Seller Reputation, Buyer Informativeness and Trust in the Market

--- How the healthiness of an Online Market Impact Price Dispersion

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Abstract

Most previous studies in price dispersion literature consider only the dispersion of post price. We build a model to study the actual sale through each post price and combine price dispersion together with the dynamic growth of the reputation system in an Internet market. We identify buyer informativeness, which measures how likely a buyer is able to search and compare multiple offers, as one essential drive of online price dispersion. We also find that both the seller side behavior (different reputation levels, as well as different in how important to be a high-reputation seller) and the market side factor (how healthy an internet market is) impact the degree of price dispersion. We argue that a healthy internet market builds trust among informed buyers, who are then willing to buy from new sellers with low reputation. This gives incentive to new, low-reputation sellers to offer lower price to generate sales and grow into a high-reputation seller. As a result, price dispersion is different among sellers with different reputation levels, and in different stages in the growth of the Internet market.

1. Introduction

Online price dispersion is always an important topic in the development of internet economy. Because internet market features fierce competitions due to the increased number of sellers (Liu etal 2009), reduced search costs (Bakos 1997, Soh et al 2006) and price transparency , it is claimed to be a frictionless market (Brynjolfsson and Smith 2000). According to the classical Bertrand model, all sellers should set the same price—the "law of one price" (LOP) in such a market. However, contrary to the theoretical prediction, researchers find substantial price dispersion in online markets (Baye et al 2004, Brynjolfsson and Smith 2000, Pan et al 2004). The causes for price dispersion of homogenous goods may come from the differences among sellers (Clay et al. 2002, Luo and Chung 2010, Rabinovich et al. 2008, Venkatesan 2006), and the differences among buyers (Alessandria 2009, Salop 1977, Varian 1980).

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This paper explains online price dispersion from the perspective of seller reputation and buyer informativeness. Internet market features huge uncertainties compared to a physical market: sellers are often hidden under the masks of meaningless electronic IDs (Kauffman and Wood 2006); it is hard to inspect the product before payment; at the same time, the payment and delivery for the products are also separated (Andrews and Benzing 2007). Fortunately, various reputation systems was developed such as buyer ratings and reviews, feedback systems, online discussion forums, etc (Ba and Pavlou 2002, Srinivasan and Sun 2009, Utz et al 2009.) Such reputation systems greatly reduce consumer uncertainties and facilitate online transactions. For example, research shows that products receiving good reviews are sold better, too (Chevalier and Mayzlin 2006).

On the seller side, reputation affects the pricing strategy of sellers. For example, Ba and Pavlou (2002) finds that consumers are willing to pay a price premium to sellers with higher reputation. From the buyer side, whether or not a buyer is informative, that is, whether or not the buyer knows how to search, compare and look for an ideal deal, determines whether or not this buyer will buy a product at a high price, which in turn affects the pricing strategy of sellers. For example, a low-reputation seller may completely give up the consumers who know how to search and compare, and set a high price just for those uninformed buyers. However, by doing so, such low-reputation sellers forego the opportunity to gain sales to grow into a seller with high reputation and they may always remain to be low-reputation sellers.

To capture this dynamic feature of reputation growth in an internet market, our model incorporate seller characteristics, buyer characteristics and the healthiness of an Internet market in the following way: (1) for sellers: the existing reputation system cannot always reflect the true "quality" of the seller. For example, a new seller in the market has "low reputation", but that does not indicate that this seller necessarily offers low-quality product. (2) for buyers, they may be "informed" or "uninformed", depending on their online shopping experience, education background, etc. (3) the reputation system not only helps consumers to find high-reputation seller. More importantly, it should also help those "high-quality" sellers with low reputation to develop into a high reputation seller. The chance that a low reputation seller may be a "good" seller gives confidence for some "informed" buyers to buy from a low-reputation seller. This gives incentive for a low-reputation seller to grow into a high reputation one, which in turn

boosts the "healthiness" of the market. How likely a low-reputation seller is a "good" seller is determined by the internet market, and this in turn determine how likely an "informed" buyer will buy from a low-reputation seller.

Our model finds that online price dispersion depends on all these three aspects: (1) online price dispersion reduces when more buyers are informative; (2) online price dispersion is smaller among sellers with higher reputation; (3) online price dispersion reduces when the internet market is more healthy, that is, when there is less chance that a low-reputation seller is a "bad" one, or, when the internet market attracts less "bad" sellers.

