Does Cap-and-Trade Enable Collusion?

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Abstract

Carbon Taxes and Cap-and-Trade are the two leading approaches for pollution regulation. Proponents of Taxes have argued that Cap-and-Trade could facilitate collusion among firms via the trading mechanism, leading to suboptimal welfare outcomes. We examine this claim using a rigorous yet rich model of production and pollution under competition that allows for the possibility of collusion among firms via trading.

1 Introduction

Pollution is an inevitable by-product of production, and an ancient problem. [Hong et al., 1996] analyze air molecules trapped in Greenland ice to track air-pollution from copper smelting over the last 5,000 years.

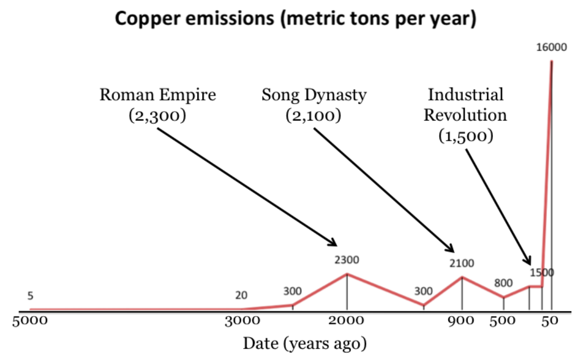


Figure 1: History of air pollution from copper smelting.

They find evidence of soaring pollution levels 2,000 and 900 years ago, coinciding precisely with the peaks of the Roman empire and the Chinese Song dynasty– two periods of bustling economic activity (See Figure 1).

More recently, concerns over climate change have thrown greenhouse gases (GHG) under the spotlight. Carbon dioxide (CO_2) emissions represent the bulk of man-made GHG emissions (83.7% in the U.S., 82.4% in Europe and 94.8% in Japan). In the U.S., business operations contribute 62% of GHG emissions, while personal vehicle use and residential buildings account for the rest ([Hockstad and Cook, 2012]). Carbon dioxide emissions "track economic growth, slowing with recessions, but essentially rising and rising." (Matthew Arnold–World Resources Institute– in [Iannuzzi, 2002], Foreword.) A reading of the carbon barometer for May 2013 puts carbon dioxide concentrations in the Earth's atmosphere at 399.89 ppm and rising, a 41 percent increase since the early 1800s ([Scripps, 2013]). Before carbon emissions became an issue, concerns crystallized around acid rains arising from sulfur dioxide discharges by fossil fuel power plants, smog caused by particulate matter and ozone emissions around our cities, and many other environmental problems originating from expanding economic activities.

A firm's pollution imposes a negative externality on society, in that the pollution affects people, wildlife and the natural environment outside the firm's boundaries. An unregulated firm does not bear the full costs of its pollution, since its incentives to control or abate its pollution are not commensurate with the pollution damage it causes. Thus, regulation is inevitable to mitigate pollution in the context of negative externalities (cf. [Baumol and Oates, 1988], [Cropper and Oates, 1992]). Perhaps the earliest recorded instance of pollution regulation was in London in 1272, when King Edward I banned the burning of sea-coal–a cheap, abundant but very smoky fuel.[1] Beginning in the 1970s, regu-

[1]http://www.epa.gov/aboutepa/history/topics/perspect/london.html

lators around the world have experimented with various mechanisms for pollution control. Centralized control mechanisms used today include technology mandates and performance standards such as the maximum permissible emissions rate for a particular technology. Often, such centralized mandates are suboptimal because (i) the regulator is unlikely to be fully informed about each firm's operating conditions (e.g., its abatement costs), (ii) efficiencies that could be achieved by tapping into firms' expertise are forgone ([Tietenberg, 1985]), (iii) the regulator incurs high monitoring and information acquisition costs, particularly when the incentives of firms and the regulator diverge (cf. [Iannuzzi, 2002]), and (iv) such mandates frequently invite litigation, with its related financial burdens and compliance delays ([Tietenberg, 1985]). To overcome these difficulties, economists have long urged the use of economic incentives, such as pollution taxes and tradable emission allowances (*i.e.*, cap-and-trade) that force firms to internalize the costs of their pollution (Stavins 1998, 2003). In this paper, we analyze and compare two widely used incentive-based mechanisms for pollution control: Tax and Cap-and-Trade.

Under the Tax mechanism, the regulator charges each firm with a tax commensurate with its pollution. [Stavins, 2003] identifies ten applications of emission taxes in Europe, including for carbon monoxide, carbon dioxide, sulfur dioxide, and nitrogen oxides. France and Sweden tax emissions of sulfur and nitrogen oxide. Finland was the first country in the world to introduce a carbon tax in 1990, with Denmark, Italy, Netherlands, Norway and Sweden following suit. In the United States, the carbon tax is being debated as an alternative to cap-and-trade.

Under Cap-and-Trade, the regulator directly imposes a pollution limit (the 'Cap') on firms with heavy fines as a deterrent for flouting, and firms can comply through some combination of three actions: (a) pollution abatement, (b) output reduction and (c) trading in emission allowances, which effectively shifts firms' pollution constraints up or down. The premise is that cap-and-trade would facilitate efficient allocation of emission allowances via the market mechanism ([Coase, 1960], [Dales, 1968], [Montgomery, 1972], [Schmalensee et al., 1998]). Since 1995, the U.S. Acid Rain Program has included a cap-and-trade system for the reduction of sulfur dioxide emissions by coal-fired power plants. In 2005, the European Union launched a large-scale cap-and-trade system for greenhouse gas emissions–the E.U. Emissions Trading Scheme. Similarly, the State of California is currently rolling out a cap-and-trade system for greenhouse gases as part of the Global Warming Solutions Act ([Barringer, 2011]). A popular argument against Cap-and-Trade is that it is vulnerable to market manipulations by firms who try to bypass pollution regulations at the expense of society ([Shapiro, 2007], [Stiglitz, 2007]). Collusion is not possible under the Tax mechanism, because the firms' responses are independent of each other. Under Cap-and-Trade, however, trading may give firms a strategic lever to alter the conditions of competition in the output market. We examine the potential for collusive behavior and its consequences using a rigorous yet rich model of production and pollution under competition that allows for the possibility of collusion among firms via trading, using the Tax mechanism as a benchmark.

We derive the Subgame-Perfect Nash equilibria for a series of games involving a regulator and two profit-maximizing firms. First the regulator chooses a mechanism (Tax or Cap-and-Trade) to achieve a specified pollution limit. Then the firms maximize profits within the constraints of the regulation. We model the processes of pollution generation, pollution abatement and pollution regulation to analyze several related questions: does cap-and-trade enable collusion between the regulated firms? If collusion is possible, what is its effect on firms, consumers and society as a whole? What can the regulator do to limit the possibly negative consequences of collusion?

2 Literature Review

From Arthur Pigou (1932) to the modern sustainability movement, the mitigation of environmental externalities has been a subject of inquiry for generations of environmental economists (see [Cropper and Oates, 1992] for a review). When the number of people impacted by pollution is large, pollution externalities cannot be resolved through bilateral transactions, and some form of government intervention is warranted. Taxation and cap-and-trade are two popular market-based mechanisms, and there is a vast literature that broadly debates the merits of these alternatives ([Baumol and Oates, 1988]). [Goulder and Parry, 2008] provide a nice and concise review of this literature in a conceptual framework that includes a broad set of criteria ranging from cost considerations (including administrative costs and the distribution of compliance costs across income groups), to the role of uncertainty, to political feasibility, to the impact of the mechanisms on R&D and technology deployment.

Several researchers have analyzed regulatory alternatives to taxes and caps, such as subsidies for pollution abatement and legal mandates requiring firms to disclose information, recycle or dispose used products. [Nault, 1996] shows the equivalence of subsidies and taxes in terms of output, pollution damage and welfare. [Levi and Nault, 2004] study how the regulator can induce firms to make a conversion in production technology to help the environment, comparing subsidies and tax-based programs when firms vary in the type, age, quality of maintenance and general condition of their production technology. [Kalkanci et al., 2012] find that voluntary disclosure of a firm's environmental footprint leads to more learning by the firm and lower environmental impact than mandatory disclosures. [Atasu et al., 2009], [Subramanian et al., 2009] and [Jacobs and Subramanian, 2012] study *extended producer responsibility*, a mechanism wherein the manufacturer is legally responsible for collecting and treating some fraction of end-of-use products, thus supporting recycling or disposal. [Plambeck and Taylor, 2010] study competitive testing and whistle-blowing as a means to achieve compliance on environmental, health and safety standards. [Keskin and Plambeck, 2011] study the effect of accounting rules on allocation of carbon emissions across coproducts serving a domestic and an export market. They find that letting the firm choose the allocation rule, as is current practice, can contribute to higher emissions, and identify the allocation rule that leads to the lowest emissions. [Subramanian et al., 2007] study the firms' reponses to a system of auctioned emission allowances, but trading is not considered.

In a seminal paper, [Weitzman, 1974] shows that in perfectly competitive markets, absent uncertainty in the costs and benefits of taxation, a price-based approach is analytically equivalent to a quota-based method like cap-and-trade in perfectly competitive markets. This basic finding underscores the conventional wisdom that, on purely efficiency grounds, Tax and Cap-and-Trade are equivalent. Both allow firms to reduce their pollution at the lowest aggregate costs. [Weitzman, 1974] goes on to reveal an important asymmetry between price and quantity controls in the presence of uncertainty, where uncertainty arises from genuine randomness, difficulties in measuring the costs and benefits of pollution control or information asymmetries between firms and regulators. Weitzman's main theorem identifies conditions under which price controls dominate quantity controls and vice versa. These conditions depend on the relative steepness of the marginal cost and benefit functions. [Adar and Griffin, 1976] derive similar results using graphical proofs in which the marginal control cost curves and the marginal damage functions are assumed linear. Our model does not include any uncertainty: we assume that the costs and benefits of pollution control are known with certainty and are common knowledge. An important departure of our paper from Weitzman's work is that we relax the assumption of perfect competition to focus on strategic behaviors by regulated firms holding some market power in the output market. We are interested in comparing emission taxes and cap-and-trade in a rich operational context in which asymmetric firms strategically choose their production quantities and exert effort to abate their pollution within the constraints of the mechanism chosen by the regulator.

2.1 Regulation of monopolies

[Barnett, 1980], [Requate, 1993a] and [Anand and Giraud-Carrier, 2012] study pollution regulations under the polar case of pure monopolies (see also [Requate, 2006]). This literature shows that, in order to comply with pollution control regulations, firms exercise their market power to reduce output (or increase their prices) even further. To compensate for this welfare loss, the regulator should set the tax rate at less than the marginal cost of pollution. [Requate, 1993a] shows the equivalence between emission taxes and cap-and-trade in a model where n local monopolies can reduce pollution only by reducing output. [Anand and Giraud-Carrier, 2012] show this equivalence when the firms can exert effort to abate pollution in addition to reducing output. Furthermore, they show that Tax and Cap-and-Trade can mimic the Groves mechanism, a theoretical benchmark from the public good literature, which forces each firm to internalize exactly the extra pollution damage (and no more) that its production and abatement decisions inflict on society. The Groves mechanism is the gold standard for fairness because it perfectly effectuates the "polluter pays" principle ([OECD, 1972]), but it is impractical.

2.2 Regulation under oligopoly

[Requate, 2006] reviews several papers studying pollution regulations (especially emission taxes) under imperfect competition, specifically Cournot and Bertrand oligopolies. [Requate, 1993b] and [von der Fehr, 1993] are closely related to ours. In [Requate, 1993b] two asymmetric firms with linear production technologies compete à la Cournot. The firms differ in their marginal production costs and emission rates. Production generates pollution, and the firms have no other abatement option than reducing output. This latter assumption means that a stringent regulation (*i.e.*, a high tax rate or a small allocation of emission allowances) could cause one of the firms to shut down. Requate compares a linear emission Tax to a Cap-and-Trade system in which firms trade to maximize their joint profits. In other words, Pareto efficiency is the driving force of the trading equilibrium. He shows that Tax and Cap-and-Trade differ from each other; neither can implement the welfare-maximizing outcome; and no mechanism is always superior to the other. However, there is a range of parameters for which Cap-and-Trade yields a higher welfare than Tax, but this result applies only in the extreme case when it is socially optimal for the more polluting firm to shut down.

[von der Fehr, 1993] specifically looks at the risk of collusion under Cap-and-Trade. He studies the firms' incentives to use Cap-and-Trade strategically to increase their market power (*i.e.*, monopolize) or exclude entry. He models a symmetric Cournot duopoly (*i.e.*, the firms have the same cost structure) in which the firms are initially allocated a different number of emission allowances on account of their different sizes. By trading, the dominant firm can influence the permit price to lower its own abatement costs, and improve its strategic position in the output market; this manipulation also impacts its rival's cost structure. Thus, strategic interactions in the trading market are likely to distort competition in the output market. Von der Fehr studies the effect on the firms' joint profits of a reallocation of emission allowances between them. If Pareto-improving trades exist, the firms have an incentive to collude. Von der Fehr shows that, if the products are homogenous, monopolization can occur (under some conditions on the firms' cost structures). However, monopolization is less likely when the products are differentiated and the cost functions exhibit diseconomies of scale. He goes on to show that, if the quantities are strategic substitutes, the exercise of market power in the output market causes firms to overinvest in emission rights. This is because buying more emission rights lowers the buyer's marginal cost, which makes it more aggressive in the output market. The rival facing a decreasing output price is forced to reduce its production quantity, which improves the profits of the first. This is an example of a Top-Dog commitment strategy in which the decision to overinvest (*i.e.*, act tough) causes the rival to behave less aggressively. This result follows from the assumption that marginal costs are decreasing in the number of emission rights.

[Hahn, 1984] was the first to study the situation in which a firm exercises power in the trading market, but the output market is perfectly competitive. He shows that the firm will tend to buy (sell) too few emission allowances in order to keep the allowance price down (up). Our contribution is threefold:

- We analyze two different trading mechanisms: in the first one, trading occurs at the time of production. This means that the production, abatement and trading decisions are simultaneous. We call this mechanism the single-stage Cap-and-Trade model. As we shall see, trading leads to the marginal abatement costs being equal across firms, which implies that the aggregate costs of compliance to the overall cap are minimum. In the second, the firms trade to maximize their joint profits before production is realized. Thus Pareto optimality is driving the trading between the firms, as in [Requate, 1993b] and [von der Fehr, 1993]. This Pareto-optimal Cap-and-Trade is called the two-stage model. This distinction offers a clean and insightful comparison of the mechanisms in a manner not previously done. By comparing two variants of Cap-and-Trade to the Tax mechanism under which no collusion is possible, we are able to generate new insights into the firms' incentives to collude and their levels of commitment. [von der Fehr, 1993] finds that collusion is possible under limited conditions. We show that it is unavoidable. Contrary to [von der Fehr, 1993], we show that firms *underinvest* in trading under collusion.

- We develop a richer model, which allows for a more detailed analysis, by disentangling the firms' production, abatement and trading decisions. In [Requate, 1993b], the firms can reduce pollution only by reducing output, and there is no alternative way to abate pollution. In [Requate, 2006, chapter 7], this assumption is relaxed but the model becomes intractable. In [von der Fehr, 1993], emission allowances serve as a proxy for production, in the sense that firms cannot produce without emission allowances; so capturing permits from the other firm restricts its production capacity, and could push the firm out of the market. Furthermore, [von der Fehr, 1993] does not explicity model the trading process. In our paper, we allow the firms to choose their production quantities, levels of abatement and trading quantity as three separate decision variables. We distinguish and explicitly model the processes of production, pollution generation, abatement and regulation. Our closed form solutions make it possible to compare outcomes, including output, abatement efforts, firm profits, consumer surplus, and welfare.

