p_r^b]\Big|_{\mu = U} = \frac{1}{12}a[5 - 4c] > 0.$$

At the lower bound, we have

$$[p_r^{se} - p_r^b]\Big|_{\mu \to L} = [p_r^{se} - p_r^b]\Big|_{\mu = L} = \frac{1}{12}[-4ac + 6a - 1].$$

Thus, we can conclude that when $-4ac + 6a - 1 \geq 0$, $[p_r^{se} - p_r^b]\Big|_{\mu \to L} \geq 0$. Accordingly, since $p_r^{se} - p_r^b$ is increasing in μ, we can conclude that $p_r^{se} - p_r^b > 0$ for all $\mu \in (L, U)$. In contrast, if $-4ac + 6a - 1 < 0$, $[p_r^{se} - p_r^b]\Big|_{\mu \to L} < 0$.

Thus, the sign of $p_r^{se} - p_r^b$ is different when μ is at the lower bound and the upper bound, which means that there is a $\mu^* \in (L, U)$, such that

- if $\mu > \mu^*$, $p_r^{se} > p_r^b$;
- if $\mu < \mu^*$, $p_r^{se} < p_r^b$;
- if $\mu = \mu^*$, $p_r^{se} = p_r^b$.

μ^* is the solution of equation $p_r^{se} = p_r^b$; and, $\mu^* = \frac{1-6a+6ac}{2}$.

Case 2: $c > 1/2$. If $c > 1/2$, we have $\mu \in (L, 1/2)$. Since

$$[p_r^{se} - p_r^b]\Big|_{\mu \to \frac{1}{2}} = \frac{1}{2}a[1 - c],$$

we have that if $c \leq 1$, $p_r^{se} - p_r^b > 0$ at $\mu = 1/2$; otherwise if $c > 1$, $p_r^{se} < p_r^b$ at $\mu = 1/2$.

When $c \in (1/2, 1]$, similar to Case 1, if $a \geq 1/[6 - 4c]$, we have $p_r^{se} - p_r^b > 0$ for all $\mu \in (L, 1/2)$. Otherwise if $a < 1/[6 - 4c]$, we can solve $p_r^{se} = p_r^b$ for μ^* such that if $\mu > \mu^*$, $p_r^{se} > p_r^b$; if $\mu < \mu^*$, $p_r^{se} < p_r^b$; if $\mu = \mu^*$, $p_r^{se} = p_r^b$.

When $c > 1$, $p_r^{se} - p_r^b < 0$ at $\mu = 1/2$. Because $p_r^{se} - p_r^b$ is increasing when $\mu \in (L, 1/2)$, we can conclude that $p_r^{se} - p_r^b < 0$ for all μ.

Part 2 (Internet prices)

In comparing p_e^{se} and p_e^b, we have

$$p_e^{se} - p_e^b = \frac{1}{6}[3a - 1 + 2\mu],$$

which is an increasing linear function of μ.

Case 1: $c \leq 1/2$. If $c \leq 1/2$, we have $\mu \in (L, U)$. Because $p_e^{se} - p_e^b$ is continuous in μ, we have

$$[p_e^{se} - p_e^b]\Big|_{\mu \to U} = [p_e^{se} - p_e^b]\Big|_{\mu = U} = \frac{a[1 + c]}{3} > 0,$$

$$\text{and } [p_e^{se} - p_e^b]\Big|_{\mu \to L} = [p_e^{se} - p_e^b]\Big|_{\mu = L} = \frac{1}{6}[2ac + 3a - 1].$$

If $a \geq 1/[3 + 2c]$, we have that $p_e^{se} - p_e^b \geq 0$ at $\mu = L$. Because $p_e^{se} - p_e^b$ is increasing in μ, we have $p_e^{se} - p_e^b \geq 0$ for all $\mu \in (L, U)$. Otherwise if $a < 1/[3 + 2c]$, there is $\mu^* = [1 - 3a]/2$ such that if $\mu < \mu^*$, $p_e^{se} < p_e^b$; if $\mu > \mu^*$, $p_e^{se} > p_e^b$; if $\mu = \mu^*$, $p_e^{se} = p_e^b$.

