Economics of Co-design: The Role of Product Lines

Amit Basu
Southern Methodist University
abasu@cox.smu.edu

Sreekumar Bhaskaran
Southern Methodist University
sbhaskar@cox.smu.edu

March 21, 2014

Abstract

In many industries including software, machinery and home construction, inputs from final customers can be very helpful in creating effective customized products. This "co-design" process requires customers who participate in it to commit significant time and effort. To make matters worse, after leveraging its customers' efforts to create a customized product, the firm may choose to overprice the product. Since this reduces a customers incentive to collaborate with the firm, co-design that involves joint development between a firm and a customer can be difficult, and in some cases, difficult to motivate. In this paper, we develop analytical models that capture these various effects. Using these models, we examine how a firm can incentivize its customers to engage in co-design, and also how offering co-design can impact the firm's product (line) strategies and the quality of its products. The effect of market and firm characteristics on the value of engaging customers in the co-design process is also examined.

1 Introduction

The traditional approach to new product development by a firm is for the firm to invest in resources that enable it to define, design, develop, produce and then commercialize each new product. In this approach, the customer plays a largely passive role. Although the firm can seek to understand the customers' needs and preferences through various market research mechanisms, the ultimate design of the product is determined by the firm itself. This approach has worked well in the context of mass-produced products, or "standard products". However, interest in offering customized products has increased, thanks to advances in production and information technologies that allow firms to cost-effectively incorporate the particular needs and preferences of customers in the design of the products delivered to them. Such trends have been observed in industries ranging from automobiles, where most major manufacturers offer wide varieties of customizable features in their products, to newspapers and magazines, where the digitization of products has led to highly personalized electronic versions of what used to be very standardized physical products.

Customization of products can be realized in various modes. One mode that is well-established and in use for millennia, is the "job shop" approach, in which customers design the product they need, and the manufacturer builds the custom product to the specifications delivered by the customer. This approach works well for highly complex products, such as heavy industrial machinery and commercial construction, which are typically at relatively low scale, as well as for custom components outsourced by OEMs, which could be at large scale. In this approach, the customer has to assemble all the necessary design resources and expertise needed to design and specify the product in terms that can be communicated to the manufacturer. A second approach is "mass customization", which has gained a lot of attention in recent years. In this approach, by leveraging the rapid and cost-effective reconfiguration capabilities of production technologies such as flexible manufacturing systems (for physical products) and active online content management systems (for digital products), variants of product designs can be produced in a cost-effective manner even at relatively high scale. In this approach, the manufacturing firm has to invest in the relevant production technologies, and possibly also in customer-facing resources that enable customers to specify their design choices, usually from a palette of preset options. A key aspect of this approach is that the customization process takes relatively little effort from the customer, since it is typically a process of selection from finite and small sets of choices. What differentiates this approach from the job-shop approach is the ability of the manufacturer to operate at scale, even though the number of units produced of each configuration is very small.

A third approach in which the customer plays a more active role in the design of the product is the "co-design" approach which the forms the basis of this research. In this mode, the manufacturer provides a significant set of design resources to the customer, who then has the opportunity to work with these resources to help design a customized product. Two key features that differentiate this approach from the earlier ones are (1) the amount of effort expended by the customer is greater than in the customization approach, and (2) the manufacturer may not commit to a price for the custom product until the customer has completed the co-design process (note that even in the mass customization approach, the custom product may have a higher price, but usually the customer is aware of the price impact in advance, because the prices of each custom feature are posted/available or quoted in advance when the customer presents the job). An early (and unsuccessful) example of this approach is the online company mymachineshop.com, which failed after a few years. The company offered a significant set of online product design and configuration tools, including sophisticated CAD software, for customers to design and upload specifications for machined components. While there may have been several reasons for the failure of the company, one likely reason is its inability to motivate sufficient numbers of paying customers to use its services.

This last point in fact relates to the central issue of this paper. Co-design is a potentially effective way to overcome the inherent information asymmetry that exists between manufacturers of products and consumers/users of those products - in that customers know what they want, but traditionally have no effective way to let the manufacturer know that, while the manufacturer knows customer preferences and wants only at the aggregate level, and therefore have to design products that are acceptable, even though likely not ideal, for a sufficient number of customers. By enabling the customer to specify their precise needs and communicate these needs to the manufacturer, co-design potentially could lead to larger numbers of (fully) satisfied customers. However, this higher level of satisfaction comes with two additional costs - the customer may first have to expend non-trivial effort on the co-design process, and second, may have to pay a higher price that is possibly unknown till after the co-design process is complete. In other words, the manufacturer may be able to extract the entire consumer surplus generated through the co-design process from the customer in the form of these additional costs, thereby demotivating the customer to engage in the effort in the first place.

Thus, the key issue addressed in this paper is how a manufacturer capable of cost-effectively producing customized products can offer customers the opportunity to co-design the products they want, in a way that motivates the customer to invest the necessary effort without fearing that they would not realize any positive benefit from the process. Using a relatively simple and stylized economic model, we show (a) that offering co-design can lead to higher profits for firms if implemented effectively, (2) that inclusion of the co-design option for (some) elements of a product line can motivate desirable changes to the quality of even standard products offered by the firm, and (3) that when there is significant uncertainty among customers about the co-design capabilities of a firm, the firm can effectively signal its co-design capabilities in a way that motivates adoption of this option by the customer.

The paper is organized in five sections. We start by discussing some relevant streams of past research in the next section. Then we present a series of models that address the first two claims above. In section 4, we address the information asymmetry issue stated in claim (3) above. Finally in section 5, we discuss the implication of our results and directions for further research.

2 Relevant Literature

A number of different streams of research reported in the literature relate to the work reported in this paper. These are discussed in turn in this section.

2.1 Customization and Mass Customization

A number of researchers have examined the feasibility of product customization relative to traditional product standardization geared towards mass production. The choice between customization and standardization in mass production contexts, has been examined as far back as the 1980s at least (Davis, 1987). The question of how to produce mass customized products has also been discussed in the literature (Anderson, 2004; Zipkin, 2001). With regard to the decision to customize, Dewan et al. (2003) use a circular market-based model (Salop, 1979) to show that a firm that offers customization can deter potential new entrants by raising its range of customization. In Syam et al.

(2005), the customization strategies of competing firms are examined to show that in equilibrium, the firms are likely to adopt similar customization choices. Mendelson and Parlaktuk (2008) consider the competitive position of a firm as a factor in its choice of whether to adopt customization, and if so, what level of customization it should choose. They show that mass customization is not an effective competitive strategy for a firm that has "an inferior cost or quality position".

Another problem in the context of customization is the challenge of understanding customer needs, and the implication of these needs in terms of the quality and/or price of customized products. Terwiesch and Loch (2004) shows how progressive prototyping, particularly iterative collaborative prototyping involving both the firm and the customer, can be used to address this problem. They study the question of how many prototypes should be built, and how they, as well as the resulting customized products, should be priced.

2.2 Product Lines

Another stream of relevant literature is work on the role and effective use of product lines. Mussa and Rosen (1978) and Moorthy (1984) show that when customers are allowed to self-select products in a product line differentiated on quality and price, the quality of the low-preference segment is lower than the optimal level. Balachander and Srinivasan (1994) considers information asymmetry regarding a firm's product quality and its impact on market entry by potential competitors, and show that when an incumbent firm attempts to signal its dominant position to potential entrants, it sets quality across its line at levels higher than when there is no uncertainty about the incumbents higher quality.