- Finally, we generalize the polar cases of monopoly and duopoly by considering markets of imperfect substitutes. As we shall see, the coefficient of substitutability is a key parameter in our analysis.

3 The Model

3.1 Modeling Pollution

Consider a firm whose production generates a harmful pollutant. We model four interrelated aspects of pollution: *(i) Pollution generation*: This relates the quantity of pollution emitted to the production quantity q as well as to the degree of pollution abatement; *(ii) Pollution abatement*: This describes, contrary to [von der Fehr, 1993], how the firm can (fully or partially) abate the pollution it generates, and the costs of abatement; *(iii) Pollution damage*: This quantifies the disutility to society from pollution; and, finally *(iv) Pollution regulation*: This describes the mechanisms that the regulator could employ to control pollution. We explicitly model these mechanisms, in particular the trading process. We discuss each of these four elements below.

3.1.1 Pollution generation

Let $\widetilde{P}$ denote the total quantity of pollution emitted by the firm. Clearly, the quantity of pollutant should be an increasing function of production. We further assume that the total pollution $\widetilde{P}$ (prior to any investment in abatement) is *proportional* to the production quantity q; *i.e.*, $\widetilde{P} = e \cdot q$ where $e \geq 0$ is the emissions rate. Several factors suggest that our linearity assumption is reasonable in the context of many industrial sectors. Pollution concentrations arise from diffusion patterns which, by the law of conservation of mass, are typically linear in the quantity of pollution released. In many industries– the power generation industry being a classical example–the output and the pollution generated are linear functions of fuel consumption, and hence, of each other. Without loss of generality, we normalize e to 1, *i.e.*, $\widetilde{P} = q$.

3.1.2 Pollution abatement

Our model of pollution abatement relies on two complementary notions: *(i)* the *abatement level*, which determines how much pollution is abated, and *(ii)* the cost of abating pollution. In our model, the firm can control the quantity of pollution it generates (albeit at a cost) by setting the abatement level, denoted by x, where $x \in [0, 1]$. The decision variable x can be interpreted as the percentage of pollution abated. In other words, $q \cdot x$ is the quantity of pollution abated. The relation between the *net* (or *residual*) pollution P, the total pollution $\widetilde{P}$ and abatement is modeled as $P = \widetilde{P} - q \cdot x = q \cdot (1 - x)$. At one extreme, when $x = 0$, the pollution is unabated (hence, $P = \widetilde{P} = q$). When $x = 1$, the pollution is completely abated and $P = 0$. Intermediate values of x correspond to partial abatement.

In our model, we assume that pollution abatement costs are *increasing* and *convex* in the quantity of pollution abated (which is $q \cdot x$). Specifically, we assume that the pollution abatement cost $C(q; x) = c \cdot (q \cdot x)^2$, where c is the abatement cost coefficient, q the production quantity, and x the percentage of pollution abated. We justify our assumption of a convex abatement cost curve on several grounds: *(i)* It is logical that the first units of pollution are easy to abate, but once the low-hanging fruits have been exploited, pollution abatement becomes increasingly difficult. *(ii)* [Hartman et al., 1997] estimate the cost of pollution abatement for 7 common air pollutants, namely particulates[2], sulfur oxides, nitrogen dioxide, carbon monoxide, hydrocarbons, lead and other hazardous emissions, using census data from 100,000 U.S. manufacturing firms across 37 industrial sectors. They find support for quadratic abatement cost curves in several industrial sectors. *(iii)* Quadratic abatement costs are commonly assumed in the extant academic literature (cf. [Subramanian et al., 2007], [Parry and Toman, 2002]). [Nault, 1996] and [Levi and Nault, 2004] assume a convex, but not necessarily quadratic, cost function.

3.1.3 Pollution damage

Pollution affects human health, wildlife habitat, and the natural environment. In a widely cited study, [Pope et al., 2002] found that a 10 $\mu g/m^3$ increase in fine particulate air pollution was associated with an increased risk of all-cause, cardiopulmonary, and lung cancer mortality, by 4%, 6%, and 8% respectively. Particulates contribute to the creation of haze, increase the acidity of lakes and rivers, and alter the balance of nutrients in waters and the soil[3]. Sulfur dioxide, another common air pollutant, contributes to acid rains, which cause widespread damage to surface waters, aquatic animals, forests, crops and buildings.

The *pollution damage function*, which we introduce next in our model, captures both present and future damage to society from emissions. Clearly, pollution damage would be increasing in the net pollu-

[2] Particulates are a mixture of fine solid particles and liquid droplets suspended in the air.

[3] http://www.epa.gov/air/particlepollution/health.html

tion generated ([Nault, 1996]; [Jacobs and Subramanian, 2012]). Furthermore, [Tietenberg and Lewis, 2011] suggests that "the marginal damage caused by a unit of pollution increases with the amount emitted" [Page 359]. Intuitively, while pollution is tolerable in small quantities, the damage from pollution increases with the quantity of pollution *at an increasing rate*. Also, the vast majority of epidemiological studies use either a log-linear or logistic functional form, suggesting that epidemiologists generally believe that the health impact of pollution is convex in the pollution concentration. Thus we model the pollution damage function $D(P)$ as an increasing, *convex* function of the net total pollution, P. Specifically, we let $D(P) = d \cdot P^2$, where $d \geq 0$, the *pollution damage factor*, varies with the pollutant under consideration. A high value of d indicates a very toxic pollutant, whereas low d suggests a pollutant with moderate, albeit still harmful, impact on society.

3.1.4 Pollution regulation

As mentioned previously, we focus on two popular mechanisms:

- **The Tax mechanism**. Under the Tax mechanism, the regulator charges the firm with a fee proportional to its emissions. Under this mechanism, the tax is equal to $\tau \cdot [q \cdot (1-x)]$ where $q \cdot (1-x)$ is the net pollution generated by the firm and $\tau \geq 0$ is the tax rate set by the regulator, common to all firms. By increasing the tax rate, the regulator makes pollution more costly to the firm causing it to reduce its emissions. Thus the regulator can strategically set the tax rate to achieve a particular emission reduction goal.

- **The Cap-and-Trade mechanism**. Under Cap-and-Trade, the regulator specifies a limit on emissions but firms are allowed to trade with each other. At the end of each year, each firm must surrender a number of allowances equal to its actual emissions, or pay a hefty fine. Based on scientific, historical and political considerations, the regulator typically assigns a cap for the entire region. Let S denote this cap. In the early stages of Cap-and-Trade implementation, each firm typically receives for free an endowment of emission rights. The sum of these initial endowments equals the overall cap S for the region. Free allocation is called "grandfathering". Most cap-and-trade programs make provisions in later stages for auctioning of the emission allowances, either in part or in full. It is important to note that the equilibrium outcome is independent of the allocation mechanism (whether grandfathered or auctioned off). The firms' optimal production schedule is unaffected by the allocation mechanism chosen by the regulator [Requate, 2006], although the firms' profits are. We will hereafter assume that the firms are initially given the same allocation. This assumption does not impact our results, but has the advantage of making the exposition clearer. Under Cap-and-Trade, the regulator does not need to know anything about firms' cost structures. Further, even if firms know their own cost curves but not necessarily those of other firms, participation in the market for emission allowances reveals the relevant information through the price mechanism ([Hayek, 1945]). So Cap-and-Trade provides an alignment of decision rights with information. Let s denote the pollution cap imposed on an arbitrary firm, *i.e.*, $s = \frac{S}{2}$, and

t be the number of emission allowances traded by the firm. Without loss of generality, $t \geq 0$ indicates that the firm is a net seller of allowances, and $t < 0$ that the firm is a net buyer. The firm's constraint is $q \cdot (1 - x) \leq s - t$.

To allow meaningful comparisons between Tax and Cap-and-Trade, we further assume that the regulator's goals are the same: that the total pollution does not exceed an amount S.

3.2 Modeling firms, the regulator and their interactions

In the model, we consider two firms, designated by subscripts i and j, and study the strategic interactions among them and a pollution-sensitive regulator. Our firms have some market power and compete with each other. We study the case of imperfect substitutes with a market demand for firm i characterized by the linear inverse demand curve $p_i = a - b \cdot q_i - \gamma \cdot b \cdot q_j$ where $a > 0,\ b > 0,\ 0 \leq \gamma \leq 1$, q_i is the quantity produced by firm i for market i, q_j the quantity serving market j, and p_i is the price. Similarly, the price p_j in market j is determined by the quantities q_j and q_i as follows: $p_j = a - b \cdot q_j - \gamma \cdot b \cdot q_i$. When $\gamma = 0$, the firms are local monopolies. The case $\gamma = 1$ is the classical Cournot duopoly. Intermediate values of γ correspond to situations in which the market price that firm i can charge is determined not only by firm i's production quantity, but also in part by the quantity produced in the other market. The parameter γ captures the intensity of the competition between the firms. Many products fit such a description. Even when the product is a commodity, there may be some heterogeneity between customers; for example, some customer segments may have a preference for products locally supplied.

Firm i has an abatement cost coefficient c_i. Thus its pollution abatement cost is $C_i = c_i \cdot (q_i \cdot x_i)^2$, where $q_i \cdot x_i$ is the quantity of pollution abated by firm i (recall section 3.1.2). Similarly for firm j. Without loss of generality, we assume that $0 < c_i \leq c_j$. Note that when $c_i = 0$, pollution abatement is costless. The firm can effortlessly ensure that the pollution constraint is not binding. This implies that the problem coincides with the unregulated case or business-as-usual. From now on, we will let $c_l = c_i$ and $c_h = c_j$, and we will use the subscript l to denote the *low-cost* firm; *i.e.*, the firm with a *low* abatement cost coefficient c_l, and the subscript h to denote the *high-cost* firm, which has a *high* abatement cost coefficient c_h. We assume that c_l and c_h are common knowledge.

Abatement cost coefficients vary across pollutants, geographic regions and industries. Even for the same pollutant and the same product, these coefficients vary across different abatement technologies, and even across abatement technologies of different vintages ([Hartman et al., 1997], [Creyts et al., 2007], [U.S. Census Bureau, 2005], [Pittman, 1981], [Swinton, 1998]). Firms maximize their profits. Firm i's profit ($i = l$ or h) $\pi_i (q_i, x_i | q_j) = q_i \cdot (a - b \cdot q_i - \gamma \cdot b \cdot q_j) - c_i \cdot (q_i \cdot x_i)^2$ is the difference between its revenues and its pollution abatement costs. We assume that all other production costs, whether fixed or variable, are zero. It is straightforward to relax this assumption in our analysis; nevertheless, this assumption enables us to minimize clutter. The firms' joint profits or industry profits are denoted $\Pi = \pi_l + \pi_h$.

3.3 Performance measures

The performance measures we use to evaluate the two mechanisms include the total output, the total abated pollution, firms' profits, consumer surplus and social welfare. We augment the concept of consumer surplus (and by extension, welfare) to include environmental effects. The traditional measure of consumer surplus focuses solely on consumers' economic surplus (CES). CES is the monetary gains enjoyed by consumers from the acquisition of a good or service, and is measured as the difference between their willingness-to-pay and the price they actually pay. A pollution-sensitive regulator should be concerned not only with the welfare of consumers measured in monetary terms, but also with society's disutility from pollution, which is the pollution damage $D(P)$. Thus we measure consumer surplus as $CS = CES - D(P)$. Social welfare, measured as the sum of producers' profits (i.e., the joint profits Π) and the consumer surplus, automatically incorporates the effects of pollution damage as well. In this, we follow the approach adopted by [Nault, 1996], [Jacobs and Subramanian, 2012] and others. Welfare, $W = \Pi + CS = \Pi + CES - D$. Using this augmented measure of social welfare, our model helps us study the trade-offs between pollution and production for society, between the benefits of pollution abatement and economic efficiency, and between consumers' monetary utility from consumption and their disutility from pollution. Our model also enables comparisons of the different pollution control mechanisms along these different dimensions.

Our notations are summarized in Table 1.

q_i	Production quantity chosen by firm i, $i = l$ or h , $q_i \geq 0$
γ	Competition coefficient, $0 \leq \gamma \leq 1$
p_i	Price in firm i's market, $p_i = a - b \cdot q_i - \gamma \cdot b \cdot q_j$, $a > 0$, $b > 0$
x_i	Pollution abatement level chosen by firm i, $0 \leq x_i \leq 1$
c_i	Abatement cost coefficient of firm i, $c_i \in \{c_l, c_h\}$, $0 < c_l \leq c_h$
P_i	Pollution generated by firm i, $P_i = q_i \cdot (1 - x_i)$
P	Total pollution generated by the firms, $P = \sum_{i=1}^{2} P_i$
π_i	Profit of firm i
Π	Firms' joint profit, $\Pi = \sum_{i=1}^{2} \pi_i$
d	Pollution damage factor, $d \geq 0$
D	Pollution damage, $D = d \cdot P^2$
CES	Consumer economic surplus
CS	Consumer surplus $CS = CES - D$
W	Social welfare $W = \Pi + CS$
s	Cap chosen by the regulator for one firm, $s \geq 0$
t_i	Number of emission allowances traded by firm i
τ	Tax rate, $\tau \geq 0$

Table 1 – Summary of Model Notations

4 Equilibrium Analysis

4.1 The Tax Mechanism

As mentioned previously, under the Tax mechanism, the regulator charges a tax *proportional* to the firm's emissions, *i.e.*, firm i pays a tax $\tau \cdot q_i \cdot (1 - x_i)$, where τ is the linear tax rate and $q_i \cdot (1 - x_i)$ is firm i's emissions. Stavins (2003) identifies seven subcategories of environmental taxes. With the exception of fixed administrative charges such as permit fees, taxes are almost universally linear in the pollution generated. Linear taxes are simple to understand and implement, and moreover, are analytically tractable. Furthermore, [Anand and Giraud-Carrier, 2012] have shown that a linear tax is optimal for pollution control, whereas a quadratic tax is not.

In choosing the tax rate τ, the regulator anticipates firms' reactions, and chooses the minimum τ to ensure that the total pollution generated by the firms is at most $2 \cdot s$. Then, each firm chooses its production quantity and pollution abatement level to maximize its profits net of pollution taxes. Firm i's objective is

$$\max_{q_i \geq 0,\ 0 \leq x_i \leq 1} \pi_i \left(q_i, x_i | q_j\right) = q_i \cdot (a - b \cdot q_i - \gamma \cdot b \cdot q_j) - c_i \cdot (q_i \cdot x_i)^2 - \tau \cdot q_i \cdot (1 - x_i)$$

We solve the two-stage game of our model using backward induction. First, we solve for the firms' optimal production quantities and abatement levels as a function of τ. Then we plug the firms' decisions into the regulator's problem, which is to find the minimum τ such that $P \leq S$. Theorem 1 shows that there is a unique Subgame-Perfect Nash Equilibrium for the two-stage Tax game. All proofs are in the technical appendix.

Theorem 1. Let $s_1 = \frac{a}{b(\gamma+2)}$ and $s_2 = \frac{a(c_h - c_l)}{2c_h(b(2+\gamma)+2c_l)}$.

Case 1 (Unfettered duopoly): $s > s_1$. The optimal tax rate is $\tau = 0$. The optimal production quantities and abatement levels are $q_l = q_h = \frac{a}{b(\gamma+2)}$, $x_l = x_h = 0$.

