Optimal Timing of Sequential Distribution: The Impact of Congestion Externalities and Day-and-Date Strategies

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Abstract

The window between a film's theatrical and video releases has been steadily declining with some studios now testing day-and-date strategies (i.e., when a film is released across multiple channels at once). We present a model of consumer choice that examines trade-offs between substitutable products (theatrical and video forms), the possibility of purchasing both alternatives, and the timing of consumption; this permits a normative study of the impact of smaller release windows (0-3 months) for which there is a scarcity of relevant data. In this setting, we demonstrate that congestion externalities can drive consumers to smooth consumption over time such that their derived equilibrium behavior is consistent with empirical observations: an exponentially time-decaying demand. Using this equilibrium characterization, we first study how day-and-date strategies impact consumption incentives and explore their optimality from a profit-maximizing perspective. We establish that day-and-date strategies are optimal for films with high content durability (i.e., films whose content tends to lead consumers to purchase both alternatives). Furthermore, in circumstances where content durability is low and congestion cannot be efficiently reduced, day-and-date strategies are optimal for hit films while a direct-to-video strategy is optimal for lower value films. Interestingly, even at lower levels of congestion, a studio can optimally use a day-and-date strategy for films with content durability within a medium-low range. Second, we characterize the optimal delayed release strategy as influenced by congestion, content durability, and movie quality. We find that the optimal release time is nonmonotonic in content durability: within an intermediate range of content durability, the optimal release time first increases in durability, then decreases. We also illustrate that, for relatively low quality movies, an increase in quality should be accompanied by a later video release time. Surprisingly, however, we observe the opposite for relatively higher quality movies: an increase in the quality can justify an earlier release of the video.

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1 Introduction

Over the past decade, a dramatic change has taken place in how movie studios manage film distribution. In those ten years, the average time between the initial theatrical release of a film and its debut on digital video has dropped thirty percent – from 179 days in 1999 to only 123 days in 2009 (Tribbey 2009). This forward shift of nearly two months has largely been driven by advances in technology, especially video quality and home theater capabilities, but also by an industry that is now challenging antiquated norms (Grover 2005, Cole 2007). Is it profit-maximizing for a release window between sequential distribution channels *always* to exist? In recent years, more films have been skipping theatrical release entirely and going directly to home video (Barnes 2008); in some cases, studios are even entertaining "day-and-date" strategies which strike at the heart of the matter. A day-and-date strategy typically means that a product is released across two or more distinct channels on the same day.[1] For example, in 2006, the film *Bubble* was simultaneously released across all channels by 2929 Entertainment, a company founded by Mark Cuban and Todd Wagner that has vertically integrated across production, distribution, and exhibition – an opportune proving ground for testing such strategies (Kirsner 2007). In fact, many argue that release windows are inherently inefficient since the positive impacts of early promotional spending are not fully captured (Gross 2006). Disney CEO Robert Iger even comments that a film should be released faster on digital video since it has "... more perceived value to the consumer because it's more fresh" (Marr 2005); perhaps not surprisingly, Disney recently announced an early video release of *Alice in Wonderland* only 12.5 weeks after its theatrical release instead of the typical 16.5 weeks (Smith and Schuker 2010). Vogel (2007) predicts that further changes in studios' sequential distribution strategies will continue to occur; as a result, there is a strong need for new research aiming to provide a better understanding of these strategies.

Although the release window is gradually narrowing and a few films have been released using day-and-date strategies, it remains difficult to ascertain the impact of a substantial reduction in the release window on profitability. From an empirical standpoint, the average release window is still approximately four months, and there is very little data on any films with windows ranging from zero to three months. In prior studies, researchers have also commented on the low variance observed in the release window measure (see, e.g., Lehmann and Weinberg 2000). Thus, accurately predicting the effect of releasing a film simultaneously on video or even one month after theatrical release continues to be quite difficult, although some studies have sought to close this gap using surveys (Grover 2006, Hennig-Thurau et al. 2007). Furthermore, as the data in Table 1 show, video release times are clearly in a state of flux which makes drawing inferences from existing data even more difficult.

[1] In this paper, our focus is on theatrical release and the video market; hence, "day-and-date" is used to signify that the video is released on the same day as the film appears in theaters. We use the term "video" since particular technologies, e.g., Betamax and VHS, have had a tendency to be short-lived. Although "video" currently refers to DVD and Blu-ray products, our model and results will apply regardless of the implementation of these home viewing options, including video on-demand.

Year	Release Window (Days)	Year	Release Window (Days)
1998	200.4	2004	145.8
1999	179.1	2005	141.8
2000	175.7	2006	129.2
2001	165.4	2007	126
2002	171.4	2008	127.8
2003	153	2009	123.2

Table 1: Shrinking of industry average video-release window from 1998 to 2009 (Tribbey 2009).

However, one can gain significant insights into how various release strategies would tend to affect consumption and hence profitability if we enhance our understanding of the economic trade-offs consumers face when choosing between theatrical and video alternatives. In this paper, we take a normative approach to studying the theater-video windowing problem. We model the primary economic incentives of consumers making film consumption decisions and subsequently analyze how consumers behave, in equilibrium, as the video release time is varied over its full span. By taking into account strategic consumer behavior, we can explore how studios should optimally set the video release time based upon market conditions, movie characteristics, and operational factors. Although consumption by moviegoers can be affected by a wide range of other considerations, in our model we restrict our attention to four primary considerations: (*i*) quality decay of the film content over time, (*ii*) the quality/price gap between theatrical and video alternatives, (*iii*) residual value of the video alternative in addition to movie consumption (multiple purchases), and (*iv*) theater congestion.

All else being equal, consumers prefer to see a film earlier instead of later (Marr 2005). Whether the content is viewed in a theater or at home on video, due to "buzz" generated by marketing, film critics, and social circles, consumers derive highest perceived value at launch, which then decreases over time (Thompson 2006, Cole 2007, Smith and Schuker 2010). Since consumers who watch a film in theaters might also purchase videos of the film, the studios can affect such multiple purchasing behavior by moving up the video release date (Moul and Shugan 2005).

On the other hand, there are two effects that can incentivize viewers to postpone consumption. First, if a film is sequentially distributed through separate channels, some consumers may prefer to wait for lower prices in the secondary channel even though the value derived from the film decays during that time. Thus, a *substitution effect* tends to shift consumption later due to lower prices in the subsequent channel. Second, a *congestion effect* can also shift consumption later in time. Many popular films sell out screenings right after theatrical release. In 2008, *The Dark Knight* was selling out so many screens at midnight on opening day that exhibitors hurried to boost capacity by adding screenings at 3 a.m. and 6 a.m. (Cieply 2008). Furthermore, when Michael Jackson's comeback concert footage was assembled into a film entitled, *This Is It*, it sold out over one thousand screens (Jurgensen 2009). Anticipating sold-out screenings, some consumers may prefer to delay their viewing. However, a film need not sell out to induce consumers to delay. Simply higher utilization of theaters leads to longer waits at ticketing and concessions, poorer seat choices in auditoriums, and other undesirable crowd externalities (e.g., crying babies, cell phones, talking, temperature problems, discomfort, and more). Hence, some consumers may prefer to avoid viewing a movie too early in its stay at the box office due to congestion-generated problems; this can help smooth out demand over time. Since congestion also strongly interacts with the studio's choice of video release time due to substitution effects, optimal management of the entire system requires careful coordination.

In this paper, we present a microeconomic model of film consumption taking into account the four critical factors identified above. Using a game-theoretic framework, we study how consumers, who are heterogeneous in their sensitivity to quality, optimally consume. In particular, when a consumer chooses to view a film in a theater, we characterize the *time* at which she consumes. We further establish properties of the consumer market structure which fully describe how consumers segment: consuming both the theatrical version and video, only one of them, or neither. We demonstrate that the demand pattern resulting from consumers' equilibrium behavior is consistent with empirical observations: an exponentially time-decaying demand. Using our equilibrium characterization, we then explore in depth how the timing of video release impacts consumption behavior and, as a result, profitability across both channels. In addition to providing implications for video release times that are relatively longer, which can be checked for consistency with other studies, our framework also permits exploration of quick release times and even day-and-date releases. Surprisingly, we find that even when the tendency for consumers to purchase both the theatrical and video versions is relatively weak, a day-and-date strategy can still be optimal. Our study of delayed release strategies also reveals that a studio's optimal timing for a video release depends on the degree of the film's durability. Thus, under relatively low durability, it is optimal for the studio to increasingly delay the time of the video release as content durability increases; however, under relatively high durability, it optimally decreases the time until video release as content durability increases.

In the remainder of this paper, we first present a brief review of the literature related to optimal timing of sequential distribution. In sections 3 and 4, we formally present our model and derive the consumer market equilibrium, respectively. In section 5, we analyze the video release timing problem faced by studios, characterize the optimal release strategy, and examine how it is affected by film and operational characteristics. Finally, we provide our concluding remarks in section 6.

2 Literature Review

Eliashberg et al. (2006) provide an extensive review of research related to the motion picture industry. In their discussion of the distribution stage, they pose several questions in regard to the substitutability of DVDs for theatrical consumption and how consumers make trade-offs between the two product forms. We investigate these topics at the consumer level in order to study day-and-date strategies while generating broader implications on how to optimally manage sequential distribution; thus, our work is closest in nature to research that examines the time window between theatrical and video release.

Frank (1994) studies the timing of sequential distribution by constructing a theoretical model in which revenue functions for both product forms linearly decrease in time. His work formalizes some of the primary trade-offs and explains how to manage the timing of a secondary channel. Lehmann and Weinberg (2000) construct a reduced-form model that captures the fact that revenues for the product forms exponentially decay over time. They empirically estimate and test their exponential decay assumption and subsequently establish the potential increase in profitability by data-driven management. Within the operations management literature, a few notable papers also demonstrate how this property of exponentially decaying demand (or arrivals) can arise in other settings. Studying the arrivals of homogeneous customers who minimize waiting times, Glazer and Hassin (1983) demonstrate that the density function of the equilibrium mixed strategy declines over time when early arrivals queue under a first-in, first-out discipline before servers open. However, when early arrivals are forbidden, the density function may increase over time, as shown in Hassin and Kleiner (2010). Furthermore, in analyzing a timing game among agents who strategically arrive to reduce delay costs, Lariviere and van Mieghem (2004) show that the equilibrium arrival pattern approaches a Poisson process as the number of agents gets large. We complement this body of work by establishing that when consumers with different quality sensitivities face congestion externalities based upon the timing of others' consumption, in equilibrium, aggregate demand for in-theater viewings exponentially decays over time.

Studying multiple purchases of product versions in a two-period model, Calzada and Valletti (2010) show that, due to multiple purchases in the motion picture industry, versioning can be optimal for information goods with zero marginal costs. They further establish that a monopolist will often set prices such that sequential release (i.e., nonimmediate release of the video) is not optimal. Sequential release may, however, be driven by the vertical structure of the industry, and by bargaining power between distributors and exhibitors. In our model, sequential release also arises in equilibrium, but it is driven by content durability and congestion. We abstract away from contracting issues between studios, exhibitors, and retailers, and focus on the continuous-time consumption and video release choices. The analysis in Luan and Sudhir (2006) is also related to our work since they account for buzz decay and multiple purchases in their utility specification. In their empirical study, they find that both highly rated films and animated films tend to be less substitutable and that, on average, the optimal release window should be 2.5 months. In another empirical study, Hennig-Thurau et al. (2007) examine multiple channels and release orderings. Although the introduction of DVD rentals is country-dependent, they similarly find that DVD sales should optimally be delayed by three months. Nelson et al. (2007) study the time gap between the end of a film's theatrical run and its release on DVD, finding that about 30 percent of films have DVD versions released while the film is still in theaters and that the time gap is generally declining.

Our work is also related to papers that study the impact of a social presence on consumers. Harrell et al. (1980) find that perceived crowding negatively affects shopping behavior as consumers employ adaptation strategies. Hui et al. (2009) study shoppers' path behavior and zone density, finding that consumers might be attracted to higher density zones but shop less in them. Argo et al. (2005) demonstrate that increasing social presence tends to positively affect emotions initially and then to have a more negative effect as the presence gets larger. In line with their finding, we focus, in our context, on the negative impact of congestion on the theater-viewing experience since theater owners maximize their profits by allocating screens based on movie demand. Specifically, theater owners have economic incentives to keep their capacity highly utilized, and consumers are more likely to be negatively affected at these higher levels of congestion.

In this paper, we build on the ideas summarized above with the goal of developing a theoretical framework in which we start from the consumer's choice problem in order to clarify the trade-offs consumers make and how demand for each product form arises as the result of equilibrium strategic behavior. Our research goals are normative in nature as we seek to explore how consumers should behave as the release time is manipulated across its full span, and, with this understanding, how studios should optimally adjust the timing of video releases. In particular, our work is complementary to the above papers because, using our model, we can further examine the impact of smaller release windows and even the viability of day-and-date strategies. Because, historically, these windows have each lasted several months, most models have not explored the simultaneous release of a film in both theatrical and video forms (see, e.g., Frank 1994 and Lehmann and Weinberg 2000). Furthermore, the lack of sufficient data makes it difficult for any empirical analysis to estimate the impact of day-and-date strategies.

The observable facts are that release windows have continued to shrink over the last decade (Cole 2007, Tribbey 2009) and that day-and-date strategies are currently being tested (Abramowitz 2004, Hofmann 2005). Thus, there is now a strong need to develop a sound theoretical understanding of the effects of such strategies. By utilizing an approach that captures individual consumer decision-making while accounting for primary influential factors (including congestion costs, quality decay, product substitutability, and content durability), we provide insights both on which product forms are consumed and on the timing of consumption. In addition to providing an analysis of the optimality of day-and-date strategies, our paper provides a deeper understanding of sequential distribution management and a foundation for further studies that can explore how to shape demand by targeting consumer incentives.

In a broader context, sequential product introduction is related to intertemporal price discrimination (see, e.g., Coase 1972, Bulow 1982, Gul et al. 1986, and Besanko and Winston 1990). The papers in this literature demonstrate that a firm competes against itself by selling across different time periods without commitment. Exploring intertemporal pricing under capacity constraint, Su

(2007) shows that markdown pricing can be optimal when high valuation customers are less patient and that, otherwise, optimal prices are increasing in time. Moorthy and Png (1992) consider a seller with two substitutable and differentiated products and show that sequential introduction can both effectively reduce cannibalization and be more profitable than simultaneous introduction. Studying films with short runs, Waterman et al. (2009) find that the release window for videos is still long and invariant to theatrical run length. Their results suggest that studios can credibly commit to release windows. We examine sequential introduction where, upon theatrical movie release, the studio commits to a video release time.

3 Model

There is a continuum of consumers who are heterogeneous in their sensitivity to the quality of a cinematic production. Each consumer's sensitivity (i.e., her type) is uniformly distributed on $\mathcal{V} \triangleq [0, \bar{v}]$. We examine the entire life cycle of a production in continuous time, inclusive of all in-theater purchases and video sales. We assume that the product can be consumed in theaters (the *movie*) for a price $p_m > 0$ starting at time zero, and consumed in digital form at home (the *video*) for $p_b > 0$. In the movie industry, there are many other channels for obtaining film content including pay-per-view on-demand services (e.g., Vudu), video rental services (e.g., Netflix, Blockbuster, and Redbox), and cable services (e.g., Time Warner and Comcast). Our model simplifies this setting and focuses on clarifying the main trade-offs between a primary and secondary channel. However, the insights derived from our analysis can be readily applied to the more complex setting.

Two of the primary factors identified in section 1 as affecting consumption are the quality/price gap between alternatives and quality decay. First, to capture the former factor, we adapt a standard model of vertical product differentiation to our specific setting (Shaked and Sutton 1983, 1987). The product consumed in theaters has an inherent level of quality given by γ_m. For example, the recent box office hit *Avatar* would be associated with a higher value for γ_m due to its special effects and 3-D features. Similarly, the inherent quality of the corresponding video is given by γ_b. Second, as discussed above, the content of the film loses its value over time. In that sense, we can consider the film to be essentially a different product at each moment in time.[2] If a consumer with quality sensitivity $v \in \mathcal{V}$ views the movie in a theater at time t, her willingness to pay is $\gamma_m(1 - \beta t)^+ v$, where β is the movie's quality decay rate.[3] Similarly, the consumer's willingness to pay for the video when released at time T is given by $\gamma_b(1 - \beta T)^+ v$.

[2] Note that the focus is not on how consumers discount the timing of consumption for the *same* product (e.g., exponential or hyperbolic discounting considerations). In our case, there is substantial decay in the quality of the film itself over time not because the content has changed but because it is steadily losing its relevance. Thus, in our model we focus on this aspect of quality decay over time and disregard discounting concerns that are relatively insignificant for film windows and, even if included, would not affect our main insights.

[3] We employ a linear quality decay in time for both alternatives. Our results do not critically depend on this functional form, and we model quality decay in this manner primarily for simplicity and tractability. For example, Luan and Sudhir (2006) similarly utilize this form for quality decay in their utility specification.

At the beginning stage, the production studio determines when to open its video distribution channel. The studio announces and commits to this video release time which is denoted by $T \in [0, \infty)$. Subsequent to the announcement, each consumer decides whether to consume the movie and, if so, at what time to consume it (e.g., $s_1 = t$), as well as whether to purchase the video ($s_2 = B$) or not ($s_2 = N$). The strategy set is thus denoted by $S \triangleq [0, \infty] \times \{N, B\}$, and each consumer chooses the action $s = (s_1, s_2) \in S$ that maximizes her payoff. For ease of notation, $s_1 = \infty$ conveys the meaning that a particular consumer does not consume the movie in a theater.