2.3 Customer Co-creation

In addition to the more established notions of customization and even mass customization, there is an emerging literature on the engagement of customers in the product design process. Wind and Rangaswamy (2001) uses the term "customerization" to describe active customer engagement though resources such as the Internet. Dewan et al. (2003) also recognize the possible active involvement of customers in the product design process, and as well as the firm's role in facilitating such engagement by offering a certain level of customization capabilities. The notion of "co-creation" is studied in Syam and Pazgal (2013), in the context of multiple customers, and examines the effect of externalities among different customers as well as between the firm and customers to determine when and to what extent co-creation is beneficial. An important factor that they consider is the effect of the firm's pricing approach on customers' incentives to engage in co-creation.

3 Model Description

We consider a market consisting of two types of customers: high-end customers who are willing to pay a premium to obtain a product that meets their specific needs, and low-end customers who are less discerning and less demanding, and have a relatively low utility from the product. We denote these segments by H and L with H representing the high-end and L representing the low-end segment. Each customer in the H segment values a product of quality q at $v_H q$ while the same product is valued by a customer in the L segment at $v_L q$. Based on the definition of the two segments, $v_H > v_L > 0$. The total number of customers in the market is $N = n_H + n_L$; of these customers, n_H are of type H and the remaining n_L customers are of type L. We use the parameter α to represent the fraction of high-end customers i.e. $\alpha = {}^{n_H}/{}_{(n_H+n_L)}$. Without any loss of generality, we normalize the value of N to 1.

The firm can choose to offer either a single product or a product line of distinct products for each customer segment. Let the quality of the product made available to each segment be q_i. We assume that the marginal cost of production incurred by the firm is convex in the level of quality of the product. Specifically, we assume that the marginal cost of production a product of quality q is given by cq^2.

3.1 No co-design

We want to examine how the firm's decision to offer a product line, as well as the qualities of the product(s) it chooses to offer, affects its customers' decision on how much effort to invest in co-designing the firm's product(s). We start, however, by considering a simple case in which there is no opportunity for customers to engage in co-design. Note that this case has already been analyzed in the literature but for completeness, we present the main results in this setting.

Mode1 1: Single Product

The profits and product quality levels offered by the firm depend upon whether the firm offers a product line or a single product to its customers. When the firm offers a single product, it can be targeted such that only high-end customers purchase it or such that both the high and low-end segments purchase it. For the case in which the product is offered only to the high-end segment, let the profits of the firm from targeting only the high-end segment be π_{1H}, where the subscript 1 is used to refer to the model. These profits then can be represented as:

$$\pi_{1H} = \alpha \left(v_H q - c q^2 \right)$$

This expression takes into consideration the fact that the firm extracts the entire surplus from the customer. The quality that maximizes the firm's profits in this scenario is given by

$$q^*_{1H} = \frac{v_H}{2c} \tag{3.1}$$

and the corresponding profits of the firm are

$$\pi^*_{1H} = \frac{\alpha v_H^2}{4c} \tag{3.2}$$

If the firm wants to cover both segments with its product, it has to ensure that the price is not greater than the willingness to pay of the low-end segment, so that the low-end customers participate in the market. However, the firm can still price such that the entire surplus from the low-end segment is extracted. Thus its profit function, when both high- and low-end segments purchase the product, can be stated as

$$\pi_{1L} = \left(v_L q - c q^2 \right)$$

The corresponding optimal product quality and profits of the firm can now be represented respectively as

$$q_{1L}^{*} = \frac{v_L}{2c} \tag{3.3}$$

$$\pi_{1L}^{*} = \frac{v_L^2}{4c} \tag{3.4}$$

Model 2: Product Line

We now our attention to the case in which the firm offers a product line consisting of a **P**remium (high-end) product and a **S**tandard (low-end) product. Let p_P and p_S be the for premium and standard products respectively. Since it is in the best interests of the firm to ensure that the high-end segment purchases the premium product, the prices should be such that the high-end customer receives greater surplus by purchasing the premium product than by purchasing the standard product. So if the quality levels of the premium and standard product are q_P and q_S respectively, the prices of these products will be

$$\begin{aligned} p_{2S} &= v_L q_S \\ p_{2P} &= v_H q_P - v_H q_S + v_L q_S \\ &= v_H q_P - (v_H - v_L)\, q_S \end{aligned}$$

Once again, the firm extracts the entire surplus from the low-end segment. Thus the profit function of the firm as a function of q_P and q_S which are the quality of the premium and standard products respectively can be stated as

$$\pi_2 = \alpha \left(v_H (q_P - q_S) + v_L q_S - c q_P^2\right) + (1-\alpha)\left(v_L q_S - c q_S^2\right)$$

Therefore the optimal quality levels for the two products in this scenario will be

$$q_{2P}^{*} = \frac{v_H}{2c} \tag{3.5}$$

$$q_{2S}^{*} = \frac{v_L - \alpha v_H}{2c\,(1-\alpha)} \tag{3.6}$$

Furthermore, the profits of the firm are given by

$$\pi_2^* = \frac{\alpha v_H^2 - 2\alpha v_H v_L + v_L^2}{4c(1-\alpha)} \tag{3.7}$$

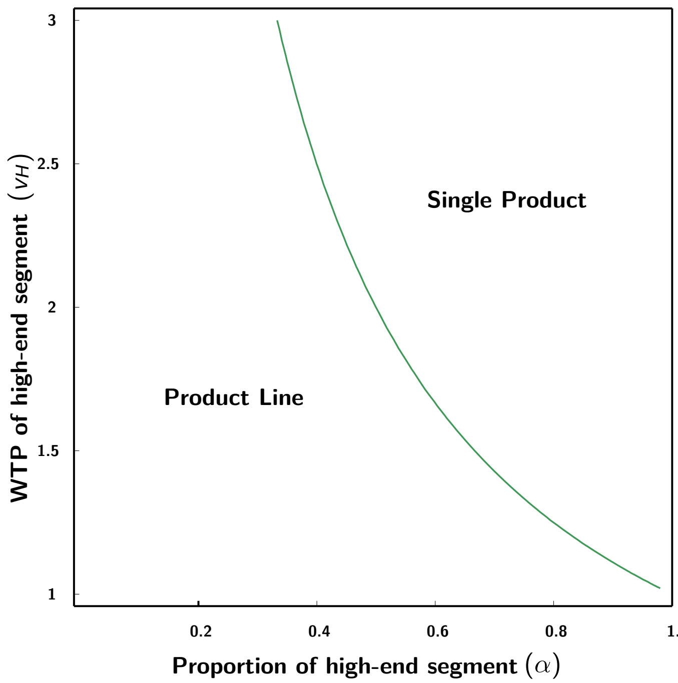


Figure 1: Effect of customer valuation of product quality

Proposition 1. *(i) There exists a threshold on the willingness to pay of the high-end segment,* $\bar{v}_H$*, above which the firm offers a single product that is targeted exclusively to the high-end. Below this threshold, the firm offers a product line.*
(ii) $\bar{v}_H$ *is decreasing in* α *and increasing in* v_L*.*

This is illustrated in Figure 1. The decision of the firm to either offer a product line or single product, quite predictably, depends on the willingness to pay of the high-end segment. A product line approach enables the firm to obtain greater market coverage, which however comes with threat of cannibalization. The presence of the standard product constrains the firm's ability to charge more for the premium product. As a result, for sufficiently high willingness to pay for the high-end customers, it is in the best interests of the firm to eliminate cannibalization by offering only the premium product, and focusing only on the high-end market segment. Consistent with intuition, an increase in the proportion of high-end customers (α) or decrease in the willingness to pay of low-end customers (v_L) increases the attractiveness of the single product strategy.