Case 2: $s_2 \leq s \leq s_1$. The optimal tax rate is $\tau = \frac{4c_l c_h(a-(2+\gamma)bs)}{(2+\gamma)b(c_l+c_h)+4c_l c_h}$. The optimal production quantities and abatement levels are $q_l = q_h = \frac{a(c_l+c_h)+4c_l c_h s}{b(2+\gamma)(c_l+c_h)+4c_l c_h}$, $x_l = \frac{2c_h(a-b(2+\gamma)s)}{a(c_l+c_h)+4c_l c_h s}$ and $x_h = \frac{2c_l(a-b(2+\gamma)s)}{a(c_l+c_h)+4c_l c_h s}$.

Case 3: $0 \leq s < s_2$. Let $\beta = 2\sqrt{(b + c_l)(b + c_h)}$. The optimal tax rate is $\tau = 2c_h \frac{a(b(2-\gamma)+2c_l)-2b\left(b\left(4-\gamma^2\right)+4c_l\right)s}{\beta^2-\gamma^2 b^2}$. The optimal production quantities and abatement levels are $q_l = \frac{a(b(2-\gamma)+2c_h)-4\gamma b c_h s}{\beta^2-\gamma^2 b^2}$, $q_h = \frac{a(b(2-\gamma)+2c_l)+8c_h(b+c_l)s}{\beta^2-\gamma^2 b^2}$, $x_l = 1$ and $x_h = \frac{a[b(2-\gamma)+2c_l]-2b\left[b\left(4-\gamma^2\right)+4c_l\right]s}{a(b(2-\gamma)+2c_l)+8c_h(b+c_l)s}$.

Above s_1, the regulatory constraint is slack, and we get the unfettered duopoly outcome. The firms do not abate any pollution. Below s_1, and as the regulator increases the regulatory stringency by increasing the tax rate (*i.e.*, s decreases), the firms reduce output and simultaneously abate pollution. The *low-cost* firm abates more pollution than the *high-cost* firm. At s_2, the *low-cost* firm reaches 100 percent abatement. Additional pollution reductions are achieved by output reduction and by the *high-cost* firm increasing its abatement effort. The *low-cost* firm produces more than the *high-cost* firm for any s. See figure 2 for a graph of the production quantities and abatement efforts as a function of s.

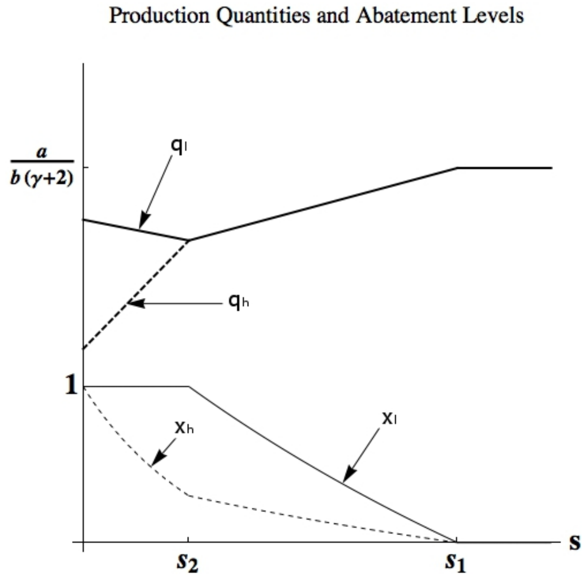


Figure 2: Production quantities and abatement levels under the Tax mechanism ($a = b = 1, c_l = .3, c_h = 1, \gamma = .5$.

4.2 The Cap-and-Trade Mechanism

In the Cap-and-Trade mechanism, the regulator assigns a cap, denoted s, to each firm, and firms have the option of trading emission allowances amongst themselves. Recall that t_i denotes the number of emission allowances sold by firm i (a negative t_i means that the firm is a net buyer of emission allowances). Clearly $t_i \le s$, because a firm can only sell allowances up to its initial endowment. Firm i's problem is given by:

$$\max_{q_i \ge 0,\ 0 \le x_i \le 1,\ t_i \le s} \pi_i\left(q_i, x_i, t_i | q_j\right) = q_i \cdot (a - b \cdot q_i - \gamma \cdot b \cdot q_j) - c_i \cdot (q_i \cdot x_i)^2 + r \cdot t_i$$

where r is the price of emission allowances at which the firms trade, *i.e.*, the market clearing price. Firm j solves a similar problem. The market clearing condition stipulates that the demand for emission allowances equals the supply, *i.e.*, $t_i + t_j = 0$. Rewrite $t_i = t$; the market clears if there exists a price $r \ge 0$ such that $t_j = -t$.

Theorem 2 shows that there is a unique Nash Equilibrium of the Cap-and-Trade game.

Theorem 2. The production quantities and abatement levels under Cap-and-Trade are the same as the Tax mechanism.

Case 1 (Unfettered duopoly): When $s > s_1$, the firms do not trade (*i.e.*, $t = 0$).

Case 2: When $s_2 \le s \le s_1$, the *low-cost* firm sells $t = \frac{(c_h - c_l)[a - b(2+\gamma)s]}{(2+\gamma)b(c_l+c_h)+4c_lc_h}$ emissions allowances at a price $r = \frac{4c_lc_h(a-(2+\gamma)bs)}{(2+\gamma)b(c_l+c_h)+4c_lc_h}$ to the *high-cost* firm.

Case 3: When $0 \leq s < s_2$, the *low-cost* firm sells all its emission allowances (*i.e.*, $t = s$) a price $r = 2c_h \frac{a(b(2-\gamma)+2c_l)-2b\left(b\left(4-\gamma^2\right)+4c_l\right)s}{\beta^2-\gamma^2 b^2}$ to the *high-cost* firm.

Because of its cost advantage, it is optimal for the *low-cost* firm to abate more pollution and sell emission allowances to the *high-cost* firms. Trading is always from the *low-cost* firm to the *high-cost* firm (*i.e.*, $\forall s,\ t \geq 0$). Trading benefits both firms: the *low-cost* firm earns additional revenues from the sale of emission allowances, and the *high-cost* firm is better off purchasing some allowances rather than abate at a high and costly level. Trading happens only when the *low-cost* firm has a distinct cost advantage (*i.e.*, $c_l < c_h$).

As expected, the Cap-and-Trade and Tax mechanisms induce identical output and abatement levels from firms. Consequently, firms' total output, total abated pollution, consumer economic surplus, consumer surplus and social welfare are identical under the two regimes. Firms' profits are greater under Cap-and-Trade than under Tax. The expressions for the firms' profits are found in the technical appendix. The difference is accounted for by the pollution tax paid to the regulator: $\forall s,\ \forall i,\ \pi_i^{ct} \geq \pi_i^t$. The difference $\pi_i^{ct} - \pi_i^t$ is exactly equal to the emission taxes paid by firm i to the regulator, viz., $\tau \cdot s,\ \forall s$; the tax rate τ is equal to the Cap-and-Trade equilibrium (market-clearing) price r.

The effect of either mechanism is to shift the burden of pollution abatement from the *high-cost* firms to the *low-cost* firms. Under Cap-and-Trade, *low-cost* firms are rewarded for bearing a higher load of the pollution reduction through the sale of surplus emission permits. Under Tax, they abate more pollution simply because abatement is cheaper than paying more emission taxes, until their marginal cost of pollution abatement equals the tax rate.

Interestingly, this equivalence under Cap-and-Trade and Tax proves that no collusion occurs under the single-stage Cap-and-Trade, because the outcome is the same as under the Tax mechanism under which collusion is impossible.

4.3 Pareto Optimality

In the previous section, we have described a Cap-and-Trade mechanism where the market-clearing price is the driving force of the equilibrium. We found a unique price at which the volume of allowances offered by the low cost firm equals the demand from the high cost firm. As it turns out, this price is equal to the marginal abatement cost of the firms, and is the shadow price of the firms' pollution constraints. We did not impose any additional condition on the equilibrium. A natural question is to determine if the equilibrium in Theorems 1 and 2 is Pareto-optimal. Remember from our discussion in the literature review that [von der Fehr, 1993] and [Requate, 1993b] use Pareto optimality as the trading objective in their Cap-and-Trade models.

In a Pareto-optimal trade, it is impossible to make one of the firms better off without hurting the other firm. Pareto optimality is a powerful criterion because, given different options, the firms are likely to prefer a Pareto optimal equilibrium. Pareto optimality requires that the firms maximize their joint profits. In other words, the firms trade to make the pie as big as possible, and then figure out a way to share the profits. The trading price may not be unique and will typically depend on the firms' bargaining power; however, the trading price being simply a transfer between firms does not affect total firms' profits, consumer surplus or welfare. With this in mind, consider the following timeline for the Cap-and-Trade mechanism:

First, the regulator chooses and assigns a cap s to each firm (as before). Second the firms trade with each other to maximize their joint profits. Third and finally, the firms play the Cournot game. This scenario differs from the previous one in the timing of the trade. In the previous scenario (section 4.2), the firms trade at the time of production. In other words, the production, competition and trading stages are simultaneously. In the alternative scenario (current section), the trading is decoupled from production/competition, and occurs before. At the time of trading, the firms anticipate each other's reaction in the final stage of the game and focus on executing a Pareto-optimal trade. By placing trading before production, we allow the firms to use the ability to trade as a strategic lever in the Cournot game. Formally, the firms solve the following problem:

1. **Trading:** choice of t

$$\max_{-s\leq t\leq s} \Pi = q_l \cdot (a - b \cdot q_l - \gamma \cdot b \cdot q_h) + q_h \cdot (a - b \cdot q_h - \gamma \cdot b \cdot q_l) - c_l \cdot (q_l \cdot x_l)^2 - c_h \cdot (q_h \cdot x_h)^2$$

where $q_l,\ q_h,\ x_l$ and x_h are determined by the Cournot game below, and are functions of t.

2. **Cournot competition**: choice of $q_l,\ q_h,\ x_l$ and x_h

$$\begin{aligned}\max_{q_l\geq 0,\ 0\leq x_l\leq 1} \pi_l\left(q_l, x_l|t, q_h\right) &= q_l \cdot (a - b \cdot q_l - \gamma \cdot b \cdot q_h) - c_l \cdot (q_l \cdot x_l)^2 \\ s.t.\ q_l \cdot (1 - x_l) &\leq s - t \\ \max_{q_h\geq 0,\ 0\leq x_h\leq 1} \pi_h\left(q_h, x_h|t, q_l\right) &= q_h \cdot (a - b \cdot q_h - \gamma \cdot b \cdot q_l) - c_h \cdot (q_h \cdot x_h)^2 \\ s.t.\ q_h \cdot (1 - x_h) &\leq s + t\end{aligned}$$

We solve by backward induction. Theorem 3 gives the unique Subgame-Perfect Nash Equilibrium of the multiple-stage Pareto-optimal Cap-and-Trade game.

Theorem 3. Recall that $\beta = 2\sqrt{(b + c_l)\,(b + c_h)}$.

Let $s_3 = \frac{a(c_h-c_l)\left(b^2\gamma^2+(1-\gamma)\beta^2\right)}{c_h\left[4\beta^2(b+(1-\gamma)c_l)-b\gamma^2((8-\gamma^2)b^2+8bc_l+12bc_h+12c_lc_h)\right]}$,

$s_4 = \frac{a\left[b(2-\gamma)^2(4+\gamma)+2c_h(8-\gamma(6+\gamma))\right]}{2b\left[b(4-\gamma^2)^2+4c_h(4-3\gamma^2)\right]}$,

$s_5 = \frac{a\left(b^2(2-\gamma)^2(c_l\gamma+c_h(4+\gamma))+2bc_h(8-\gamma(6+\gamma))(c_l+c_h)+16c_lc_h^2(1-\gamma)\right)}{2bc_h\left(b^2(4-\gamma^2)^2+2b(c_l(8-\gamma(4+\gamma(2+\gamma)))+2(4-3\gamma)c_h)+8c_lc_h(2-\gamma-\gamma^2)\right)}$

There exists a unique $\overline{s} \in [s_4, s_5]$ such that:

Case 1: $s \geq \overline{s}$. The optimal trade is $t = \frac{2a\left[b(2-\gamma)^2+4(1-\gamma)c_h\right]}{b\left[b(4-\gamma^2)^2+4c_h(4-3\gamma^2)\right]} - s$. The optimal production quantities and abatement levels are $q_l = \frac{a\left[b(2-\gamma)^2(2+\gamma)+2c_h[4-\gamma(2+\gamma)]\right]}{b\left[b(4-\gamma^2)^2+4c_h(4-3\gamma^2)\right]}$, $q_h = \frac{a\left[b(2-\gamma)^2(2+\gamma)+8(1-\gamma)c_h\right]}{b\left[b(4-\gamma^2)^2+4c_h(4-3\gamma^2)\right]}$, $x_l = 0$ and $x_h = \frac{b(2-\gamma)^2\gamma}{b(2-\gamma)^2(2+\gamma)+8(1-\gamma)c_h}$.

Case 2: $s_3 \leq s < \overline{s}$. The optimal trade is $t = \frac{(c_h-c_l)\left[2a\left(b^2\gamma^2+(1-\gamma)\beta^2\right)-bs\left(4\beta^2-\gamma^2\left(\left(8-\gamma^2\right)b^2+12b(c_l+c_h)+12c_lc_h\right)\right)\right]}{4\beta^2(b(c_l+c_h)+2(1-\gamma)c_lc_h)-b\gamma^2\left((8-\gamma^2)b^2(c_l+c_h)+4b\left(3c_l^2+4c_lc_h+3c_h^2\right)+12c_lc_h(c_l+c_h)\right)}$

The optimal production quantities and abatement levels are

$q_l = \frac{(a(c_l+c_h)+4c_lc_hs)\left(2\beta^2-b\gamma^2(2(1-\gamma)b+2c_h)-4\gamma(b+2c_l)(b+c_h)\right)}{4\beta^2(b(c_l+c_h)+2(1-\gamma)c_lc_h)-b\gamma^2\left((8-\gamma)b^2(c_l+c_h)+4b\left(3c_l^2+4c_lc_h+3c_h^2\right)+12c_lc_h(c_l+c_h)\right)}$,

$q_h = \frac{(a(c_l+c_h)+4c_lc_hs)\left(2\beta^2-b\gamma^2(2(1-\gamma)b+2c_l)-4\gamma(b+c_l)(b+2c_h)\right)}{4\beta^2(b(c_l+c_h)+2(1-\gamma)c_lc_h)-b\gamma^2\left((8-\gamma)b^2(c_l+c_h)+4b\left(3c_l^2+4c_lc_h+3c_h^2\right)+12c_lc_h(c_l+c_h)\right)}$,

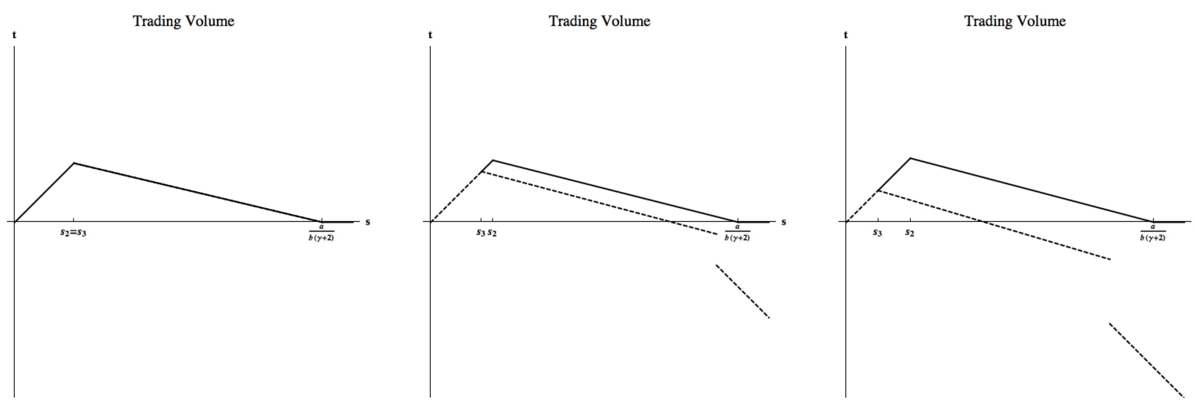
$x_l = 1 - \frac{s-t}{q_i}$ and $x_h = 1 - \frac{s+t}{q_h}$.