Case 2: $c > 1/2$. If $c > 1/2$, we have $\mu \in (L, 1/2)$. At the upper bound,

$$[p_e^{se} - p_e^b]\Big|_{\mu \to \frac{1}{2}} = [p_e^{se} - p_e^b]\Big|_{\mu = \frac{1}{2}} = \frac{a}{2} > 0.$$

Similar to Case 1, we have that if $a \geq 1/[3+2c]$, $p_e^{se} - p_e^b \geq 0$ at $\mu = L$. Thus, $p_e^{se} - p_e^b \geq 0$ for all $\mu \in (L, 1/2)$. Otherwise if $a < 1/[3+2c]$, there is $\mu^* = [1-3a]/2$ such that if $\mu < \mu^*$, $p_e^{se} - p_e^b < 0$; if $\mu > \mu^*$, $p_e^{se} - p_e^b > 0$; otherwise if $\mu = \mu^*$, $p_e^{se} - p_e^b = 0$.

Q.E.D.

A.2. Proof of Corollary 1

In comparing π_r^b and π_r^{se}, we have

$$\pi_r^{se} - \pi_r^b = \kappa_1 \mu^2 + \kappa_2 \mu + \kappa_3,$$

where

$$\kappa_1 = -\frac{[16a+20]}{72[a-1]},$$
$$\kappa_2 = -\frac{u[-72ac+8a-8]}{72[a-1]},$$
$$\text{and } \kappa_3 = -\frac{36a^2c^2 - 18a^2 + 19a - 1}{72[a-1]}.$$

Observe that $\pi_r^{se} - \pi_r^b$ is a parabola open up (or convex) with respect of μ, where $\mu \in (L, \min\{1/2, U\})$. Denoting the axis of symmetry as Ω, we get

$$\Omega = -\frac{\kappa_2}{2\kappa_1} = \frac{9ac - a + 1}{4a + 5}.$$

When $c \leq 1/2$, $U \leq 1/2$ which means $\mu \in (L, U)$. Note $\pi_r^{se} - \pi_r^b$ is continuous in μ and the value of $\pi_r^{se} - \pi_r^b$ at the upper bound of μ, $\mu \to U$, is

$$\begin{aligned} [\pi_r^{se} - \pi_r^b]\Big|_{\mu \to U} &= [\pi_r^{se} - \pi_r^b]\Big|_{\mu = U} \\ &= \frac{1}{72} a\left[-4a[1-2c]^2 - 24c + 21\right] \\ &> a\left[-4 - 24 \times \frac{1}{2} + 21\right] > 0. \end{aligned}$$

When $a \in (0,1)$ and $c \in (1/4, 1/2]$, we obtain $L < \Omega < U$. Thus, if $[\pi_r^{se} - \pi_r^b]|_{\mu=\Omega} \geq 0$, we have $\pi_r^{se} - \pi_r^b \geq 0$ for $\mu \in (L, U)$. Otherwise, there are two solutions for $\pi_r^{se} - \pi_r^b = 0$, denoted by μ_1^* and μ_2^*, where

$$\mu_1^* = \frac{-3\sqrt{-[a-1]\left[8a^2\left[2c^2-1\right] + 2a[4c-5] + 1\right]} + 2a[9c-1] + 2}{8a+10}$$

$$\text{and } \mu_2^* = \frac{3\sqrt{-[a-1]\left[8a^2\left[2c^2-1\right] + 2a[4c-5] + 1\right]} + 2a[9c-1] + 2}{8a+10}.$$

Since $[\pi_r^{se} - \pi_r^b]|_{\mu=U} > 0$, we have $L < \Omega < \mu_2^* < U$. Hence, we can conclude:

- When $\mu_1^* \leq L$,
  - —if $\mu < \mu_2^*$, $\pi_r^{se} - \pi_r^b < 0$;
  - —if $\mu > \mu_2^*$, $\pi_r^{se} - \pi_r^b > 0$;
  - —if $\mu = \mu_2^*$, $\pi_r^{se} - \pi_r^b = 0$.

- When $\mu_1^* > L$,
  —if $\mu_1^* < \mu < \mu_2^*$, $\pi_r^{se} - \pi_r^b < 0$;
  —if $\mu < \mu_1^*$ or $\mu > \mu_2^*$, $\pi_r^{se} - \pi_r^b > 0$;
  —if $\mu = \mu_1^*$ or $\mu = \mu_2^*$, $\pi_r^{se} - \pi_r^b = 0$.

When $c > 1/2$, $U > 1/2$ which means $\mu \in (L, 1/2)$. It is difficult to definitively characterize the difference in π_r^b and π_r^{se}. However, the general trend continues.