When the firm indeed offers a product line, it crimps the quality of the standard product to make it an unattractive choice for the high-end segment. So while the high-end segment receives efficient quality (used to denote the quality level a firm would offer a customer segment if it were the only one targeted, unconstrained by cannibalization from other segments (Krishnan and Zhu, 2006)), the low-end segment is offered a product whose quality is strictly below the efficient level.[1]

3.2 Co-design

We now consider the scenario in which the firm offers its customers the opportunity to co-design the product along with the firm. By engaging in co-design, the customer is able to develop a product that uniquely suits her requirements; consequently, the customer will be willing to pay more for such a product.

We capture this effect as follows: In the absence of the co-design option, the willingness of each customer segment i to pay for a given level of product quality is v_i. However, if the customer engages in co-design, the willingness to pay increases by a factor θ; thus the willingness to pay of the customer in segment i becomes $(1+\theta)\,v_i$. This increase in willingness to pay for a co-designed product depends on the *co-design effort* committed by the customer as well as the firm's *co-design capability* — which represents the set of co-design support features and facilities offered by the firm to facilitate the customer's co-design efforts. It is reasonable to assume that θ is increasing in both customer effort as well as firm capability, and for analytical tractability we assume the specific functional form $\theta = \phi \times \gamma$ where ϕ is the firm's co-design capability and γ is the co-design effort of the customer. We also assume that the cost incurred by the customer engaging in a co-design effort γ is convex of the form $\kappa\gamma^2$.

The sequence of decision making is as follows: The firm decides whether to engage the customers in co-design, and determines the quality of the products it would like to make available to the customers. The customer then decides how much effort to put into the co-design process, and this determines the willingness to pay for the products that they choose to buy. Finally after the co-designed product(s) is(are) produced, the firm announces the price(s) for the co-designed product(s)

[1] In our formulation, q^*_{1H} (eq. 3.1) and q^*_{1L}(eq. 3.3) are the efficient quality levels for the high- and low-end segments respectively.

and the customer decides which of the products she would like to purchase from the firm.

Model 3: Single Product

First let us consider a scenario where the firm offers a single product of quality q and in addition, allows the customer the option to customizing it through co-design. When the customer of segment $i \in [H, L]$ purchases the co-designed product, the utility she receives would increase from $v_i q$ to $(1+\theta)\, v_i q$. Since the firm can defer its pricing decision for the co-designed product till after the customer has incurred the cost $\kappa\gamma^2$ to co-design the product, it can extract (almost) all the surplus from the customer. The customer, anticipating this opportunistic behavior of the firm, will therefore choose not to engage in co-design. It follows that the profits that the firm is able to generate for this scenario would be identical to those in model 1.

Model 4: Product Line

Now we consider the scenario in which the firm, in addition to the premium product, also offers a standard product. As in model 2, let the quality of the premium product be q_P and that of the standard product be q_S. The effect of the co-design process is to increase the customer's willingness to pay for the product they have co-designed. Thus, when a customer of type $i \in [H, L]$ participates in the design process, her willingness to pay for the co-designed product increases to $(1+\theta)\, v_i$. As a result, the utility that she would obtain from the product $j \in [P, S]$ can be written as

$$u_{ij} = (1+\theta)\, v_i q_j$$

In contrast, when the same customer does not engage in co-design and purchases a standard product, her utility will be

$$u_{ij} = v_i q_j$$

Again, when the firm offers the product line, it would like the customers to self-select such that the high-end customer chooses the premium product while the low-end customer purchases the standard product. This can be ensured if the price of the standard product is no greater than the utility that the low-end customer obtains from it (participation constraint), while the price of the premium product is such that high-end segment obtains greater utility from purchasing it instead of the standard product (incentive compatibility constraint). As in the single product case, this implies that entire surplus of the low-end customers would be extracted out by the firm. This results in a situation wherein the low-end segment does not have an incentive to engage in co-design. Hence

$$\begin{aligned} p_{4S} &= u_S \\ &= v_L q_S \end{aligned}$$

If the high-end segment engages in co-design, the above incentive compatibility constraint will imply that the price of the premium product will be

$$\begin{aligned} p_{4P} &= (1+\theta)\, v_H q_P - (1+\theta)\, v_H q_S + v_L q_S \\ &= (1+\theta)\, v_H (q_P - q_S) + v_L q_S \end{aligned}$$

Note that the entire surplus of the high-end segment $((1+\theta)\, v_H q_P)$ is not captured by by the firm through this pricing decision; hence the high-segment will always find it to be valuable to engage in co-design. The extent of this engagement is reflected in the customer's co-design effort.

The net surplus of the customer for a given level of effort γ is the difference between the utility she derives from the product and the price she has to pay for it. The customer will choose an effort level that will maximize the net surplus she is able to obtain from the transaction while taking into consideration the cost of effort $\kappa\gamma^2$. Let $\lambda(\gamma)$ be the net surplus of the high-end customer; it follows that

$$\begin{aligned} \lambda(\gamma) &= u_{HP} - p_{4P} - \kappa\gamma^2 \\ &= (1+\phi\gamma)\, v_H q_P - p_{4P} - \kappa\gamma^2 \\ &= q_S\left((1+\phi\gamma)\, v_H - v_L\right) - \kappa\gamma^2 \qquad (3.8) \end{aligned}$$

Proposition 2. *The optimal effort of the customer is given by*

$$\gamma^* = \frac{q_S v_H \phi}{2\kappa} \tag{3.9}$$

In addition, γ^* is increasing in v_H, q_S and ϕ.

Proof. In the appendix □

Unlike in model 3 where the firm offers only a single product, the customer now finds it to be worthwhile to engage in co-design when both a premium as well as a standard product are offered. In effect, the standard product serves as a fallback option for the customer in case the firm chooses to gouge the customer by over-pricing the co-designed product. It is interesting to note that the optimal effort of the customer is increasing in q_S which is the quality of the standard product. As q_S increases, the value of the fallback option from the standard product increases as result of which the customer increases her effort on co-design. Consistent with intuition, this effort is also increasing in v_H which is the willingness of the customer to pay for quality, as well as ϕ, which is the firm's co-design capability.

Having determined the optimal co-design effort of the customer, we now turn our attention to the firm's decision with respect to the quality level of the standard and premium products. The firm's profit function when it offers both products and customers self-select as above can be represented as

$$\pi_4 = \alpha \left(p_{4P} - c q_P^2\right) + (1 - \alpha) \left(p_{4S} - c q_S^2\right)$$

Substituting the optimal effort and the corresponding prices for both products into the above profit function, we obtain that

$$\pi_4 \left(q_P, q_S\right) = q_S \left(v_L - c q_S\right) - \frac{\alpha(q_P - q_S) \left(2c\kappa(q_P + q_S) - v_H \left(2\kappa + q_S v_H \phi^2\right)\right)}{2\kappa}$$

The optimal quality levels for both products can be determined by differentiating the profits *w.r.t.*

q_P and q_S and these are as below

$$q_{4P}^{*}(\phi) = \frac{2\kappa v_H\left(v_H\phi^2(\alpha v_H+v_L)+4(1-\alpha)c\kappa\right)}{16(1-\alpha)c^2\kappa^2+8\alpha c\kappa v_H^2\phi^2-\alpha v_H^4\phi^4} \tag{3.10}$$

$$q_{4S}^{*}(\phi) = \frac{2\kappa\left(\alpha v_H^3\phi^2+4c\kappa v_L-4\alpha c\kappa v_H\right)}{16(1-\alpha)c^2\kappa^2+8\alpha c\kappa v_H^2\phi^2-\alpha v_H^4\phi^4} \tag{3.11}$$

Having determined the optimal quality level for the premium and standard products, we now examine how these quality levels are affected by changes in market and product characteristics.