Case 3: $0 \leq s < s_3$. The optimal trade is $t = s$. The optimal production quantities and abatement levels are the same as the Tax mechanism.

4.4 The Evidence for Collusion

As can be seen from Theorems 1-3, the three mechanisms (Tax, single-stage Cap-and-Trade and multi-stage Pareto-optimal Cap-and-Trade) are equivalent when $s \leq s_3$, except for the firms' profits. The firms' profits under Tax are lower than under Cap-and-Trade by $\tau \cdot S$, which is the tax payment to the regulator. The situation $s \leq s_3$ corresponds to a very stringent pollution cap requiring the *low-cost* firm to abate all its pollution (*i.e.*, $x_l = 1$) and, under Cap-and-Trade, to sell all its emission allowances to the *high-cost* firm. When $s \leq s_3$, the Tax and Cap-and-Trade mechanisms are Pareto optimal.

Assume now that $s > s_3$ and consider the special case $\gamma = 0$. This corresponds to the firms being local monopolies, *i.e.*, the firms do not compete and have full market power in their respective markets. As can be seen from Theorems 1-3, the Tax mechanism is Pareto optimal if and only if $\gamma = 0$. When $\gamma > 0$, the firms' response under Cap-and-Trade when their goal is to achieve Pareto optimality is different from the Tax mechanism if $s > s_3$. Figure 3 compares the trading volume under both Cap-and-Trade mechanisms.



$\gamma = 0$ $\qquad$ $\gamma = .5$ $\qquad$ $\gamma = 1$

Figure 3: Trading volume under the single-stage (solid line) and multi-stage (dashed line) Cap-and-Trade mechanisms.

When implementing Cap-and-Trade, the regulator does not have any say in the timing of the trade. It is the firms' choice. By definition, the firms' joint profits in the Pareto-optimal condition are greater than under any other trading arrangement; this means that the Pareto-optimal equilibrium is a dominant strategy. In other words, we can expect collusion under Cap-and-Trade when $s > s_3$. Proposition 1 summarizes these results.

Proposition 1. The firms will collude under Cap-and-Trade if and only if $\gamma > 0$ and $s > s_3$.

5 Performance Measures and Comparisons

The remainder of the paper consists in a comparison and discussion of outcomes and performance measures under the simultaneous Cap-and-Trade (which is equivalent to the Tax mechanism) and the two-stage Pareto-optimal Cap-and-Trade mechanism in order to understand the conditions under which collusion can occur, and its consequences on firms, consumers and society.

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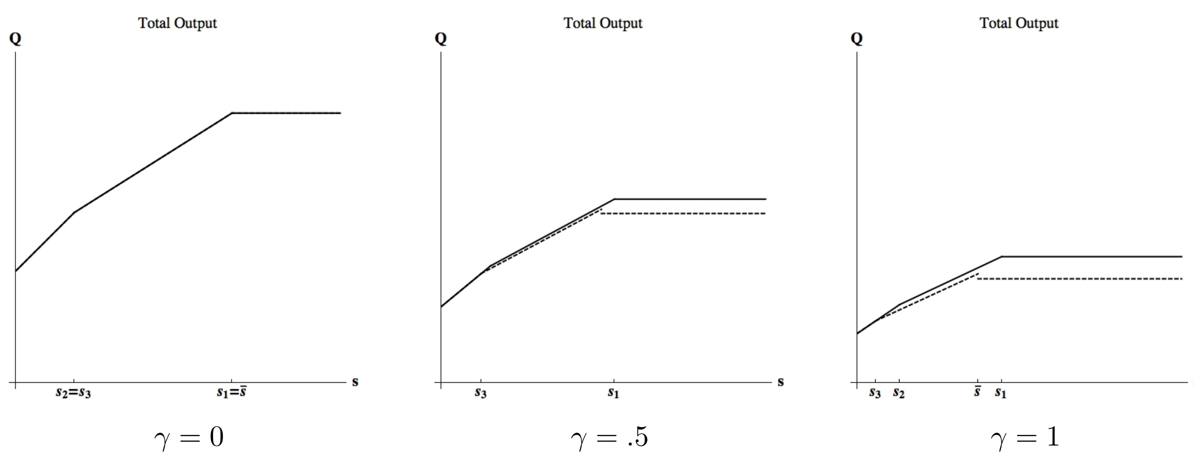
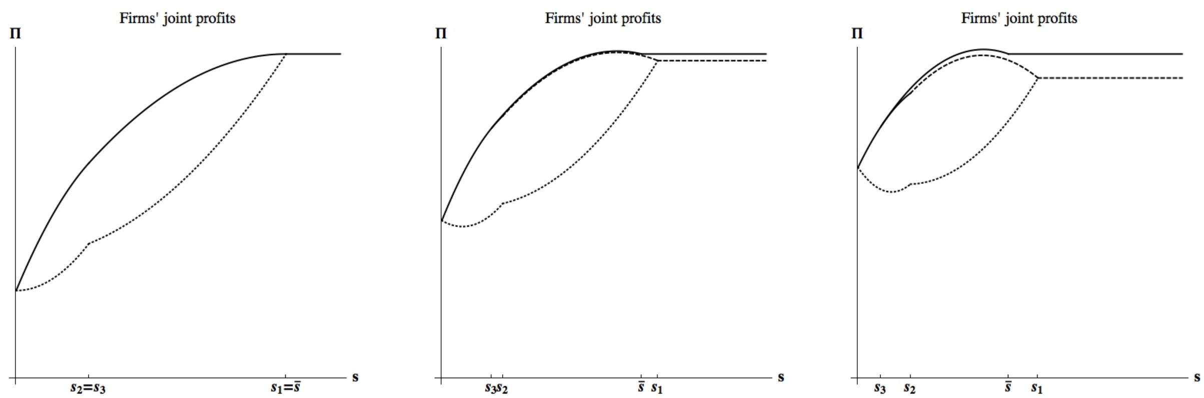


Figure 4: Total output under single-stage (solid line) and multi-stage (dashed line) Cap-and-Trade mechanisms.

6 Conclusion

Several years ago, research into the field of 'supply chain management' exploded in response to the increasing importance of inter-firm operational issues. This research integrated inter-firm coordination, information and agency issues within the framework of traditional research in Operations. Similarly, sustainable operations is an increasingly important research area that expands the scope of traditional



$\gamma = 0$ $\gamma = .5$ $\gamma = 1$

Figure 5: Firm joint profits under single-stage (solid line) and multi-stage (dashed line) Cap-and-Trade mechanisms.

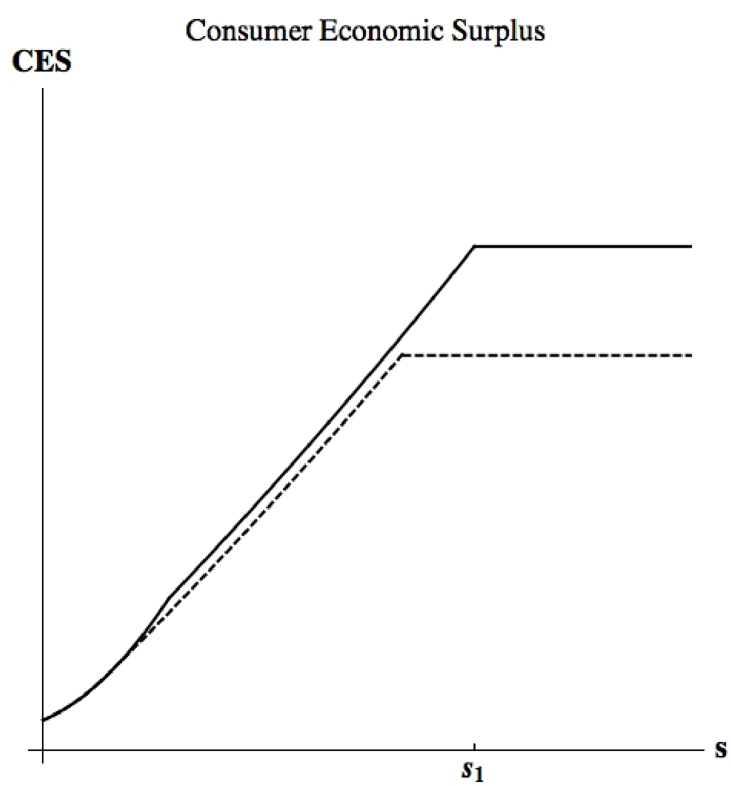


Figure 6: Consumer economic surplus under single-stage (solid line) and multi-stage (dashed line) Cap-and-Trade mechanisms.

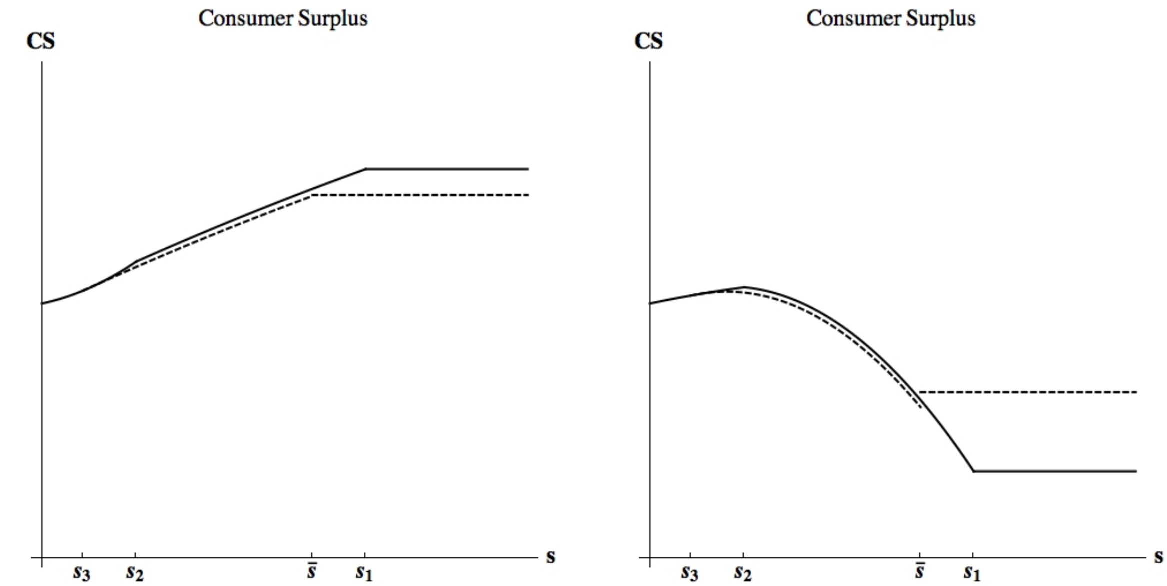


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Figure 7: Consumer surplus under single-stage (solid line) and multi-stage (dashed line) Cap-and-Trade mechanisms.

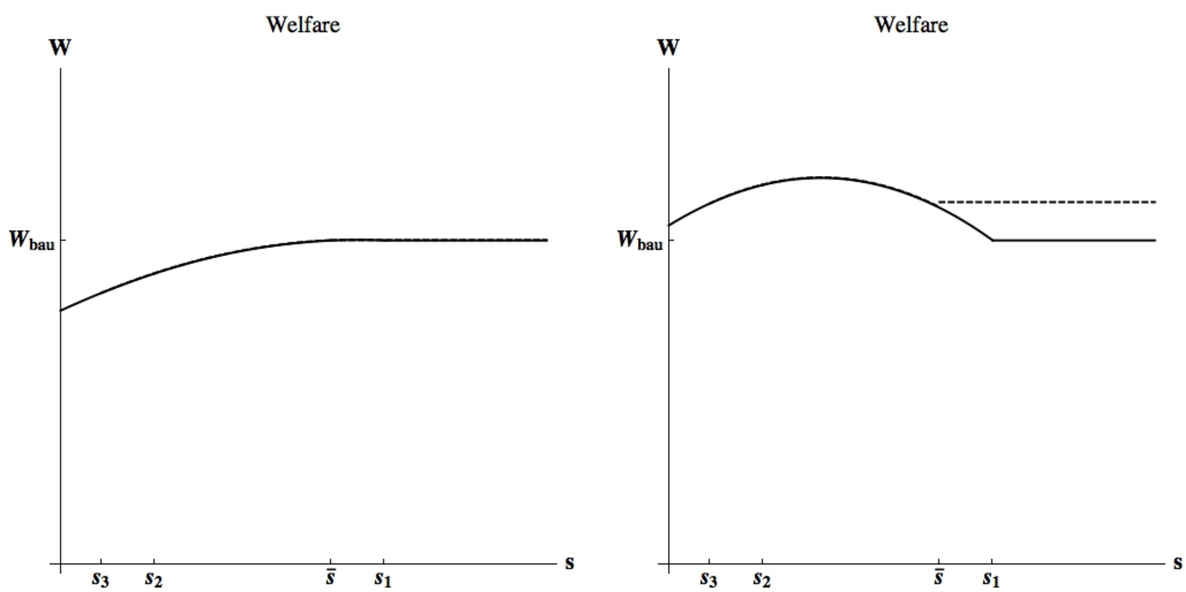

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Figure 8: Welfare under single-stage (solid line) and multi-stage (dashed line) Cap-and-Trade mechanisms.

supply chain management to include environmental considerations. [Kleindorfer et al., 2005] succinctly summarize the case for research in sustainable operations, "The modelers (the OR-based OM population) must *revisit the classical models... to reformulate the objective function and the set of constraints...* in the new context." [Emphasis added] In this spirit, our model contributes several analytical "building-blocks"– notably of pollution generation, abatement, damage and regulation– that can be integrated into traditional supply chain models for use by future researchers in sustainable operations.

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7 Appendix

7.1 Proofs

7.1.1 Proof of Theorem 1

The regulator charges a tax proportional to the firm's emissions, *i.e.*, firm i pays a tax $\tau \cdot q_i \cdot (1 - x_i)$, where τ is the linear tax rate and $q_i \cdot (1 - x_i)$ is firm i's emissions. In choosing the tax rate τ, the regulator anticipates firms' reactions, and chooses the minimum τ to ensure that the total pollution generated by the firms is at most S. Then, each firm chooses its production quantity and pollution abatement level to maximize its profits net of pollution taxes. Firm i's objective is

$$\max_{q_i \geq 0,\ 0 \leq x_i \leq 1} \pi_i^t (q_i, x_i | q_j) = q_i \cdot (a - b \cdot q_i - \gamma \cdot b \cdot q_j) - c_i \cdot (q_i \cdot x_i)^2 - \tau \cdot q_i \cdot (1 - x_i)$$

where the superscript t is for tax. We solve the two-stage game of our model using backward induction. First, we solve for the firms' optimal production quantities and abatement levels as a function of τ. Then we plug the firms' decisions into the regulator's problem, which is to find the minimum τ such that $P \leq 2s$.