2. Benchmark Model

Consider an Internet market selling one homogeneous product. The product costs 0 to the sellers, and offers a utility of 1 to consumers. There are two sellers with established reputation through their past transaction and services. Denote these two sellers as high-reputation sellers. For simplicity, we normalize their reputation level to be consistent with the product quality they offer. That is, we normalize their reputation level to be $r_H = 1$. Now consider one new seller entering the market without any reputation $r_L = 0$. Denote this seller as the low-reputation seller. Assume that there is a probability r that this seller offers a product with quality 1, and a probability of $1 - \mathrm{r}$ this seller offers a product with quality 0.

There are two types of consumers in the market. Following Varian (1980), we assume that k percent are informed consumers and $(1-k)$ percent are uninformed consumers. An informed buyer will compare the products from each seller and purchase from the seller who offers the highest expected utility; while an uninformed buyer will randomly visit one seller without knowledge of the seller's true quality, and purchase if the expected utility is non-negative. We assume that an informed buyer will make judgment about a low-reputation seller's true quality level at $r < 1$, while an uninformed buyer believes that any seller they transact offers a product quality of 1. So, an informed buyer's expected utility in a transaction with seller i can be specified as $1 - p_i$ if purchasing from a high-reputation seller, and $\mathrm{r} - p_i$ if purchased from a low-reputation seller; while an uninformed buyer's expected utility is always $1 - p_i$. Buyers will purchase the product only when the expected utility is non-negative. In this model, each high-

reputation seller has to compete not only with the low-reputation sellers, but also with the other high-reputation seller. The low-reputation seller competes with the two high-reputation sellers. The three sellers set their optimal prices P_L , P_{H1} and P_{H2} optimally to compete in the market.

Proposition 1:

In equilibrium, the low-reputation seller sets a price at 1, while the high-reputation sellers play mixed strategy in pricing, where $F_H(p) = 1 - \frac{(1-k)(1-p)}{3kp}$.

We present all the proofs in the Appendix. Proposition 1 shows that, the existence of the new, low-reputation seller places little risk to the existing established sellers. The new seller cannot set a price lower than 1 because two high-reputation seller can quickly undercut and compete away those consumers.

Corollary 1.

In equilibrium, the low-reputation seller cannot sell to any informed consumers.

Corollary 1 indicates that, since the low-reputation seller only sells to uninformed buyers, she will never reach a reputation level similar or higher than the two established sellers and in turn, will never sell to any informed consumers again. Such a market does not grant any opportunity for the low-reputation seller to grow.

Model with Growth Opportunity for the Low-reputation Seller.

Now assume that part of the informed consumers is willing to trust the new seller offering a product with the same quality as is offered by high-reputation sellers, even though the seller has no/low reputation score. The trust of consumers may inherit from their trust to the specific internet market (for example, the market may promote that they strictly select the set of sellers who sells on their platform;) or from signals that they observe and trust (for example, the seller is their friend or family members in real life;) or their past purchasing experience (they may have purchased from an unknown seller in their life before;) or simply because these buyers are risk taking and willing to try new opportunities when information is limited. Specifically, assume that r percent of the informed buyers believe that the new seller is trustworthy (which offers a

quality level of 1, the same as high-reputation sellers), while the remaining $(1-r)$ percent of them believe that the new seller cannot be trusted (with quality 0).

In this way, the low-reputation seller may earn a chance to sell to the experienced buyers, thus a chance to sell more than a high-reputation seller and grow into a high-reputation seller. To characterize this different objective of a low-reputation seller, we assume that (1) the objective of a new seller is to maximize the number of sales generated; (2) to achieve such an objective, the new sellers may be willing to sacrifice certain profit. Let $e \in [0,1]$ represent the percertage of profit that a low-reputation seller is willing to sacrifice to obtain a sale. When $e = 1$, the seller's objective is to maximize its market share without any profit; when $e = 0$, the seller's objective is to maximize its short-term profit as in the benchmark case.

Proposition 2: [The equilibrium pricing strategies for sellers with different reputation levels]

For short-term profit maximizing low-reputation seller (e $= 0$): *the low-reputation seller sets a price at 1, while the high-reputation seller plays a mixed strategy in pricing, where* $p_H \in \left(\frac{1-k}{1+2k-3kr}, 1\right)$, *and the probability distribution function of the equilibrium price is* $F_H(p) = 1 - \frac{(1-k)(1-p)}{3krp}$.

For transaction maximizing low-reputation seller $e = 1$*: the low-reputation seller sets a price at 0, while the high-reputation seller plays mixed strategy in pricing, where* $p_H \in \left(\frac{1-k-3kre}{1-k+3kr}, 1\right)$, *and the probability distribution function of the price is* $F_H(p) = 1 - \frac{(1-k)(1-p)}{3kp}$.