Q.E.D.

A.3. Proof of Proposition 2

When both b and br models are considered together, from Definition 1, we obtain $\mu \in (L, U')$. In comparing π_r^{br} and π_r^b, we have

$$\pi_r^b - \pi_r^{br} = \kappa_1 \mu^2 + \kappa_2 \mu + \kappa_3,$$

where

$$\kappa_1 = \frac{16a + 20}{72[a-1]},$$

$$\kappa_2 = \frac{u[-72ac + 8a - 8]}{72[a-1]},$$

$$\text{and } \kappa_3 = \frac{20a^2c^2 - 8a^2c - a^2 + 16ac^2 + 8ac + 2a - 1}{72[a-1]}.$$

Thus, with respect to μ, $\pi_r^b - \pi_r^{br}$ is a parabola open down (or concave).

Since $\pi_r^{br} - \pi_r^b$ is continuous in μ, we can derive that when $a \in (0,1)$ and $c \in (1/8, 1/2)$,

$$\begin{aligned}
[\pi_r^b - \pi_r^{br}]\Big|_{\mu \to U'} &= [\pi_r^b - \pi_r^{br}]\Big|_{\mu = U'} \\
&= \frac{1}{162}[1-a][1-2c][2ac - a + 10c + 4] \\
&> 0
\end{aligned}$$

$$\text{and } [\pi_r^b - \pi_r^{br}]\Big|_{\mu \to L} = [\pi_r^b - \pi_r^{br}]\Big|_{\mu = L} = \frac{[1-a][1-16ac^2]}{72}.$$

Hence, we can conclude:

- If $a \le 1/[16c^2]$, $[\pi_r^b - \pi_r^{br}]\Big|_{\mu \to L} \ge 0$, which implies that $\pi_r^b > \pi_r^{br}$ for all μ in the feasible region, $\mu \in (L, U')$.
- If $a > 1/[16c^2]$, $[\pi_r^b - \pi_r^{br}]\Big|_{\mu \to L} < 0$, which implies that there is a $\mu^* \in (L, U')$ such that
  —if $\mu < \mu^*$, then $\pi_r^b < \pi_r^{br}$;
  —if $\mu > \mu^*$, then $\pi_r^b > \pi_r^{br}$;
  —otherwise if $\mu = \mu^*$, then $\pi_r^b = \pi_r^{br}$.

μ^* is the smaller solution of $\pi_r^b = \pi_r^{br}$; and,

$$\mu^* = \frac{-\sqrt{-80a^3c^2 + 32a^3c + 4a^3 + 160a^2c^2 - 64a^2c + a^2 - 80ac^2 + 32ac - 14a + 9} + 18ac - 2a + 2}{2[4a+5]}.$$

Q.E.D.

A.4. Proof of Proposition 4

In comparing ω^{se} and ω^{b}, we have

$$\omega^{se}-\omega^{b}=\kappa_1\mu^2+\kappa_2\mu+\kappa_3,$$

where

$$\kappa_1=\frac{16a+20}{72[a-1]},$$

$$\kappa_1=\frac{-72ac+32a-32}{72[a-1]},$$

$$\text{and } \kappa_3=\frac{36a^2c^2-72a^2c+45a^2+72ac-56a+11}{72[a-1]}.$$

Thus, with respect to μ, $\omega^{se}-\omega^{b}$ is a parabola open down (or concave). Denoting the axis of symmetry as Ω, then we get

$$\Omega=-\frac{\kappa_2}{2\kappa_1}=\frac{9ac-4a+4}{4a+5}.$$

When se and b models are considered together, from Definition 1, we get $\mu\in(L,\max\{1/2,U\})$. If $c\leq 1/2$, $U\leq 1/2$; if $c>1/2$, $U>1/2$.