The third critical factor we identified relates to the residual value of the video for consumers who make multiple purchases. Since these repeat buyers have a significant effect on the profitability of day-and-date strategies, it is important to permit consumption of both movie and video alternatives in the model. Should a consumer opt for multiple purchases by consuming both the movie and the video, her willingness to pay for the video is modified to $\delta\gamma_b(1 - \beta T)^+ v$, where $\delta \in [0, 1]$ represents the *durability* of the film in terms of its content. For example, a larger value for δ indicates that consumers derive considerable value from watching the film on video in addition to viewing it in a theater. At the boundary, i.e., when $\delta = 1$, a consumer deciding whether to purchase both the movie and video will consider each form of the product independently.[4] However, when $\delta < 1$, the video serves as a substitute for the movie since the residual value associated with multiple purchasing diminishes. That is, smaller values for δ imply that a consumer has less incentive to consume both alternatives; instead, she tends toward the one providing higher net utility. In particular, at $\delta = 0$, a consumer purchases at most one form of the product: either a movie in a theater or a video, but not both. In our model, the content durability is a film characteristic, but it directly interacts with consumers' heterogeneous types. Specifically, consumers with the highest types are the ones with the greatest incentive to engage in multiple purchases. However, the durability of the content itself is dependent on the type of film produced. For instance, a children's movie would likely be associated with a higher δ since its content maintains large residual value for repeated viewings. Similarly, videos of films that have established subcultures (e.g., *Star Wars* and *Star Trek*) would also have higher durability. On the other hand, documentary films, in which the focus is on being more informative, may have relatively lower residual value after a first viewing than a highly entertaining film.

When the product is consumed in a theater, concurrent consumption externalities arise due to the theater's fixed capacity and limited resources. For example, as the number of patrons seeing a movie at a theater grows large, there is an increased risk of screenings being sold out, having only poor seats remaining, and the viewing experience being degraded due to congestion externalities, the fourth and final factor we capture in our model that critically affects moviegoer consumption timing. We use the term "congestion" in a vein similar to that of Vickrey (1955) in his study of New York City's subway system where he states, "...where congestion occurs, the fare may

[4] Specifically, when $\delta = 1$, a consumer still derives full value from viewing the content on video even though she also sees the content in a theater. Because there is no loss in value, she can treat each decision independently. We restrict attention to $\delta \in [0, 1]$ since $\delta > 1$ is unlikely, and the intuition gained by studying high δ will still apply.

fail to reflect the relatively high cost either of providing additional service at such times, or of the added discomfort to existing passengers occasioned by the crowding in of additional passengers." We highlight this point since traditional congestion costs in the operations management literature stem from longer waiting times in service processes. In our context, however, consumers do not simply wait at theaters and incur costs until a screen becomes free; rather, they adapt by altering their consumption timing. Nevertheless, they still incur congestion costs associated with inconvenience and the crowding factors described above. Ceteris paribus, a consumer strives to consume earlier to increase her surplus due to quality decay, but congestion provides incentives to delay. For these reasons, we model congestion costs at any given time as proportional to the rate of consumers viewing in a theater at that time.[5] We let $\sigma : \mathcal{V} \to S$ be a strategy profile of consumer actions and denote the associated rate of in-theater consumption at time t with $\eta(t)$.[6] With the congestion cost parameter denoted as $\alpha_m \geq 0$, a consumer with quality sensitivity v obtains a net payoff of $\gamma_m(1-\beta t)^+ v - \alpha_m\,\eta(t) - p_m$ if she consumes in a theater at time t. Fixing all other consumers to the strategy prescribed by σ_{-v}, we can summarize the net payoff to the consumer with quality sensitivity v when undertaking action s by:

$$V(v,s,\sigma_{-v}) \triangleq \begin{cases} \gamma_m(1-\beta t)^+ v - \alpha_m\,\eta(t) - p_m + \delta\gamma_b(1-\beta T)^+ v - p_b & if \quad s=(t,B)\,; \\ \gamma_m(1-\beta t)^+ v - \alpha_m\,\eta(t) - p_m & if \quad s=(t,N)\,; \\ \gamma_b(1-\beta T)^+ v - p_b & if \quad s=(\infty,B)\,; \\ 0 & if \quad s=(\infty,N)\,. \end{cases} \tag{1}$$

Box office and retailer data often suggest that video demand decays much faster in time than theater demand.[7] Consequentially, we assume that all video consumption occurs at time T which provides a simpler, more accessible model while providing the same insights. For model relevance, we restrict the parameter space to $T \leq 1/\beta$, $p_m/\gamma_m \geq p_b/\gamma_b$, and $\bar{v} > p_m/\gamma_m$ so that there exist consumers who can obtain surplus from both theater and video alternatives.[8] We also focus our

[5]Note that a proportional congestion cost assumption can be used in our context and is seen in other papers (see, e.g., Shy 2001). For tractability, we include only a first-order effect although most of our results would be robust to the inclusion of a second-order term. Since our context is not a classical queueing system, moviegoers do not incur nonlinear congestion costs associated with waiting. There can also exist positive crowding effects as seen at sports venues. However, when a given theater is filled close to its capacity, the negative aspects of crowding in our setting (e.g., discomfort, poor seat choices, possibly sold-out, etc.) would tend to dominate the marginal positive effect of adding a few consumers to an already near capacity theater. Since most theater demand occurs early on when theaters are indeed operating at near capacity, our congestion externality term can be considered the aggregate negative effect, net of any small positive crowding effect.

[6]Consumers form rational expectations on $\eta(t)$ which are fulfilled in equilibrium. A closed-form expression for the equilibrium level of $\eta(t)$ is given in the next section.

[7]For *The Dark Knight*, for example, weekly theater sales dropped by 2 percent from week 1 to week 2, in contrast to DVD sales where there was a 64 percent decline. For another, less popular 2008 movie, *Role Models*, the percentage changes were 42 and 76 percent, respectively. One driving force for this observation is that consumers are not exposed to the same crowd externalities and sometimes sold-out screenings (e.g., on opening weekend) at theaters when purchasing readily available DVDs which are easily reproducible.

[8]These restrictions can be relaxed but will give rise only to either trivial equilibrium consumer market structures or to regions that can already be well understood in the restricted parameter space. A full characterization of the consumer market equilibrium when $p_m/\gamma_m < p_b/\gamma_b$ is available from the authors upon request.

study on the more commonly–observed region where $\gamma_m > \gamma_b$ and $p_m > p_b$ are satisfied.[9]

4 Consumer Market Equilibrium

In this section, taking the video release time T and other model parameters as given, we derive the consumer market equilibrium. Its characterization consists of two components. First, we classify consumers by the product forms they consume: both the movie and the video (*both*), only the movie (*movie*), only the video (*video*), or nothing (*none*).[10] Second, for each consumer who, in equilibrium, consumes the movie, we further characterize the time at which she consumes. Our goal is to first develop an understanding of what types of consumption outcomes occur under the various market conditions. For example, if a highly–valued blockbuster movie is coupled with a fast video release time, to what extent will theater demand be cannibalized, particularly for high content durability films? By gaining insight into how moviegoers adjust their consumption patterns in response to durability, video release times, and congestion, we can more clearly see how release timing can be optimized toward profitability, a subject we address in the next section.

Thus, taking into account a film's quality decay over time, negative congestion externalities, and the availability of a video alternative, each consumer chooses an action that maximizes her own surplus.[11] An equilibrium strategy profile σ^* must satisfy the following for each $v \in \mathcal{V}$:

$$V(v, \sigma^*(v), \sigma^*_{-v}) \geq V(v, s, \sigma^*_{-v}) \quad \text{for all } s \in S\,. \tag{2}$$

We provide a complete characterization of the consumer market equilibrium in appendix A. Essentially, one of six consumer market structures will arise depending on the film characteristics; the equilibrium strategy profiles for these cases are provided in section A.2, and the conditions under which each profile arises in equilibrium are presented in section A.3. We begin by discussing one important structure in which *all* consumer segments are represented in equilibrium.

Proposition 1 *For any $\bar{v}$, there exists $\underline{\delta} > 0$ and $\bar{T} > \underline{T} \geq 0$ such that if $\delta > \underline{\delta}$ and $\underline{T} \leq T \leq \bar{T}$, the equilibrium consumer strategy profile σ^* is given by the following:*

$$\sigma^*(v) = \begin{cases} (f^{-1}(v),\, B) & if \quad \frac{p_b}{\delta\gamma_b(1-\beta T)} \leq v \leq \bar{v}\,; \\ (f^{-1}(v),\, N) & if \quad v_n \leq v < \frac{p_b}{\delta\gamma_b(1-\beta T)}\,; \\ (\infty,\, B) & if \quad \frac{p_b}{\gamma_b(1-\beta T)} \leq v < v_n\,; \\ (\infty,\, N) & if \quad v < \frac{p_b}{\gamma_b(1-\beta T)}\,, \end{cases} \tag{3}$$

[9] For most cases, the inherent quality of the theatrical experience is higher, and, typically, several viewers can consume a single video. Thus, p_b more realistically reflects the average price per viewer (Vogel 2007). As an example, a family of four will usually find it cheaper to consume a video at home than in a theater; this price difference is what motivates our region of focus.

[10] Throughout the rest of the paper, we use the italicized words: *both*, *movie*, *video*, and *none* to refer to each of these specific consumer market segments.

[11] We clearly focus on wide-release strategies. However, we provide additional discussion of "sleeper" hits in the concluding remarks.

where $v_n \triangleq \bar{v}/(\cosh \lambda\tau(T))$, $\tau(T)$ *satisfies*

$$\frac{\gamma_m(1-\beta\tau)-\gamma_b(1-\beta T)}{\cosh \lambda\tau} = \frac{p_m - p_b}{\bar{v}}\,, \tag{4}$$

and

$$f(t) = \frac{\bar{v}\left(e^{\lambda(\tau-t)} + e^{-\lambda(\tau-t)}\right)}{e^{\lambda\tau}+e^{-\lambda\tau}}\,. \tag{5}$$

Proposition 1 formally presents the consumer market characterization when all consumer segments are represented in equilibrium. This equilibrium arises whenever film durability δ is sufficiently high and the video release time T is within an appropriate range to induce some consumers to strategically substitute one alternative for the other. Proposition 1 establishes that, in equilibrium, consumers segment across all product forms for a substantial range of video release times. Specifically, consumers with lower quality sensitivity purchase only the video alternative; consumers with slightly higher sensitivity choose to view the movie in theaters; and consumers with the highest quality sensitivity optimally select both alternatives.

We next explore the consumer's problem of when to watch a movie in a theater, if ever, by heuristically deriving the structure of f for the case in which there exists a segment that consumes both alternatives in equilibrium. When we examine the optimal timing problem that a consumer with type v faces if she optimally purchases both movie and video product forms, it follows that her timing choice for movie consumption must satisfy

$$t = \arg\max_{\xi}\left\{\gamma_m(1-\beta\xi)v - \alpha_m\,\eta(\xi) - p_m + \gamma_b\delta(1-\beta T)v - p_b\right\}. \tag{6}$$

If we define $\lambda \triangleq \sqrt{\gamma_m\beta/\alpha_m}$, by (6) and the corresponding first order condition, it follows that

$$\lambda^2 v = -\eta'(t) \tag{7}$$

is satisfied when the solution is interior. Given the rate of in-theater consumption $\eta(\cdot)$, a consumer of type v chooses the optimal time t to see a movie in a theater from (7). It is useful to define a function $f:\mathbb{R}^+ \to \mathcal{V}$ which maps a time t to the consumer who views the movie at that time, i.e., consumer v as given by (7). We can then conveniently express (7) as $\lambda^2 f(t) = -\eta'(t)$. Note that $\eta(\cdot)$, the rate of in-theater consumption (i.e., the demand rate at time t) is determined endogenously by consumers' timing choices in equilibrium. Since f is decreasing, which is easily established in equilibrium and means that higher type customers consume a theatrical movie first, it then follows that the demand rate is given by $\eta(t) = \lim_{\triangle\to 0}\,(f(t) - f(t+\triangle))/\triangle = -f'(t)$. Therefore, in equilibrium, the consumption path for movies satisfies this fundamental differential equation:

$$f''(t) - \lambda^2 f(t) = 0\,, \tag{8}$$

where the applicable boundary conditions are $f(0) = \bar{v}$ and $f'(\tau) = 0$, due to the threshold nature of consumption strategies and incentive compatibility constraints. We use τ to denote the time after which no consumer ever finds it incentive compatible to still consume the film in a theater. Solving this equation, we obtain the characterization of f presented in (5). Lastly, τ is determined by the consumer's choice between *movie* and *video*, in equilibrium, as given in (4). In other words, the consumer with type v_n is essentially indifferent between consuming the movie at time τ and consuming the video at time T, i.e., $v_n = f(\tau)$.

Consumers with higher types are therefore the ones who visit theaters earlier, which is consistent with the industry observation found in Experian Simmons (2009). More significantly, the optimal consumption time for a movie-viewing consumer with type v is governed by $f^{-1}(v)$ which is heuristically derived above. An immediate and important consequence of this result is the following corollary.

Corollary 1 *When there exist strictly positive congestion costs, consumers optimally shift consumption such that their equilibrium choice behavior generates an aggregate in-theater demand rate* $|f'(t)|$ *that exponentially decays over time.*[12]

Congestion is a significant factor in consumer decision-making in many cases, and controlling congestion is frequently an important objective, particularly in traditional service processes. In our setting, congestion is the result of aggregate consumers' timing decisions. However, unlike traditional service processes, the costs are not due to waiting to be serviced; here, consumers can freely choose to arrive later, which is endogenous in our model. Rather, the costs are associated with discomfort due to highly utilized theaters (Vickrey 1955). Thus, each individual consumer must strategically select her own optimal viewing/arrival time while taking others' strategic behavior into account. Corollary 1 establishes that, in the presence of congestion, consumers will optimally smooth their consumption over time to manage these externalities as efficiently as possible. In equilibrium, their rational smoothing behavior gives rise to aggregate consumption being characterized by demand rates exponentially decaying over time. Empirical evidence has consistently demonstrated that actual movie demand, indeed, often decays exponentially (Sawhney and Eliashberg 1996).[13] Naturally then, in many models (see, e.g., Krider and Weinberg 1998, Lehmann and Weinberg 2000, and Eliashberg et al. 2009), demand is usually assumed to exhibit this property. In our model, we also demonstrate that this exponential decay property can be explained as the *outcome* of rational consumption decisions, whose driving force is the presence of congestion externalities.[14]

[12] Technically, we mean "exponential decay" in the sense that there exists c and $r > 0$ such that $\lim_{t\to\infty} |f'(t)|/e^{-rt} = c$.

[13] Furthermore, Sawhney and Eliashberg (1996) show that when a consumer's consumption time is the sum of a stochastic decision time and an action time, under certain assumptions on the underlying stochastic processes, the expected rate of consumption has an exponentially decaying functional form. Also, in a model of DVD preorder and sales, Hui et al. (2008) establish that, when a customer's interarrival times between visits to a DVD ordering site are exponentially distributed, then optimal consumption behavior gives rise to exponentially decaying sales after the DVD is released.

[14] From a modeling perspective, we have modeled quality decay and congestion costs as linear and therefore not

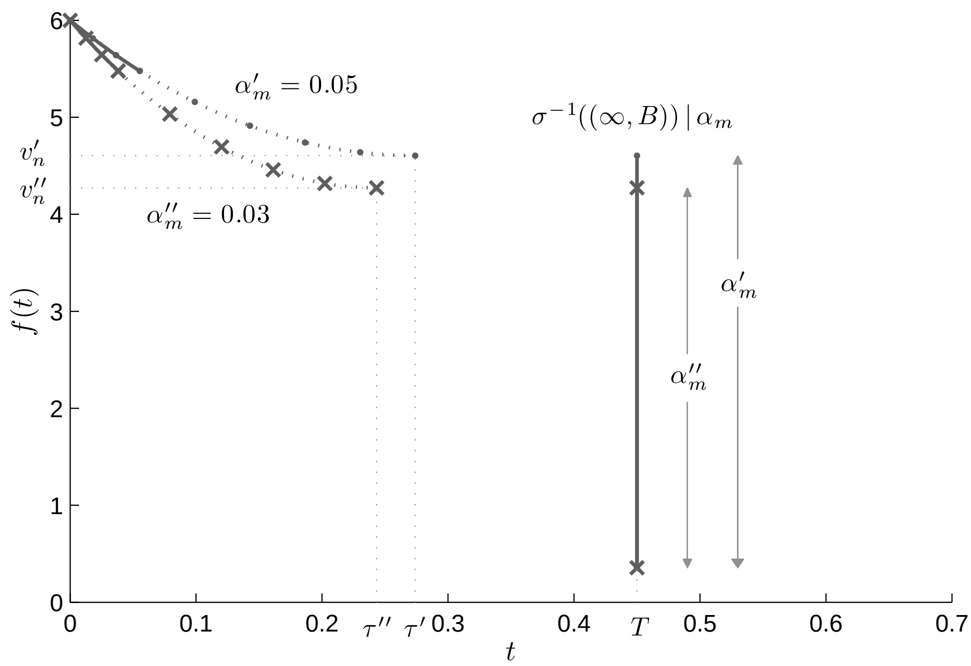

Figure 1: Consumer market structure and path of movie consumption. Dots and x-marks indicate congestion parameters of $\alpha'_m = 0.05$ and $\alpha''_m = 0.03$, respectively. For each case, the length of the film's run in theaters is denoted by τ' and τ'', respectively. The consumer indifferent between viewing the movie at the end of its run and waiting for the video is labeled with v'_n and v''_n. Solid curves represent consumer types who make multiple purchases; dotted curves represent consumer types who consume only the movie; and solid vertical lines represent consumer types who only purchase the video. The other parameter values are $\beta = 0.35$, $\bar{v} = 6$, $\gamma_m = 1.1$, $p_m = 1$, $\gamma_b = 1$, $p_b = 0.30$, $\delta = 0.065$, and $T = 0.45$.