Model analysis:

1. Negative Price Premium

   One interesting question to ask is whether or not a high-reputation seller sets a higher price than a low-reputation seller because consumers have high willingness to pay for a product with higher quality. Some literature finds that this is true, while others find the reverse. In our paper, since the high-reputation seller plays mixed strategy in pricing, we can easily find that the high-reputation seller can set either a higher or lower price than a

low-reputation seller. To explore whether a high-reputation seller sets a higher price on average, we compare the average price of the high-reputation sellers against the price of the low-reputation seller.

Proposition 3:

It is more likely to observe negative price premium when the proportion of informed buyers is large (k is large), and/or when an informed buyer strongly trust the market price (r is large).

Figure: to be inserted.

2. Market Share

   As from before, a high-reputation seller does not necessarily enjoy a price premium. Then what are the incentives for a low-reputation seller to grow into a high-reputation one? Understanding the market share of each seller is crucial to answer this question.

   Proposition 4:

   A high-reputation seller always occupies a larger market share relative to a low-reputation seller. The difference is larger when the proportion of informed buyers is high, and/or when the market lacks trust to low-reputation seller (r is small).

   Figure: to be inserted.

3. The proportion of “Experienced” buyer in the customer base.

   We define the experienced buyer as the ones that are both informed and have trust in the Internet market. Understanding the composition of the customer base, especially the proportion of the “experienced” buyers in the customer base can help the seller to better position their products and set prices.

   Proposition 5:

   Figure: to be inserted.

4. Price dispersion

   Our understanding of the price dispersion does not only consider the post price, but more importantly, the actual sale generated by the price, including the consideration of who

sells the product (high or low-reputation sellers) and who made the purchase (informed vs uninformed buyers, experienced buyers, etc.)

Proposition 6:

The price dispersion is higher for low-reputation seller than for high-reputation seller, is higher when there is less informed buyers, and/or when the market lacks trust.

3. Empirical Analysis

To confirm our theoretical predictions, we obtain a dataset from one of the largest internet market place in China. Our dataset includes the product list, the price the product, seller reputation, sales volume and the consumers who made the purchase. There are two important unique features of this dataset are, (1) it not only includes the post price of each product, but also how many transactions have been undertaking with that price. This feature is useful because some seller may post an abnormally high price but cannot sell at all. We are able to separate those prices out in our dataset. (2) This dataset contains the detailed information about the buyers of a certain product. For example, through the "experience" measure of a buyer, we are able to infer whether that buyer is an informed buyer who establishes trust towards the Internet market, as well as a low-reputation seller.

The type of product we collected data include: Game card 600 points, Canon 450 set camera, butter-taste nuts 500g, and Adidas male Sports Clothes set. Since game card 600 points and Canon 450 set camera are standardized products (search-type product) with clear market guide price, thus the buyers are more likely to be informed buyers; while butter-taste nuts 500g and Adidas male set sport clothes are experience-type product, thus buyers have to collect more information to know the quality of the product, thus buyers are more likely to be uninformed buyers.

The Negative Price Premium (Price Comparison)

To test our finding about negative price premium, we calculate the median of seller reputation in each product category, and then mark the data points with seller reputation greater than the median as High, and mark the data points with seller reputation less than the median as Low. The direct comparison shows intuitive proof on our propositions. Moreover, this method shows the differences of the high and low reputation sellers.

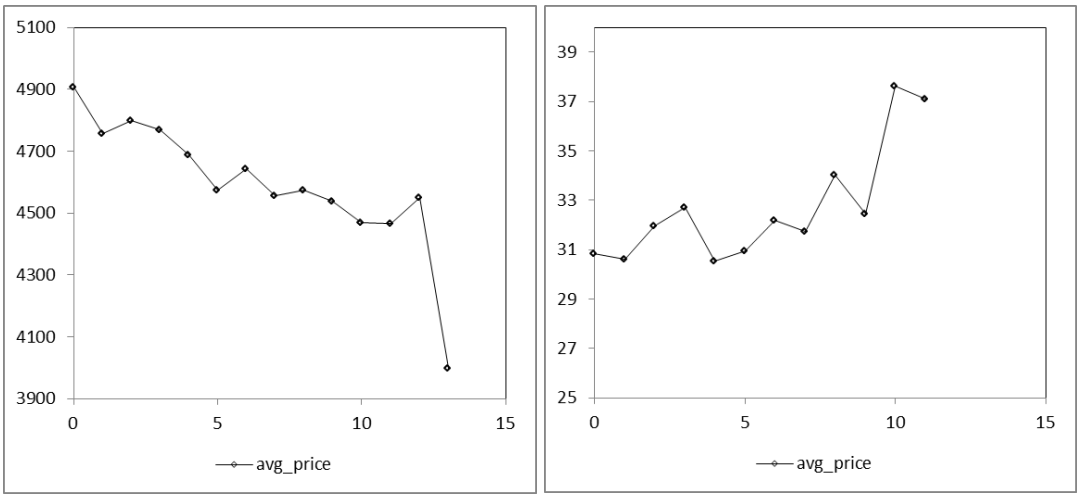
Patch	Record Date	Product	N	Median Seller RP	N (High)	N (Low)
1	2008/3/7	Nokia N73 Cell Phone	1211	79	605	603
2	2008/10/8	WOW Card 600	2918	400.5	1459	1459
2	2008/10/8	Nokia N73 Cell Phone	1081	274	539	540
3	2008/11/28	WOW Card 600	2913	353	1454	1453
3	2008/11/28	Nokia N73 Cell Phone	1144	275	570	570
3	2008/12/6	Nuts 500g without Package	1442	579	713	713
4	2010/2/28	Nokia 5800 Cell Phone	2274	359.5	1137	1137
4	2010/2/28	Nokia N73 Cell Phone	1803	319	901	901
5	2010/5/29	Faked Nokia 5800 Cell Phone	390	645.5	195	195
5	2010/5/29	Nokia 5800 Cell Phone	1289	714	644	644
5	2010/5/30	Nokia 5530 Cell Phone	1324	677	660	659