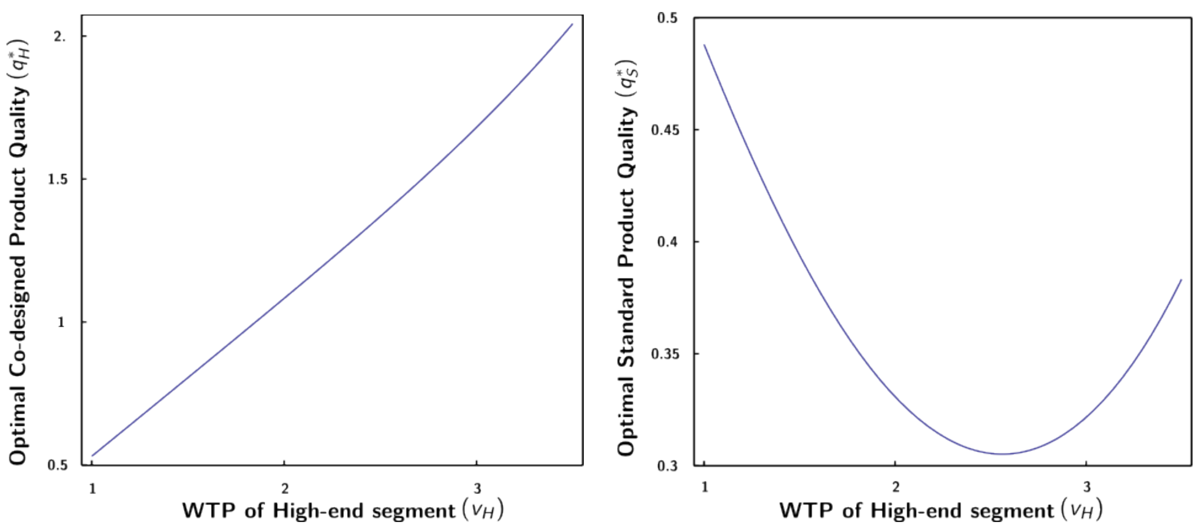


Figure 2: Effect of customer valuation of product quality

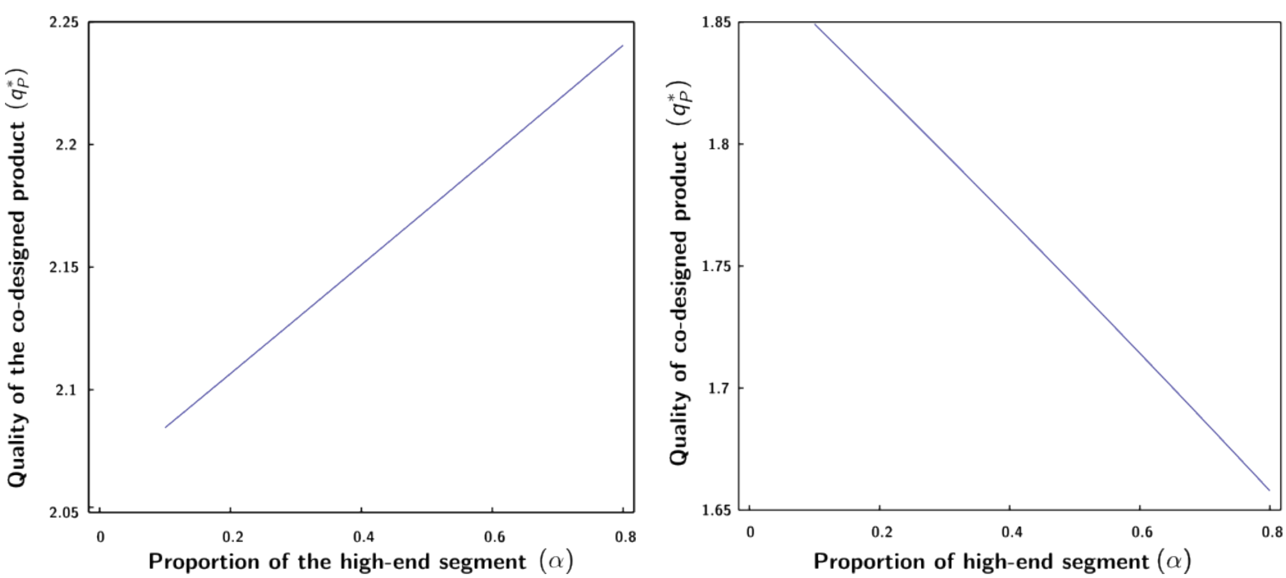
Proposition 3. ***Effect of customer valuation***

1. q_{4P}^* is increasing in v_H and v_L

2. q_{4S}^* is increasing in v_L. In contrast, q_{4S}^* is non-monotonic in v_H; in addition, q_{4S}^* is decreasing in v_H when v_H is low and increasing in v_H when v_H is high.

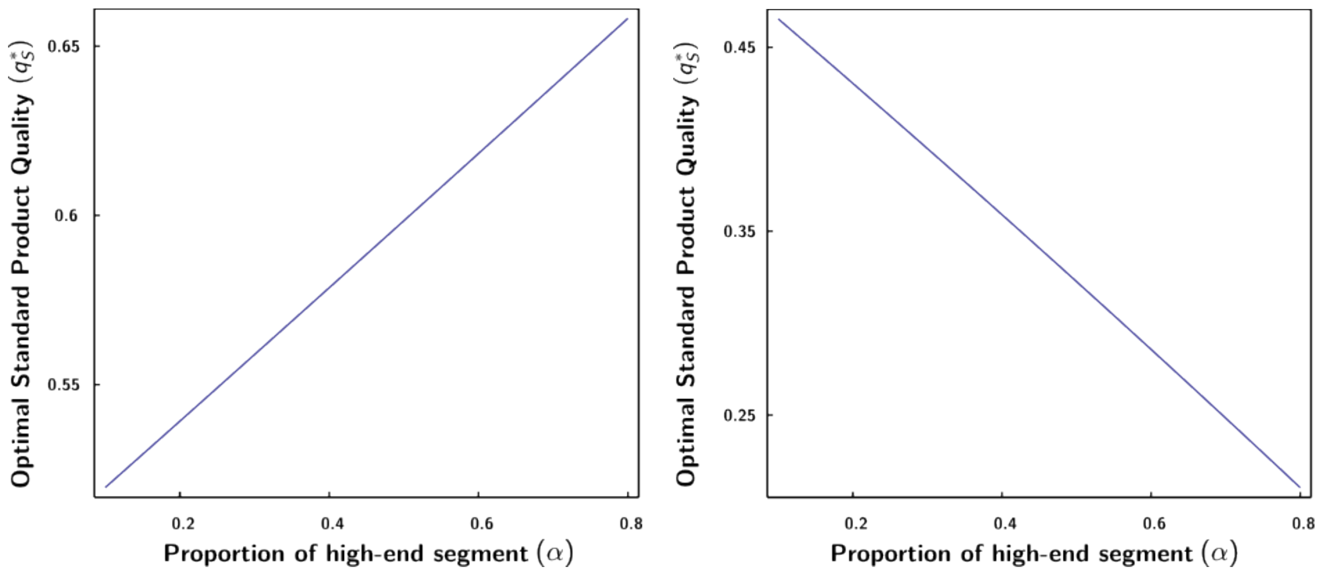
This proposition is illustrated in Figure 2. It implies that the optimal quality of the premium product is increasing in both the valuation of the high and low-end customer segments. A higher valuation of both these segments serves two objectives: an increase in the marginal return from an increase in product quality as well as a reduction in the level of cannibalization from the low-end segment. First, because the high-end customer segment is willing to spend more on quality, it becomes worthwhile for the firm to offer a better quality for the premium product. Second the higher valuation of the low-end customers (v_L) increases the price that a firm can charge for the standard product, which in turn mitigates the effect of cannibalization from that product. This same increase in low-end customer valuations similarly increases the optimal product quality of the standard product as well.