Write the Lagrangian

$$\mathfrak{L} = q_i (a - bq_i - \gamma bq_j) - c_i (q_i x_i)^2 - \tau q_i (1 - x_i) + \lambda_i q_i + \mu_{i1} x_i - \mu_{i2} (x_i - 1)$$

where $\lambda_i,\ \mu_{i1}$ and μ_{i2} are Lagrange multipliers. The Kuhn-Tucker (KT) necessary conditions are:

$$\begin{aligned} a - \gamma bq_j - 2q_i \left(b + c_i x_i^2\right) - \tau (1 - x_i) + \lambda_i &= 0 \quad (1) \\ q_i (\tau - 2c_i q_i x_i) + \mu_{i1} - \mu_{i2} &= 0 \quad (2) \end{aligned}$$

with the complementary slackness conditions $\lambda_i q_i = 0,\ \mu_{i1} x_i = 0$ and $\mu_{i2} (x_i - 1) = 0$, and the feasibility constraints $0 \leq x_i \leq 1$ and $q_i,\ \lambda_i,\ \mu_{i1},\ \mu_{i2} \geq 0$.

It is not possible for μ_{i1} and μ_{i2} to be simultaneously > 0.

First we show that $\mu_{i1} = 0$. *Proof.* (By contradiction.) Suppose on the contrary that $\mu_{i1} > 0$. Then $\mu_{i2} = 0$ and $x_i = 0$, and by equation (4) $\mu_{i1} = -\nu_i q_i \leq 0$. ■

We next show that $\lambda_i = 0$. *Proof.* If $\lambda_i > 0$ (*i.e.*, $q_i = 0$) the firm makes a profit of 0. If $\lambda_i = 0$, it is easy to verify that, if q_i and x_i satisfy (1), $\pi_i^t = \left(b + c_i x_i^2\right) q_i^2 \geq 0$. This shows that in equilibrium $\lambda_i = 0$. ■

There are 2 cases:

- $\mu_i = 0$. Then $\tau = 2c_i q_i x_i$. Substituting in (1), the term τx_i cancels out with $-2c_i q_i x_i^2$ and we get

$$q_i = \frac{a - \tau - \gamma bq_j}{2b}$$

We also have

$$x_i = \frac{\tau}{2c_i q_i}$$

provided that $q_i > 0$ and

$$\tau \leq 2c_i q_i$$

- $\mu_i > 0$. Then $x_i = 1$. Equation (1) gives

$$q_i = \frac{a - \gamma b q_j}{2(b + c_i)}$$

$$\mu_i > 0 \Rightarrow \tau > 2c_i q_i$$

Next we solve the joint maximization problem corresponding to the Cournot game. There are 4 cases:

1. $\tau \leq \min\{2c_l q_l, 2c_h q_h\}$. The firms solve

$$\begin{cases} 2bq_l + \gamma b q_h = a - \tau \\ \gamma b q_l + 2bq_h = a - \tau \end{cases}$$

From which we derive

$$\begin{aligned} q_l &= q_h = \frac{a - \tau}{b(2+\gamma)} \\ x_l &= \frac{\tau}{2c_l q_l} = \frac{b(2+\gamma)\tau}{2c_l(a-\tau)} \\ x_h &= \frac{b(2+\gamma)\tau}{2c_h(a-\tau)} \end{aligned}$$

Feasibility conditions are

$$\tau \leq \min\left\{\frac{2ac_l}{b(2+\gamma)+2c_l}, \frac{2ac_h}{b(2+\gamma)+2c_h}\right\} = \frac{2ac_l}{b(2+\gamma)+2c_l}$$

If the above condition is satisfied, then $q_l > 0$ and $q_h > 0$.

The firms' total pollution is

$$P = q_l(1-x_l) + q_h(1-x_h) = \frac{4ac_l c_h - \tau\left[b(2+\gamma)(c_l+c_h) + 4c_l c_h\right]}{2(2+\gamma)bc_l c_h}$$

It is linear decreasing in τ. Thus the smallest τ such that $P \leq 2s$ is

$$\tau_1^* = \frac{4c_l c_h(a - (2+\gamma)bs)}{(2+\gamma)b(c_l+c_h) + 4c_l c_h}$$

$$0 \leq \tau_1^* \leq \frac{2ac_l}{b(2+\gamma)+2c_l} \iff s_2 \leq s \leq s_1$$

where $s_1 \equiv \frac{a}{b(\gamma+2)}$ and $s_2 \equiv \frac{a(c_h - c_l)}{2c_h(b(2+\gamma)+2c_l)}$.

The equilibrium production quantities and abatement levels are

$$\begin{aligned} q_l^* &= q_h^* = \frac{a\left(c_l + c_h\right) + 4c_l c_h s}{b\left(2+\gamma\right)\left(c_l + c_h\right) + 4c_l c_h} \\ x_l &= \frac{2c_h\left(a - b\left(2+\gamma\right)s\right)}{a\left(c_l + c_h\right) + 4c_l c_h s} \\ x_j &= \frac{2c_l\left(a - b\left(2+\gamma\right)s\right)}{a\left(c_l + c_h\right) + 4c_l c_h s} \end{aligned}$$

The firms' optimal profits are

$$\begin{aligned} \pi_l^t &= \left(b + c_l x_l^2\right) q_l^2 = bq_l^2 + c_l\left(q_l x_l\right)^2 = bq_l^2 + \frac{\tau^2}{4c_l} \\ &= \frac{b\left[a\left(c_l + c_h\right) + 4c_l c_h s\right]^2 + 4c_l c_h^2\left(a - \left(2+\gamma\right)bs\right)^2}{\left[b\left(2+\gamma\right)\left(c_l + c_h\right) + 4c_l c_h\right]^2} \\ \pi_h^t &= \frac{b\left[a\left(c_l + c_h\right) + 4c_l c_h s\right]^2 + 4c_l^2 c_h\left(a - \left(2+\gamma\right)bs\right)^2}{\left[b\left(2+\gamma\right)\left(c_l + c_h\right) + 4c_l c_h\right]^2} \end{aligned}$$

Call this equilibrium point E_1.

2. $2c_h q_h < \tau \leq 2c_l q_l$. The production quantities must solve

$$\begin{cases} 2bq_l + \gamma b q_h = a - \tau \\ \gamma b q_l + 2\left(b + c_h\right) q_h = a \end{cases}$$

From which we derive

$$\begin{aligned} q_l &= \frac{a\left[b\left(2-\gamma\right) + 2c_h\right] - 2\left(b + c_h\right)\tau}{b\left[b\left(4-\gamma^2\right) + 4c_h\right]} \\ q_h &= \frac{a\left(2-\gamma\right) + \gamma\tau}{b\left(4-\gamma^2\right) + 4c_h} \end{aligned}$$

Let $\beta^2 = 4\left(b + c_l\right)\left(b + c_h\right)$. The feasibility conditions are

$$2c_h q_h < \tau \leq 2c_l q_l \iff \frac{2ac_h}{b\left(2+\gamma\right) + 2c_h} < \tau \leq \frac{2ac_l\left[b\left(2-\gamma\right) + 2c_h\right]}{\beta^2 - \gamma^2 b^2}$$

This is impossible because the RHS is less than the LHS.

3. $2c_l q_l < \tau \leq 2c_h q_h$. The production quantities must solve

$$\begin{cases} 2\left(b + c_l\right) q_l + \gamma b q_h = a \\ \gamma b q_l + 2bq_h = a - \tau \end{cases}$$

From which we derive

$$\begin{aligned}
q_l &= \frac{a(2-\gamma)+\gamma\tau}{b(4-\gamma^2)+4c_l} \\
q_h &= \frac{a[b(2-\gamma)+2c_l]-2(b+c_l)\tau}{b[b(4-\gamma^2)+4c_l]} \\
x_l &= 1 \\
x_h &= \frac{\tau}{2c_h q_h} = \frac{b\left[b\left(4-\gamma^2\right)+4c_l\right]\tau}{2c_h[a(b(2-\gamma)+2c_l)-2(b+c_l)\tau]}
\end{aligned}$$

The feasibility conditions are

$$2c_l q_l < \tau \le 2c_h q_h \iff \frac{2ac_l}{b(2+\gamma)+2c_l} < \tau \le \frac{2ac_h[b(2-\gamma)+2c_l]}{\beta^2-\gamma^2 b^2}$$

The firms' total pollution is

$$P = q_h(1-x_h) = \frac{2ac_h[b(2-\gamma)+2c_l]-\tau\left[\beta^2-\gamma^2 b^2\right]}{2bc_h[b(4-\gamma^2)+4c_l]}$$

The smallest τ such that $P \le 2s$ is

$$\tau_2^* = 2c_h \frac{a(b(2-\gamma)+2c_l)-2b\left(b\left(4-\gamma^2\right)+4c_l\right)s}{\beta^2-\gamma^2 b^2}$$

$$\begin{aligned}
2c_l q_l &< \tau_2^* \le 2c_h q_h \\
\iff \frac{2ac_l}{b(2+\gamma)+2c_l} &< \tau_2^* \le \frac{2ac_h[b(2-\gamma)+2c_l]}{\beta^2-\gamma^2 b^2} \\
\iff 0 &\le s < s_2
\end{aligned}$$

The equilibrium production quantities and abatement levels are

$$\begin{aligned}
q_l &= \frac{a(b(2-\gamma)+2c_h)-4\gamma bc_h s}{\beta^2-\gamma^2 b^2} \\
q_h &= \frac{a(b(2-\gamma)+2c_l)+8c_h(b+c_l)s}{\beta^2-\gamma^2 b^2} \\
x_l &= 1 \\
x_h &= \frac{a[b(2-\gamma)+2c_l]-2b\left[b\left(4-\gamma^2\right)+4c_l\right]s}{a(b(2-\gamma)+2c_l)+8c_h(b+c_l)s}
\end{aligned}$$

The firms' optimal profits are

$$\begin{aligned}
\pi_i^t &= \frac{(b+c_l)[a(b(2-\gamma)+2c_h)-4\gamma bc_h s]^2}{\left(\beta^2-\gamma^2 b^2\right)^2} \\
\pi_j^t &= \frac{1}{\left(\beta^2-\gamma^2 b^2\right)^2} \times \left[\begin{array}{c} a^2(b+c_h)(b(2-\gamma)+2c_l)^2+4\gamma^2 ab^2 c_h(b(2-\gamma)+2c_l)s \\ +4bc_h\left(b^3\gamma^4+4(b+c_l)\left(\beta^2-2b^2\gamma^2\right)\right)s^2 \end{array}\right]
\end{aligned}$$

Call this equilibrium point E_2.

4. $\tau > \max\{2c_l q_l, 2c_h q_h\}$. The production quantities must solve

$$\begin{cases} 2(b+c_l)q_l + \gamma b q_h = a \\ \gamma b q_l + 2(b+c_h)q_h = a \end{cases}$$

From which we derive

$$\begin{aligned} q_l &= \frac{a(b(2-\gamma)+2c_h)}{\beta^2-\gamma^2 b^2} > 0 \\ q_h &= \frac{a(b(2-\gamma)+2c_l)}{\beta^2-\gamma^2 b^2} > 0 \\ x_l &= x_h = 1 \end{aligned}$$

The feasibility conditions are

$$\tau > \max\{2c_l q_l, 2c_h q_h\} \Rightarrow \tau > \frac{2ac_h[b(2-\gamma)+2c_l]}{\beta^2-\gamma^2 b^2}$$

The firms emit no pollution. The minimum τ that satisfies the pollution constraint is simply

$$\tau_3^* = \frac{2ac_h[b(2-\gamma)+2c_l]}{\beta^2-\gamma^2 b^2}$$

It is straightforward to verify that this equilibrium (case 4) is dominated by E_2 because $\tau_3^* \geq \tau_2^*$ for all $s \geq 0$.

τ_1^* is decreasing in s. We have

$$\tau_1^* \leq \frac{ac_l}{b(2+\gamma)+2c_l} \text{ (value at } s = s_2)$$

Case 4 is also dominated by E_1 because τ_3^* is greater than this upper bound of τ_1^*.

The second order condition verification is the same as the Cap-and-Trade model. (See the proof of Theorem 2).

To summarize, the unique equilibrium is

E_2 if $0 \leq s < s_2$

E_1 if $s_2 \leq s \leq s_1$

and $\tau^* = 0$ if $s > s_1$

■

7.1.2 Proof of Theorem 2

We assume that, if several trades are feasible and give the same payoff, the firms prefer the smallest trading volume, *i.e.*, $t = \arg\min\{|t|, \ \forall t \text{ feasible}\}$. In particular, if $t = 0$ if feasible, the firms will choose not to trade.

The firms' problem is to solve the following joint-maximization problem:

$$\max_{q_l \geq 0,\ 0 \leq x_l \leq 1,\ t_l \leq s} \pi_l^1 (q_l, x_l, t_l | q_h) = q_l \cdot (a - b \cdot q_l - \gamma \cdot b \cdot q_h) - c_l \cdot (q_l \cdot x_l)^2 + r \cdot t_l$$
$$\text{subject to } q_l \cdot (1 - x_l) \leq s - t_l$$
$$\max_{q_h \geq 0,\ 0 \leq x_h \leq 1,\ t_h \leq s} \pi_h^1 (q_h, x_h, t_h | q_l) = q_h \cdot (a - b \cdot q_h - \gamma \cdot b \cdot q_l) - c_h \cdot (q_h \cdot x_h)^2 + r \cdot t_h$$
$$\text{subject to } q_h \cdot (1 - x_h) \leq s - t_h$$

and subject to the market clearing condition which stipulates that the demand for emission allowances equals the supply, *i.e.*, $t_l + t_h = 0$. Rewrite $t_l = t$; the market clears if there exists a price r such that $t_h = -t$. Note that since $q_l \geq 0$ and $x_l \leq 1$, the pollution constraint guarantees that $t \leq s$ (and similarly $q_h \geq 0$ and $x_h \leq 1$ and the pollution constraint $q_h (1 - x_h) \leq s + t$ imply that $t \geq -s$).

Consider an arbitrary firm i. Write the Lagrangian

$$\mathfrak{L} = q_i (a - bq_i - \gamma b q_j) - c_i (q_i x_i)^2 + r t_i + \lambda_i q_i + \mu_{i1} x_i - \mu_{i2} (x_i - 1) - \nu_i [q_i (1 - x_i) - s + t_i]$$

where $\lambda_i,\ \mu_{i1},\ \mu_{i2},\ \nu_i$ and ξ_i are Lagrange multipliers. The KT conditions are:

$$a - \gamma b q_j - 2q_i \left(b + c_i x_i^2\right) + \lambda_i - \nu_i (1 - x_i) = 0 \tag{3}$$
$$q_i (\nu_i - 2 c_i q_i x_i) + \mu_{i1} - \mu_{i2} = 0 \tag{4}$$
$$r = \nu_i \tag{5}$$

with the complementary slackness conditions $\lambda_i q_i = 0,\ \mu_{i1} x_i = 0,\ \mu_{i2} (x_i - 1) = 0$, and $\nu_i [q_i (1 - x_i) - s + t_i] = 0$, and the feasibility constraints $0 \leq x_i \leq 1$, and $q_i,\ \lambda_i,\ \mu_{i1},\ \mu_{i2},\ \nu_i \geq 0$. These conditions are identical to equations (1) and (2) of the Tax model with $\nu_i = \tau$. Thus we know that $\mu_{i1} = 0$.

Note that if $s = 0$, there are no emission allowances for sale, and we have $t = t_l = t_h = 0$, because although there may be some buyers, the supply is inexistent. The firms must abate all their pollution by setting $x_l = x_h = 1$. The profit maximizing quantities satisfy the following:

$$\begin{cases} 2 (b + c_l) q_l + \gamma b q_h = a \\ \gamma b q_l + 2 (b + c_h) q_h = a \end{cases}$$

Recall that $\beta^2 = 4 (b + c_l)(b + c_h)$. The unique solution is

Production quantities	*Profits*
$q_l = \frac{a(b(2-\gamma)+2c_h)}{\beta^2 - \gamma^2 b^2}$	$\pi_l = a^2 (b + c_l) \frac{(b(2-\gamma)+2c_h)^2}{\left(\beta^2 - \gamma^2 b^2\right)^2}$
$q_h = \frac{a(b(2-\gamma)+2c_l)}{\beta^2 - \gamma^2 b^2}$	$\pi_h = a^2 (b + c_h) \frac{(b(2-\gamma)+2c_l)^2}{\left(\beta^2 - \gamma^2 b^2\right)^2}$

Note that the production quantities and the profits are positive, which means that the firms are better abating all their pollution and competing than shutting down. We will henceforth assume that $s > 0$.