As shown in Table1, in the game card 600 points and canon 450 set camera cases, the average price of the high reputation sellers are lower than that of the low reputation sellers, which shows the negative price premium effect (in the case where the proportion of informed consumers is high). In the butter taste nuts 500g and Adidas male set cases, inversely, the average price of the high reputation sellers are higher than that of the low reputation sellers, which shows the positive price premium effect (when the proportion of informed consumers is low). This confirms to our theoretical findings.

Patch	Product	Avg Price (High)	Avg Price (Low)	t	Sig	Price Premium
1	Nokia N73 Cell Phone	2407.99	2440.32	-3.086	***	*NPP*
2	WOW Card 600	26.93	27.1	-8.221	***	*NPP*
2	Nokia N73 Cell Phone	1766.06	1838.42	-6.174	***	*NPP*
3	WOW Card 600	26.93	27.09	-7.413	***	*NPP*
3	Nokia N73 Cell Phone	1649.74	1731.54	-7.869	***	*NPP*
3	Nuts 500g without Package	31.76	30.86	4.547	***	*PP*
4	Nokia 5800 Cell Phone	1896.21	1945.27	-6.328	***	*NPP*
4	Nokia N73 Cell Phone	1186.82	1227.52	-4.156	***	*NPP*
5	Faked Nokia 5800 Cell Phone	515.51	483.26	2.094	**	*PP*
5	Nokia 5800 Cell Phone	1669.61	1734.94	-5.325	***	*NPP*
5	Nokia 5530 Cell Phone	1380.65	1449.43	-6.656	***	*NPP*

Table 1: Price Comparison

The following figures plot the average seller price against seller reputation. The left panel shows the Canon 450 camera set case, and the right panel shows the butter-taste nuts 500g case.



It can be shown that for products with a large proportion of informed buyers (Canon 450 camera set), negative price premium is more significant than for products with a small proportion of informed buyers (butter-taste nut 500g).

Market Share Comparison

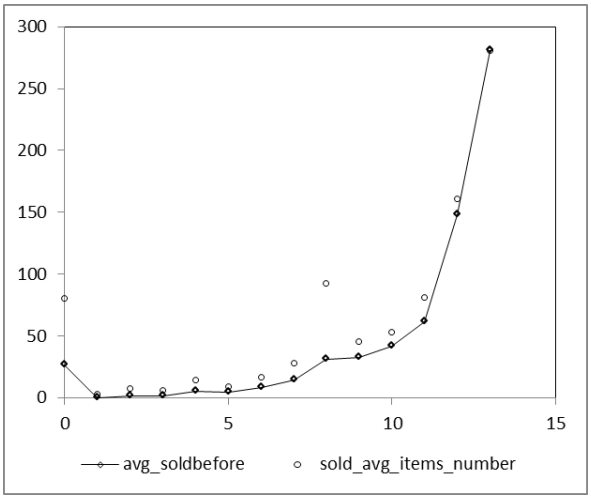
We compare the market share using the measure of "sales volume in a 30-day period." The data shows that high-reputation sellers enjoy a much higher market share than those low-reputation sellers. This result indicates that, whether the high reputation seller enjoys price premium or not, the high reputation sellers enjoy a higher market share. This finding confirms our proposition 2.