In contrast, the effect of higher v_H on the optimal quality of the standard product is less straightforward. Depending on whether the valuation of the high-end customer segment is high or low, a further increase in their valuation can increase or decrease the optimal quality of the standard product, due to two opposing effects. On the one hand, an increase in v_H increases the value of the high-end segment for the firm; as v_H increases, the firm seeks to consolidate this value by crimping the quality of the standard product which helps to reduce the effect of cannibalization. As a result, as v_H increases, the firm is tempted to reduce the quality of the standard product (Mussa and Rosen, 1978; Moorthy, 1984). However, in the presence of the co-design option, the firm also needs to consider the effect of the quality of the standard product on the customer's incentive to participate in the co-design process. As detailed earlier, the optimal effort of the customer is increasing in the quality of the standard product. The value of this higher effort is also greater when the willingness to pay of the high-end segment is higher. Due to this, above a sufficiently high value of v_H, a further increase in v_H calls for a higher quality standard product. Indeed, this result illustrates the tension between the need to mitigate cannibalization in conjunction with the imperative to incentivize the customers to participate in the co-design process.



Low κ High κ

Figure 3: Effect of α on optimal co-designed product quality



Low κ High κ

Figure 4: Effect of α on optimal standard product quality

Proposition 4. ***Effect of*** α

1. The optimal qualities q_{4j}^* *is increasing in* α *only if* κ *is below a threshold.* q_{4j}^* *is decreasing in* α *otherwise.*

This is illustrated in Figures 3 and 4. It characterizes the effect of α, which represents the relative size of the high-end segment of the market, on the optimal quality of both products. Interestingly, we find that its effect depends on the co-design cost of the customer, i.e., whether it is high or low. When this cost is low (low κ), the firm should optimally increase the quality of both products when there is an increase in the size of the high-end segment. In contrast, when the customer's co-design cost is high (high κ), an increase in α pushes the optimal qualities in the opposite direction. To understand these results, it is useful to examine the effect of an increase in α on the relative significance of the high and low-end customer segments.

As α increases, the firm's decisions are increasingly focused on the value that can be derived from the high-end segment. An increase in the quality of the standard product has two diametrically opposing effects. On the one hand, a higher standard product quality serves to increase the high-end customer's incentive to engage in co-design and invest in greater co-design effort. At the same time, this higher standard product quality can also intensify the cannibalization of the premium product by the standard product.

The relative strength of these two effects determine the firm's optimal response to an increase in α. When κ is sufficiently low, the marginal return from a co-design effort of the customer is much higher. As a result, the firm finds it to be more profitable to increase the quality of the standard product and incentivize the customer to commit a higher level of effort on co-design. This higher co-design effort also increases the value from the premium product and hence the quality of the premium product as well. On the other hand, when κ is sufficiently high, the return from customers' effort into co-design is too low to overcome the effect of cannibalization. As a result, an increase in α pushes the firm to reduce the quality of the standard product as a means to reduce cannibalization. The lower standard product quality now allows the firm to charge more for the premium product. However, since the customer's effort is also lower in this scenario, the quality of the premium product also suffers. Thus, as α increases, the optimal quality of both premium and standard products go down.

Proposition 5. ***Effect of co-design capability*** ϕ

1. q^*_{4j} *is increasing in* ϕ.

The implication of this proposition is relatively easy to see, and the effect of the firm's co-design capability on the optimal quality levels of both products is consistent with intuition. Higher co-design capability supported by a firm encourages a greater level of co-design effort from the customers, which in turn translates to higher quality for both products.

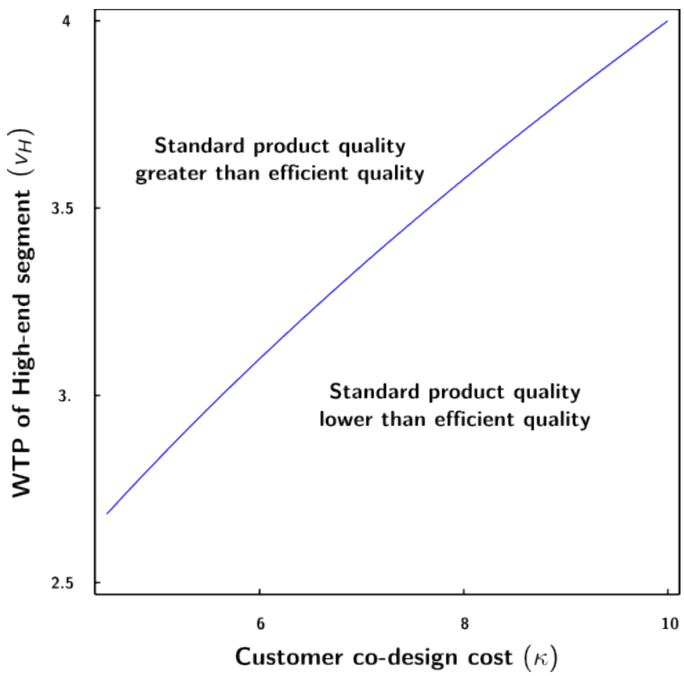


Figure 5: Effect of co-design on standard product quality

Proposition 6. ***Effect of co-design on product quality***

1. $q^*_{4j} \geq q^*_{2j}$ i.e. the firm offers higher standard and premium quality products when customers have the option of co-design.

2. There exists a threshold on κ below which $q^*_{4S} > q^*_{1L}$ i.e. low-end customers obtain a quality that is even greater than the efficient quality for that segment.

We find that when the firm offers the co-design option to customers, the optimal quality of both products is higher. The option to co-design allows a customer to fine-tune the premium product to her particular needs and increases her valuation for that product. This in turn increases the marginal value that the firm can derive from its quality investments for the premium product and thus increases the optimal quality of the premium product.

In addition to the direct effect on the premium product quality, the option of co-design also indirectly affects the quality of the standard product. A higher quality of the standard product can encourage customers to invest more effort into the co-design process; this effect increases the optimal quality of the standard product. Indeed, this effect can be so strong that the firm might even offer a standard product whose quality can be higher than the efficient quality. However, for this to be the case, we find that the customer's co-design cost needs to be sufficiently low. In this region, the high marginal return from customers' co-design effort mitigates the cannibalization threat from the standard product. As a result, the firm finds it optimal to raise the standard product quality to a level that is even beyond the efficient quality.