Case 1: $c\leq 1/2$. If $c\leq 1/2$, we have $\mu\in\{L,U\}$. Because $\omega^{se}-\omega^{b}$ is continuous in μ, we have

$$\begin{aligned}\left.[\omega^{se}-\omega^{b}]\right|_{\mu\to U} &= \left.[\omega^{se}-\omega^{b}]\right|_{\mu=U}\\ &= \frac{1}{36}a\left[2a[1-2c]^2-12c+15\right]\\ &\geq \frac{1}{36}a\left[-12\times\frac{1}{2}+15\right]>0.\end{aligned}$$

$$\begin{aligned}\left.[\omega^{se}-\omega^{b}]\right|_{\mu\to L} &= \left.[\omega^{se}-\omega^{b}]\right|_{\mu=L}\\ &= \frac{1}{72}\left[16a^2c^2-5a[8c-9]-11\right].\end{aligned}$$

When $a\in(0,1)$ and $c\in(1/4,1/2]$, we can derive that $\Omega>U$. Thus, $\omega^{se}-\omega^{b}$ is monotone increasing in μ when $\mu\in(L,U)$. Accordingly, we can conclude:

- If $16a^2c^2-5a[8c-9]-11\geq 0$, $\omega^{se}-\omega^{b}\geq 0$ at $\mu=L$, which means that $\omega^{se}-\omega^{b}>0$ for all $\mu\in\{L,U\}$.
- If $16a^2c^2-5a[8c-9]-11<0$, $\omega^{se}-\omega^{b}<0$ at $\mu=L$, which means that there is a $\mu^*\in(L,U)$ such that
  —if $\mu<\mu^*$, $\omega^{se}-\omega^{b}<0$;
  —if $\mu>\mu^*$, $\omega^{se}-\omega^{b}>0$;
  —if $\mu=\mu^*$, $\omega^{se}-\omega^{b}=0$.

μ^* is the smaller solution of equation $\omega^{se}=\omega^{b}$; and,

$$\mu^*=\frac{-3\sqrt{-[a-1]\left[4a^2\left[4c^2-8c+5\right]+a[13-8c]+1\right]}+2a[9c-4]+8}{8a+10}.$$

Case 2: $c > 1/2$. When $c > 1/2$, $U > 1/2$ and, therefore, $\mu \in (L, 1/2)$. We can also derive that when $a \in (0,1)$ and $c > 1/2$, $\Omega > U$, which indicates that $\omega^{se} - \omega^b$ is monotone increasing in μ when $\mu \in (L, 1/2)$.

$$\begin{aligned}[\omega^{se} - \omega^b]\Big|_{\mu \to \frac{1}{2}} &= [\omega^{se} - \omega^b]\Big|_{\mu = \frac{1}{2}} \\ &= \frac{a\,[4ac^2 - 8ac + 5a + 4c - 4]}{8[a-1]}.\end{aligned}$$

Thus, we can conclude:

- If $4ac^2 - 8ac + 5a + 4c - 4 \geq 0$, $[\omega^{se} - \omega^b]|_{\mu=1/2} \leq 0$, which indicates that $\omega^{se} - \omega^b < 0$ for all $\mu \in (L, 1/2)$.
- If $4ac^2 - 8ac + 5a + 4c - 4 < 0$, $[\omega^{se} - \omega^b]|_{\mu=1/2} > 0$, then we have

—If $16a^2c^2 - 5a[8c-9] - 11 \geq 0$, $\omega^{se} - \omega^b \geq 0$ at $\mu = L$, which means that $\omega^{se} - \omega^b > 0$ for all $\mu \in \{L, 1/2\}$.

—If $16a^2c^2 - 5a[8c-9] - 11 < 0$, $\omega^{se} - \omega^b < 0$ at $\mu = L$, which means that there is a $\mu^* \in (L, 1/2)$ such that

  * if $\mu < \mu^*$, $\omega^{se} - \omega^b < 0$;
  * if $\mu > \mu^*$, $\omega^{se} - \omega^b > 0$;
  * if $\mu = \mu^*$, $\omega^{se} - \omega^b = 0$.

μ^* is the smaller solution of equation $\omega^{se} = \omega^b$; and,

$$\mu^* = \frac{-3\sqrt{-[a-1]\,[4a^2\,[4c^2 - 8c + 5] + a[13 - 8c] + 1]} + 2a[9c-4] + 8}{8a + 10}.$$

Q.E.D.