The difference of the average prices between high-reputation sellers and low-reputation sellers is also much larger when there are a large percentage of informed buyers than when the proportion of informed buyers is low.

Table 3, Market-Share Comparison

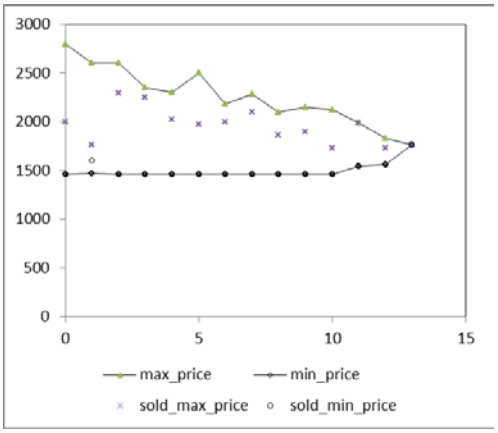
Patch	Product	Avg Price (High)	Avg Price (Low)	t	Sig	Market Premium
1	Nokia N73 Cell Phone	2.395	0.509	2.714	***	*MP*
2	WOW Card 600	67.638	18.683	2.818	***	*MP*
2	Nokia N73 Cell Phone	3.245	1.143	3.792	***	*MP*
3	WOW Card 600	47.331	12.355	7.898	***	*MP*
3	Nokia N73 Cell Phone	3.239	1.030	4.752	***	*MP*
3	Nuts 500g without Package	35.578	6.210	3.654	***	*MP*
4	Nokia 5800 Cell Phone	3.737	0.209	7.180	***	*MP*
4	Nokia N73 Cell Phone	1.907	0.368	3.155	***	*MP*
5	Faked Nokia 5800 Cell Phone	0.759	0.277	1.749	**	*MP*
5	Nokia 5800 Cell Phone	5.590	0.318	3.389	***	*MP*
5	Nokia 5530 Cell Phone	3.585	0.158	3.647	***	*MP*

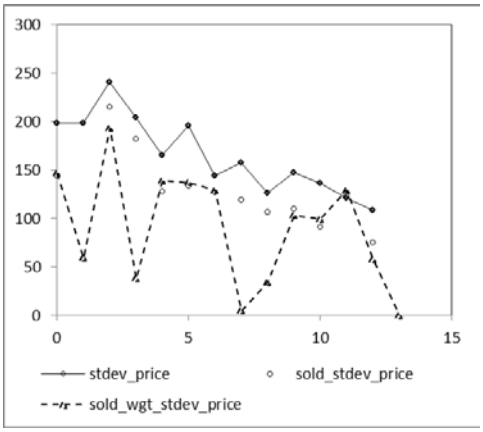
The following figure plot the average market share against seller reputation.

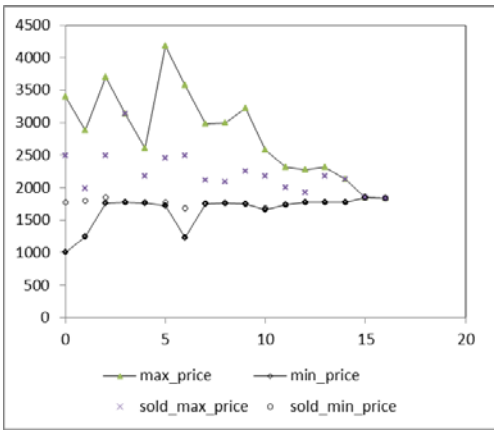


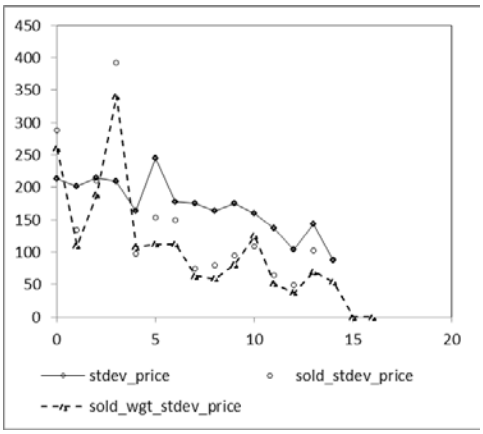
Price Dispersion:

The following figures show that, even though price dispersion seems to be huge on the internet, if we exclude those prices at which no transaction happens, the price dispersion is not as big. In addition, price dispersion is higher with low-reputation sellers than with high-reputation sellers. This may be caused by the heterogeneity among the low-reputation sellers: some see great values if growing into a high-reputation seller, so they are willing to set a lower price; some are more short-term player and satisfied with a random purchase with a high price.