This result is particularly interesting because it runs counter to the conventional wisdom from the product line literature. As described in (Moorthy, 1984; Moorthy and Png, 1992; Mussa and Rosen, 1978) , when a firm offers a product line, it seeks to mitigate the potential cannibalization effect by reducing the quality of the low-end offering. The lower quality discourages the high-end segment from settling for this option. In contrast, when a firm offers co-design options to its customers, the same motivation to increase the profitability of the high-end segment encourages the firm to increase the quality of the low-end offering (standard product). Thus the cannibalization effect is superseded by the benefit derived through customer co-design.

It is also useful to compare our results with (Dewan et al., 2003) who show that a monopolist manufacturer might find it optimal to offer both standard and customized products as a means to enable higher but efficient market coverage. They also show that the firm might find it optimal to offer a level of customization far greater than what the customers themselves might prefer. In contrast in our paper, the level of co-design, is decided by the customer herself and her incentives therein are influenced by the firm's co-design capability as well as its product line choices. Moreover, the customers are strictly better off both because they get a better quality product and because their greater effort leads to higher willingness to pay for quality.

Optimal Strategy of the Firm

As we showed earlier, in order to motivate co-design, the firm has to offer a product line. This is because without a product line, the ability of the firm to extract all surplus from the customer after she puts in the co-design effort, discourages her to even engage in this process. However, although a product line allows a firm to enagage the customer, the resulting cannibalization still constrains the firm's pricing power. One option for the firm in this situation would be to offer just a single product that is targeted only towards the high-end segment as in model 1. While this option lacks the involvement from the customers through co-design, it also does not suffer from the cannibalization effect in a product line. In the following proposition, we determine when the firm would find it optimal to offer a product line and furthermore, when to engage the customer in the co-design process.

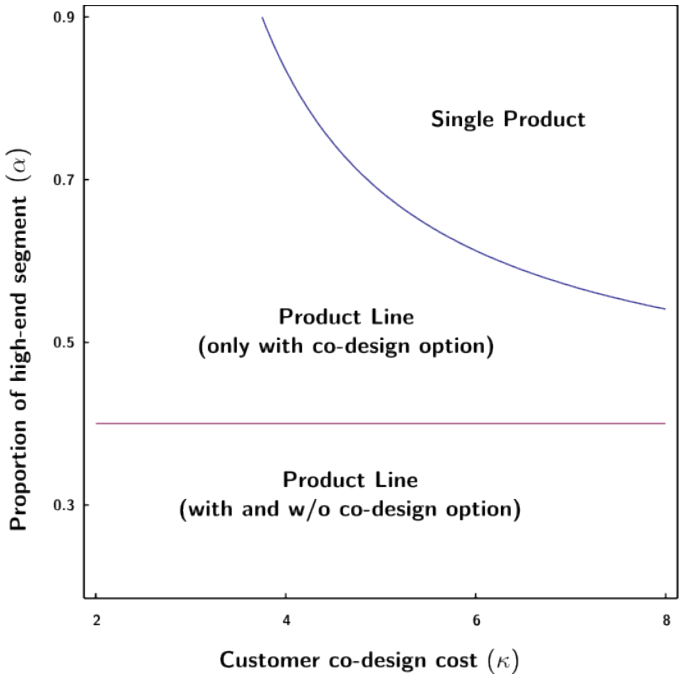


Figure 6: Optimal co-design strategy

Proposition 7. ***Optimal co-design strategy***

1. There exists a threshold on κ above which single product strategy dominates co-designed product line. This threshold is decreasing in α.

2. The range of parameters for which a product line is optimal increases when the firm offers customers the option of co-design.

The first part of the proposition underscores the critical role played by the customer's co-design cost (κ) in the firm's decision to offer a product line. When this cost is sufficiently low, co-design encouraged through the offering of product lines becomes a viable strategy for the firm. However, if this cost is too high, a single product strategy becomes the preferred option. Essentially, the presence of the low-end standard option helps motivate the customer to participate in the co-design process and its value increases as the customer's co-design cost decreases. This effect is further moderated by the size of the high-end segment. As the proportion of high-end customers increases, the single product option becomes preferable at lower levels of co-design cost. The firm in this situation is balancing the need to encourage its customers to participate in co-design with its desire to manage cannibalization. When α becomes larger, a single product helps the firm eliminate cannibalization and derive higher profits from the larger high-end segment.

The second part of the proposition illustrates an important effect of co-design in a firm's product strategy. The presence of co-design option increases the range of parameters for which a firm should offer a product line to its customers. Figure 6 shows that the co-design option motivates the firm to offer a product line in the intermediate region in which it would otherwise have offered only a single product targeted at the high-end segment. Thus, the possibility of co-design encourages the firm to offer broader coverage of the market. In addition, the value of this higher coverage is greater when the customer's cost to co-design is lower.

4 Informational Asymmetry about Firm's Co-design Capability

We have so far assumed that customers know the co-design capability (henceforth referred to as capability) of the firm. In reality, until the customer invests significant effort into the co-design process, they might not be aware of the true level of a firm's capability. At the same time, the firm might also be unable to reliably communicate its capability level to customers since every firm might find it to be beneficial to claim that it has high capability.

In this section, we explicitly account for this possibility that customers are not able to reliably distinguish between a high and low capability firm. We model this by assuming that the firm can be one of two types: an h type firm that has a high co-design capability ϕ_h or an l type firm with low co-design capability ϕ_l where $\phi_h > \phi_l > 0$. The customer does not know with certainty which type of firm she is interacting with (high capability or low capability) and knows only the probability associated with the different types. In particular, she knows that the firm is of h type with probability ω (and correspondingly, of type l with probability$(1-\omega)$). Let us define $\bar{\phi} = \omega\phi_h + (1-\omega)\,\phi_l$, defined as the expected value of the firm's capability.

First let us consider the "full-information" scenario in which the customer is able to identify the type of the firm that she is dealing with. In this scenario, when a customer knows that she is dealing with a high (low) capability firm, she would invest in a high (low) level of effort that is consistent with the analysis in the previous section (eq. 3.9). The firm, anticipating this response from the customer would also choose to offer quality levels in accordance with its capability level, as determined earlier (and repeated below for ease of exposition).

$$q_P^*(\phi_i) = \frac{2\kappa v_H\left(v_H\phi_i^2(\alpha v_H+v_L)+4(1-\alpha)c\kappa\right)}{16(1-\alpha)c^2\kappa^2+8\alpha c\kappa v_H^2\phi_i^2-\alpha v_{Hi}^4\phi^4} \quad (4.12)$$

$$q_S^*(\phi_i) = \frac{2\kappa\left(\alpha v_H^3\phi_i^2+4c\kappa v_L-4\alpha c\kappa v_H\right)}{16(1-\alpha)c^2\kappa^2+8\alpha c\kappa v_H^2\phi_i^2-\alpha v_H^4\phi_i^4} \quad (4.13)$$

where $i \in [h, l]$.

Now consider the scenario in which the customer is not certain about the firm's capability. In the absence of any information that resolves this uncertainty, the decision of the customer with respect to the effort on co-design depends on the expected surplus that she obtains through the co-design process. Recall from Proposition 2 that the optimal co-design effort is increasing in the firm's capability; as a result the co-design effort under informational asymmetry would also depend on the customer's perception of the firm's capability.

Given the uncertainty about the firm's capability, the customer will be hesitant to invest much effort in the co-design process due to the fear that it might be dealing with a low-capability firm. This leads to the natural question of whether a high-capability firm can reliably signal its type to encourage the customer to put in a high level of effort into the co-design process. In this section, we examine how and when the firm is able to signal this information.