Figure 1 illustrates equilibrium consumption strategies with a numerical example. The curve plotted in the figure is $f(t)$ which identifies the consumer type who views the movie at each time t. The demand rate at time t, i.e., $-f'(t)$, necessarily has a similar exponential shape. Plotting $f(t)$ is more informative than plotting the demand rate $-f'(t)$ since whether a consumer purchases *both* or *movie* can also be indicated using a solid curved line and dotted curved line, respectively, as seen in the figure. Figure 1 demonstrates that when congestion is decreased from $\alpha'_m = 0.05$ down to $\alpha''_m = 0.03$, the rate of movie demand is initially much higher and exhibits faster decay, leading to a shorter run in theaters. The vertical line indicates the consumer types who consume only the video at time T. The entire segment applies under α'_m, whereas only the consumer types between the x-marks consume the video under α''_m. Lehmann and Weinberg (2000) make this interesting observation: "In the summer of 1997, in contrast to historical practice, movies were released with

introduced any particular form that would give rise to this property. One could also model congestion costs as nonlinear in which case the result would be partially driven by a stronger assumption. However, in a more focused way, we use our model to clearly demonstrate that congestion externalities can play a critical role in explaining this commonly observed empirical property.

a much larger initial number of screens, which resulted in relatively high opening weekends but fast decay rates and less time in theatres." This is also consistent with our model prediction: A decrease in congestion effects (α_m) reduces the film's run in theaters (τ) while simultaneously boosting theater demand ($\bar{v} - v_n$).

Having discussed in detail the consumer market structure in (3), we turn our attention to the other structures as presented in appendices A.2 and A.3. First, we examine the conditions under which, if the video were released early enough, no consumer would find it optimal to view the movie in a theater even if it were available.

Proposition 2 *For lower value films that possess low durability, under a wide range of video release times, consumers will optimally forgo in-theater consumption. Technically, if* $\bar{v} \leq \frac{p_m - p_b}{\gamma_m - \gamma_b}$, $\delta \leq \delta_n$, *and* $T < T_n$ *are satisfied, then the equilibrium strategy profile is*

$$\sigma^*(v) = \begin{cases} (\infty,\, B) & if \quad \frac{p_b}{\gamma_b(1-\beta T)} \leq v \leq \bar{v}\,; \\ (\infty,\, N) & if \quad v < \frac{p_b}{\gamma_b(1-\beta T)}\,, \end{cases} \tag{9}$$

where $\delta_n = \frac{p_b}{\gamma_m \bar{v} - p_m + p_b}$ *and* $T_n = \frac{1}{\beta}\left(1 - \frac{\gamma_m \bar{v} - p_m + p_b}{\gamma_b \bar{v}}\right)$.

Proposition 2 highlights the fact that the substitution effect can be sufficiently strong that no consumer finds it optimal to consume the movie in a theater, even in the absence of any congestion effects. For such an equilibrium to arise, two conditions should be satisfied: (*i*) the video should be preferred over the theatrical movie, which requires a lower-value niche-type of film ($\bar{v} \leq (p_m - p_b)/(\gamma_m - \gamma_b)$) that is released early ($T < T_n$) and priced cheaply on video; (*ii*) the video should be preferred over selecting multiple purchases (*both*), which requires sufficiently low content durability ($\delta \leq \delta_n$). Note that the consumer market structure characterized in (9) is the one induced by a *direct-to-video* strategy. If the studio chooses T such that an equilibrium characterized by (9) arises, it would be strictly better off by setting $T = 0$ since the consumer market size is completely governed by the lower bound on quality sensitivity $p_b/(\gamma_b(1 - \beta T))$. By delaying the release time, the studios lose video demand with no gain. In the section to follow, a direct-to-video strategy will therefore correspond to studio selection of $T = 0$ under certain conditions such that the consumer market structure in (9) results.

In contrast, under higher content durability or for higher value films, a strategic video release at $T = 0$ can still give rise to in-theater demand. As long as there exists a segment of consumers who still watch the movie, we call the studio's choice of $T = 0$ a genuine *day-and-date* strategy. In the next section, we will examine the profitability of day-and-date strategies and highlight when they can be most effective. In addition to the consumer market structures (*both, movie, video, none*) and (*video, none*) presented in Propositions 1 and 2, respectively, four other structures can arise: (*both, video, none*), (*both, movie, none*), (*movie, video, none*), and (*movie, none*). We include their complete formal presentation in appendix A.3 while providing a higher level commentary here. If we start from the structure (*both, movie, video, none*) described in (3), there are two forces that can induce the *both* segment to wane, leading to (*movie, video, none*). First, a later video release time reduces the value associated with the video and can negatively impact *both* demand. Second, lower content durability can reduce the size of this segment as well. In contrast, the consumer market structure described by (*both, video, none*) requires higher durability and earlier video release times. However, this structure can only arise for films of sufficiently low and high value. Moderate value films always have a *movie* segment with a positive measure. Finally, the remaining two structures (*both, movie, none*) and (*movie, none*) arise when the video release time is significantly delayed. These consumer market structures are of less interest since in most cases, studios optimally choose T to generate some video demand.

5 Optimal Release Time Characterization

In section 4, we examined how quality decay, congestion, and film durability affect the consumption of a film both in theaters and on video. As demonstrated in Propositions 1 and 2, the video release time strongly interacts with these three factors and can consequently induce diverse consumer market structures in equilibrium. In this section, taking into consideration the characterization of consumers' responses developed in section 4, we formally examine the optimal release time choice problem faced by studios. We then characterize the market environments under which day-and-date strategies are effective as well as how an appropriate video release time strategy can be tailored to fit film-specific marketing and operational characteristics.

We denote the demand for the movie by

$$D_m \triangleq \int_{\mathcal{V}} \mathbf{1}_{\{\sigma^*(v) \in \{(f^{-1}(v),B),(f^{-1}(v),N)\}\}} \, dv \,, \tag{10}$$

which measures the population of consumers whose equilibrium strategy includes viewing the film in a theater. Similarly, we denote the aggregate demand for the video by

$$D_b \triangleq \int_{\mathcal{V}} \mathbf{1}_{\{\sigma^*(v) \in \{(f^{-1}(v),B),(\infty,B)\}\}} \, dv \,. \tag{11}$$

Since the marginal cost of satisfying an extra consumer, whether in a theater or by providing a video, is fairly small, we make a simplifying assumption that it is zero. Recall that the main objective of this paper is to develop an understanding of how consumers adapt their purchasing strategies to changing video release times, and, subsequently, to characterize how studios can optimize their release timing. Hence, we abstract away from the vertical contracting and determination of prices, treating them as exogenous parameters in our model.[15] We denote the studio's share of movie revenues with $\tilde{p}_m \leq p_m$ and of video profit margins with $\tilde{p}_b \leq p_b$. The studio's profit function and

[15]Taking prices as exogenous is common when pricing is not the focus of the study (see, e.g., Lehmann and Weinberg 2000). In our concluding remarks, we provide some sensitivity analysis for our results.

its optimal release time problem can be written as

$$\max_{T\in[0,\infty)} \quad \Pi(T) \triangleq \tilde{p}_m D_m + \tilde{p}_b D_b \tag{12}$$

$$s.t. \quad \sigma^*(\cdot\,|\,T) \text{ satisfies } (2)\,.$$

As can be seen in (12), prices have a direct effect on studio profits while all parameters, including release time and prices, indirectly influence profitability through their impact on the strategic consumption behavior of consumers. Also, given video release time T, social welfare can be measured by

$$W(T) = p_m D_m + p_b D_b + \int_{v\in\mathcal{V}} V(v, \sigma^*(v\,|\,T))\,dv\,. \tag{13}$$

In figure 2, which illustrates how release time affects consumer market structure and profitability, we examine the case where film durability falls within an intermediate range (e.g., $\delta = 0.64$ in the plot). In panel (a), we plot the typical shape of the studio's profit curve, which demonstrates that the optimal video release time can be either zero or one of two local interior solutions. Panel (b) of figure 2 shows the corresponding equilibrium consumer market structure also as a function of the release time to clearly demonstrate its underlying effect on profit. When T is low and the video is a close substitute for the movie in terms of quality, consumers are not willing to make multiple purchases, provided that δ is low to intermediate in magnitude. Consequently, the resulting consumer market structure (*video*, *none*) is characterized by a sizable population of consumers who only purchase the video, as depicted in region A (i.e., $T < 7.428$) of panel (b). Within this region, an increase in T reduces only the value of the video substitute, which in turn causes profits to decrease, as indicated in the corresponding portion of panel (a).

As the video release time further increases to region B (i.e., $7.428 < T < 9.104$) in panel (b), the quality of the video is decayed so that the most quality-sensitive consumers now prefer to watch the movie in theaters as well as on the video, which then results in the consumer market structure (*both*, *video*, *none*). This shift in the consumer market structure is associated with increasing profits under a lower T (< 8.83) due to the increase in the *both* segment as illustrated in panel (a). However, as we begin to sharply lose video demand in the right hand portion of region B (higher T) and then subsequently lose repeat purchasers in region C (i.e., $9.104 < T < 9.132$), profits diminish rapidly. In region D (i.e., $9.132 < T < 9.414$), which corresponds to the consumer market structure (*movie*, *video*, *none*), a continued increase in T can again have a profitable effect since it can now induce video purchasers to become movie viewers; this substitution result is illustrated in the corresponding region of panel (a). Finally, for sufficiently large T, the video is no longer a rational alternative and the studio's profits are not affected by changes in release time.

Note that when content durability (δ) is low, it is relatively difficult for the studio to induce multiple purchases; in this case, the market structures corresponding to regions B and C from panel (b) would not occur. As a consequence, the studio only considers either releasing the product

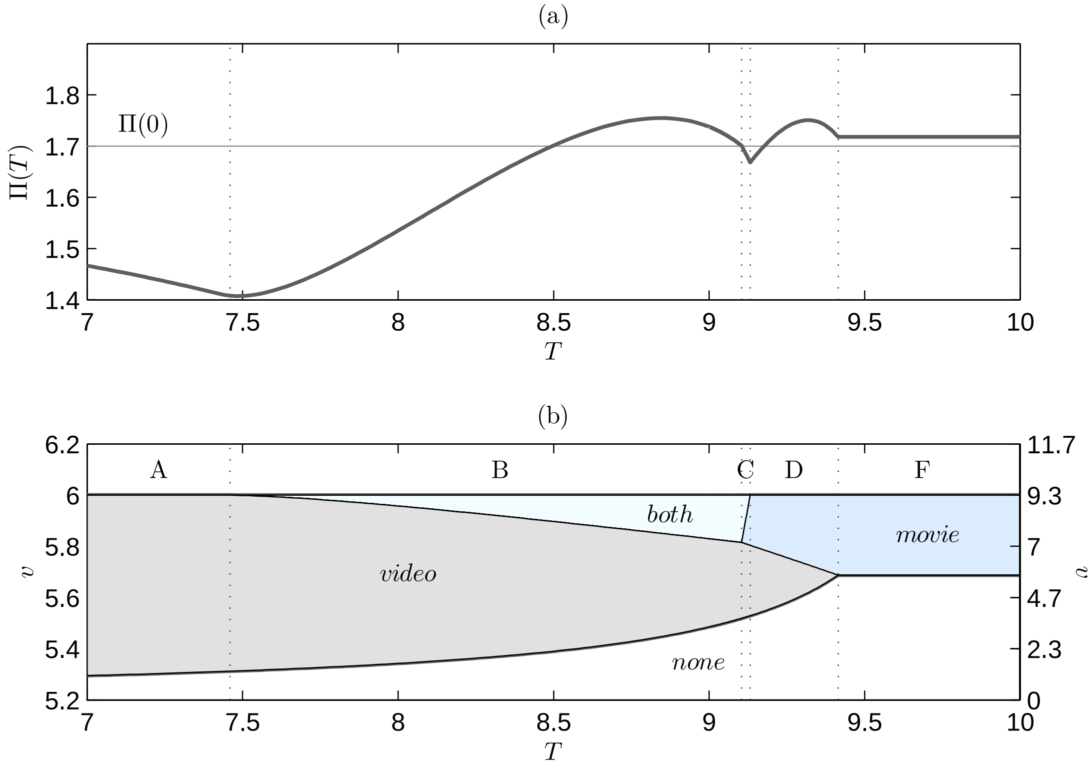

Figure 2: The effect of video release time on the studio's profit and the consumer market structure. In panel (a), the studio's profit is plotted as a function of video release time with its direct-to-video profits $\Pi(0)$ also displayed. In panel (b), consumers' optimal strategies are identified by the two-dimensional regions labeled *video*, *both* (these consumers make multiple purchases), *movie*, and *none*. We separate regions in T by consumer market structure: A - (*video*, *none*); B - (*both*, *video*, *none*); C - (*both*, *movie*, *video*, *none*); D - (*movie*, *video*, *none*); E - (*both*, *movie*, *none*); and F - (*movie*, *none*). Throughout the paper, the labeling of these regions is consistent, e.g., "C" always identifies the consumer market structure where all strategies are represented in equilibrium as formally characterized in section A.2 in the appendix. To produce a clear illustration, the curve that governs the bottom of the shaded regions, *video* and *movie* (or upper boundary of the region labeled *none*), is plotted against the secondary (right-hand) y-axis while the other curves use the primary y-axis. The parameter values for both panels are $\alpha_m = 0.1$, $\beta = 0.1$, $\bar{v} = 6$, $p_m = 5.5$, $p_b = 0.3$, $\tilde{p}_m = 5.5$, $\tilde{p}_b = 0.3$, $\gamma_m = 1$, $\gamma_b = 0.9$, and $\delta = 0.64$.[16]

at time zero or choosing a time long enough to operate in an analog of region D. This difference highlights how film durability affects the studio's release time decision.

At this point, we have completely specified how consumers segment in response to congestion and film characteristics, including content durability, quality, and video release times. Furthermore, we have formulated the studio's optimal release timing problem and demonstrated how its release time

[16]In all figures presented in this paper, the set of numerical parameters listed in the caption simply reflects the conditions of the proposition being illustrated. As formalized in the proposition statements, the essence of each illustration will hold over wide parameter ranges and in no way critically depends on the representative set of parameters employed.

interacts with the consumer market structure and profitability. Having built up this understanding, we now turn our attention to answering our next primary research question by characterizing when day-and-date strategies are effective.

5.1 When to employ day-and-date and direct-to-video strategies

One open question that our framework enables us to explore is whether day-and-date strategies can ever maximize profits. In this section, we investigate how congestion constraints and content durability can influence the profitability of day-and-date strategies, and we clarify the conditions under which simultaneous release in theaters and videos can be an effective tool to manage moviegoers' consumption incentives.

We begin by discussing the case in which the content durability of a film is high. In this case, whether a consumer has viewed the movie in a theater has little negative effect on her incentive to purchase the video. Thus, the incentive for consumers to choose *movie* instead of *both* decreases as the release time shifts earlier, and, as we see in the following proposition, a day-and-date release is then an optimal strategy.

Proposition 3 *When the content durability of a film is sufficiently high, a studio should optimally pursue a day-and-date strategy. Technically, there exists* $0<\underline{\delta}<1$ *such that* $T^*=0$ *whenever* $\delta>\underline{\delta}$*, and, in equilibrium,* $D_m>0$*.*

If content durability is sufficiently high, then studios should release their films concurrently in theaters and on video since movie demand will not be significantly cannibalized. For example, when $\delta=1$, consumers treat the purchase of both alternatives as independent decisions; there is thus no cannibalization of movie demand. One practical implication is that a studio may want to consider a day-and-date strategy for both children's movies and films with established subcultures such as *Star Wars*, which often have high content durability. However, as we discussed in the introduction, there are also many cases where durability is not high. Going forward, we focus on a lower range of durability and examine other driving forces that, taken together, can sway a studio to pursue a day-and-date strategy. For example, large congestion costs also lead to immediate release strategies, as formalized in the following proposition:

Proposition 4 *When the content durability of a film is relatively low and movie congestion cannot be cost-efficiently reduced, then a studio should optimally pursue either a day-and-date or a direct-to-video strategy. Technically, if* $\delta\leq\delta_n$ *and* $\tilde{p}_b/p_b\geq\tilde{p}_m/p_m$*, then the following holds:*

(i) If $\bar{v}<(p_m-p_b)/(\gamma_m-\gamma_b)$*, then there exists a threshold value* $\bar{\alpha}_m$ *such that* $T^*>0$ *if and only if* $\alpha_m<\bar{\alpha}_m$*. Furthermore, if* $\alpha_m\geq\bar{\alpha}_m$*, then* $D_m=0$ *in equilibrium.*

(ii) If $\bar{v}\geq(p_m-p_b)/(\gamma_m-\gamma_b)$*, then there exists* $\underline{\alpha}_m>0$ *such that* $T^*=0$ *whenever* $\alpha_m\geq\underline{\alpha}_m$ *is satisfied. Furthermore,* $D_m>0$ *in equilibrium.*

In Proposition 4, we study the video release timing problem under conditions in which the movie and video versions are highly substitutable, i.e., the film has low durability ($\delta \leq \delta_n$). When δ is low, there is less incentive for people to consume both product forms by making multiple purchases; in this case, the focus lies on the substitution effect between a film's theatrical and video forms. Under these conditions, the viability of an early release strategy hinges on how congestion is controlled at the theatrical exhibition level. Part (i) demonstrates that for lower value films, the studio optimally sets $T^* = 0$ when the congestion cost parameter is sufficiently high. Furthermore, in equilibrium, the *both* and *movie* consumer segments are absent, i.e., $D_m = 0$. In this case, immediate release corresponds to a direct-to-video strategy since the induced consumer market structure consists of video demand only, as given in Proposition 2. This implies that low value movies with low durability should be released direct-to-video. Indeed, movies actually released direct-to-video, such as *American Pie Presents: Beta House*, often belong to this category (Barnes 2008).