Conclusion

Online price dispersion is an important phenomenon to study. This paper examines price dispersion from both the perspective of the seller side (reputation) and the buyer side (informativeness.) More importantly, we make the first attempt to connect price dispersion with the growth of an internet market which impacts both sellers and buyers through reputation systems. A "healthy" Internet market that can sustain in the long run should have a reputation system that helps to differentiate "good" low-reputation sellers from "bad" low reputation sellers. This way, the "good", low-reputation seller is able to grow into a high reputation seller and this help promote a more healthy internet market. We find that the existence of such a growth potential increases the price dispersion for low-reputation seller, while reduces the price dispersion for high-reputation sellers.

Future direction includes the consideration of more than 2 buyers at each reputation level. It is also worthwhile to perform a more detailed empirical study, for example, a longitude study for a set of sellers, to analyze the dynamic evolution about internet sellers with different reputation levels.

References:

G. Alessandria, Consumer Search, Price Dispersion, and International Relative Price Fluctuations. International Economic Review, 2009. 50(3): p. 803-829.

T. Andrews and C. Benzing, *The Determinants of Price in Internet Auctions of Used Cars.* Atlantic Economic Journal, 2007. **35**: p. 43-57.

S.L. Ba and P.A. Pavlou, Evidence of the effect of trust building technology in electronic markets: Price premiums and buyer behavior. MIS Quarterly, 2002. 26(3): p. 243-268.

J.Y. Bakos, Reducing buyer search costs: Implications for electronic marketplaces. Management Science, 1997. 43(12): p. 1676-1692.

M.R. Baye, J. Morgan, and P. Scholten, Price dispersion in the small and in the large: Evidence from an internet price comparison site. Journal of Industrial Economics, 2004. 52(4): p. 463-496.

E. Brynjolfsson and M.D. Smith, Frictionless commerce? A comparison of Internet and conventional retailers. Management Science, 2000. 46(4): p. 563-585.

J. Chevalier and D. Mayzlin (2006). The Effect of Word of Mouth on Sales: Online Book Reviews. Journal of Marketing Research: Vol. 43, No. 3, pp. 345-354.

K. Clay, et al., Retail Strategies on the Web: Price and Non-Price Competition in the Online Book Industry. Journal of Industrial Economics, 2002. L(3).

R.J. Kauffman and C.A. Wood, *Doing Their Bidding: An Empirical Examination of Factors that Affect a Buyer's Utility in Internet Auctions.* Information Technology Management, 2006. **7**: p. 171-190.

S.B. Li, K. Srinivasan, and B.H. Sun, Internet Auction Features as Quality Signals. Journal of Marketing, 2009. 73(1): p. 75-92.

Y.W. Liu, K.K. Wei, and H.P. Chen, Pricing Strategy in Online Retailing Marketplaces of Homogeneous Goods: Should High Reputation Seller Charge More? AMCIS 2009 Proceedings, 2009: p. 404.

W.H. Luo and Q.B. Chung, Retailer Reputation and Online Pricing Strategy. Journal of Computer Information Systems, 2010. 50(4): p. 50-56.

X. Pan, B.T. Ratchford, and V. Shankar, Can price dispersion in online markets be explained by differences in e-tailer service quality? Journal of the Academy of Marketing Science, 2002. 30(4): p. 433-445.

E. Rabinovich, A. Maltz, and R.K. Sinha, Assessing Markups, Service Quality, and Product Attributes in Music CDs' Internet Retailing. Production & Operations Management, 2008.

S. Salop, The Noisy Monopolist - Imperfect Information, Price Dispersion and Price-Discrimination. Review of Economic Studies, 1977. 44(3): p. 393-406.

C. Soh, M.L. Markus, and K.H. Goh, Electronic marketplaces and price transparency: Strategy, information technology, and success. MIS Quarterly, 2006. 30(3): p. 705-723.

S. Utz, U. Matzat, and C. Snijders, On-line Reputation Systems: The Effects of Feedback Comments and Reactions on Building and Rebuilding Trust in On-line Auctions. International Journal of Electronic Commerce, 2009. 13(3): p. 95-118.

H.R. Varian, A Model of Sales. American Economic Review, 1980. 70(4): p. 651-659.

R. Venkatesan, K. Mehta, and R. Bapna, Understanding the confluence of retailer characteristics, market characteristics and online pricing strategies. Decision Support Systems, 2006. 42(3): p. 1759-1775.

M. Zhou, M. Dresner, and R. Windle, Revisiting feedback systems: Trust building in digital markets. Information & Management, 2009. 46(5): p. 279-284.

Appendix.

Proof of Corollary 1:

Let $F_H(P)$ denote the distribution of the H sellers in equilibrium, and $F_H(P)$ denote that of the L seller. Let $H = 1 - F_H(p)$, $L = 1 - F_L(p)$.