Let us first consider the case in which the firm is not able to reliably signal its capability. Since the customer is not able to distinguish between the two firm-types, her decision on how much to invest in the co-design process will take into consideration the expected surplus she will obtain through her efforts. This surplus can be represented as

$$\begin{aligned}\lambda_E(\gamma) = {} & \omega\left(q_S\left((1+\phi_h\gamma)v_H-v_L\right)\right)+(1-\omega)q_S\left((1+\phi_l\gamma)v_H-v_L\right) \\ & -\kappa\gamma^2\end{aligned} \quad (4.14)$$

The optimal co-design effort of the customer in this scenario will be

$$\underline{\gamma} = \frac{q_S v_H\bar{\phi}}{2\kappa} \quad (4.15)$$

To figure out the quality levels that the firm might choose to offer, we need to determine the profits that the firm might be able to obtain, when it anticipates this effort from a customer. These profits, would in turn, depend on the capability level of the firm. If the firm is of high capability, its profits will be[2]

$$\pi_{mh}\left(q_P, q_S, p_P, p_S\right) = \alpha\left(p_P\left(\phi_h, \underline{\gamma}\right) - cq_P^2\right) + (1-\alpha)\left(p_S - cq_S^2\right) \tag{4.16}$$

The above expression takes into account the fact that the customer would be benefiting from her opportunity of having worked with a high capability firm; hence, the price she is willing to pay for the co-designed product would be higher (due to the higher return for her effort). Substituting the prices and quality levels into the profit function, we have that

$$\pi_{mh} = \frac{2\kappa q_S\left(v_L - cq_S\right) - \alpha\left(q_P - q_S\right)\left(2c\kappa\left(q_P + q_S\right) - v_H\left(2\kappa + q_S v_H \bar{\phi}\right)\right)}{2\kappa} \tag{4.17}$$

Differentiating the above profit function with respect to q_P and q_S gives us the optimal quality levels that the high-type firm would like to offer

$$q_{mhP}^* = \frac{2\kappa v_H\left(v_H \phi_h \bar{\phi}\left(\alpha v_H + v_L\right) + 4\left(1-\alpha\right)c\kappa\right)}{16\left(1-\alpha\right)c^2\kappa^2 + 8\alpha c\kappa v_H^2 \phi_h \bar{\phi} - \alpha v_H^4 \phi_h^2 \bar{\phi}^2} \tag{4.18}$$

$$q_{mhS}^* = \frac{2\kappa\left(\alpha v_H\left(4c\kappa - v_H^2 \phi_h \bar{\phi}\right) - 4c\kappa v_L\right)}{16(1-\alpha)c^2\kappa^2 + 8\alpha c\kappa v_H^2 \phi_h \bar{\phi} - \alpha v_H^4 \phi_h^2 \bar{\phi}^2} \tag{4.19}$$

Note that the above analysis assumes that the high capability firm is not able to differentiate itself from the low capability firm. For this to be an equilibrium, it should be optimal for the low capability firm to mimic the high capability firm's quality levels. This will lead to a situation in which the customer will not be able to distinguish between the two firms and hence she will determine her optimal effort based on the average of the two capability levels. It follows the optimal quality level

[2]The first subscript m refers to the fact that the resulting equilibrium will a pooling equilibrium where the low-capability firm **m**imics the high-capabiltiy firms product quality levels, and d refers to a separating equilibrium where high and low capability firms offer **d**ifferent quality levels.

of the low-capability firm would be

$$
\begin{aligned}
q^*_{mlP} &= q^*_{mhP} \\
q^*_{mlS} &= q^*_{mhS}
\end{aligned}
$$

Substituting the optimal quality levels into the profit function gives us the optimal profits of the firm as a function of the capability as well as the customers' beliefs about the capability.

Now consider the scenario in which the firm tries to signal its type through its product quality levels. In such a scenario, the customer will be able to make an informed decision on the level of effort that she should invest in co-design. Let q_{dhj} be the quality levels offered by the high capability firm and q_{dlj} be that of the low capability firm where $j \in [P, S]$.

Given the different quality levels, when the customer decides how much effort to invest in co-design, she takes into account both the implied capability level of the firm as well as the quality of the standard product it offers. As detailed in Proposition 2, her optimal effort would then be

$$
\gamma_j = \frac{q_{djS} v_H \phi_j}{2\kappa} \tag{4.20}
$$

where $j \in [h, l]$ indicates the capability of the firm as inferred from the quality level.

Now consider the decision of a firm whose capability is low. Under the assumption that the quality levels of the low-capability firm are different from that of the high-capability firm, its profits as a function of prices and quality can be characterized as

$$
\pi_{dl1}\left(q_{dlP}, q_{dlS}, p_P, p_S\right) = \alpha\left(p_P\left(\phi_l, \gamma_l\right) - cq^2_{dlP}\right) + (1-\alpha)\left(p_S - cq^2_{dlS}\right) \tag{4.21}
$$

If however, the low-capability firm mimics the quality levels chosen by the high-capability firm, its profits will be

$$
\pi_{dl2}\left(q_{dhP}, q_{dhS}, p_P, p_S\right) = \alpha\left(p_P\left(\phi_l, \underline{\gamma}\right) - cq^2_{dhP}\right) + (1-\alpha)\left(p_S - cq^2_{dhS}\right) \tag{4.22}
$$

For the high-capability firm to be able to credibly signal its type, it has to ensure that the low-capability firm finds it optimal to not mimic its quality levels. Hence the only feasible quality levels for the high capability firm would be those levels that ensure that $\pi_{dl2} < \pi_{dl1}$.

Proposition 8. *If $\pi_{dl2}\left(q_P^*\left(\phi_h\right), q_S^*\left(\phi_h\right)\right) > \pi_{dl1}\left(q_P^*\left(\phi_l\right), q_S^*\left(\phi_l\right)\right)$, then the optimal product quality levels that are offered under a separating equilibrium for both firms will be as follows*

1. $q_{dhP} > q_P^*\left(\phi_h\right)$ and $q_{dhS} > q_S^*\left(\phi_h\right)$

2. $q_{dlP} = q_P^*\left(\phi_l\right)$ and $q_{dlS} = q_S^*\left(\phi_l\right)$.

Proposition 8 characterizes the optimal response of firms when a separating equilibrium in which a high capability firm is able to signal its type becomes possible. When $\pi_{dl2}\left(q_P^*\left(\phi_h\right), q_S^*\left(\phi_h\right)\right) > \pi_{dl1}\left(q_P^*\left(\phi_l\right), q_S^*\left(\phi_l\right)\right)$, the low-capability firm finds it optimal to mimic the high type firm's quality levels if the high-capability firm were to offer a quality level that is optimal under the full information scenario. In such a situation, the high-capability firm signals its type by choosing a quality which is even higher than the quality under the full information case (i.e. $q_{dhP} > q_P^*\left(\phi_h\right)$ and $q_{dhS} > q_S^*\left(\phi_h\right)$). Although the low-capability firm would be able to induce a higher level of effort from the customer by mimicking the high-capability firm's quality levels, the cost of this strategy does not justify the benefit it provides. The customer, after having designed the product, would be able to decipher the true capability and its impact on her willingness to pay, and consequently would not be willing to pay a premium for the product. As a result, the low-capability firm would find it optimal to stick with the quality levels under a full information scenario ($q_{dlP} = q_P^*\left(\phi_l\right)$ and $q_{dlS} = q_S^*\left(\phi_l\right)$).