A.5. Proof of Proposition 6

In comparing γ^{se} and γ^s, we have

$$\gamma^{se} - \gamma^s = \kappa_1 \mu^2 + \kappa_2 \mu + \kappa_3,$$

where

$$\kappa_1 = \frac{3}{2[a-1]},$$
$$\kappa_2 = \frac{-24ac + 8a - 8}{8[a-1]},$$
$$\text{and } \kappa_3 = \frac{12a^2c^2 - 8a^2c + a^2 + 8ac - 2a + 1}{8[a-1]}.$$

Thus, with respect to μ, $\gamma^{se} - \gamma^s$ is a parabola open down (or concave). Denoting the axis of symmetry as Ω, we get

$$\Omega = \frac{1}{3}[3ac - a + 1].$$

Using Definition 1, we can show $L < \Omega < U$. At $\mu = \Omega$, we have

$$\gamma^{se} - \gamma^s\Big|_{\mu=\Omega} = \frac{1-a}{24} > 0.$$

Because $\gamma^{se} - \gamma^s$ is continuous in μ, we obtain

$$[\gamma^{se} - \gamma^s]\Big|_{\mu \to L} = [\gamma^{se} - \gamma^s]\Big|_{\mu = L} = \frac{a-1}{8} < 0,$$

$$\text{and } [\gamma^{se} - \gamma^s]\Big|_{\mu \to U} = [\gamma^{se} - \gamma^s]\Big|_{\mu = U} = 0.$$

Hence, we can conclude that there is a μ^* in (L, U), such that

- if $\mu > \mu^*$, $\gamma^{se} > \gamma^s$;
- if $\mu < \mu^*$, $\gamma^{se} < \gamma^s$;
- if $\mu = \mu^*$, $\gamma^{se} = \gamma^s$.

μ^* is the smaller solution of equation $\gamma^{se} = \gamma^s$; and, $\mu^* = \frac{1}{6}[6ac - a + 1]$.

Q.E.D.

A.6. Proof of Proposition 8

In comparing γ^{se} and γ^b, we have

$$\gamma^{se} - \gamma^b = \kappa_1 \mu^2 + \kappa_2 \mu + \kappa_3,$$

where

$$\kappa_1 = \frac{80a + 28}{72[a-1]},$$
$$\kappa_2 = \frac{[-216ac + 16a - 16]}{72[a-1]},$$
$$\text{and } \kappa_3 = \frac{108a^2c^2 - 72a^2c + 9a^2 + 72ac - 10a + 1}{72[a-1]}.$$

Thus, with respect to μ, $\gamma^{se} - \gamma^b$ is a parabola open down (or concave). The axis of symmetry, denoted by Ω, is

$$\Omega = \frac{27ac - 2a + 2}{20a + 7}.$$

When *se* and *b* models are considered together, we have $\mu \in (L, \min\{1/2, U\})$. Accordingly, if $c \leq 1/2$, $U \leq 1/2$; otherwise, if $c > 1/2$, $U > 1/2$.

If $c \leq 1/2$, $U \leq 1/2$ which means $\mu \in (L, U)$. Because $\pi_r^{se} - \pi_r^b$ is continuous in μ, we have

$$\begin{aligned} [\gamma^{se} - \gamma^b]\Big|_{\mu \to U} &= [\gamma^{se} - \gamma^b]\Big|_{\mu = U} \\ &= \frac{1}{18} a[2c-1][10ac - 5a + 3] \\ &< \frac{1}{18} a[2c-1][10a \times \frac{1}{4} - 5a + 3] \\ &< 0 \end{aligned}$$

$$\begin{aligned} \text{and } [\gamma^{se} - \gamma^b]\Big|_{\mu \to L} &= [\gamma^{se} - \gamma^b]\Big|_{\mu = L} \\ &= \frac{1}{72}\left[80a^2c^2 - 56ac + 9a - 1\right]. \end{aligned}$$

When $a \in (0,1)$ and $c \in (1/4, 1/2]$, we have $80a^2c^2 - 56ac + 9a - 1 < 0$, which means $\gamma^{se} - \gamma^b < 0$ when $\mu = L$. We can also show $L < \Omega < U$ when $a \in (0,1)$ and $c \in (1/4, 1/2]$. At $\mu = \Omega$, we have

$$[\gamma^{se} - \gamma^b]\Big|_{\mu = \Omega} = \frac{20a^2\left[12c^2 - 8c + 1\right] + a[3 - 8c] + 1}{8[20a + 7]}$$

Hence, we can conclude:

- If $[\gamma^{se} - \gamma^b]|_{\mu=\Omega} \leq 0$, $\gamma^{se} - \gamma^b \leq 0$ for all $\mu \in (L, U)$.
- If $[\gamma^{se} - \gamma^b]|_{\mu=\Omega} > 0$, there are μ_1^* and μ_2^* in (L, U), $\mu_1^* < \mu_2^*$, such that
  —if $\mu_1^* < \mu < \mu_2^*$, $\gamma^{se} - \gamma^b > 0$;

—if $\mu < \mu_1^*$ or $\mu > \mu_2^*$, $\gamma^{se} - \gamma^b < 0$;

—if $\mu = \mu_1^*$ or $\mu = \mu_2^*$, $\gamma^{se} - \gamma^b = 0$.