Now back to the arbitrary firm i. Assume for now that $\lambda_i = 0$. (We analyze the case $\lambda_i > 0$, *i.e.*, $q_i = 0$ later.)

Rewrite $\mu_{i2} = \mu_i$. There are 3 Lagrange multipliers leading to the following cases:

- $\mu_i = \nu_i = 0$ (The pollution constraint is slack):

  Then $r = 0$.

  $$(4) \Rightarrow x_i = 0 \text{ (assume for now that } q_i > 0.)$$

  By equation (3)

  $$q_i = \frac{a - \gamma b q_j}{2b}$$

  We also have

  $$q_i \leq s - t_i$$

- $\mu_i = 0, \ \nu_i > 0$ (The pollution constraint binds):

  $$\nu_i > 0 \Rightarrow q_i (1 - x_i) = s - t_i \iff q_i x_i = q_i - (s - t_i) \tag{6}$$

  $$(4) \Rightarrow \nu_i = 2 c_i q_i x_i \tag{7}$$

  Note that this case is feasible only if $q_i > 0$ and $x_i > 0$.

  Combining (6) and (7) into (3), we get

  $$q_i = \frac{a - \gamma b q_j + 2c_i (s - t_i)}{2(b + c_i)}$$

  Then

  $$x_i = 1 - \frac{s - t_i}{q_i} \leq 1$$

  provided that

  $$q_i > s - t_i \text{ (this condition guarantees that } x_i > 0)$$

  We also have

  $$r = \nu_i = 2 c_i q_i x_i$$

- $\mu_i > 0$. Then $x_i = 1$ and by equation (3), $q_i = \frac{a - \gamma b q_j}{2(b + c_i)}$. We must have $\nu_i > 0$, otherwise by (4) $\mu_i = -2 c_i q_i^2 \leq 0$.

  $$\nu_i > 0 \Rightarrow q_i (1 - x_i) = 0 = s - t_i \Rightarrow t_i = s$$

  This is a special case of the previous situation where the pollution constraint binds.

To summarize, we have the following cases:

The pollution constraint is slack:

$$q_i \leq s - t_i : \; q_i = \frac{a - \gamma b q_j}{2b}, \; q_i \geq 0, \; x_i = 0, \; r = 0$$

The pollution constraint binds:

$$q_i > s - t_i : \; q_i = \frac{a - \gamma b q_j + 2c_i (s - t_i)}{2(b + c_i)}, \; q_i \geq 0, \; x_i = 1 - \frac{s - t_i}{q_i}, \; t_i \leq s, \; r = 2c_i q_i x_i > 0$$

Next we solve the joint maximization problem subject to the market clearing condition $t_l = t = -t_h$. First we show that the pollution constraints are either slack or binding for both firms.

Proof. This follows immediately from the condition that $r = 0$ if the pollution constraint is slack and $r > 0$ otherwise. ■

We first consider the cases where the pollution constraints are slack for both firms. The firms solve the following system of equations:

$$\begin{cases} 2bq_l + \gamma b q_h = a \\ \gamma b q_l + 2bq_h = a \end{cases} \tag{8}$$

From which we derive

$$q_l = q_h = \frac{a}{b(\gamma + 2)}$$

We have $r = 0$. The feasibility conditions are

$$\frac{a}{b(\gamma + 2)} - s \leq t \leq s - \frac{a}{b(\gamma + 2)}$$

In particular, we must have $s \geq s_1$ (s_1 is defined in the proof of the Tax mechanism). Call this stationary point S_1.

The firms' profits are

$$\pi_l^1 = \pi_h^1 = \frac{a^2}{b(\gamma + 2)^2}$$

The firms will choose the trading volume corresponding to the smallest feasible $|t|$, *i.e.*, $t^* = 0$.

The final case is for both pollution constraints to bind. (See figure below for a representation of the feasible region in the $\{s, t\}$ space.)

The firms solve:

$$\begin{cases} 2(b + c_l) q_l + \gamma b q_h = a + 2c_l (s - t) \\ \gamma b q_l + 2(b + c_h) q_h = a + 2c_h (s + t) \end{cases} \tag{9}$$

From which we derive

$$\begin{aligned} q_l &= \frac{a[b(2-\gamma) + 2c_h] - 2\gamma b c_h (s + t) + 4c_l (b + c_h)(s - t)}{\beta^2 - \gamma^2 b^2} \\ q_h &= \frac{a[b(2-\gamma) + 2c_l] - 2\gamma b c_l (s - t) + 4c_h (b + c_l)(s + t)}{\beta^2 - \gamma^2 b^2} \end{aligned}$$

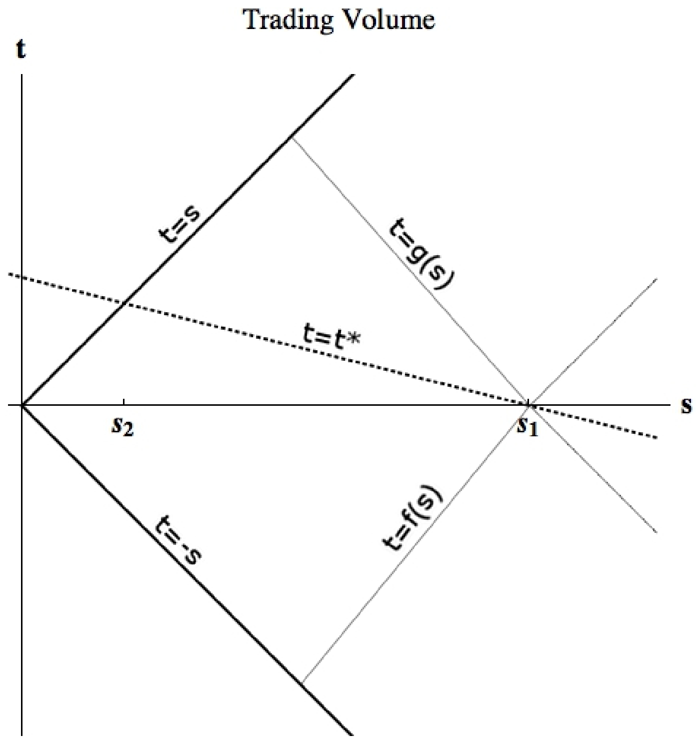

Figure 9: Representation of the feasible region in the $\{s,t\}$ space.

The feasibility conditions

$$q_l > s - t \iff t > \frac{(b(2-\gamma)+2c_h)(b(2+\gamma)s-a)}{b(2-\gamma)(b(2+\gamma)+2c_h)} \equiv f(s) \tag{10}$$

$$q_h > s + t \iff t < \frac{(b(2-\gamma)+2c_l)(a-b(2+\gamma)s)}{b(2-\gamma)(b(2+\gamma)+2c_l)} \equiv g(s) \tag{11}$$

A necessary condition is that $s < s_1$. (This is because f and g are linear functions of s that intersect at s_1.)

f intersects the line $-s$ at $\overline{s} = \frac{a(b(2-\gamma)+2c_h)}{2b(b(4-\gamma^2)+4c_h)}$. g intersects the line s at $\underline{s} = \frac{a(b(2-\gamma)+2c_l)}{2b(b(4-\gamma^2)+4c_l)}$.

Next we solve for the price r at which supply equals demand.

$$r = 2c_l q_l x_l = 2c_h q_h x_h \Rightarrow c_l q_l x_l = c_h q_h x_h \tag{12}$$

Since the pollution constraints are binding, $q_h x_h = q_h - s - t$ and $q_l x_l = q_l - s + t$. Thus equation (12) is equivalent to

$$c_l q_l - c_h q_h + (c_h - c_l)s + (c_l + c_h)t = 0 \tag{13}$$

Equation (13) is a linear function of t. Thus there is a unique t^* that clears the market:

$$t^* = \frac{(c_h - c_l)[a - b(2+\gamma)s]}{b(2+\gamma)(c_l+c_h) + 4c_l c_h}$$

It is straightforward to verify that $t^* > 0$ for $s < s_1$. t^* intersects the line s at s_2 (s_2 is defined in the proof of the Tax mechanism). Since

$$0 \leq s_2 \leq \underline{s}$$

we have that t^* is feasible if and only if

$$s_2 \leq s \leq s_1$$

The optimal production quantities and abatement levels are

$$\begin{aligned}
q_l^* &= q_h^* = \frac{a(c_l + c_h) + 4c_l c_h s}{b(2+\gamma)(c_l + c_h) + 4c_l c_h} \\
x_l^* &= \frac{2c_h(a - b(2+\gamma)s)}{a(c_l + c_h) + 4c_l c_h s} \\
x_h^* &= \frac{2c_l(a - b(2+\gamma)s)}{a(c_l + c_h) + 4c_l c_h s} \\
r &= \frac{4c_l c_h(a - b(2+\gamma)s)}{b(2+\gamma)(c_l + c_h) + 4c_l c_h} \\
\pi_l^1 &= \frac{1}{[b(2+\gamma)(c_l + c_h) + 4c_l c_h]^2} \times \left[\begin{array}{c} a^2\left(b(c_l + c_h)^2 + 4c_l c_h^2\right) + \\ 4ac_l c_h s(\gamma b(c_l - c_h) + 4c_l(b + c_h)) - \\ 4bc_l^2 c_h s^2\left(b(2+\gamma)^2 + 4(1+\gamma)c_h\right) \end{array} \right] && (14) \\
\pi_h^1 &= \frac{1}{[b(2+\gamma)(c_l + c_h) + 4c_l c_h]^2} \times \left[\begin{array}{c} a^2\left(b(c_l + c_h)^2 + 4c_l^2 c_h\right) + \\ 4ac_l c_h s(\gamma b(c_h - c_l) + 4c_h(b + c_l)) - \\ 4bc_l c_h^2 s^2\left(b(2+\gamma)^2 + 4(1+\gamma)c_l\right) \end{array} \right] && (15)
\end{aligned}$$

When $0 \leq s \leq s_2$, t^* hits a boundary of the feasible set.

$$\begin{aligned}
t^* &= s \\
q_l^* &= \frac{a[b(2-\gamma) + 2c_h] - 4\gamma b c_h s}{\beta^2 - \gamma^2 b^2} \\
q_h^* &= \frac{a[b(2-\gamma) + 2c_l] + 8c_h(b + c_l)s}{\beta^2 - \gamma^2 b^2} \\
x_l^* &= 1 \\
x_h^* &= 1 - \frac{2s}{q_h^*} = \frac{a[b(2-\gamma) + 2c_l] - 2b\left[b\left(4-\gamma^2\right) + 4c_l\right]s}{a[b(2-\gamma) + 2c_l] + 8c_h(b + c_l)s} \\
r &= 2c_h \frac{a(b(2-\gamma) + 2c_l) - 2b\left(b\left(4-\gamma^2\right) + 4c_l\right)s}{\beta^2 - \gamma^2 b^2} \\
\pi_i^1 &= \frac{1}{\left(\beta^2 - \gamma^2 b^2\right)^2} \times \left[\begin{array}{c} a^2(b + c_l)(b(2-\gamma) + 2c_h)^2 + \\ 2ac_h s\left(b^3\gamma^3 + 2b^2\gamma^2(b + c_l) + \beta^2(b(2-3\gamma) + 2c_l)\right) - \\ 4bc_h s^2\left(b^3\gamma^4 + 2\beta^2\left(b\left(2-\gamma^2\right) + 2c_l\right)\right) \end{array} \right] && (16) \\
\pi_j^1 &= \frac{[a(b(2-\gamma) + 2c_l) + 8c_h(b + c_l)s]\left[a(b + c_h)(b(2-\gamma) + 2c_l) + 2b^2 c_h \gamma^2 s\right]}{\left(\beta^2 - \gamma^2 b^2\right)^2} && (17)
\end{aligned}$$

Call this stationary point S_2. It is straightforward to show that all the decision variables are continuous at s_1, which is at the junction between S_1 and S_2.

We now check the second order condition. We need only show the optimality in terms of (q^*, x^*) for each individual firm because the objective functions are separable in (q, x) and t, and linear in t. For an arbitrary firm i, the Hessian H is the following 2×2 matrix.

$$H = \begin{pmatrix} -2\left(b + c_i x_i^2\right) & -4c_i q_i x_i + v_i \\ -4c_i q_i x_i + v_i & -2c_i q_i^2 \end{pmatrix}$$

Note that the diagonal elements are negative.

At stationary point S_1 we have

$$H = -2 \begin{pmatrix} b & 0 \\ 0 & c_i q_i^2 \end{pmatrix}$$

whose determinant is $\det(H) = 4bc_i q_i^2 > 0$.

At stationary point S_2

$$H = -2 \begin{pmatrix} b + c_i x_i^2 & c_i q_i x_i \\ c_i q_i x_i & c_i q_i^2 \end{pmatrix}$$

and $\det(H) = 4bc_i q_i^2$ also. ■

Proof that $q_l > 0$ Assume that $\lambda_l > 0$ and $q_l = 0$. This corresponds to the situation where firm l withdraws from the market and sells all its emission allowances to firm h who becomes a monopolist. The firms jointly maximize

$$\max \pi_l^0(t) = r \cdot t$$

$$\text{subject to } t \leq s$$

$$\max_{q_h \geq 0,\ 0 \leq x_h \leq 1} \pi_h^m(q_h, x_h, t) = q_h \cdot (a - b \cdot q_h) - c_h \cdot (q_h \cdot x_h)^2 - r \cdot t$$

$$\text{subject to } q_h \cdot (1 - x_h) \leq s + t$$

The KT conditions are:

$$r = \xi_l \tag{18}$$

$$a - 2q_h\left(b + c_h x_h^2\right) - \nu_h(1 - x_h) = 0 \tag{19}$$

$$q_h(\nu_h - 2c_h q_h x_h) - \mu_h = 0 \tag{20}$$

$$r = \nu_h \tag{21}$$

Where we use the same notations as previously and ξ_l is the Lagrange multiplier associated with the constraint $t \leq s$. A necessary condition is

$$r = \xi_l = \nu_h$$

- Suppose $\xi_l = 0$. Then we must also have $\nu_h = 0$. Equation (20) implies that $\mu_h = 0$ and $q_h x_h = 0$. Equation (19) forbids that $q_h = 0$ and implies that

$$q_h = \frac{a}{2b} \text{ and } x_h = 0$$

The firm will set $t = \arg\min \left\{ |t| \text{ such that } \frac{a}{2b} - s \leq t \leq s \right\}$. A necessary condition is

$$s \geq \frac{a}{4b}$$

Firm l's profits are

$$\pi_l^0 = 0$$

- Suppose $\xi_l > 0$. Then $t = s$ and $r = \xi_l = \nu_h > 0$, which implies $q_h (1 - x_h) = s + t = 2s$.

  If $\mu_h > 0$, $x_h = 1$ and we have the contradiction $t = -s$. Thus $\mu_h = 0$.