For a high-reputation seller, she has to charge lower than both the low-reputation seller and another high-reputation seller to win the informed buyers: $\left(\frac{1-k}{3} + HLk\right)p = \frac{1-k}{3}$

For the low-reputation seller, she has to charge lower than the two high-reputation sellers to win the informed buyers: $\left(\frac{1-k}{3} + H^2k\right)p = \frac{1-k}{3}$

We now show that the high reputation sellers play a mixed strategy without considering the low-reputation seller, i.e.,

$\left(\frac{1-k}{3} + Hk\right)p = \frac{1-k}{3}$, and $F_H(p) = 1 - \frac{(1-k)(1-p)}{3kp}$, while the low reputation seller will not charge any price less than 1:

Since $\left(\frac{1-k}{3}+k\right)p \geq \frac{1-k}{3}$, we have that $p \geq \frac{1-k}{1+2k}$; Note that $p \leq 1$. If the low reputation seller charges any price less than 1, then the probability of winning the game is:

$\left(1-F_H(p)\right)^2 = \left(\frac{(1-k)(1-p)}{3kp}\right)^2$. We have to show that:

$\left(\frac{1-k}{3}+\left(1-F_H(p)\right)^2 k\right)p = \left(\frac{1-k}{3}+\left(\frac{(1-k)(1-p)}{3kp}\right)^2 k\right)p < \frac{1-k}{3}$. This is equivalent to show that $\left(\frac{(1-k)(1-p)}{3kp}\right)^2 kp < \frac{1-k}{3}(1-p)$, or $(1-k)(1-p) < 3kp$, or $3kp-(1-k)(1-p) > 0$, or $p \geq \frac{1-k}{1+2k}$, which is already shown to be true.

Done.

Proof of Proposition 2:

The high-reputation sellers try to win the $k(1-r)$ informed consumers, since she cannot win the kr informed consumers if the low-reputation seller sets a lower price. So we have

$\left(\frac{1-k}{3}+Hk(1-r)\right)p = \frac{1-k}{3}$, which gives $H = \frac{(1-k)(1-p)}{3kp(1-r)}$ and $F_H(p) = 1-\frac{(1-k)(1-p)}{3k(1-r)p}$.

The lower-bound price is determined by:

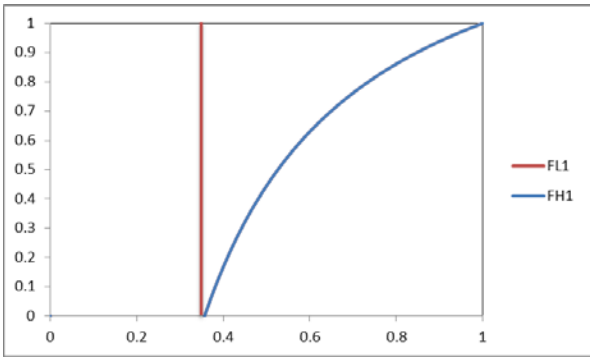
$\left(\frac{1-k}{3}+k(1-r)\right)p = \frac{1-k}{3}$, that is: $p = \frac{1-k}{1+2k-3kr}$

The low-reputation seller sticks to $p = \frac{1-k}{1+2k-3kr}$, since the high reputation seller will charge no less than $p = \frac{1-k}{1+2k-3kr}$. The threshold value of eis obtained by:

$\left(\frac{1-k}{3}+kr\right)\left(\frac{1-k}{1+2k-3kr}+e\right) > \frac{1-k}{3}(1+e)$, that is, by setting a price to win the kr informed buyer, the low-reputation seller gets higher profit than charging the upper-bound price and only serves those uninformed buyers. This gives $e > \frac{(1-k)(1-2r)}{r(1+2k-3kr)}$, or equivalently, $\overline{e}=\frac{(1-k)(1-2r)}{r(1+2k-3kr)}$.

It is obvious that, when $r \geq 1/2$, the critical value $e < 0$; i.e., if there are at least 50% of informed buyers believe the new sellers are good, the new sellers will definitely charge a low price to win the kr informed buyers.

When $r < 1/2$, only when the low-reputation seller with a high e finds it profitable to pursue the kr informed buyers.