When $c > 1/2$, $U > 1/2$ which means $\mu \in (L, 1/2)$. It is difficult to definitively characterize the difference in π_r^b and π_r^{se}. However, the general trend continues.

Q.E.D.

A.7. Proof of Proposition 9

In comparing γ^{br} and γ^b, we have

$$\gamma^{br} - \gamma^b = \kappa_1 \mu^2 + \kappa_2 \mu + \kappa_3,$$

where

$$\kappa_1 = \frac{80a + 28}{72[a-1]},$$
$$\kappa_2 = \frac{16a - 16}{72[a-1]},$$
$$\text{and } \kappa_3 = \frac{28a^2c^2 - 16a^2c + a^2 + 80ac^2 - 200ac - 2a + 1}{72[a-1]}.$$

Thus, with respect to μ, $\gamma^{br} - \gamma^b$ is a parabola open down (or concave). Since $\gamma^{br} - \gamma^b$ is continuous in μ, we get

$$\begin{aligned} [\gamma^{br} - \gamma^b]\Big|_{\mu \to U'} &= [\gamma^{br} - \gamma^b]\Big|_{\mu = U'} \\ &= \kappa_1' c^2 + \kappa_2' c + \kappa_3', \end{aligned}$$

where

$$\kappa_1' = \frac{[20a^3 + 150a^2 + 288a + 28]}{162[a-1]},$$
$$\kappa_2' = \frac{[-20a^3 - 51a^2 - 405a - 10]}{162[a-1]},$$
$$\text{and } \kappa_3' = \frac{5a^3 - 12a^2 + 9a - 2}{162[a-1]}.$$

Observe that $[\gamma^{br} - \gamma^b]|_{\mu \to U'}$ is a concave function of c. Thus, we only need $[\gamma^{br} - \gamma^b]|_{\mu \to U'} > 0$ when $c = 1/8$ and $c = 1/2$ to show $[\gamma^{br} - \gamma^b]|_{\mu \to U'} > 0$ for all $c \in (1/8, 1/2)$. Because

$$[\gamma^{br} - \gamma^b]\Big|_{\mu = U', c = \frac{1}{8}} = \frac{10[a^3 - 1] - 57a^2 - 132a}{576[a-1]} > 0$$
$$\text{and } [\gamma^{br} - \gamma^b]\Big|_{\mu = U', c = \frac{1}{2}} = \frac{-3a}{4[a-1]} > 0,$$

we can conclude that $\gamma^{br} > \gamma^b$ when $\mu \to U'$ for all $a \in (0,1)$ and $c \in (1/8, 1/2)$.

At the lower bound of μ, we have

$$[\gamma^{br} - \gamma^b]\Big|_{\mu = L} = \frac{80a^3c^2 + 56a^2c^2 + a^2 + 80ac^2 - 216ac - 2a + 1}{72[a-1]}.$$

Since $\gamma^{br} - \gamma^b$ is concave in μ and $[\gamma^{br} - \gamma^b]|_{\mu = U'} > 0$, we can conclude:

- If $[\gamma^{br} - \gamma^b]|_{\mu = L} \geq 0$, then $\gamma^{br} - \gamma^b > 0$ for all $\mu \in (L, U')$.
- If $[\gamma^{br} - \gamma^b]|_{\mu = L} < 0$, then there is μ^* in (L, U') such that

—if $\mu < \mu^*$, $\gamma^{br} - \gamma^b < 0$;

—if $\mu > \mu^*$, $\gamma^{br} - \gamma^b > 0$;

—if $\mu = \mu^*$, $\gamma^{br} - \gamma^b = 0$.

μ^* is the smaller solution of $\gamma^{br} = \gamma^b$; and,

$$\mu^* = \frac{-\sqrt{[16a-16]^2 - 4[80a+28]\,[28a^2c^2 - 16a^2c + a^2 + 80ac^2 - 200ac - 2a + 1]} - 16a + 16}{8[20a+7]}.$$

Q.E.D.