  Equations (19) and (20) imply

$$\begin{aligned} \nu_h &= 2c_h q_h x_h \\ q_h &= \frac{a + 4c_h s}{2(b + c_h)} \\ x_h &= 1 - \frac{2s}{q_h} = \frac{a - 4bs}{a + 4c_h s} \end{aligned}$$

  Since $\nu_h > 0$ then $x_h > 0$ which implies that

$$s < \frac{a}{4b}$$

  We can calculate the price of emission allowances

$$r = \nu_h = \frac{c_h (a - 4bs)}{b + c_h}$$

Firm l's profits are

$$\pi_l^0 = \frac{c_h (a - 4bs) s}{b + c_h}$$

We immediately have

$$\pi_l^0 (0) = \pi_l^0 \left(\frac{a}{4b} \right) = 0$$

The maximum value of π_l^0 is at the middle of $\left[0, \frac{a}{4b}\right]$:

$$\pi_l^0 \left(\frac{a}{8b} \right) = \frac{a^2 c_h}{16b (b + c_h)}$$

For $s \geq \frac{a}{4b}$, it is clear that firm l will choose to produce $q_l > 0$.
For $s \leq \frac{a}{4b}$, since $s_2 < \frac{a}{4b}$, we also need to verify that for $s_2 < s < \frac{a}{4b}$

$$\pi_l^0 < \pi_l^1 \text{ (given in equation 14)}$$

and that for $0 \le s \le s_2$

$$\pi_l^0 < \pi_l^1 \text{ (given in equation 16)}$$

π_l^0 and π_l^1 are quadratic concave functions of s.
Let us start with $s_2 < s < \frac{a}{4b}$. We have

$$\begin{aligned}
\pi_l^1(0) &= \frac{a^2\left(b\left(c_l+c_h\right)^2+4c_lc_h^2\right)}{\left(b\left(2+\gamma\right)\left(c_l+c_h\right)+4c_lc_h\right)^2} > 0 \\
\pi_l^1\left(\frac{a}{4b}\right) &= \frac{a^2\left[4b^2\left(c_l+c_h\right)^2+bc_lc_h\left[c_l\left(12-\gamma^2\right)+4c_h\left(4-\gamma\right)\right]+4\left(3-\gamma\right)c_l^2c_h^2\right]}{4b\left(b\left(2+\gamma\right)\left(c_l+c_h\right)+4c_lc_h\right)^2} > 0
\end{aligned}$$

It is enough to show that $\pi_l^1 \ge \pi_l^0$ at $\frac{a}{8b}$.

$$\pi_l^1\left(\frac{a}{8b}\right) = \frac{a^2\left(16b^2\left(c_l+c_h\right)^2+bc_lc_h\left[c_l\left(28+\gamma\left(4-\gamma\right)\right)+8\left(8-\gamma\right)c_h\right]+4\left(7-\gamma\right)c_l^2c_h^2\right)}{16b\left(b\left(2+\gamma\right)\left(c_l+c_h\right)+4c_lc_h\right)^2}$$

Then

$$\begin{aligned}
& \pi_l^1\left(\frac{a}{8b}\right) > \pi_l^0\left(\frac{a}{8b}\right) \\
\Longleftrightarrow\quad & 16b^3\left(c_l+c_h\right)^2+b^2c_h\left(2c_l^2\left(20-\gamma^2\right)+2c_lc_h\left(44-\gamma\left(8+\gamma\right)\right)+c_h^2\left(2-\gamma\right)\left(6+\gamma\right)\right)+ \\
& bc_lc_h^2\left(c_l\left(40-\gamma\left(\gamma+8\right)\right)+16c_h\left(3-\gamma\right)\right)+4\left(3-\gamma\right)c_l^2c_h^3 > 0
\end{aligned}$$

Now $0 \le s \le s_2$. We have

$$\begin{aligned}
\pi_l^1(0) &= \frac{a^2\left(b+c_l\right)\left(b\left(2-\gamma\right)+2c_h\right)^2}{\left(\beta^2-\gamma^2b^2\right)^2} > 0 \\
\pi_l^1\left(\frac{a}{4b}\right) &= \frac{a^2\left[b^2c_h\gamma^3\left(2-\gamma\right)+\left(2-\gamma\right)\beta^2\left(b\left(2-\gamma\right)+2c_l\left(1-\gamma\right)\right)\right]}{4\left(\beta^2-\gamma^2b^2\right)^2} > 0 \\
\pi_l^1\left(\frac{a}{8b}\right) &= \frac{a^2}{16b\left(\beta^2-\gamma^2b^2\right)} \times \left[\begin{array}{c} b^3c_h\gamma^3\left(4-\gamma\right)+8\gamma^2b\left(b+c_l\right)\left(2b^2+2bc_h+c_h^2\right)+ \\ 4\beta^2\left(4\left(1-\gamma\right)b^2+\left(5-3\gamma\right)bc_h+c_lc_h\right) \end{array}\right]
\end{aligned}$$

Then

$$\begin{aligned}
& \pi_l^1\left(\frac{a}{8b}\right) > \pi_l^0\left(\frac{a}{8b}\right) \\
\Longleftrightarrow\quad & b\left(2b\left(2-\gamma\right)+c_h\left(4-\gamma\right)\right)\left(b^2c_h\gamma^3+2\beta^2\left(2-\gamma\right)\left(b+c_h\right)\right) > 0
\end{aligned}$$

■

Proof that $q_h > 0$ The case $q_h = 0$ is the symmetric of $q_l = 0$. The proof is obtained by swapping l and h, except in the last step when comparing firm h's profits if $q_h = 0$ to its profits if $q_h > 0$ when $0 \leq s \leq s_2$. In this case, $x_l = 1$ and we have

$$\begin{aligned}
\pi_h^1(0) &= \frac{a^2(b+c_h)(b(2-\gamma)+2c_i)^2}{\left(\beta^2-\gamma^2 b^2\right)^2} > 0 \\
\pi_h^1\left(\frac{a}{4b}\right) &= \frac{a^2\left(\beta^2-2\gamma b^2\right)\left[2(b+c_h)(b(2-\gamma)+2c_l)+\gamma^2 bc_h\right]}{4b\left(\beta^2-\gamma^2 b^2\right)^2} > 0 \\
\pi_h^1\left(\frac{a}{8b}\right) &= \frac{a^2\left((b+c_l)(2b+c_h)-\gamma b^2\right)\left[4(b+c_h)(b(2-\gamma)+2c_l)+\gamma^2 bc_h\right]}{4b\left(\beta^2-\gamma^2 b^2\right)^2}
\end{aligned}$$

Then

$$\begin{aligned}
& \pi_h^1\left(\frac{a}{8b}\right) > \pi_h^0\left(\frac{a}{8b}\right) \\
\iff \quad & 4\beta^2(b+c_l)\left(4(1-\gamma)b^2+3bc_l+b(2-\gamma)c_h+c_lc_h\right)+ \\
& 4\gamma^2 b\left[\begin{array}{c}\left(2b^2+c_lc_h\right)^2+b^2\left(2c_l^2+c_h^2+4(6-\gamma)c_lc_h\right)+ \\ 2bc_lc_h(2c_l+c_h)+b^3\left(\left(6-\frac{\gamma^2}{4}\right)c_l+(6-\gamma)c_h\right)\end{array}\right] > 0
\end{aligned}$$

■

7.1.3 Proof of Theorem 3

In the two-stage model, the firms first trade emission allowances with each other to maximize their joint profits, then produce and compete in the market. The solution is derived by backward induction, starting from the production/competition stage, and working backwards to the trading stage. In the first stage, the firms' problem is to solve the following joint-maximization problem:

$$\begin{aligned}
& \max_{q_l>0,\ 0\leq x_l\leq 1} \pi_l^2(q_l, x_l|t, q_h) = q_l\cdot(a-b\cdot q_l-\gamma\cdot b\cdot q_h)-c_l\cdot(q_l\cdot x_l)^2 \\
& \text{subject to } q_l\cdot(1-x_l)\leq s-t \\
& \max_{q_h>0,\ 0\leq x_h\leq 1} \pi_h^2(q_h, x_h|t, q_l) = q_h\cdot(a-b\cdot q_h-\gamma\cdot b\cdot q_l)-c_h\cdot(q_h\cdot x_h)^2 \\
& \text{subject to } q_h\cdot(1-x_h)\leq s+t
\end{aligned}$$

This problem is the same as the single-stage model, except that the firms do not choose t at that point. Let Π denote the firms' joint profits. Without any conditions on the price of emission allowances (r) and the trading volume, four production equilibria are feasible:

1. The pollution constraints are slack for both firms (equation 8 of the single-stage model):

$$\begin{aligned}
q_l &= q_h = \frac{a}{b(\gamma+2)} \\
x_l &= x_h = 0 \\
\Pi_1^* &= \frac{2a^2}{b(\gamma+2)^2}
\end{aligned}$$

provided that

$$\frac{a}{b(\gamma+2)} - s \leq t \leq s - \frac{a}{b(\gamma+2)}$$

In particular, we must have $s \geq s_1$. Call this stationary point S_1. In the second stage, the firms choose t to maximize their joint profits. Since the profits are independent of t, the firms will choose the smallest $|t|$.

$$t_1^* = 0$$

2. The pollution constraint is slack for firm l but binds for firm h. The firms must solve

$$\begin{cases} 2bq_l + \gamma bq_h = a \\ \gamma bq_l + 2(b+c_h)q_h = a + 2c_h(s+t) \end{cases}$$

The solution is

$$\begin{aligned}
q_l &= \frac{a[b(2-\gamma)+2c_h] - 2\gamma bc_h(s+t)}{b[b(4-\gamma^2)+4c_h]} \\
q_h &= \frac{a(2-\gamma)+4c_h(s+t)}{b(4-\gamma^2)+4c_h} \\
x_l &= 0 \\
x_h &= \frac{(2-\gamma)[a-b(\gamma+2)(s+t)]}{a(2-\gamma)+4c_h(s+t)} \\
\Pi_2 &= \frac{1}{b[b(4-\gamma^2)+4c_h]^2} \times \begin{bmatrix} a^2\left[2b^2(2-\gamma)^2 + bc_h(6-\gamma)(2-\gamma) + 4c_h^2\right] \\ +4abc_h\left[b(2-\gamma)^2 + 4c_h(1-\gamma)\right](s+t) \\ -b^2c_h\left[b\left(4-\gamma^2\right)^2 + 4c_h\left(4-3\gamma^2\right)\right](s+t)^2 \end{bmatrix}
\end{aligned}$$

Call this stationary point S_2. Feasibility conditions are

$$\begin{aligned}
-s &\leq t < \frac{a}{b(\gamma+2)} - s \\
t &\leq f(s)
\end{aligned}$$

where the function f is defined in the proof of the single-stage model (equation 10).

In the second stage, the firms choose t to maximize Π_2.

Define $s_4 = \frac{a\left[b(2-\gamma)^2(4+\gamma)+2c_h(8-\gamma(6+\gamma))\right]}{2b\left[b(4-\gamma^2)^2+4c_h(4-3\gamma^2)\right]}$.

Π_2 is quadratic and concave in $s+t$, thus there is a unique maximum

$$t_2^* = \begin{cases} f(s) & \text{if } \frac{a[b(2-\gamma)+2c_h]}{2b[b(4-\gamma^2)+4c_h]} \leq s < s_4 \\ \frac{2a\left[b(2-\gamma)^2+4(1-\gamma)c_h\right]}{b\left[b(4-\gamma^2)^2+4c_h(4-3\gamma^2)\right]} - s & \text{if } s \geq s_4 \end{cases}$$

When $s \geq s_4$, the optimal production quantities and abatement levels are:

$$
\begin{aligned}
q_l^* &= \frac{a\left[b(2-\gamma)^2(2+\gamma)+2c_h\left[4-\gamma(2+\gamma)\right]\right]}{b\left[b\left(4-\gamma^2\right)^2+4c_h\left(4-3\gamma^2\right)\right]} \\
q_h^* &= \frac{a\left[b(2-\gamma)^2(2+\gamma)+8(1-\gamma)c_h\right]}{b\left[b\left(4-\gamma^2\right)^2+4c_h\left(4-3\gamma^2\right)\right]} \\
x_l^* &= 0 \\
x_h^* &= \frac{b(2-\gamma)^2\gamma}{b(2-\gamma)^2(2+\gamma)+8(1-\gamma)c_h} \\
\Pi_2^* &= \frac{a^2\left[2b(2-\gamma)^2+c_h\left[8-\gamma(8-\gamma)\right]\right]}{b\left[b\left(4-\gamma^2\right)^2+4c_h\left(4-3\gamma^2\right)\right]}
\end{aligned}
$$

It is straightforward to verify that $\Pi_2^* \geq \Pi_1^*$ so that S_1 is dominated by S_2. We will later show that S_2 is dominated when $s < s_4$ and $t^* = f(s)$, so we can ignore that case.

3. The pollution constraint is binding for firm l but slack for firm h. The firms must solve

$$
\begin{cases}
2(b+c_l)q_l+\gamma b q_h = a+2c_l(s-t) \\
\gamma b q_l + 2bq_h = a
\end{cases}
$$

The solution is

$$
\begin{aligned}
q_l &= \frac{a(2-\gamma)+4c_l(s-t)}{b(4-\gamma^2)+4c_l} \\
q_h &= \frac{a\left[b(2-\gamma)+2c_l\right]-2\gamma b c_l(s-t)}{b\left[b(4-\gamma^2)+4c_l\right]} \\
x_l &= \frac{(2-\gamma)\left[a-b(\gamma+2)(s-t)\right]}{a(2-\gamma)+4c_l(s-t)} \\
x_h &= 0 \\
\Pi_3 &= \frac{1}{b\left[b\left(4-\gamma^2\right)+4c_l\right]^2}\times\left[\begin{array}{c}
a^2\left[2b^2(2-\gamma)^2+bc_l(6-\gamma)(2-\gamma)+4c_l^2\right] \\
+4abc_l\left[b(2-\gamma)^2+4c_l(1-\gamma)\right](s-t) \\
-b^2c_l\left[b\left(4-\gamma^2\right)^2+4c_l\left(4-3\gamma^2\right)\right](s-t)^2
\end{array}\right]
\end{aligned}
$$

Call this stationary point S_3. Feasibility conditions are

$$
\begin{aligned}
s-\frac{a}{b(\gamma+2)} &< t \leq s \\
t &\geq g(s)
\end{aligned}
$$

where the function g is defined in the proof of the single-stage model (equation 11).

Define $s_6 = \frac{a\left[b(2-\gamma)^2(4+\gamma)+2c_l(8-\gamma(6+\gamma))\right]}{2b\left[b(4-\gamma^2)^2+4c_l(4-3\gamma^2)\right]}$. Π_3 is quadratic and concave in $s-t$; thus

$$t_3^* = \begin{cases} g(s) & \text{if } \frac{a[b(2-\gamma)+2c_h]}{2b[b(4-\gamma^2)+4c_h]} \le s \le s_6 \\ s - \frac{2a\left[b(2-\gamma)^2+4(1-\gamma)c_l\right]}{b\left[b(4-\gamma^2)^2+4c_l(4-3\gamma^2)\right]} & \text{if } s \ge s_6 \end{cases}$$

The optimal production quantities and abatement levels are:

$$\begin{aligned}
q_l^* &= \frac{a\left[b(2-\gamma)^2(2+\gamma)+8(1-\gamma)c_l\right]}{b\left[b(4-\gamma^2)^2+4c_l(4-3\gamma^2)\right]} \\
q_h^* &= \frac{a\left[b(2-\gamma)^2(2+\gamma)+2c_l\left[4-\gamma(2+\gamma)\right]\right]}{b\left[b(4-\gamma^2)^2+4c_l(4-3\gamma^2)\right]} \\
x_l^* &= \frac{b(2-\gamma)^2\gamma}{b(2-\gamma)^2(2+\gamma)+8(1-\gamma)c_l} \\
x_h^* &= 0 \\
\Pi_3^* &= \frac{a^2\left[2b(2-\gamma)^2+c_l\left[8-\gamma(8-\gamma)\right]\right]}{b\left[b(4-\gamma^2)^2+4c_l(4-3\gamma^2)\right]}
\end{aligned}$$

It is straightforward to verify that $\Pi_2^* \ge \Pi_3^*$, meaning that S_3 is dominated by S_2. We will later show that S_3 is also dominated when $s < s_6$ and $t^* = g(s)$, so we can ignore that case.