To understand the finding in part (i) and to gain insight into the underlying structure of the analysis, the profit maximization problem given earlier in (12) is first transformed in terms of τ, and its first-order condition then has either no solution or two solutions. If there is no solution, then $T^* = 0$. Otherwise, the larger solution yields a local maximum and is mapped back to a strictly positive release time, which is optimal if the corresponding profit is greater than that under immediate release. From the characterization of these potential solutions of the optimal release time problem, part (i) of Proposition 4 establishes that whether T^* equals zero or lies in the interior critically depends on the impact of congestion on consumers at the theater. If congestion externalities are sufficiently weak, it is optimal to delay the release of the video. On the other hand, if the cost of congestion is significant, the studio is better off releasing the video immediately, i.e., taking a direct-to-video approach. For lower value films, the potential to generate substantial revenues in the theaters is limited. Furthermore, it is relatively more difficult to satisfy the condition $\alpha_m \leq \bar{\alpha}_m$ since the threshold $\bar{\alpha}_m$ increases in $\bar{v}$. If we assume that costs increase in order to reduce the effective congestion parameter, whether through additional screenings or other means, and that these costs are partially absorbed by the studio, our result indicates that, for a lower value film, a studio is more likely to benefit by releasing it straight to video, skipping a theatrical release.[17]

For hit films, an immediate video release is still optimal when theatrical movie congestion is high, as can be seen in part (ii) of Proposition 4. In this case, an early video release is a genuine day-and-date strategy since there exists a segment of consumers who choose *both*. However, for hit films, the congestion condition is relatively easier to satisfy due to the higher $\bar{v}$; in those cases, delaying video release is more likely to be optimal. But there are still circumstances where a studio might have a hit film and find this constraint difficult to satisfy. For example, if a studio expects

[17]In the remainder of the paper, we have simply taken $\tilde{p}_m = p_m$ and $\tilde{p}_b = p_b$ since all of the results and intuitions we present hold for a wide range of revenue shares. Although contracting between the studios, retailers, and theaters is important, it falls outside the scope of this study, which focuses on identifying the main effects of video release time on consumption and the optimality of earlier releases. We discuss several extensions related to contracting in the concluding remarks.

to have to compete against an even stronger film in theaters, especially during peak seasons such as late December, it may choose to alter its theatrical release time; several examples of this have occurred in the past (Krider and Weinberg 1998, Cole 2007). One interesting result suggested by our model is that a day-and-date strategy may also be effective in this type of scenario. In situations with other strong films playing in theaters at the same time, it can be atypically costly for the studio to ensure it has enough screens. In this case, $\alpha_m \geq \underline{\alpha}_m$ is likely to be satisfied, and a day-and-date strategy can be optimal since the studio can substantially boost its profits from video sales while still satisfying the segment of theatrical demand that can tolerate a higher congestion externality due to their higher willingness to pay.

Thus far, we have established that day-and-date strategies are effective for (*i*) films with high content durability, and (*ii*) hit films with low content durability when congestion cannot be cost-efficiently reduced. However, for lower value films, direct-to-video strategies tend to be optimal under the conditions stated in (*ii*). We next explore the remaining range of durability to see if day-and-date strategies can also be effective for hit films when congestion is at a lower level. We find that there is a distinct lower region of durability disconnected from the higher region discussed in Proposition 3, where consumer incentives can be targeted in a different way by employing a day-and-date strategy.

Proposition 5 *When congestion is low and content durability is not too high, there exists an intermediate range of durability where a day-and-date strategy is also optimal. Outside of this range, video release times should be strictly positive. Technically, there exist* $\hat{\alpha}_m$, $\bar{v}_u$, $\bar{\delta}_1$, $\bar{\delta}_2$, *and* $\bar{\delta}_3$ *satisfying* $0 < \bar{\delta}_1 < \bar{\delta}_2 < \bar{\delta}_3$ *such that if* $\alpha_m \leq \hat{\alpha}_m$, $\bar{v} > \bar{v}_u$, *and* $\delta \leq \bar{\delta}_3$, *then a day-and-date strategy is optimal if and only if* $\delta \in [\bar{\delta}_1, \bar{\delta}_2]$.[18]

As we have established, it is quite difficult to induce multiple purchases when content durability is low. However, there are still cases where a studio can strengthen consumers' multiple purchasing incentives by strategically changing the timing of its video release. In Proposition 5, we formally establish that there is another important instance where a studio should consider utilizing a day-and-date strategy. Specifically, when film durability is within a lower-to-intermediate range, there is a small region where studios have incentives to optimally implement such a strategy. Within this range of film durability, consumers do not have naturally strong incentives to make multiple purchases. Nonetheless, studios can benefit in two ways from an early video release. By selecting $T = 0$, they can expand the market at the low end. Also, since durability is slightly higher, an early release strategy can actually provide sufficient incentives to induce the high end of the market to consume both the movie and the video. In aggregate, these two effects increase revenues to an extent that outweighs the losses associated with cannibalization of movie revenues when some consumers substitute to the video. Therefore, even though durability is within a lower range in this case, the studio should still optimally employ a day-and-date strategy.

[18] The characterization of $\bar{\delta}_1$, $\bar{\delta}_2$, $\bar{\delta}_3$, and $\bar{v}_u$ is provided in the appendix.

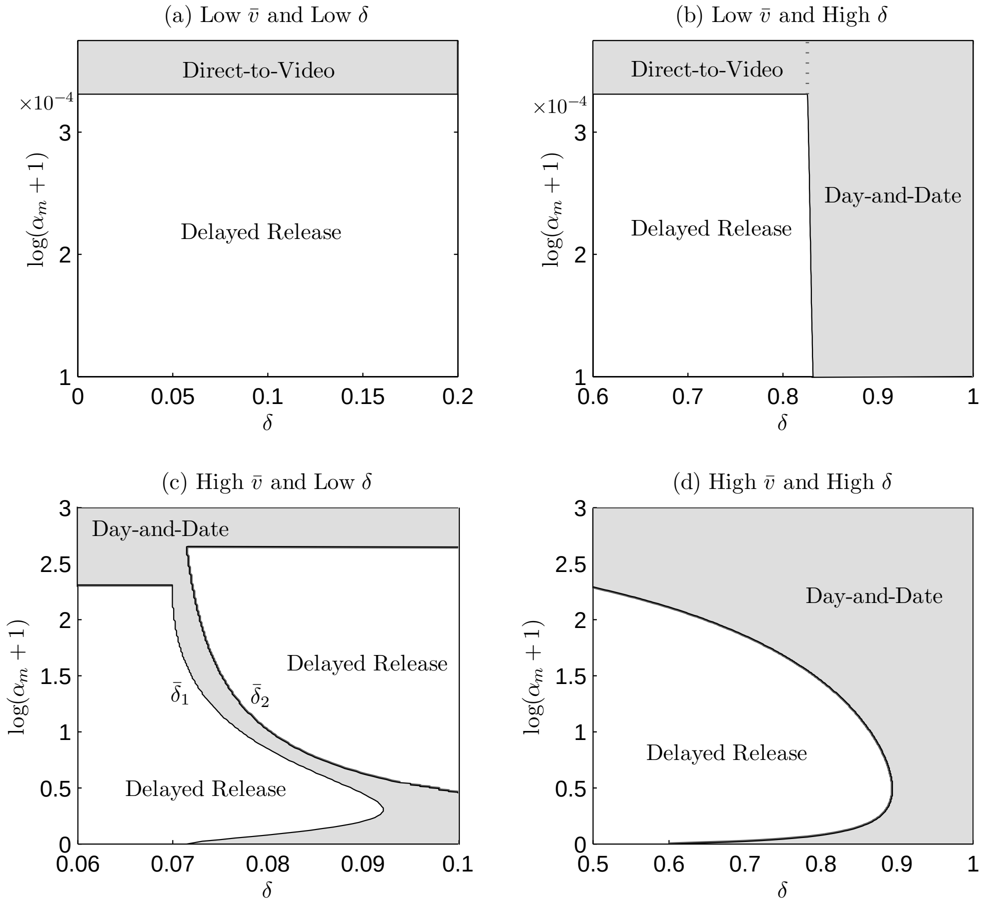

Figure 3: The optimal video release strategies. The parameter values are $\beta=3$, $p_m=20$, $p_b=7$, $\gamma_m=5$, $\gamma_b=2$, $\bar{v}=50$ for high $\bar{v}$, and $\bar{v}=4.3$ for low $\bar{v}$.

One film genre that can have content durability falling within this range is the documentary. Although documentaries have traditionally been made for television or focused on direct-to-video releases, in modern times some documentaries have been mildly successful at the box office (e.g., *Fahrenheit 9/11*, *Super Size Me*, *An Inconvenient Truth*). Since the box office revenues generated by even the most popular documentaries are orders of magnitude less than those from blockbuster action films, a day-and-date can be a profitable strategy that generates much larger video revenues while still providing short runs in theaters.

Figure 3 graphically summarizes the results from this section. Here, we classify the optimal release time policy into three categories: day-and-date, direct-to-video, and delayed release. Each category of the policy is presented as determined by the level of congestion and the content durability of a film. Panels (b) and (d) illustrate the wide regions in which day-and-date strategies are optimal under high content durability for lower value and hit films, respectively, as formally established in Proposition 3. The results from Proposition 5 can be seen clearly in panel (c). Although delayed release strategies ($T^*>0$) are optimal for sufficiently low and intermediate δ ($\delta<\bar{\delta}_1$ and $\delta>\bar{\delta}_2$), there exists a medium-low range ($\delta\in[\bar{\delta}_1,\,\bar{\delta}_2]$) where a day-and-date strategy is effective.

Together, panels (c) and (d) show how the day-and-date region forms an annulus that splits the delayed release region into two disconnected portions. However, under low content durability, lower value films should be treated differently. As can be seen in panel (a) of figure 3, a day-and-date strategy is never optimal under lower durability. Furthermore, under high congestion costs, a direct-to-video strategy is taken instead, which is consistent with part (i) of Proposition 4. In fact, figure 3 suggests that a delayed release strategy is optimal for a significant portion of the parameter space. Thus, in the next section, we turn our attention toward building an understanding of how the delayed release time should be adjusted within these regions.

5.2 Delayed release strategies

In Proposition 5, we saw that when congestion is low, there exists a lower interval of film durability in which a day-and-date strategy is optimal despite the fact that delayed releases are preferable locally outside of the interval. However, when release times are strictly positive, one important question is how they should be set to engender higher profits. Moreover, the way that optimal video release times are affected by congestion, durability and other factors can also have significant implications beyond the release time decision itself. For instance, social welfare can be negatively affected by these factors, a phenomenon we highlight in the following proposition.

Proposition 6 *A small decrease in film durability can result in significant welfare losses.*

Although day-and-date strategies are not always profit-maximizing, they can certainly enhance social welfare. Since these strategies make both theatrical and video versions of the film available early, consumer surplus can be much higher because (i) more consumers make multiple purchases, (ii) consumers who would have purchased the video anyway can now purchase it earlier, before its quality decays, and (iii) consumers who were left out of the market can now consume the video and obtain positive surplus. However, as Proposition 6 indicates, reducing film durability even slightly can cause a shift from the optimality of a day-and-date strategy to one characterized by a much delayed release.

The essence of Proposition 6 is illustrated in figure 4. When a studio cannot effectively induce multiple purchasing behavior by consumers, then it should usually delay its video release for a long enough time to prevent excessive substitution by would-be movie consumers. However, a day-and-date strategy means releasing the video immediately at $T = 0$. Therefore, the optimal release time (T^*) can be discontinuous when viewed as a function of either content durability or congestion. In panel (b) of figure 4, we illustrate how T^* jumps from zero to a significantly higher level as durability decreases out of the channel described by Proposition 5, since the studio completely alters its release strategy. Because welfare greatly suffers when a studio adapts in this manner, policy can play an important role. For example, social planners might consider subsidizing day-and-date releases or strategically taxing/subsidizing video purchases to expand the region of lower film durability in which studios find it optimal to pursue day-and-date strategies. Of course, how

(a) Optimal release strategy for low $\bar{v}$ movies (b) Optimal release strategy for high $\bar{v}$ movies

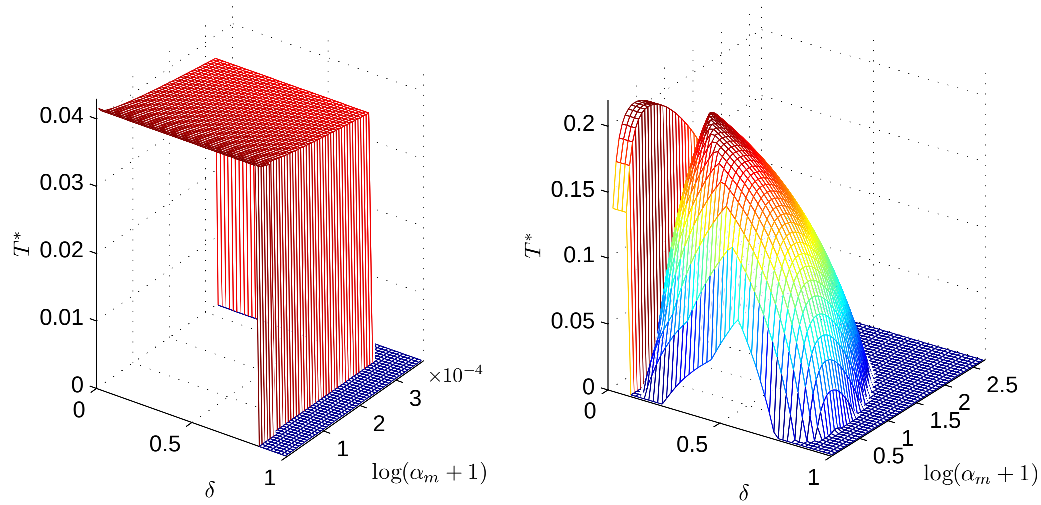

Figure 4: The impact of congestion externalities and consumer incentives toward making multiple purchases (viewing the movie in theaters and buying the video) on the studio's optimal video release time. The parameter values are the same as those in figure 3.

to efficiently implement policy to encourage better outcomes as studios move toward earlier release times will be a delicate issue, but our results clearly indicate its potential. For example, if $\delta = .09$, $\alpha_m = 0.65$, and if we take other parameters as in panel (b) of figure 4, then a small decrease in film durability causes a substantially delayed video release time, and consequently induces welfare to decrease by approximately 35 percent.

Similar welfare losses can result for lower value films, as can be seen in panel (a) of figure 4. In this case, a small reduction in either durability or congestion can lead to substantially delayed video release times, which hurts welfare. However, this means that a higher congestion level can sometimes be indirectly beneficial to welfare through its effect on the studio's selection of T^*. Since congestion itself has negative welfare consequences, it may be useful to provide other incentives for studios to adopt day-and-date releases. Figure 4 also suggests that the manner in which a delayed release strategy is managed differs significantly for lower value and hit films. For hit films, moreover, the shape of the optimal release time surface presented in panel (b) of figure 4 suggests that a studio must carefully adjust its video release time when considering how the durability of the film affects consumers' incentives. We next explore the optimal release time T^* when it falls into the interior of the *delayed release* strategy region (i.e., an intermediate range of durability lying between the two regions of durability in which a day-and-date strategy is optimal), in order to provide more insight into how the studio adapts its video release time. In particular, we clarify what drives it first to delay release as durability improves, and then to strategically advance the release time as durability continues to improve.

Proposition 7 *For a film characterized by content durability on the lower end of the intermediate range, the higher the content durability of a film, the longer its video release time should be delayed.*

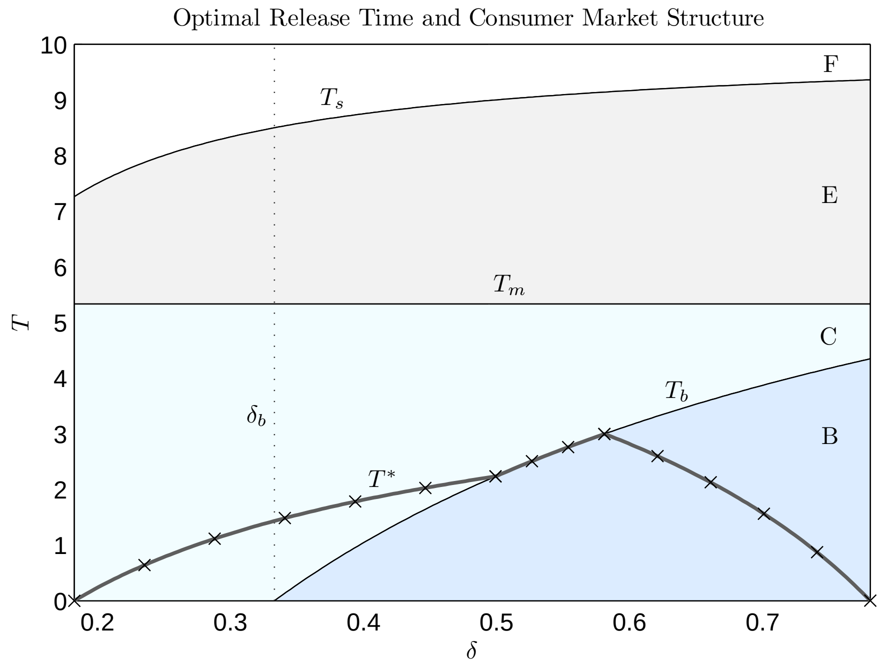

Figure 5: The impact of content durability on a studio's optimal delayed release strategy. Regions B, C, E, and F correspond to the consumer market structures explained in figure 2 and in section A.2 in the appendix. The boundary values for release times (T_s, T_m, T_b) and film durability (δ_b) are characterized in section A.1 in the appendix. The parameter values are $\alpha_m = 0.5$, $\beta = 0.1$, $\bar{v} = 10$, $p_m = 2$, $p_b = 0.5$, $\gamma_m = 3$, and $\gamma_b = 1$.