References

Balasubramanian, Sridhar. 1998. Mail versus mall: A strategic analysis of competition between direct marketers and conventional retailers. *Marketing Science* **17**(3) 181–195.

Baye, M.R., J. Morgan, P. Scholten. 2004. Price dispersion in the small and the large: Evidence from an internet price comparison site. *Journal of Industrial Economics* **52**(4) 463–496.

Baylis, K., J. M. Perloff. 2002. Price dispersion on the internet: Good firms and bad firms. *Review of Industrial Organization* **21**(3) 305–xxx.

Brynjolfsson, E., Y. J. Hu, M. S. Rahman. 2009. Battle of the retail channels: How product selection and geography drive cross-channel competition. *Management Science* **55**(11) 1755–1765.

Chevalier, J., A. Goolsbee. 2003. Measuring prices and price competition online: Amazon and barnes and noble. *Quantitative Marketing and Economics* **1** 203–222.

Clay, K., R. Krishnan, E. Wolff, D. Fernandes. 2002. Retail strategies on the web: Price and non-price competition in the on-line book industry. *Journal of Industrial Economics* **50**(3) 351–367.

Clemons, E., H. Il-Horn, L. Hitt. 2002. Price dispersion and differentiation in online travel: An empirical investigation. *Management Science* **48**(4) 534–549.

de Frutos, M. A., H. Hamoudi, X. Jarque. 1999. Equilibrium existence in the circle model with linear quadratic transport cost. *Regional Science and Urban Economics* **29**(5) 605–615.

Engel, J., R. Blackwell, P. Winiard. 1990. *Consumer behavior*. Dryden Press, Hinsdale, IL.

Forman, Chris, Anindya Ghose, Avi Goldfarb. 2009. Competition between local and electronic markets: How the benefit of buying online depends on where you live. *Management Science* **55**(1) 47–57.

Ghemawat, P. 1991. Market incumbency and technological inertia. *Marketing Science* **10**(2) 161–171.

Goolsbee, A. 2001. Competition in the computer industry: Online versus retail. *Journal of Industrial Economics* **49**(4) 487–499.

Hoffman, D.L., T.P. Novak, M. Peralta. 1999. Building consumer trust online. *Communications of the ACM* **42**(4) 80–85.

Hotelling, H. 1929. Stability in competition. *Economic Journal* **95** 41–57.

Iyer, G., A. Pazgal. 2003. Internet shopping agents: Virtual co-location and competition. *Marketing Science* **22**(1) 85–106.

Jarvenpaaa, S.L., N. Tractinsky, M. Vitale. 2000. Customer trust in an internet store. *Information Technology and Management* **1** 45–71.

Jeffers, Patrick I., Barrie R. Nault. 2011. Why competition from a multi-channel e-tailer does not always benefit consumers. *Decision Sciences* **42**(1) 69–91.

Judd, K. 1985. Credible spatial preemption. *Rand Journal of Economics* **16**(1) 153–166.

Kotler, P. 2002. *Marketing management*. Prentice Hall, Upper Saddle River, NJ.

Lal, R., M. Sarvary. 1999. When and how the internet is likely to decrease price competition. *Marketing Science* **18**(4) 485–503.

Liang, T.P., J.S. Huang. 1998. An empirical study on consumer acceptance of products in electronic markets: A transaction cost model. *Decision Support Systems* **24**(1) 29–43.

Nault, B.R. 1997. Quality differentiation and adoption costs: The case for interorganizational information systems pricing. *Annals of Operations Research* **71** 115–142.

Salop, Steven C. 1979. Monopolistic competition with outside goods. *The Bell Journal of Economics* **10**(1) 141–156.

Stewart, Katherine J. 2003. Trust transfer on the world wide web. *Organization Science* **14**(1) 5–17.

Tang, F., X. Xing. 2001. Will the growth of multi-channel retailing diminish the pricing efficiency of the web? *Journal of Retailing* **77**(3) 319–333.

Tirole, Jean. 1988. *The Theory of Industrial Organization*. The MIT Press, Cambridge, MA.

Verhoef, P.C., S.A. Neslin, B. Vroomen. 2007. Multichannel customer management: Understanding the research-shopper phenomenon. *International Journal of Research in Marketing* **24**(2) 129–148.

Yoo, Weon S., Eunkyu Lee. 2011. Internet channel entry: A strategic analysis of mixed channel structures. *Marketing Science* **30**(1) 29–41.