(2) When e $<$ $\underline{e}$, for a high reputation seller, she defeats the other high-reputation seller to win the whole market if the following equation is satisfied: $\left(\frac{1-k}{3}+Hk\right)p=\frac{1-k}{3}$. This gives

$$F_H(p)=1-\frac{(1-k)(1-p)}{3kp}$$

If the low reputation seller charges any price less than 1, then the probability of winning the informed consumer is: $\left(1-F_H(p)\right)^2=\left(\frac{(1-k)(1-p)}{3kp}\right)^2$

We have to prove that:

$$\left(\frac{1-k}{3}+\left(1-F_H(p)\right)^2 kr\right)(p+e)=\left(\frac{1-k}{3}+\left(\frac{(1-k)(1-p)}{3kp}\right)^2 kr\right)(p+e)<\frac{1-k}{3}(1+e)$$, or

$$(1-k)(1-p)^2 r(p+e)<3kp^2(1-p)$$

i.e., the low reputation seller will have no chance to win the kr informed buyers.

Special Case: when $e=0$, we have $(1-k)(1-p)^2 rp < 3kp^2(1-p)$, which is equivalent to $p > \frac{(1-k)r}{3k+(1-k)r}$. Since $p \geq \frac{1-k}{1-k+3kr} = \frac{(1-k)r}{(1-k)r+3kr^2} > \frac{(1-k)r}{(1-k)r+3k}$, thus, when $e=0$, the new seller L charges 1.

The threshold e satisfies the following condition :

$e < \frac{3kp^2}{(1-k)(1-p)r} - p$. First we will show that the RHS of this inequality is increasing in p.

Taking the derivative, we need to show that $\frac{6kp(1-k)(1-p)r+(1-k)r3kp^2}{\left((1-k)(1-p)r\right)^2} - 1 > 0$.

$\leftrightarrow 6kp(1-p)+3kp^2 > (1-k)r(1-p)^2$

$\leftrightarrow 6kp-3kp^2 > (1-k)r(1-p)^2$

$\leftrightarrow (r-kr+3k)p^2-(2r-2kr+6k)p+(1-k)r<0$

$\leftrightarrow (r-kr+3k)(1-p)^2-3k<0$

$\leftrightarrow 1-p < \frac{3kr+3kre}{1-k+3kr} = \frac{3krr+3krre}{r-kr+3krr}$, or $3kp^2-(1-k)(1-p)r(p+e)>0$

Or $\frac{dLHS}{dp} = 6kp-(1-k)r(-p-e+1-p) = 6kp-(1-k)r(1-2p-e)$

$= 6kp+2p(1-k)r-(1-k)r(1-e)$

The lower bound of price is:

$\left(\frac{1-k}{3}+kr\right)(p+e) = \frac{1-k}{3}(1+e)$ or $(1-k+3kr)(p+e) = (1-k)(1+e)$

We have $p \geq \frac{1-k-3kre}{1-k+3kr}$

Join this with $e < \frac{3kp^2}{(1-k)(1-p)r} - p$, we can solve the threshold value $\underline{e}$.

(3) Otherwise, both the high reputation seller and the low reputation seller play mixed strategy.

For high reputation seller (note that defeated by the other H will lose the whole k informed consumers):

$$\left(\frac{1-k}{3}+H(1-L)k(1-r)+HLk\right)p=\frac{1-k}{3}$$

For Low reputation seller:

$$\left(\frac{1-k}{3}+H^2kr\right)(p+e)=\frac{1-k}{3}(1+e)$$

Then we can solve the equations, and get:

$$F_H(p)=1-\sqrt{\frac{(1-k)(1-p)}{3kr(p+e)}}$$

Note that $F_H(r)=1$, $F_H\left(\frac{1-k+3kre}{1-k+3kr}\right)=0$

$\left(\frac{1-k}{3}+(1+rL-r-L)Hk+HLk\right)p=\frac{1-k}{3}$, which gives $L=\frac{(1-k)(1-p)}{3prHk}+1-\frac{1}{r}$, and

$F_L(p)=\frac{1}{r}-\frac{(1-k)(1-p)}{3prHk}$, which is equivalent to $F_L(p)=\frac{1}{r}-\sqrt{\frac{(p+e)(1-k)(1-p)}{3krp^2}}$

Let $\frac{1}{r}=\sqrt{\frac{(p+e)(1-k)(1-p)}{3krp^2}}$, we have $(p+e)(1-k)(1-p)r=3kp^2$, or

$(3k+(1-k)r)p^2-(1-e)(1-k)rp-(1-k)re=0$. This gives the lower boundary of p.

Let $\frac{1-r}{r}=\sqrt{\frac{(p+e)(1-k)(1-p)}{3krp^2}}$, we have $(p+e)(1-k)(1-p)r=3kp^2(1-r)^2$. So the upper boundary of p is determined by:

$$\left(3k+3kr^2-7kr+r\right)p^2-\left(1-e\right)\left(1-k\right)rp-\left(1-k\right)re=0\,.$$