Technically, there exist $\hat{\alpha}_m$, $\hat{\delta}$, and $\bar{\gamma}_b$ such that if $\alpha_m \leq \hat{\alpha}_m$, $\delta \in [\delta_b, \hat{\delta}]$, $\bar{v} \in [(p_m - p_b)/(\gamma_m - \gamma_b), \bar{v}_s]$, $\gamma_b < \bar{\gamma}_b$, and $p_b/\gamma_b > (\sqrt{5} - 1)(p_m - p_b)/(2(\gamma_m - \gamma_b))$ are satisfied, then T^* is increasing in δ.

One important result analytically demonstrated in Proposition 7 is that as δ increases away from the lower channel of day-and-date optimality, T^* also increases, i.e., *increasing film durability can actually encourage a studio to delay release.* Figure 5 helps to explain the intuition behind this result. In this figure, the shaded regions marked by capital letters indicate different consumer market structures.[19] For fixed $\delta < \delta_b$, one of three consumer market structures (C, E, or F) can be induced depending on the studio's choice of T; for $\delta > \delta_b$, there is an additional possibility (B). When prices are fixed, a studio's top preference is for the consumer to purchase both forms; its second preference is a movie purchase alone, and third is a video purchase alone. When δ is low, it is relatively difficult to induce substantial multiple purchasing behavior. In fact, the studio must decrease the timing of its video release to near zero to prevent quality decay of the video and thereby make this option more attractive to consumers. However, decreasing T cannibalizes the demand of consumers who may have been willing only to purchase the movie but who are now incentivized

[19] As in the caption of figure 2, the labeling is consistent with the consumer market structures presented in section A.2 of the appendix. Furthermore, the numerical values for the parameters are such that the equilibrium strategy profile satisfies either (A.31) or (A.34).

to consume the video option instead. As δ increases slightly, for the same release time, consumers naturally have greater incentives to make multiple purchases, which relaxes this pressure on the studio's release time. In this case, the studio is able to increase T in order to incentivize some consumers to purchase the movie instead of waiting for the video substitute. Since the movie is more profitable, the studio optimally increases T^* but only to a limited extent, since delaying the video release also hurts the studio at the tail end of consumer type space where it loses potential customers for the video alternative. As δ increases further ($\delta > \delta_b$), the multiple purchase option becomes more attractive for consumers. Thus, if the studio releases the video too early ($T \leq T_b$), there will not be a segment of consumers who consume only the movie in equilibrium (see region B in figure 5) since the alternatives (consuming both or just the video) provide much more surplus. Because inducing some consumers to purchase just the movie instead of just the video is profitable, the studio must delay the release time even further.

In the left-hand portion of figure 5, not only has the release time once again become strictly positive, but it also continuously increases away from a day-and-date strategy. This result has an important empirical implication. Due to high variability on the many dimensions that characterize films, we can expect that δ will vary significantly film by film. As the movie industry moves toward having more day-and-date releases, our model suggests that we should see video release times spanning the feasible range. As Disney CEO Robert Iger commented, "we're not doing a one-size-fits-all approach" (quoted in Smith and Schuker 2010); we should therefore not expect that videos will be released according to the current binary standard: either immediately or 3–4 months later. Instead, we expect to see the full timespan utilized due to the diversity of film characteristics, and this prediction is a potentially testable implication of the model. Furthermore, if we look at the α_m cross-sections (i.e., plotting T^* as a function of δ for a given α_m) in panel (b) of figure 4, our results suggest that the movies whose videos should be released the latest in time are the ones that have both higher content durability and low congestion upon theatrical release. However, as congestion increases, it is films with lower content durability which should be the ones delayed the longest.

Studying the optimal release timing of sequential distribution, Frank (1994) establishes that video windows should be longer when the substitutability for video over movie is higher. In our model, the impact of this substitutability is greater under low content durability than under high durability. In that sense, one may interpret higher content durability as lower substitutability. Under this interpretation, our finding in Proposition 7 states that as substitutability increases, the video window should be shorter. This result stands in contrast to the findings in Frank (1994), which highlights the fact that the combined effects of multiple purchasing behavior and congestion can provide powerful incentives to reduce the video window. On the other hand, as film durability increases further, other driving forces fundamentally push the release time in the opposite direction in response to an increase in durability, which is more consistent with Frank (1994). The following proposition demonstrates this result and clarifies how a studio's management of congestion externalities hinges on the consumer market structure being induced.

Proposition 8 *For a film characterized by content durability on the higher end of the intermediate range, the higher the content durability, the earlier its video release time should be. Technically, there exist* $\hat{\alpha}_m$*,* $\bar{\delta}_4$*,* $\bar{\gamma}_b$*, and* $\bar{p}_b$ *such that if* $\alpha_m \le \hat{\alpha}_m$*,* $\bar{v} \in [(p_m - p_b)/(\gamma_m - \gamma_b),\, \bar{v}_s]$*,* $\gamma_b < \bar{\gamma}_b$*,* $p_b < \bar{p}_b$*, and* $\delta \in [\delta_b,\, \bar{\delta}_4]$*, then*

$$T^* = \frac{1}{\beta}\left(1 - \frac{\gamma_m p_b}{\gamma_b\sqrt{1-\delta}\,\left(p_m + p_b\sqrt{1-\delta}\right)}\right) + O(\sqrt{\alpha_m})\,, \tag{14}$$

which is decreasing in δ*.*

Proposition 8 establishes that when δ gets large, the marginal effect of having a film with higher durability is substantially different than what we found for films at the lower end of this intermediate range of content durability. In contrast to delaying the video release time in order to provide incentives for consumers who would otherwise purchase only the movie option, in this case the studio should release the video earlier, as indicated by the proposition; this is also illustrated by the way the optimal T^* curve cuts across region B, which corresponds to the market structure (*both*, *video*, *none*), in figure 5.

When δ is low, the optimal release time is in the interior of region C as illustrated in the figure. Moreover, region C is characterized by having all the various consumer segments represented. In that case, for a fixed T, an increase in δ has no effect on the theater run of the movie, $\tau(T)$. That is, theatrical movie demand does not depend on δ. If the studio attempted to decrease T for slightly higher δ, the net effect would be increased congestion and a shorter run in theaters. However, when the consumer market structure falls into region B, increasing δ has a direct and positive effect on the run of the film. As a result, the studio is able to advance the video release time while still maintaining a longer run in theaters for the film. When a film's durability is sufficiently high, consumers have quite strong incentive to make multiple purchases. The studio can thus advance the video release time without causing too many consumers to abandon making purchases in favor of consuming only the video. Furthermore, by reducing T^*, it also reduces the quality decay of the video which increases the potential population of video purchasers. It is important to note that under this consumer market structure, an increase in δ provides the studio with some slack by directly increasing the run in theaters; the studio is thus able to reduce T^* to benefit from the increased incentives of the two segments of consumers (*both* and *video*) while avoiding a relative increase in congestion and a shorter run. Therefore, for this range of film durability, we see that the studio should decrease T^* as δ increases, as illustrated in figure 5.

As technology rapidly evolves, its applications to theater equipment often precedes applications to consumer electronics. For example, the last few years have seen a rebirth of 3-D theatrical releases driven by improvements in 3-D technology, as seen with *Avatar*, *Alice in Wonderland*, and *Clash of the Titans*. However, 3-D televisions have just recently become available and are not yet

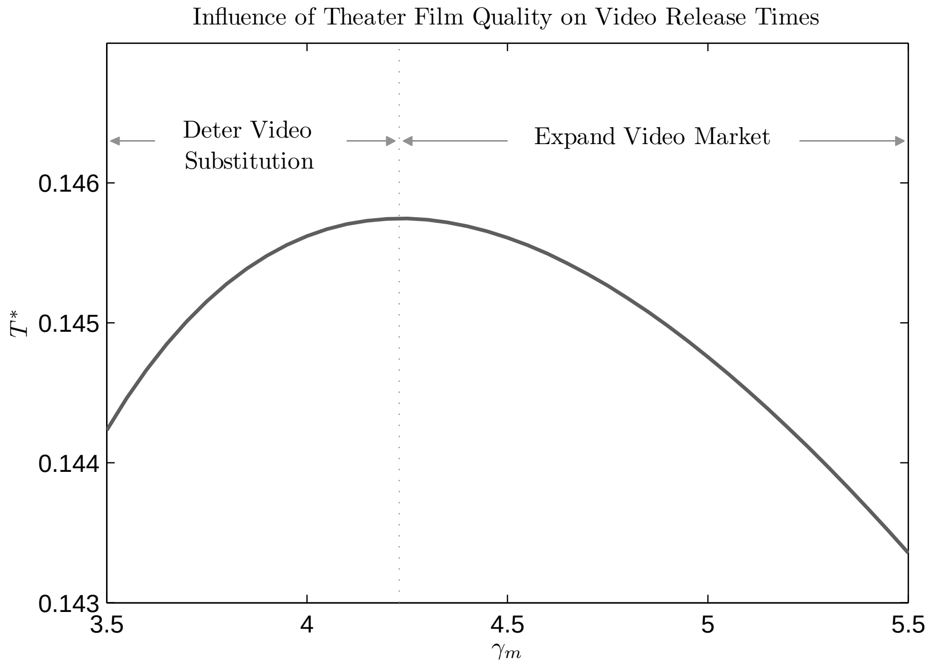

Figure 6: Nonmonotonicity of the optimal release time in theater quality. The parameter values are $\alpha_m = 3$, $\beta = 3$, $\bar{v} = 50$, $p_m = 20$, $p_b = 7$, $\gamma_b = 2$, and $\delta = 0.4$.

in widespread use. This example demonstrates that the relative quality of theatrical and video offerings can sometimes vary over time. To better understand the effects of shifts in technology, we explore how changes in γ_m impact optimal release timing.

People commonly believe that low quality films are the ones that should be released earlier to video (Frank 1994, Epstein 2005). This intuition is often associated with the observation that very low quality films sometimes bypass theatrical release entirely and appear directly on home video. Although it seems reasonable that higher quality movies are more likely to have longer release windows, this conclusion is not always justified. One important implication of our model is that an increase in the inherent quality of a movie should sometimes be coupled with an *earlier* release time. Thus, the optimal release time is actually nonmonotonic in the inherent quality of the theatrical version of the film.

Figure 6 illustrates how the optimal release time changes with the quality of the movie. An increase in γ_m has several effects. First, consumers have increased incentive to both consume the film in theaters and become multiple purchasers. Second, if more consumers view the film in theaters, in equilibrium, they must smooth out their timing of consumption to manage congestion externalities. However, if the run in theaters lengthens, consumers who would optimally view the film toward the end of its run now have increased incentive to consume the video substitute instead because it is available after a shorter wait. For lower levels of γ_m, consumers do not have particularly strong incentive to consume the movie in theaters; as a result, when γ_m increases, a studio must optimally delay the video release time in order to deter substitution, as illustrated in the left-hand portion of figure 6.

On the other hand, one surprising implication of our model is that higher quality theater experiences can also lead to earlier optimal video release times. The right-hand portion of figure 6 illustrates this behavior. An increase in γ_m leads to effects similar to those described above; however, at larger values of γ_m, consumers naturally already have greater incentives to consume the movie. In this case, the substitution effect is not as strong, and a studio does not need to increase T^* to deter consumers from substituting. Rather, advancing the release time can be profitable to the studio since it increases the size of the video market at the lower end. Moreover, since the studio can advance the release time without inducing too much cannibalization, it optimally does so to expand the video market in this case.

6 Concluding Remarks

In this paper, we presented a model of film distribution and consumption to gain insight into how studios should optimally time the release of video versions of their films when accounting for strategic behavior of consumers in product choice and timing. We studied the video release timing problem using a normative approach in order to highlight the critical factors that can motivate studios to release videos substantially earlier in time, or alternatively, purposefully delay their release. In developing the consumer model, we incorporated several important factors: (*i*) quality decay of the film content over time; (*ii*) the quality/price gap between theater and video alternatives; (*iii*) consumption of both the theatrical and video version and quantification of these preferences using a notion of content durability; and (*iv*) negative consumption externalities associated with congestion at theaters. These factors, which are in turn influenced by release timing, together affect substitution by consumers of one product version for the other. Using this model, we fully characterized the consumer market equilibrium and discussed the possible structures that can arise endogenously. In equilibrium, consumers making incentive compatible product choices and consumption timing decisions in continuous time naturally give rise to an aggregate demand that is exponentially decaying over time. Our consumer equilibrium is thus consistent with empirically observed consumption behavior and provides a good starting point for analyzing a variety of topics related to film distribution.

Using our equilibrium characterization, we addressed the main objective of our work, which was to examine the optimal video release timing problem faced by studios. We analyzed a variety of dimensions, including whether a film is a hit, how durable its content is in terms of repeat viewings, and whether congestion in theaters can be effectively controlled. We demonstrated that a day-and-date release can be an optimal strategy for both medium-low and high levels of content durability. However, since studios have incentives to significantly delay video release at sufficiently low durability, we highlighted the negative welfare implications of this behavior and suggested that policy may play an important role in expanding regions of day-and-date optimality in these cases. We also formally proved that video release time first increases and then decreases in durability, and provided insights into how studios can strategically use release time to control consumer incentives to make both multiple and substitute purchases. Lastly, we established that for relatively low quality movies, an increase in theatrical movie quality makes the optimal video release window longer, whereas for relatively high quality movies, an increase in quality tends to shorten the optimal release window.

A number of important questions remain for future research. For example, in this work, we focused our attention on the optimal video release time for a single movie. Studying a single movie setting is important since, in many cases, studios have some degree of freedom in the timing of video releases; in addition, such a model more readily clarifies the central trade-offs. This is why we did not consider competition between different movies. Competition is, however, certainly an important topic and can even be a critical factor for deciding when to schedule theatrical release, particularly during peak seasons, but the impact of competition on video release times is restricted to a few cases (Goldberg 1991). Consequently, the insights we provide in this work should remain valid for most movies. Another aspect that remains to be investigated is the release of multiple films by the same studio over time, i.e., the repeated game aspect. Along these lines, Prasad et al. (2004) specifically consider the impact of consumer expectations over time in an aggregate model. In that context, the authors commented that an interesting avenue of future research would be to start from a consumer utility model and capture the impact of consumer expectations which arise endogenously. Our paper now can serve as a building block for further research in this direction.

In this paper, we study the video release problem in the context of a fixed simple revenue sharing contract between studios and exhibitors. In practice, conventional contracts involve time-dependent revenue sharing (i.e., a sliding, increasing percentage of revenues to exhibitors) after allowance for exhibitors' expenses. But recently, some studios and exhibitors have begun to implement "aggregate settlement," a simple revenue split without sliding scales (Vogel 2007). Our model can serve as a reasonable approximation for that scenario. However, we have made simplifying assumptions in order to focus primarily on the underlying timing problems. Nevertheless, our results are quite robust and are satisfied for wide ranges of prices. For example, one of our primary findings, as stated in Propositions 3 and 5, is that day-and-date strategies are optimal for both regions of high durability and a lower, interior interval of durability.[20] Looking ahead, we believe one fruitful extension would be to rigorously analyze how the various contracts between studios, theater owners, and video retailers should be designed, particularly in light of our results on the profitability of day-and-date release strategies which are not yet to be widely employed in industry.

[20] Using the parameters for the graphical illustration in figure 3, we find these regions are indeed insensitive to the prices. In fact, the regions of day-and-date optimality continue to exist in the following ranges: $7 < p_m < \infty$ and $0 < p_b < 20$. Furthermore, as we analytically establish in the paper, all results concerning the optimality of day-and-date strategies (i.e., Propositions 3–6) only require that $p_m/\gamma_m \geq p_b/\gamma_b$ be satisfied. In addition, most intuitions extend to the region of $p_m/\gamma_m < p_b/\gamma_b$ and similar analysis can be done for that region with few technical conditions.

We assumed here that movie quality is certain and common knowledge, noting that empirical evidence concerning demand uncertainty associated with this quality is not as strong as popularly argued (see, e.g., Orbach 2004, Orbach and Einav 2007). Furthermore, most of the uncertainty is revealed after the first weekend of release. Consequently, when distributors set video release dates after uncertainties are almost resolved and consumers are well-informed, most of our results are preserved. If consumers are not informed about the quality of films, then video windows determined by distributors may signal quality, a scenario which could be a useful extension of our work. In this direction, our paper provides an interesting observation: Contrary to conventional wisdom, longer video release windows do not necessarily signal higher movie quality in a context of multiple purchases, as we demonstrated at the end of section 5.2.

We studied only wide release films that are available to the entire consumer population upon release. In contrast, platform release strategies would require a slightly different model in which the film's availability may be restricted (and may affect consumer strategy sets) over time. Since one of our primary goals here has been to characterize when day-and-date strategies are optimal, the analysis of a platform release strategy would stand in stark contrast. However, an interesting extension of our work would be to study such strategies to better understand how to manage "sleeper" hits (see, e.g., Ainslie et al. 2005) in addition to the wide release hits we focus on in this work. In the case of sleeper hits, studios may need to delay video release times further to reduce substitution effects caused by the extended theater runs.

Our work can be extended on several additional fronts to gain a richer understanding of optimal video release timing. First, although studios can predict demand fairly well, based upon pre-screenings and even the first weekend's box office performance, there is always some demand uncertainty. Our model can be extended to examine the effects of demand uncertainty on day-and-date strategies, particularly through the parameter $\bar{v}$. Second, in our study, we assumed that the studio announces the video release time fairly early on and commits to its announced dates which is reasonably credible given the repeated nature of the context. For simplicity, the announcement takes place immediately in our model; this timing is a nonissue when either day-and-date or direct-to-video strategies are optimal since the video channel is opened immediately. However, a studio's inability to announce immediately can affect consumption early on during the run of a film in theaters. Extending the model to permit slightly delayed announcements could be worth studying, especially for cases where video release times are optimally determined to be less than a month out.

This paper is a first step to increasing our fundamental understanding of why the video release window has become shorter, as well as just how short it is likely to become. Studios are demonstrably interested in the prospects of earlier release times and even the possibility of implementing day-and-date strategies. Moreover, some are considering extending these strategies across additional channels beyond these of theaters and videos, e.g., pay-per-view, cable networks, and more. Given the speed of technological advances and the enduring use of a channeling system and rules of thumb, the film industry now has a significant opportunity to design products and make decisions on delivery systems that cater more effectively to consumer preferences. Such developments have the potential to increase welfare substantially, and we hope that our work helps to initiate part of this progress.