4. The final case is for both pollution constraints to bind (equation 9 of the single-stage model):

$$\begin{aligned}
q_l &= \frac{a(b+2c_h)-2\gamma bc_h(s+t)+4c_l(b+c_h)(s-t)}{\beta^2-\gamma^2b^2} \\
q_h &= \frac{a(b+2c_l)-2\gamma bc_l(s-t)+4c_h(b+c_l)(s+t)}{\beta^2-\gamma^2b^2} \\
x_l &= 1-\frac{s-t}{q_l} \\
x_h &= 1-\frac{s+t}{q_h}
\end{aligned}$$

The expression for the joint profits is complicated.

$$\Pi_4 = -\frac{1}{\left(\beta^2-\gamma^2b^2\right)^2} \times$$
$$\left[\begin{array}{c}
\left[a(b(2-\gamma)+2c_h)+2b(2c_l(s-t)-c_h\gamma(s+t))+4c_lc_h(s-t)\right] \times \\
\left[a(b+2c_l)(b\gamma-2(b+c_h))+2b\left(b\left(c_h\gamma(s+t)+c_l\left(2-\gamma^2\right)(s-t)\right)+2c_lc_h((1+\gamma)s-(1-\gamma)t)\right)\right] \\
+\left[a(b(2-\gamma)+2c_l)-2(b(c_l\gamma(s-t)-2c_h(s+t))+2c_lc_h(s+t))\right] \times \\
\left[a(b+2c_h)(b\gamma-2(b+c_l))+2b\left(2c_lc_h(s-t+\gamma(s+t))+b\left(c_l\gamma(s-t)+c_h\left(2-\gamma^2\right)(s+t)\right)\right)\right] \\
+c_h\left[(b(2-\gamma)+2c_l)(a-b(2+\gamma)s)-b(2-\gamma)t(b(2+\gamma)+2c_l)\right]^2 \\
+c_l\left[(b(2-\gamma)+2c_h)(a-b(2+\gamma)s)+b(2-\gamma)t(b(2+\gamma)+2c_h)\right]^2
\end{array}\right]$$

The feasibility conditions $q_i > s - t$ and $q_j > s + t$ lead to the following conditions on t

$$g(s) < t < f(s)$$

A necessary condition is that $s < s_1$. Call this stationary point S_4. Π_4 is quadratic and concave in t.

The unique maximum is at

$$t_4^* = \frac{(c_h - c_l)\left[2a\left(b^2\gamma^2 + (1-\gamma)\beta^2\right) - bs\left(4\beta^2 - \gamma^2\left(\left(8-\gamma^2\right)b^2 + 12b(c_l + c_h) + 12c_lc_h\right)\right)\right]}{4\beta^2\left(b\left(c_l + c_h\right) + 2\left(1-\gamma\right)c_lc_h\right) - b\gamma^2\left(\left(8-\gamma^2\right)b^2\left(c_l + c_h\right) + 4b\left(3c_l^2 + 4c_lc_h + 3c_h^2\right) + 12c_lc_h\left(c_l + c_h\right)\right)}$$

t^* is a linear, increasing function of s. We need to check that it is feasible.

Define

$$\begin{array}{lll} s_3 & \text{as the point at which } t_4^* \text{ intersects} & t = s \\ s_5 & \text{as the point at which } t_4^* \text{ intersects} & t = f(s) \end{array}$$

We have

$$\begin{aligned} s_3 &= \frac{a\left(c_h - c_l\right)\left(b^2\gamma^2 + (1-\gamma)\beta^2\right)}{c_h\left[4\beta^2\left(b + (1-\gamma)c_l\right) - b\gamma^2\left(\left(8-\gamma^2\right)b^2 + 8bc_l + 12bc_h + 12c_lc_h\right)\right]} \\ s_5 &= \frac{a\left(b^2\left(2-\gamma\right)^2\left(c_l\gamma + c_h\left(4+\gamma\right)\right) + 2bc_h\left(8 - \gamma\left(6+\gamma\right)\right)\left(c_l + c_h\right) + 16c_lc_h^2(1-\gamma)\right)}{2bc_h\left(b^2\left(4-\gamma^2\right)^2 + 2b\left(c_l\left(8 - \gamma\left(4 + \gamma\left(2+\gamma\right)\right)\right) + 2\left(4-3\gamma\right)c_h\right) + 8c_lc_h\left(2-\gamma-\gamma^2\right)\right)} \end{aligned}$$

Note that

$$0 \leq s_3 \leq s_5 \leq s_1$$

See figure 10 below.

$$t_4^* \text{ is feasible} \iff s_3 \leq s \leq s_5$$

The optimal production quantities and abatement levels are

$$\begin{aligned} q_i^* &= \frac{\left(a\left(c_l + c_h\right) + 4c_lc_hs\right)\left(2\beta^2 - b\gamma^2\left(2\left(1-\gamma\right)b + 2c_h\right) - 4\gamma\left(b + 2c_l\right)\left(b + c_h\right)\right)}{4\beta^2\left(b\left(c_l + c_h\right) + 2\left(1-\gamma\right)c_lc_h\right) - b\gamma^2\left(\left(8-\gamma\right)b^2\left(c_l + c_h\right) + 4b\left(3c_l^2 + 4c_lc_h + 3c_h^2\right) + 12c_lc_h\left(c_l + c_h\right)\right)} \\ q_j^* &= \frac{\left(a\left(c_l + c_h\right) + 4c_lc_hs\right)\left(2\beta^2 - b\gamma^2\left(2\left(1-\gamma\right)b + 2c_l\right) - 4\gamma\left(b + c_l\right)\left(b + 2c_h\right)\right)}{4\beta^2\left(b\left(c_l + c_h\right) + 2\left(1-\gamma\right)c_lc_h\right) - b\gamma^2\left(\left(8-\gamma\right)b^2\left(c_l + c_h\right) + 4b\left(3c_l^2 + 4c_lc_h + 3c_h^2\right) + 12c_lc_h\left(c_l + c_h\right)\right)} \\ x_i^* &= 1 - \frac{s - t_4^*}{q_i^*} \\ x_j^* &= 1 - \frac{s + t_4^*}{q_j^*} \\ \Pi_4^* &= \frac{1}{4\beta^2\left(b\left(c_l + c_h\right) + 2\left(1-\gamma\right)c_lc_h\right) - b\gamma^2\left(\left(8-\gamma^2\right)b^2\left(c_l + c_h\right) + 4b\left(3c_l^2 + 4c_lc_h + 3c_h^2\right) + 12c_lc_h\left(c_l + c_h\right)\right)} && (22) \\ &\times \left[\begin{array}{c} a\left(2\left(1-\gamma\right)\beta^2 + b\gamma^2\left(2b + c_l + c_h\right)\right)\left(a\left(c_l + c_h\right) + 8c_lc_hs\right) \\ -4bc_lc_hs^2\left(4\beta^2 - \gamma^2\left(\left(8-\gamma^2\right)b^2 + 12b(c_l + c_h) + 16c_lc_h\right)\right) \end{array}\right] && (23) \end{aligned}$$

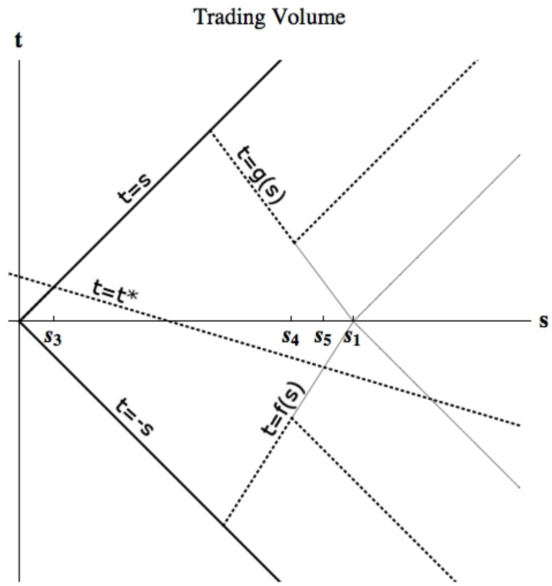

Figure 10: Representation of the feasible region in the $\{s,t\}$ space.

When $0 \leq s \leq s_3$, t_4^* hits a boundary of the feasible set.

$$\begin{aligned}
t_4^* &= s \\
q_i^* &= \frac{a\left(b\left(2-\gamma\right)+2c_h\right)-4\gamma b c_h s}{\beta^2-\gamma^2 b^2} \\
q_j^* &= \frac{a\left(b\left(2-\gamma\right)+2c_l\right)+8c_h\left(b+c_l\right)s}{\beta^2-\gamma^2 b^2} \\
x_i^* &= 1 \\
x_j^* &= 1-\frac{2s}{q_j^*} \\
\Pi_4^* &= \frac{1}{\left(\beta^2-\gamma^2 b^2\right)^2} \times \left[\begin{array}{c} a^2\left[b^2\gamma^2\left(2b+c_l+c_h\right)+\beta^2\left(2b\left(1-\gamma\right)+c_l+c_h\right)\right] \\ +8ac_h s\left(b+c_l\right)\left(\beta^2-b\gamma\left(\left(4-\gamma\right)b+4c_h\right)\right) \\ -4bc_h s^2\left[b^3\gamma^4-4b\gamma^2\left(b+c_l\right)\left(2b+3c_h\right)+4\beta^2\left(b+c_l\right)\right] \end{array}\right]
\end{aligned} \tag{24}$$

When $s_5 \leq s \leq s_1$, t_4^* hits another boundary of the feasible set.

$$t_4^* = f\left(s\right)$$

We now show that this solution is dominated. The firms' production and abatement decisions are continuous along the boundary $t = f\left(s\right)$ and so are the joint profits. We have

$$s_5 \geq s_4$$

which implies that the joint profits under S_4 along $t = f\left(s\right)$ (when t^* hits the boundary of the feasible set) are less than the interior solution of S_2. In other words, S_2 dominates S_4. We have the following result:

$$S_2 \text{ is the global maximum for } s \geq s_5 \tag{25}$$

A similar reasoning is used to prove that S_4 dominates S_2 when $s \leq s_4$ (in this case S_2 sits on the boundary and S_4 is the interior point solution), and that S_4 dominates S_3 when $s \leq s_6$ (in this case S_3 sits on the boundary and S_4 is the interior point solution). Since $s_4 \leq s_5$, we have the following result:

$$S_4 \text{ is the global maximum for } s \leq s_4 \tag{26}$$

Combining results (25) and (26) and the fact that Π_2^* is independent of s and Π_4^* is strictly concave in s, we have the following result:

There exists a unique $\overline{s} \in [s_4, s_5]$ such that S_4 is the unique global maximum for $s < \overline{s}$ and S_2 is the unique global maximum for $s \geq \overline{s}$.

Can $q_h = 0$? When $q_h = 0$, firm l is a monopolist. The firms jointly maximize

$$\begin{aligned}
&\max_{q_l \geq 0,\ 0 \leq x_l \leq 1} \Pi^m (q_l, x_l) = q_l \cdot (a - b \cdot q_l) - c_l \cdot (q_l \cdot x_l)^2 \\
&\text{subject to } q_l \cdot (1 - x_l) \leq s - t \text{ and } t \geq -s
\end{aligned}$$

Using the same notations as previously, the KT conditions are:

$$a - 2q_l \left(b + c_l x_l^2\right) - \nu_l (1 - x_l) = 0 \tag{27}$$

$$q_l (\nu_l - 2c_l q_l x_l) - \mu_l = 0 \tag{28}$$

- Suppose $\nu_l = 0$ (slack pollution constraint). Equation (28) implies that $\mu_l = 0$ and $q_l x_l = 0$. Equation (27) implies that

  $$q_l = \frac{a}{2b}$$

  Since $q_l > 0$, $x_l = 0$. We have the following conditions

  $$-s \leq t \leq s - \frac{a}{2b}$$

  A necessary condition is that $s \geq \frac{a}{4b}$. The optimal joint profits are

  $$\Pi^{m*} = \frac{a^2}{4b}$$

- Suppose $\nu_l > 0$ (binding pollution constraint). Then $q_l (1 - x_l) = s - t$.

– If $\mu_l = 0$, equations (27) and (28) imply

$$\begin{aligned}\nu_l &= 2c_l q_l x_l \\ q_l &= \frac{a + 2c_l (s - t)}{2(b + c_l)} \\ x_l &= \frac{a - 2b(s - t)}{a + 2c_l (s - t)}\end{aligned}$$

provided that

$$\begin{aligned} s - \frac{a}{2b} &< t \leq s \\ t &\geq -s \end{aligned}$$

The firms' joint profits are

$$\Pi^m = \frac{a^2 + 4ac_l (s - t) - 4bc_l (s - t)^2}{4(b + c_l)}$$

If $s \geq \frac{a}{4b}$, to maximize their joint profits, the firms will choose

$$t^* = s - \frac{a}{2b}$$

and

$$\Pi^{m*} = \frac{a^2}{4b}$$

If $s < \frac{a}{4b}$, then $t^* = -s$ and the joint profits are

$$\Pi^{m*} = \frac{a^2 + 8ac_l s - 16bc_l s^2}{4(b + c_l)}$$

– $\mu_i > 0$ is a special case of the previous case with $t = s = 0$, leading to a joint profit $= \frac{a^2}{4(b+c_l)}$.

We conclude by comparing the firms' joint profits when $q_h = 0$ and when $q_h > 0$.
Note that $s_3 \leq \frac{a}{4b} \leq s_4$. This implies that $\frac{a}{4b} \leq \overline{s}$.

- When $s > \overline{s}$, firm h will shut down ($i.e.$, $q_h = 0$) if and only if

$$\frac{a^2}{4b} > \Pi_2^* \tag{29}$$

The condition (29) is equivalent to

$$b\gamma^4 - 16(b + c_h)\gamma^2 + 32(b + c_h)\gamma - 16(b + c_h) = 0$$

This equation in γ has a unique root $\overline{\gamma_h}$ between 0 and 1. Define $\alpha_h = \frac{c_h}{b}$.

$$\overline{\gamma_h} = 2\sqrt{1 + \alpha_h}\left[\sqrt{1 + \sqrt{\frac{1}{1 + \alpha_h}}} - 1\right]$$

$\overline{\gamma_h}$ ranges from $2\left(\sqrt{2}-1\right) \approx .83$ when $\alpha_h = 0$ to 1 when $\alpha_h \to +\infty$.

$$\text{The condition } (29) \iff \gamma \leq \overline{\gamma_h}$$

If $\gamma > \overline{\gamma_h}$ firm h will shut down (i.e., $q_h = 0$) and let firm l take the whole market.

- When $\frac{a}{4b} < s < \overline{s}$, firm h will shut down if and only if

$$\frac{a^2}{4b} > \Pi_4^*$$

where Π_4^* is given by equation (22).

- When $s_3 \leq s \leq \frac{a}{4b}$, firm h will shut down if and only if

$$\frac{a^2 + 8ac_l s - 16bc_l s^2}{4\left(b + c_l\right)} > \Pi_4^*$$

where Π_4^* is given by equation (22).

- When $0 \leq s \leq s_3$, firm h will shut down if and only if

$$\frac{a^2 + 8ac_l s - 16bc_l s^2}{4\left(b + c_l\right)} > \Pi_4^*$$

where Π_4^* is given by equation (24). $\square$