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Appendix for “Optimal Timing of Sequential Distribution: The Impact of Congestion Externalities and Day-and-Date Strategies”

We first present the consumer market equilibrium taking the video release time as given in section A. Subsequently, in section B, we provide the proofs of the lemmas and propositions presented in the paper.

A Consumer Market Equilibrium Characterization

In this section, we first present some notation used throughout the paper to aid in the exposition. Next, we summarize the potential equilibrium consumer strategy profiles that may arise as the video release time varies over its domain. Lastly, we provide a complete characterization of the endogenous consumer market equilibrium.

A.1 Definitions

$$T_r \triangleq \frac{1}{\beta}\left(1 - \frac{\gamma_m \bar{v} - p_m}{(1-\delta)\gamma_b \bar{v}}\right); \tag{A.1}$$

$$T_n \triangleq \frac{1}{\beta}\left(1 - \frac{\gamma_m \bar{v} - p_m + p_b}{\gamma_b \bar{v}}\right); \tag{A.2}$$

$$T_s \triangleq \frac{1}{\beta}\left(1 - \frac{p_b}{\delta \gamma_b \bar{v}}\right); \tag{A.3}$$

$$\hat{\tau}_r(T) \triangleq \left\{\tau \;\middle|\; \frac{\gamma_m(1-\beta\tau) - \gamma_b(1-\delta)(1-\beta T)}{\cosh \lambda\tau} = \frac{p_m}{\bar{v}}\right\}; \tag{A.4}$$

$$\hat{\tau}_n(T) \triangleq \left\{\tau \;\middle|\; \frac{\gamma_m(1-\beta\tau) - \gamma_b(1-\beta T)}{\cosh \lambda\tau} = \frac{p_m - p_b}{\bar{v}}\right\}; \tag{A.5}$$

$$\tau_r(T) \triangleq \begin{cases} 0, & if \quad T \le T_r\,; \\ \hat{\tau}_r(T), & if \quad T > T_r\,. \end{cases} \tag{A.6}$$

$$\tau_n(T) \triangleq \begin{cases} 0, & if \quad T \le T_n\,; \\ \hat{\tau}_n(T), & if \quad T > T_n\,. \end{cases} \tag{A.7}$$

$$\tau_b \triangleq \left\{\tau \,\middle|\, \frac{1-\beta\tau}{\cosh\lambda\tau} = \frac{\delta p_m + (1-\delta)p_b}{\delta\gamma_m\bar{v}}\right\} (=\tau_r(T_b) = \tau_n(T_b))\,; \tag{A.8}$$

$$\tau_m(\bar{v}) \triangleq \left\{\tau(\bar{v}) \,\middle|\, \frac{1-\beta\tau}{\cosh\lambda\tau} = \frac{p_m}{\gamma_m\bar{v}}\right\} (=\tau_n(T_m))\,; \tag{A.9}$$

$$\tau_s(\bar{v}) \triangleq \left\{\tau(\bar{v}) \,\middle|\, \frac{\gamma_m(1-\beta\tau)-\gamma_b}{\cosh\lambda\tau} = \frac{p_m - p_b}{\bar{v}}\right\} (=\tau_b(\delta=\delta_b) = \tau_n(T=0))\,; \tag{A.10}$$

$$T_b \triangleq \frac{1}{\beta}\left(1 - \frac{p_b\cosh\lambda\tau_b}{\delta\gamma_b\bar{v}}\right); \tag{A.11}$$

$$T_m \triangleq \frac{1}{\beta}\left(1 - \frac{p_b\cosh\lambda\tau_m}{\gamma_b\bar{v}}\right); \tag{A.12}$$

$$v_r \triangleq \frac{\bar{v}}{\cosh\lambda\tau_r(T)}; \tag{A.13}$$

$$v_n \triangleq \frac{\bar{v}}{\cosh\lambda\tau_n(T)}; \tag{A.14}$$

$$\tag{A.15}$$

$$\delta_b \triangleq \frac{p_b\cosh\lambda\tau_s}{\gamma_b\bar{v}}; \tag{A.16}$$

$$\delta_m \triangleq \frac{1}{\cosh\lambda\tau_m}; \tag{A.17}$$

$$\delta_r \triangleq 1 - \frac{\gamma_m\bar{v} - p_m}{\gamma_b\bar{v}}; \tag{A.18}$$

$$\delta_n \triangleq \frac{p_b}{\gamma_m\bar{v} - p_m + p_b}; \tag{A.19}$$

$$\bar{v}_m \triangleq \left\{\bar{v} \,\middle|\, \frac{p_b\cosh\lambda\tau_m(\bar{v})}{\gamma_b\bar{v}} = 1\right\}; \tag{A.20}$$

$$\bar{v}_s \triangleq \left\{\bar{v} \,\middle|\, \frac{p_b\cosh\lambda\tau_s(\bar{v})}{\gamma_b\bar{v}} = \frac{1}{\cosh\lambda\tau_m(\bar{v})}\right\}; \tag{A.21}$$

A.2 Potential Equilibrium Consumer Strategy Profiles

In the following, *both* means there is a segment of consumers who purchase both alternatives in equilibrium, *movie* means there is a segment of consumers who purchase only the movie, *video* means there is a segment of consumers who purchase only the video, and, finally, *none* reflects that there is a segment of consumers who opt out of the market.

A. (*video, none*):

$$\sigma^*(v) = \begin{cases} (\infty,\, B) & if \quad \frac{p_b}{\gamma_b(1-\beta T)} \le v \le \bar{v}\,; \\ (\infty,\, N) & if \quad v \le \frac{p_b}{\gamma_b(1-\beta T)}\,. \end{cases} \tag{A.22}$$

B. (*both, video, none*):

$$\sigma^*(v) = \begin{cases} (f^{-1}(v),\, B) & if \quad v_r \le v \le \bar{v}\,; \\ (\infty,\, B) & if \quad \frac{p_b}{\gamma_b(1-\beta T)} \le v \le v_r\,; \\ (\infty,\, N) & if \quad v \le \frac{p_b}{\gamma_b(1-\beta T)}\,. \end{cases} \tag{A.23}$$

C. (*both, movie, video, none*):

$$\sigma^*(v) = \begin{cases} (f^{-1}(v),\, B) & if \quad \frac{p_b}{\delta\gamma_b(1-\beta T)} \le v \le \bar{v}\,; \\ (f^{-1}(v),\, N) & if \quad v_n \le v \le \frac{p_b}{\delta\gamma_b(1-\beta T)}\,; \\ (\infty,\, B) & if \quad \frac{p_b}{\gamma_b(1-\beta T)} \le v \le v_n\,; \\ (\infty,\, N) & if \quad v \le \frac{p_b}{\gamma_b(1-\beta T)}\,. \end{cases} \tag{A.24}$$

D. (*movie, video, none*):

$$\sigma^*(v) = \begin{cases} (f^{-1}(v),\, N) & if \quad v_n \le v \le \bar{v}\,; \\ (\infty,\, B) & if \quad \frac{p_b}{\gamma_b(1-\beta T)} \le v \le v_n\,; \\ (\infty,\, N) & if \quad v \le \frac{p_b}{\gamma_b(1-\beta T)}\,. \end{cases} \tag{A.25}$$

E. (*both, movie, none*):

$$\sigma^*(v) = \begin{cases} (f^{-1}(v),\, B) & if \quad \frac{p_b}{\delta\gamma_b(1-\beta T)} \le v \le \bar{v}\,; \\ (f^{-1}(v),\, N) & if \quad v_n(T = T_m) \le v \le \frac{p_b}{\delta\gamma_b(1-\beta T)}\,; \\ (\infty,\, N) & if \quad v \le v_n(T = T_m)\,. \end{cases} \tag{A.26}$$

F. (*movie, none*):

$$\sigma^*(v) = \begin{cases} (f^{-1}(v),\, N) & if \quad v_n(T = T_m) \le v \le \bar{v}\,; \\ (\infty,\, N) & if \quad v \le v_n(T = T_m)\,. \end{cases} \tag{A.27}$$

A.3 Complete Characterization of Consumer Equilibrium Outcome:

(a) $\frac{p_m}{\gamma_m} \le \bar{v} \le \frac{p_m - p_b}{\gamma_m - \gamma_b}$:

(i) $0 \le \delta \le \delta_n$:

$$\sigma^* = \begin{cases} A : (A.22) & if \quad 0 \le T \le T_n; \\ D : (A.25) & if \quad T_n \le T \le T_m; \\ F : (A.27) & if \quad T_m \le T\,. \end{cases} \tag{A.28}$$

(ii) $\delta_n \le \delta \le \delta_r$:

$$\sigma^* = \begin{cases} A : (A.22) & if \quad 0 \le T \le T_r; \\ B : (A.23) & if \quad T_r \le T \le T_b; \\ C : (A.24) & if \quad T_b \le T \le T_s; \\ D : (A.25) & if \quad T_s \le T \le T_m; \\ F : (A.27) & if \quad T_m \le T\,. \end{cases} \tag{A.29}$$

(iii) $\delta_r \le \delta \le \delta_m$:

$$\sigma^* = \begin{cases} B : (A.23) & if \quad 0 \le T \le T_b; \\ C : (A.24) & if \quad T_b \le T \le T_s; \\ D : (A.25) & if \quad T_s \le T \le T_m; \\ F : (A.27) & if \quad T_m \le T\,. \end{cases} \tag{A.30}$$

(iv) $\delta_m \le \delta \le 1$:

$$\sigma^* = \begin{cases} B : (A.23) & if \quad 0 \le T \le T_b; \\ C : (A.24) & if \quad T_b \le T \le T_m; \\ E : (A.26) & if \quad T_m \le T \le T_s; \\ F : (A.27) & if \quad T_s \le T\,. \end{cases} \tag{A.31}$$

(b) $\frac{p_m - p_b}{\gamma_m - \gamma_b} \le \bar{v} \le \bar{v}_s$:

(i) $0 \le \delta \le \frac{p_b}{\gamma_b \bar{v}}$ $(\delta_n \le \frac{p_b}{\gamma_b \bar{v}})$:

$$\sigma^* = \begin{cases} D : (A.25) & if \quad 0 \le T \le T_m; \\ F : (A.27) & if \quad T_m \le T\,. \end{cases} \tag{A.32}$$

(ii) $\frac{p_b}{\gamma_b \bar{v}} \le \delta \le \delta_m$:

$$\sigma^* = \begin{cases} C : (A.24) & if \quad 0 \le T \le T_s; \\ D : (A.25) & if \quad T_s \le T \le T_m; \\ F : (A.27) & if \quad T_m \le T\,. \end{cases} \tag{A.33}$$

(iii) $\delta_m \le \delta \le \delta_b$:

$$\sigma^* = \begin{cases} C : (A.24) & if \quad 0 \le T \le T_m; \\ E : (A.26) & if \quad T_m \le T \le T_s; \\ F : (A.27) & if \quad T_s \le T\,. \end{cases} \tag{A.34}$$

(iv) $\delta_b \le \delta \le 1$: (A.31).

(c) $\bar{v}_s \le \bar{v}$:

(i) $0 \le \delta \le \frac{p_b}{\gamma_b \bar{v}}$ $(\delta_n \le \frac{p_b}{\gamma_b \bar{v}})$: (A.32).

(ii) $\frac{p_b}{\gamma_b \bar{v}} \leq \delta \leq \delta_b$: (A.33).

(iii) $\delta_b \leq \delta \leq \delta_m$: (A.30).

(iv) $\delta_m \leq \delta \leq 1$: (A.31).

B Proofs of Propositions and Lemmas

Lemma A.1 *Suppose $p_m > p_b$, $\eta(\cdot) \geq 0$, $\eta'(\cdot) \leq 0$, and $\eta''(\cdot) \geq 0$ are satisfied. Then, there exist $\omega_1 \geq \omega_2 \geq \omega_3$ such that*

$$\sigma^*(v) = \begin{cases} (t(v),\, B) & if \quad \omega_1 \leq v \leq \bar{v}\,; \\ (t(v),\, N) & if \quad \omega_2 \leq v < \omega_1\,; \\ (\infty,\, B) & if \quad \omega_3 \leq v < \omega_2\,; \\ (\infty,\, N) & if \quad v < \omega_3\,; \end{cases} \tag{A.35}$$

where

$$t(v) = \begin{cases} \{t \leq 1/\beta : \eta'(t) + \lambda^2 v = 0\} & if \quad v < -\eta'(0)/\lambda^2\,; \\ 0 & if \quad v \geq -\eta'(0)/\lambda^2\,. \end{cases} \tag{A.36}$$

Proof of Lemma A.1: For the consumer with quality sensitivity v to find it optimal to consume both forms of the product, by (1) and (6), he will view the movie in a theater at time $t(v)$ as given in (A.36), which follows from the assumptions on $\eta(\cdot)$, and, by (1), at time $t(v)$, the following inequalities must be satisfied:

$$v \geq \frac{p_b}{\gamma_b \delta (1 - \beta T)}\,, \tag{A.37}$$

$$F \triangleq v\left(\gamma_m(1 - \beta t(v)) - \gamma_b(1-\delta)(1-\beta T)\right) - \alpha_m \eta(t(v)) - p_m \geq 0\,, \tag{A.38}$$

and

$$G \triangleq v\left(\gamma_m(1 - \beta t(v)) + \gamma_b \delta (1-\beta T)\right) - \alpha_m \eta(t(v)) - p_m - p_b \geq 0\,. \tag{A.39}$$

Suppose that for consumer $\hat{v}$, by (1), $s(\hat{v}) = (t(\hat{v}),\, B)$ such that (A.37), (A.38), and (A.39) are satisfied. If $\hat{v} \geq -\eta'(0)/\lambda^2$, then for any $v \geq \hat{v}$, by (A.36), $t(v) = t(\hat{v}) = 0$. Hence (A.37), (A.38), and (A.39) are satisfied for all $v \geq \hat{v}$. Suppose $\hat{v} < -\eta'(0)/\lambda^2$. If $\hat{v} \leq v < -\eta'(0)/\lambda^2$, by (A.38) and the envelope theorem, we obtain

$$\frac{dF}{dv} = \gamma_m(1 - \beta t(v)) - \gamma_b(1-\delta)(1-\beta T)\,, \tag{A.40}$$

and similarly, by (A.39),

$$\frac{dG}{dv} = \gamma_m(1 - \beta t(v)) + \gamma_b \delta (1-\beta T)\,. \tag{A.41}$$

Further, by (A.36), $dt/dv = -\lambda^2/\eta''(t) \leq 0$, and since (A.40) and (A.41) are positive at $\hat{v}$, these expressions are positive when $\hat{v} \leq v < -\eta'(0)/\lambda^2$, hence (A.37), (A.38), and (A.39) are satisfied. The argument for $v > -\eta'(0)/\lambda^2$ is the same as given previously. Therefore, there exists ω_1 such that $\sigma^*(v) = (t(v),\, B)$ for all $\omega_1 \leq v \leq \bar{v}$.

Similarly, if a consumer only views the movie in a theater, then his consumption time is given by (A.36). Thus, if

$$v \leq \frac{p_b}{\gamma_b \delta(1-\beta T)}\,, \tag{A.42}$$

$$H \triangleq v\left(\gamma_m(1-\beta t(v)) - \gamma_b(1-\beta T)\right) - \alpha_m \eta(t(v)) - p_m + p_b \geq 0\,, \tag{A.43}$$

and

$$I \triangleq v\gamma_m(1-\beta t(v)) - \alpha_m \eta(t(v)) - p_m \geq 0 \tag{A.44}$$

are satisfied, then $s^*(v) = (t(v),\, N)$. Suppose $\hat{v}$ satisfies (A.42), (A.43) and (A.44). As before, $\hat{v} \geq -\eta'(0)/\lambda^2$ implies for all $\hat{v} \leq v \leq p_b/(\gamma_b\delta(1-\beta T))$, $t(v)=0$, (A.43) and (A.44) are satisfied. Suppose $\hat{v} < -\eta'(0)/\lambda^2$. If $\hat{v} \leq v < \min(-\eta'(0)/\lambda^2, p_b/(\gamma_b\delta(1-\beta T)))$, by (A.36), (A.43) and the envelope theorem, we obtain

$$\frac{dH}{dv} = \gamma_m(1-\beta t(v)) - \gamma_b(1-\beta T)\,, \tag{A.45}$$

and similarly, by (A.44),

$$\frac{dI}{dv} = \gamma_m(1-\beta t(v))\,. \tag{A.46}$$

Further, by (A.36), $dt/dv = -\lambda^2/\eta''(t) \leq 0$, and since (A.45) and (A.46) are positive at $\hat{v}$, these expressions are positive when $\hat{v} \leq v < \min(-\eta'(0)/\lambda^2, p_b/(\gamma_b\delta(1-\beta T)))$, hence (A.42), (A.43), and (A.44) are satisfied. The argument for $v > -\eta'(0)/\lambda^2$ is the same as given previously. Therefore, by (A.42), there exists ω_2 such that $\sigma^*(v) = (t(v),\, N)$ for all $\omega_2 \leq v < \omega_1$.

By the properties of F and H and the fact that a consumer with sensitivity v prefers the video to not consuming if

$$v \geq \frac{p_b}{\gamma_b(1-\beta T)}\,, \tag{A.47}$$

we can similarly establish there exists ω_3 such that $\sigma^*(v) = (\infty,\, B)$ for all $\omega_3 \leq v < \omega_2$. For all $v < \omega_3$, it follows that $\sigma^*(v) = (\infty,\, N)$ which completes the proof. $\square$

Proof of Proposition 1: Suppose $\bar{v} \leq -\eta'(0)/\lambda^2$. For (A.35) to be satisfied with all nonempty segments, by (A.37), we require $\bar{v}\gamma_b\delta(1-\beta T) \geq p_b$ which is satisfied if and only if $T \leq T_s$. Further, by (A.36), a consumer who optimally consumes in a theater at time t has sensitivity $f(t) \triangleq -\eta'(t)/\lambda^2$. By (A.35), the rate of demand at time t is given by $-f'(t)$. Substituting, we obtain $\eta(t) = -f'(t)$, and hence

$$f''(t) - \lambda^2 f(t) = 0\,. \tag{A.48}$$

By (A.35), $f(0) = \bar{v}$, and, in equilibrium, $\eta(t(\omega_2)) = 0$ must be satisfied. Denoting $\tau_1 = t(\omega_2)$ gives rise to a second boundary condition, $f'(\tau_1) = 0$, and, by solving (A.48), we obtain the characterization in (5). By (A.43), evaluating H at τ_1 and $v = f(\tau_1)$ must yield $H = 0$. Thus, τ_1 satisfies (A.7).

By (A.43) and (A.44), we also require $f(\tau_1) \geq p_b/(\gamma_b(1-\beta T))$ which is satisfied if and only if

$$\frac{1-\beta T}{\cosh \lambda \tau_1} \geq \frac{p_b}{\bar{v}\gamma_b} . \tag{A.49}$$

By (A.7), it follows that τ_1 increases in T, hence (A.49) is satisfied if and only if $T \leq T_m$. Finally, by (A.42) and (A.43), we require $f(\tau_1) \leq p_b/(\gamma_b \delta(1-\beta T))$, which is satisfied whenever $T \geq T_b$. Moreover, note that $T_b \leq \min(T_s,\, T_m)$, and that $T_s \leq T_m$ if and only if $\delta \leq \delta_m$. This completes the proof. ■

Proof of Corollary 1: Taking the derivative of $f(t)$ in (5) with respect to t, we obtain

$$|f'(t)| = \frac{\lambda \bar{v}(e^{\lambda(\tau - t)} - e^{-\lambda(\tau-t)})}{e^{\lambda\tau} + e^{-\lambda\tau}} . \tag{A.50}$$

Given τ, it is proportional to $\sinh \lambda(\tau - t)$, which is exponentially decaying over t. □

Proof of Proposition 2: $\eta(\cdot)$ is measured as the rate of in-theater movie consumption at time t, and Lemma A.1 establishes the equilibrium strategy profile has a threshold structure under general properties of $\eta(\cdot)$. For $\min(\omega_1, \omega_2) \geq \bar{v}$ to be satisfied, neither (A.38) nor (A.43) can be satisfied for any $v \in \mathcal{V}$ when $\eta = 0$. By Lemma A.1, it is sufficient to evaluate F and H at $\bar{v}$ and $t=0$, in which case $H \leq 0$ if and only if $T \leq T_n$ and $F \leq 0$ if and only if $T \leq T_r$. By (A.2) and (A.1), it follows that $T_n \leq T_r$ if and only if $\delta \leq \delta_n$. Under the given conditions, $\nexists v \in \mathcal{V}$ such that $\sigma^*(v) \in \{(t(v), B), (t(v), N)\}$, and, by the proof of Lemma A.1, $\sigma^*(v) = (\infty, B)$ if and only if (A.47) is satisfied which proves (9). ■

Proof of Proposition 3: Note that when $\delta \geq \max(\delta_m,\, \delta_b)$, the distributor's profit becomes

$$\Pi(T) = \begin{cases} \tilde{p}_m(\bar{v} - v_r(T)) + \tilde{p}_b\left(\bar{v} - \frac{p_b}{\gamma_b(1-\beta T)}\right) & if \quad 0 \leq T \leq T_b\,; \\ \tilde{p}_m(\bar{v} - v_n(T)) + \tilde{p}_b\left(\bar{v} - \frac{p_b}{\delta\gamma_b(1-\beta T)} + v_n(T) - \frac{p_b}{\gamma_b(1-\beta T)}\right) & if \quad T_b \leq T \leq T_m\,; \\ \tilde{p}_m(\bar{v} - v_n(T_m)) + \tilde{p}_b\left(\bar{v} - \frac{p_b}{\delta\gamma_b(1-\beta T)}\right) & if \quad T_m \leq T \leq T_s\,; \\ \tilde{p}_m(\bar{v} - v_n(T_m)) & if \quad T_s \leq T\,. \end{cases} \tag{A.51}$$

First, in the region $T \in [0,\, T_b]$, taking derivative with respect to T, we obtain

$$\frac{\partial \Pi(T)}{\partial T} = \frac{(1-\delta)\beta\lambda\gamma_b\tilde{p}_m\bar{v}^2 \sinh \lambda\tau_r}{\cosh^2 \lambda\tau_r(\beta\gamma_m\bar{v} + \lambda p_m \sinh \lambda\tau_r)} - \frac{\beta p_b \tilde{p}_b}{\gamma_b(1-\beta T)^2} . \tag{A.52}$$

It then follows that there exists $\underline{\delta}_1 < 1$ such that for all $\delta > \underline{\delta}_1$, $\Pi(T)$ is strictly decreasing in $T \in [0,\, T_b]$. Consequently, the maximum profit within this range of T is achieved at $T = 0$. Furthermore, note that if δ is close to 1, T_b becomes close to T_m. Then from the continuity of the profit function $\Pi(T)$, there exists $\underline{\delta}_2 < 1$ such that for all $\delta > \underline{\delta}_2$, $\Pi(T=0) > \Pi(T)$ for all $T \in [T_b,\, T_m]$.

Finally, when $T \geq T_m$, $\Pi(T)$ is weakly decreasing in T. Hence, if $\delta > \underline{\delta} = \max(\underline{\delta}_1, \underline{\delta}_2, \delta_m, \delta_b)$, $T^* = 0$. Furthermore, in this case, the equilibrium market structure contains (*both*) segment, and, hence, $D_m > 0$. This completes the proof. ■

Proof of Proposition 4: For part (i), from the characterization of consumer market equilibrium under the given conditions, we obtain the corresponding profit as

$$\Pi(T) = \begin{cases} \tilde{p}_b\left(\bar{v} - \frac{p_b}{\gamma_b(1-\beta T)}\right) & if \quad 0 \leq T \leq T_n\,; \\ \tilde{p}_m(\bar{v} - v_n(T)) + \tilde{p}_b\left(v_n(T) - \frac{p_b}{\gamma_b(1-\beta T)}\right) & if \quad T_n \leq T \leq T_m\,; \\ \tilde{p}_m(\bar{v} - v_n(T_m)) & if \quad T_m \leq T\,. \end{cases} \tag{A.53}$$

Note that $v_n(T = T_n) = \bar{v}$ and $v_n(T = T_m) = p_b/(\gamma_b(1 - \beta T_m))$. Hence, $\Pi(T)$ in (A.53) is continuous in T. Further, if $0 \leq T \leq T_n$, $\Pi(T)$ is decreasing in T, and consequently, the optimal T within this interval is $T = 0$ with the corresponding profit of $\Pi(T = 0) = \tilde{p}_b(\bar{v} - p_b/\gamma_b)$. If $T_m \leq T$, $\Pi(T)$ is constant, i.e., does not depend on T, and it is equal to the boundary case within the interval of $T_n \leq T \leq T_m$. From the continuity of $\Pi(T)$, we can ignore this region $T_m \leq T$ to find the optimal T. If $T_n \leq T \leq T_m$, $\tau_n(T)$ is increasing in T, and from this one-to-one relationship between τ_n and T, we can write Π as a function of τ_n:

$$\Pi(\tau_n) = \tilde{p}_m\bar{v} - \bar{v}\left(\frac{\tilde{p}_m - \tilde{p}_b}{\cosh\lambda\tau_n} + \frac{\tilde{p}_b p_b}{\gamma_m\bar{v}(1 - \beta\tau_n) - (p_m - p_b)\cosh\lambda\tau_n}\right). \tag{A.54}$$

Taking the derivative with respect to τ_n, we obtain the first order condition

$$\Pi'(\tau_n) = \frac{\lambda\bar{v}(\tilde{p}_m - \tilde{p}_b)\sinh\lambda\tau_n}{\cosh^2\lambda\tau_n} - \frac{\tilde{p}_b p_b\bar{v}(\beta\gamma_m\bar{v} + (p_m - p_b)\lambda\sinh\lambda\tau_n)}{(\gamma_m\bar{v}(1 - \beta\tau_n) - (p_m - p_b)\cosh\lambda\tau_n)^2}\,. \tag{A.55}$$

Note that $\tau_n(T = T_n) = 0$ and $\tau_n(T = T_m) = \tau_m$. Together with monotonicity of $\tau_n(T)$, it follows that $T \in [T_n, T_m]$ maps to $\tau_n \in [0, \tau_m]$. Further, we obtain

$$\Pi'(\tau_n = 0) = -\frac{\beta\gamma_m\tilde{p}_b p_b\bar{v}^2}{(\gamma_m\bar{v} - (p_m - p_b))^2} < 0\,, \tag{A.56}$$

and

$$\Pi'(\tau_n = \tau_m) = -\frac{\beta\gamma_m\tilde{p}_b\bar{v}^2 + \lambda\bar{v}(p_m\tilde{p}_b - p_b\tilde{p}_m)\sinh\lambda\tau_m}{p_b\cosh^2\lambda\tau_m} < 0\,, \tag{A.57}$$

from $\tilde{p}_b/p_b \geq \tilde{p}_m/p_m$. Taking the derivative of (A.55) with respect to τ_n, it follows

$$\begin{aligned}\Pi''(\tau_n) = \frac{\lambda^2\bar{v}(\tilde{p}_m - \tilde{p}_b)(1 - \sinh^2\lambda\tau_n)}{\cosh^3\lambda\tau_n} &- \frac{\lambda^2\bar{v}\tilde{p}_b p_b(p_m - p_b)\cosh\lambda\tau_n}{(\gamma_m\bar{v}(1 - \beta\tau_n) - (p_m - p_b)\cosh\lambda\tau_n)^2} \\ &- \frac{2\bar{v}\tilde{p}_b p_b(\beta\gamma_m\bar{v} + \lambda(p_m - p_b)\sinh\lambda\tau_n)^2}{(\gamma_m\bar{v}(1 - \beta\tau_n) - (p_m - p_b)\cosh\lambda\tau_n)^3}\,.\end{aligned} \tag{A.58}$$

Note that $\gamma_m\bar{v}(1-\beta\tau_n)-(p_m-p_b)\cosh\lambda\tau_n=\gamma_b\bar{v}(1-\beta T)>0$. Denote $\bar{\tau}_m$ as the unique $\tau_n>0$ that solves $\sinh^2\lambda\tau_n=1$. Because $\sinh^2\lambda\tau_n$ is strictly increasing in $\tau_n>0$ with $\sinh^2 0=0$ and $\lim_{\tau_n\to\infty}\sinh^2\lambda\tau_n=\infty$, the existence and the uniqueness of $\bar{\tau}_m$ are guaranteed. Further, if $\tau_n\geq\bar{\tau}_m$, it follows that $1-\sinh^2\lambda\tau_n<0$. Consequently, if $\tau_n\geq\bar{\tau}_m$, $\Pi''(\tau_n)<0$ and hence $\Pi'(\tau_n)$ is strictly decreasing. Furthermore, if $\tau_n\leq\bar{\tau}_m$, $\Pi''(\tau_n)$ is decreasing in τ_n, which implies that $\Pi'(\tau_n)$ is concave in τ_n. Therefore, $\Pi'(\tau_n)=0$ has either no solution or two solutions in $[0,\,\tau_m]$. If $\Pi'(\tau_n)=0$ has no solution, $\Pi(\tau_n)$ is decreasing in τ_n in the interval $[0,\,\tau_m]$, which also implies that $\Pi(T)$ is decreasing in T in the interval $[T_n,\,T_m]$, and hence the optimal home video release time in this case is $T^*=0$. In this case, the corresponding $\bar{\alpha}_m$ is equal to 0. If $\Pi'(\tau_n)=0$ has two solutions, the smaller one is the local minimum, and the larger one is local maximum. In this case, denote the larger solution as τ_n^*. The corresponding home video release time is given as

$$T^*=\frac{1}{\beta}\left(1-\frac{\gamma_m(1-\beta\tau_n^*)}{\gamma_b}+\frac{(p_m-p_b)\cosh\lambda\tau_n^*}{\gamma_b\bar{v}}\right). \tag{A.59}$$

Thus, the optimal release time is 0 if $\tilde{p}_b(\bar{v}-p_b/\gamma_b)>\Pi(\tau_n=\tau_n^*)$ in (A.54), otherwise it is T^*. Taking the total derivative of (A.54) at $\tau_n=\tau_n^*$ and using the implicit function theorem, we obtain

$$\frac{d\Pi(\tau_n^*(\alpha_m))}{d\alpha_m}=\frac{\tau_n\lambda\bar{v}\sinh\lambda\tau_n}{2\alpha_m}\left(\frac{\tilde{p}_b p_b(p_m-p_b)}{(\gamma_m\bar{v}(1-\beta\tau_n)-(p_m-p_b)\cosh\lambda\tau_n)^2}-\frac{\tilde{p}_m-\tilde{p}_b}{\cosh^2\lambda\tau_n}\right). \tag{A.60}$$

Note that (A.60) is negative if $\tau_n^*(\alpha_m)<\tau_m$. Because $\tau_n^*(\alpha_m)<\tau_m$ from (A.57), $\Pi(\tau_n^*(\alpha_m))$ is strictly decreasing in α_m. When α_m approaches to 0, τ_n^* that solves (A.55) approaches to 0, and correspondingly $\cosh\lambda\tau_n^*$ to $\gamma_m\bar{v}/p_m$, and T^* in (A.59) to $1/\beta(1-p_b\gamma_m/(p_m\gamma_b))$. Consequently, the profit in (A.54) approaches to $\tilde{p}_m(\bar{v}-p_m/\gamma_m)$. When α_m approaches to ∞, which implies that λ approaches to 0, it follows that τ_m approaches to $1/\beta(1-p_m/(\gamma_m\bar{v}))$, which is finite, and hence τ_n is bounded above. Thus, as α_m approaches to ∞, (A.55) becomes negative. Consequently, τ_n^* in this interval becomes 0, and the corresponding optimal profit is $\tilde{p}_b\bar{v}(1-p_b/(\gamma_m\bar{v}-p_m+p_b))$, which is less than $\tilde{p}_b(\bar{v}-p_b/\gamma_b)$. As a result, if $\tilde{p}_m(\bar{v}-p_m/\gamma_m)<\tilde{p}_b(\bar{v}-p_b/\gamma_b)$, i.e., $(\tilde{p}_m-\tilde{p}_b)\bar{v}<\tilde{p}_m p_m/\gamma_m-\tilde{p}_b p_b/\gamma_b$, then the optimal home video release time is equal to 0 with the corresponding $\bar{\alpha}_m=0$. Otherwise, there exists the unique $\bar{\alpha}_m$, which solves

$$\Pi(\tau_n=\tau_n^*(\bar{\alpha}_m))=\tilde{p}_b\left(\bar{v}-\frac{p_b}{\bar{v}}\right), \tag{A.61}$$

and the optimal home video release time is strictly positive if and only if $\alpha_m<\bar{\alpha}_m$.

For part (ii), the equilibrium consumer market structure is given in (A.32), and the corresponding profit of the studio can be written as

$$\Pi(T)=\begin{cases}\tilde{p}_m(\bar{v}-v_n(T))+\tilde{p}_b\left(v_n(T)-\frac{p_b}{\gamma_b(1-\beta T)}\right) & if \quad 0\leq T\leq T_m\,;\\ \tilde{p}_m(\bar{v}-v_n(T_m)) & if \quad T_m\leq T\,.\end{cases} \tag{A.62}$$

Note that (A.62) is continuous in T and is constant if $T \geq T_m$. In the interval of $T \in [0, T_m]$, using the one-to-one relationship between τ_n and T, we can write $\Pi(T)$ as a function of τ_n, which is given in (A.54). The interval of T, $[0, T_m]$, corresponds to the interval for $\tau_n \in [\tau_s, \tau_m]$. Since $\Pi'(\tau_n = \tau_m) < 0$ in (A.57), we can focus on the interval of $\tau_n \in [\tau_s, \tau_m]$ to find the optimal home video release time. From (A.7) and (A.55), we observe that when α_m becomes large, i.e., λ becomes small, $\Pi'(\tau_n)$ goes to negative. Hence, there exists $\underline{\alpha}_m > 0$ such that $T^* = 0$ when $\alpha_m \geq \underline{\alpha}_m$. Furthermore, in this case, from (A.32), it follows that $D_m > 0$. ■

Proof of Proposition 5: Define $\bar{v}_u \triangleq \max(\bar{v}_s, \bar{v}_{u_1}, \bar{v}_{u_2})$, where

$$\bar{v}_{u_1} = \frac{\gamma_m^2 (p_m - p_b)^2}{\gamma_b p_b (\gamma_m - \gamma_b)^2} + \frac{2p_m}{\gamma_b} + \frac{p_m^2 \gamma_b^2 - p_b^2 \gamma_m^2}{p_b \gamma_m \gamma_b (\gamma_m - \gamma_b)}\,, \tag{A.63}$$

$$\bar{v}_{u_2} = \frac{2p_m(\gamma_m(p_m - p_b) + p_b(\gamma_m - \gamma_b))}{(p_m - p_b)\gamma_m^2 + p_b(\gamma_m - \gamma_b)(2\gamma_m - \gamma_b)}\,, \tag{A.64}$$

and $\bar{v}_s$ is given at (A.21). Given that $\bar{v} \geq \bar{v}_u$, when $\delta \leq p_b/(\gamma_b \bar{v})$, the distributor's profit is given in (A.62). Similar to the proof of part (*ii*) of Proposition 4, after the transformation of variable from T to τ_n, we can focus on the interval of $\tau_n \in [\tau_s, \tau_m]$ to find the optimal home video release time. Substituting $\tau_n = \tau_s$ in (A.55), we obtain

$$\Pi'(\tau_n = \tau_s) = -\frac{\beta \gamma_m p_b^2}{\gamma_b^2} - (p_m - p_b)\lambda \bar{v} \sinh \lambda \tau_s \left(\frac{p_b^2}{\gamma_b^2 \bar{v}^2} - \frac{1}{\cosh^2 \lambda \tau_s} \right). \tag{A.65}$$

Note that from (A.65), there exists $\epsilon_1 > 0$ such that for all $\alpha_m \leq \epsilon_1$, $\Pi'(\tau_n = \tau_s) > 0$ since $\lambda \tau_s$ is bounded above and

$$\frac{1}{\cosh \lambda \tau_s} = \frac{p_m - p_b}{(\gamma_m(1 - \beta \tau_s) - \gamma_b)\bar{v}} \geq \frac{p_m - p_b}{(\gamma_m - \gamma_b)\bar{v}} > \frac{p_b}{\gamma_b \bar{v}}\,, \tag{A.66}$$

from (A.10). As a result, T^* is strictly positive when $\delta \leq p_b/(\gamma_b \bar{v})$. When $\delta \in [p_b/(\gamma_b \bar{v}), \delta_b]$, the distributor's profit becomes

$$\Pi(T) = \begin{cases} p_m(\bar{v} - v_n(T)) + p_b \left(\bar{v} - \frac{p_b}{\delta \gamma_b (1 - \beta T)} + v_n(T) - \frac{p_b}{\gamma_b(1-\beta T)} \right) & if \quad 0 \leq T \leq T_s\,; \\ p_m(\bar{v} - v_n(T)) + p_b \left(v_n(T) - \frac{p_b}{\gamma_b(1-\beta T)} \right) & if \quad T_s \leq T \leq T_m\,; \\ p_m(\bar{v} - v_n(T_m)) & if \quad T_m \leq T\,. \end{cases} \tag{A.67}$$

First, in the region $T \in [0, T_s]$, we transform the problem into the optimization over τ_n, which becomes

$$\Pi(\tau_n) = p_m \bar{v} - \bar{v} \left(\frac{p_m - p_b}{\cosh \lambda \tau_n} + \frac{1 + \delta}{\delta} \cdot \frac{p_b^2}{\gamma_m \bar{v}(1 - \beta \tau_n) - (p_m - p_b)\cosh \lambda \tau_n} \right). \tag{A.68}$$

Taking the derivative of (A.68) with respect to τ_n, we obtain

$$\Pi'(\tau_n) = \frac{\lambda \bar{v}(p_m - p_b)\sinh \lambda\tau_n}{\cosh^2 \lambda\tau_n} - \frac{1+\delta}{\delta} \cdot \frac{p_b^2 \bar{v}(\beta\gamma_m \bar{v} + (p_m - p_b)\lambda \sinh \lambda\tau_n)}{(\gamma_m \bar{v}(1-\beta\tau_n) - (p_m - p_b)\cosh \lambda\tau_n)^2}\,. \tag{A.69}$$

Under small α_m, i.e., large λ, observe that τ_n becomes small, and $\lambda\tau_n$ is bounded from (A.7). As α_m becomes small, we can rewrite (A.69) as

$$\begin{aligned}\Pi'(\tau_n) = \frac{\lambda \bar{v}(p_m - p_b)\sinh \lambda\tau_n}{\delta(\gamma_m \bar{v} - (p_m - p_b)\cosh \lambda\tau_n)^2} \left(\delta \frac{\gamma_m \bar{v} - p_m}{\cosh \lambda\tau_n}\left(\frac{\gamma_m \bar{v} - p_m}{\cosh \lambda\tau_n} + 2p_b\right) - p_b^2\right) \\ - \frac{(1+\delta)\beta\gamma_m p_b^2 \bar{v}^2}{\delta(\gamma_m \bar{v} - (p_m - p_b)\cosh \lambda\tau_n)^2} + O(\sqrt{\alpha_m})\,.\end{aligned} \tag{A.70}$$

Note that in this regime of small α_m, if $\delta < \bar{\delta}_2$, $\Pi'(\tau_n)$ is strictly negative for all $\tau_n \geq \tau_s$, where τ_s is given in (A.10), and

$$\bar{\delta}_2 \triangleq \frac{(\gamma_m - \gamma_b)p_b^2 \bar{v}}{(p_m - p_b)(\gamma_m \bar{v} - p_m)} \left(\frac{(p_m - p_b)(\gamma_m \bar{v} - p_m)}{(\gamma_m - \gamma_b)\bar{v}} + 2p_b\right)^{-1}\,. \tag{A.71}$$

Further, if $\bar{v} > \bar{v}_{u_2}$, $\bar{\delta}_2 < \delta_b$, where δ_b is given in (A.16). In this case, the optimal release time within $T \in [0,\, T_s]$ becomes 0, with the corresponding distributor's profit as

$$\Pi(T=0) = (p_m + p_b)\bar{v} - \frac{(p_m - p_b)^2}{\gamma_m - \gamma_b} - \frac{(1+\delta)p_b^2}{\delta\gamma_b} + O(\sqrt{\alpha_m})\,. \tag{A.72}$$

Next, when $T \in [T_s,\, T_m]$, from (A.55), we obtain that for all $T < T_m$, there exists $\epsilon_2 > 0$, such that for all $\alpha_m \leq \epsilon_2$, (A.55) is strictly positive. Then, from the continuity and the upper bounds of $\Pi(T)$ in (A.67), it implies that for any $\xi > 0$, ϵ_2 can be refined to guarantee that the optimal profit within the region $T \in [T_s,\, T_m]$ is strictly less than $\Pi(\tau_n = \tau_m) + \xi$ for all $\alpha_m \leq \epsilon_2$, i.e.,

$$\Pi(T = T_m) = p_m\left(\bar{v} - \frac{\bar{v}}{\cosh \lambda\tau_m}\right) \leq p_m\left(\bar{v} - \frac{p_m}{\gamma_m}\right) + \xi\,. \tag{A.73}$$

Consequently, under small α_m, when $\delta \in [p_b/(\gamma_b \bar{v}),\, \bar{\delta}_2]$, T^* becomes 0, if and only if $\delta \geq \bar{\delta}_1$, where

$$\bar{\delta}_1 \triangleq \frac{p_b^2}{\gamma_b}\left(p_b \bar{v} - \frac{(p_m - p_b)^2}{\gamma_m - \gamma_b} + \frac{p_m^2}{\gamma_m} - \frac{p_b^2}{\gamma_b}\right)^{-1}\,. \tag{A.74}$$

Note that from $(p_m - p_b)^2/(\gamma_m - \gamma_b) - p_m^2/\gamma_m + p_b^2/\gamma_b \geq 0$, $\bar{\delta}_1 \geq p_b/(\gamma_b \bar{v})$ holds. Moreover, it follows that $\bar{\delta}_1 < \bar{\delta}_2$, under $\bar{v} > \bar{v}_{u_1}$. Lastly, if $\delta \in (\bar{\delta}_2,\, \delta_b)$, (A.69) is strictly positive at $\tau_n = \tau_s$, i.e., $\Pi'(T=0) > 0$, and hence $T^* > 0$. Therefore, when $\delta \leq \bar{\delta}_3 \triangleq \delta_b$, for $\alpha_m \leq \hat{\alpha}_m \triangleq \min(\epsilon_1,\, \epsilon_2)$ and $\bar{v} > \bar{v}_u$, $T^* = 0$ if and only if $\delta \in [\bar{\delta}_1,\, \bar{\delta}_2]$. This completes the proof. ■

Proof of Proposition 6: First, note that given τ, the equilibrium demand rate is $\eta(t) = -f'(t)$, which is increasing τ for all $t \le \tau$, since

$$\frac{\partial \eta(t)}{\partial \tau} = \frac{\lambda^2 \bar{v} \cosh \lambda t}{\cosh^2 \lambda \tau} > 0\,, \tag{A.75}$$

for all t. Consider the same parameter region in Proposition 5 with $\delta \in [\bar{\delta}_1 - \epsilon,\, \bar{\delta}_1 + \epsilon]$ for small ϵ. From the proof of Proposition 5, it follows that $T^* > 0$ for $\delta \in [\bar{\delta}_1 - \epsilon,\, \bar{\delta}_1)$, and $T^* = 0$ for $\delta \in [\bar{\delta}_1,\, \bar{\delta}_1 + \epsilon]$. Further, under $T^* > 0$ for $\delta \in [\bar{\delta}_1 - \epsilon,\, \bar{\delta}_1)$, the equilibrium consumer strategy profile is given at (A.25) with consumer segments (*movie*, *video*, *none*), whereas under $T^* = 0$ for $\delta \in [\bar{\delta}_1,\, \bar{\delta}_1 + \epsilon]$, it is given at (A.24) with (*both*, *movie*, *video*, *none*). Note that the corresponding τ for both cases is τ_n, given in (A.7), which is independent of δ and increases in T. Consequently, the corresponding τ_n for $\delta \in [\bar{\delta}_1 - \epsilon,\, \bar{\delta}_1)$ is larger than that for $\delta \in [\bar{\delta}_1,\, \bar{\delta}_1 + \epsilon]$. Then from (A.75), the demand rate at any time t is larger for $\delta \in [\bar{\delta}_1 - \epsilon,\, \bar{\delta}_1)$ than for $\delta \in [\bar{\delta}_1,\, \bar{\delta}_1 + \epsilon]$. Under the given prices p_m and p_b, the consumers can then obtain larger utilities from watching a movie in a theater at any time due to smaller congestion externalities for $\delta \in [\bar{\delta}_1,\, \bar{\delta}_1 + \epsilon]$ (under $T^* = 0$) than for $\delta \in [\bar{\delta}_1 - \epsilon,\, \bar{\delta}_1)$ (under $T^* > 0$). In addition, home video also provides higher utilities due to earlier release for $\delta \in [\bar{\delta}_1,\, \bar{\delta}_1 + \epsilon]$ than for $\delta \in [\bar{\delta}_1 - \epsilon,\, \bar{\delta}_1)$. Hence, from the optimality of customers' choices, all consumers are better off at $\delta \in [\bar{\delta}_1,\, \bar{\delta}_1 + \epsilon]$ than at $\delta \in [\bar{\delta}_1 - \epsilon,\, \bar{\delta}_1)$. Furthermore, the day-and-date strategy brings a strictly positive mass of additional home video customers with strictly positive additional welfare increase. Since the prices p_m and p_b are fixed, it leads to strictly positive increase in social welfare. Therefore, a small decrease in δ results in T^* from zero to strictly positive delay, which in turn yields strictly positive mass of social welfare loss. ■

Proof of Proposition 7: Consider the region, $\bar{v} \in [(p_m - p_b)/(\gamma_m - \gamma_b),\, \bar{v}_s]$ and $\delta \in [\delta_b,\, \hat{\delta}]$, where $\hat{\delta} \triangleq \min(\bar{\delta}_4, (\sqrt{5}-1)/2)$, $\bar{\delta}_4 \triangleq (p_m p_b \gamma_m \bar{v})^2/\psi$, and

$$\psi \triangleq (\gamma_m \bar{v} - p_m)(p_m^2 + (\gamma_m \bar{v} - p_m)p_b)\left((\gamma_m \bar{v} - p_m)(p_m^2 + (\gamma_m \bar{v} - p_m)p_b) + 2 p_m p_b \gamma_m \bar{v}\right)\,. \tag{A.76}$$

Note that for $\gamma_b < \bar{\gamma}_b$, where

$$\bar{\gamma}_b \triangleq \gamma_m \frac{\psi}{p_m^2 p_b \gamma_m^2 \bar{v}(p_m - p_b) + \psi}\,, \tag{A.77}$$

the inequality, $\bar{\delta}_4 > \delta_b$, holds under small α_m. In addition, for $p_b/\gamma_b > (\sqrt{5}-1)(p_m - p_b)/\{2(\gamma_m - \gamma_b)\}$, $\delta_b < (\sqrt{5}-1)/2$ holds under small α_m. In this region, the distributor's profit can be written as

$$\Pi(T) = \begin{cases} p_m(\bar{v} - v_r(T)) + p_b\left(\bar{v} - \frac{p_b}{\gamma_b(1-\beta T)}\right) & if \quad 0 \le T \le T_b\,; \\ p_m(\bar{v} - v_n(T)) + p_b\left(\bar{v} - \frac{p_b}{\delta\gamma_b(1-\beta T)} + v_n(T) - \frac{p_b}{\gamma_b(1-\beta T)}\right) & if \quad T_b \le T \le T_m\,; \\ p_m(\bar{v} - v_n(T_m)) + p_b\left(\bar{v} - \frac{p_b}{\delta\gamma_b(1-\beta T)}\right) & if \quad T_m \le T \le T_s\,; \\ p_m(\bar{v} - v_n(T_m)) & if \quad T_s \le T\,. \end{cases} \tag{A.78}$$

First, for $T \in [0,\, T_b]$, the profit function can be written as a function of τ_r, which becomes

$$\Pi(\tau_r) = (p_m + p_b)\bar{v} - \bar{v}\left(\frac{p_m}{\cosh\lambda\tau_r} + \frac{(1-\delta)p_b^2}{\gamma_m\bar{v}(1-\beta\tau_r) - p_m\cosh\lambda\tau_r}\right). \tag{A.79}$$

Note that $T \le T_b$ corresponds to $\tau_r \le \tau_b$. Differentiating (A.79) with respect to τ_r, we obtain

$$\Pi'(\tau_r) = \frac{\lambda p_m\bar{v}\sinh\lambda\tau_r}{\cosh^2\lambda\tau_r} - \frac{p_b^2\bar{v}(1-\delta)(\beta\gamma_m\bar{v} + \lambda p_m\sinh\lambda\tau_r)}{(\gamma_m\bar{v}(1-\beta\tau_r) - p_m\cosh\lambda\tau_r)^2}. \tag{A.80}$$

Under small α_m, i.e., large λ, (A.80) becomes strictly positive for $\delta < (\sqrt{5}-1)/2$. Hence, in this case, within $T \in [0,\, T_b]$, the optimal home video release time is T_b with the corresponding profit as

$$\Pi(T = T_b) = (p_m + p_b)\bar{v} - \frac{(p_m + \delta p_b)\bar{v}}{\cosh\lambda\tau_b}. \tag{A.81}$$

Next, for $T \in [T_b,\, T_m]$, the profit function can be rewritten as a function of τ_n, as given in (A.68), with the corresponding domain of $\tau_n \in [\tau_b,\, \tau_m]$. Note that under small α_m, using the similar argument in the proof of Proposition 5, we obtain that if $\delta < \bar{\delta}_4$, $\Pi'(\tau_n)$ in (A.70) is strictly negative for all $\tau_n \ge \tau_b$. As a result, within $T \in [T_b,\, T_m]$, the distributor's profit is strictly decreasing in T. In addition, for $T \in [T_m,\, T_s]$, the profit is also strictly decreasing in T. Lastly, the distributor's profit is constant if $T \ge T_s$. Therefore, we obtain that $T^* = T_b$. From (A.8), it follows that

$$\frac{\partial\tau_b}{\partial\delta} = \frac{p_b}{\delta^2\gamma_m\bar{v}} \cdot \frac{\cosh^2\lambda\tau_b}{\beta\cosh\lambda\tau_b + \lambda\sinh\lambda\tau_b(1-\beta\tau_b)}. \tag{A.82}$$

Further, from (A.11), we obtain

$$\frac{\partial T_b}{\partial\delta} = \frac{p_b}{\beta\delta^2\gamma_b\bar{v}}\left(\cosh\lambda\tau_b - \lambda\delta\sinh\lambda\tau_b\,\frac{\partial\tau_b}{\partial\delta}\right). \tag{A.83}$$

Plugging (A.82) into (A.83), and using (A.8), we obtain that (A.83) is strictly positive under small α_m. Consequently, $T^* = T_b$ is increasing in δ under small α_m. ■

Proof of Proposition 8: Consider the region, $\bar{v} \in [(p_m - p_b)/(\gamma_m - \gamma_b),\, \bar{v}_s]$, $\delta \in [\delta_b,\, \bar{\delta}_4]$, and $p_b < \bar{p}_b$, where $\bar{p}_b \triangleq \min(\bar{p}_{b_1},\, \bar{p}_{b_2})$,

$$\bar{p}_{b_1} \triangleq \frac{(\sqrt{5}-1)\gamma_b p_m}{2\gamma_m - (3-\sqrt{5})\gamma_b}, \quad \text{and} \quad \bar{p}_{b_2} \triangleq \frac{\sqrt{1-\delta}\,\gamma_b p_m}{\gamma_m - (1-\delta)\gamma_b}. \tag{A.84}$$

In this case, the distributor's profit function is given as (A.78). First, consider the region in which $T \in [0,\, T_b]$. Note that if $p_b < \bar{p}_{b_1}$, $\delta_b > (\sqrt{5}-1)/2$ holds under small α_m. Furthermore, when $\delta > (\sqrt{5}-1)/2$, (A.80) is strictly negative at $\tau_r = \tau_b$ under small α_m. In addition, for $p_b < \bar{p}_{b_2}$, (A.80) is strictly positive at $\tau_r(T=0)$ under small α_m. Moreover, in this case, under small α_m, (A.78) is quasi-concave in τ_r, and there exists the unique solution that solves first order condition within the corresponding range of τ_r, which then yields the optimal home video release time within $T \in [0, T_b]$ as given in (14). Next, from the proof of Proposition 7, we have that for $\delta \leq \bar{\delta}_4$, the profit function is decreasing in $T \in [T_b, T_m]$. Note that $\bar{\delta}_4 > \delta_b$ for $\gamma_b < \bar{\gamma}_b$ under small α_m. Consequently, in this case, the optimal home video release time is as given in (14). Lastly, it follows that T^* in (14) is decreasing in δ under small α_m. ■