Thus we find that the role that a product line plays in encouraging customers to participate in the co-design process becomes greater when customers are uncertain about the co-design capability of the firm before they engage in co-design. The implication of this result is that under informational asymmetry, there will be more circumstances under which the low-end customers will be offered a higher quality standard product (and sometimes, a quality level that is even higher than the efficient quality) and where the high-capability firm will be able to reliably signal its type to the customer interested in co-design. This in turn will motivate a higher level of co-design effort from the customer which will result in a more valuable custom product for that customer.

5 Discussion and Conclusion

Although there is a considerable body of research on the consideration of individual customer preferences in product design, mostly in the context of customization and mass customization, a tacit assumption common in this literature is that the primary cost of any customization effort is borne by the manufacturer of the product. In this paper, we have examined the issue of co-design, which involves the investment of a non-trivial amount of effort by customers in their contribution to the product design process. In other words, we have modeled the co-design process as defined by both the co-design capability supported by the manufacturer and made available to the customer, as well as the co-design effort invested by the customer engaging in co-design. We start with the premise that the customer facing the prospect of investing significant effort on co-design may balk at such an effort if she perceives all the consumer surplus resulting from this effort will be captured by the firm through over-pricing of the custom product. We examine the problem of overcoming this resistance on the customer's part to engage in co-design. We show that the customer is unlikely to engage in a potentially expensive co-design process unless she also has the option of buying a standard product that does not involve co-design, and therefore may provide lower value to the customer. In other words, co-design is feasible only if the firm offers a product line that includes both standard products and custom products. Furthermore, we show that when a firm offers a product line including both standard products as well as customizable products that could generate higher utility to the customer through co-design, it may be profitable for the firm to actually offer higher quality standard products than it might otherwise. This is an interesting result, which counteracts the cannibalization effect that might motivate the firm to lower the quality of its low-end products.

We also address the question of the firm's optimal strategy when it has the option of supporting co-design. We show that depending on the composition of the market, in terms of the distribution of high-end versus low-end customers, as well as the cost faced by customers in engaging in co-design, the firm's optimal strategy can range from offering only high quality standard products to high-end customers to offering a product line of standard products at different levels of quality, to offering a product line that includes co-designed products as well as standard products. An interesting result in this context is that the range of conditions under which a product line is preferable to a single product strategy increases when the firm offers the option of co-design.

Finally, we examine the very real problem of information asymmetry that arises when customers considering the option of engaging in co-design are unsure of the co-design capability of the firm offering that option. We show that a firm interested in leveraging co-design and therefore providing a high level of co-design capability can signal this higher capability to the interested customer by raising the quality of its products relative to the full information situation in which the customer knows the firm's co-design capability before investing any effort. In effect, when there is this information asymmetry, the firm may find it profitable to raise the quality of not only its customizable products but also that of its standard products, in some cases even beyond its efficient quality level.

The interplay of the product line planning decision and the co-design capability planning decision that we show through our analysis can be valuable in the product planning process. As we have mentioned in the introduction section, firms that want to pursue co-design as a feature of their product offerings cannot simply adopt a "field of dreams" approach (based on the notion "build it and they will come"). Rather, it is important to view the co-design capability decision as a part of their product portfolio planning.

While we believe that our model provides useful insights, it is still a first step in understanding the economics of co-design. There are a number of refinements and extensions that would be worth exploring in future research. For instance, we have so far ignored the cost of co-design to the firm. Realistically, the firm faces two kinds of costs in supporting co-design. First, the firm may incur non-trivial costs in developing and delivering its co-design capability to its customer base. While this may be quite high in some contexts, particularly in traditional brick and mortar businesses, the use of technologies such as the Internet and a variety of computer-aided software tools that are increasingly becoming more user-friendly, can make these costs more manageable. In addition, however, the firm also faces the cost of producing the custom products that result from the co-design process. In this paper, we have assumed that the extent of co-design is limited only by the firm's co-design capability and the customer's co-design costs. Factoring in this "mass customization" cost can potentially lead to some significant additional insights.

References

Anderson, D.M. 2004. *Build-to-order & Mass Customization: The Ultimate Supply Chain Management and Lean Manufacturing Strategy for Low-cost On-demand Production Without Forecasts Or Inventory*. CIM Press.

Balachander, Subramanian, Kannan Srinivasan. 1994. Selection of product line qualities and prices to signal competitive advantage. *Management Science* **40**(7) 824–841.

Davis, S. M. 1987. *Future Perfect*. Addison-Wesley.

Dewan, Rajiv, Bing Jing, Abraham Seidmann. 2003. Product customization and price competition on the internet. *Management Science* **49**(8) 1055–1070.

Krishnan, V., W. Zhu. 2006. Designing a family of development-intensive products. *Management Science* **52**(6) 813–825.

Mendelson, Haim, Ali K. Parlaktuk. 2008. Competitive customization. *Manufacturing & Service Operations Management* **10**(3) 377–390.

Moorthy, K. Sridhar. 1984. Market segmentation, self-selection, and product line design. *Marketing Science* **3**(4) 288–307.

Moorthy, Sridhar R., I. P. L. Png. 1992. Market Segmentation, Cannibalization, and the Timing of Product Introductions. *Management Science* **38**(3) 345–359.

Mussa, Michael, Sherwin Rosen. 1978. Monopoly and product quality. *Journal of Economic Theory* **18**(2) 301 – 317.

Salop, S. 1979. Monopolistic competition with outside goods. *Bell Journal of Economics* **10**(9) 141–156.

Syam, Niladri B., Amit Pazgal. 2013. Co-creation with production externalities. *Marketing Science* **32**(5) 805–820.

Syam, Niladri B., Ranran Ruan, James D. Hess. 2005. Customized products: A competitive analysis. *Marketing Science* **24**(4) 569–584.

Terwiesch, Christian, Christoph H. Loch. 2004. Collaborative prototyping and the pricing of custom-designed products. *Management Science* **50**(2) 145–158.

Wind, Jerry, Arvind Rangaswamy. 2001. Customerization: The next revolution in mass customization. *Journal of Interactive Marketing* **15**(1) 13–32.

Zipkin, P.H. 2001. The limits of mass customization **42**(3) 81–88.

A Proofs

A.1 Proof of Proposition 1

A product line strategy dominates single product when $\pi_2^* > \pi_{1H}^*$ and $\pi_2^* > \pi_{1L}^*$. Comparing the profit functions, this boils down to

$$\begin{aligned} \frac{\alpha v_H^2 - 2\alpha v_H v_L + v_L^2}{4c\,(1-\alpha)} &> \frac{\alpha v_H^2}{4c} \\ \Rightarrow v_H &< \frac{v_L}{\alpha} \end{aligned}$$

and that

$$\begin{aligned} \frac{\alpha v_H^2 - 2\alpha v_H v_L + v_L^2}{4c\,(1-\alpha)} &> \frac{v_L^2}{4c} \\ \Rightarrow v_H &> v_L \end{aligned}$$

This implies that $\bar{v}_H = \dfrac{v_L}{\alpha}$. In addition, $\bar{v}_H$ is decreasing in α.

A.2 Proof of Proposition 2

The optimal effort can be determined by differentiating eq. 3.8 w.r.t γ, which gives us

$$q_S v_H \phi - 2\kappa\gamma = 0$$

Solving for γ in the above equation shows that the optimal effort will be

$$\gamma^* = \frac{q_S v_H \phi}{2\kappa}$$

In addition,

$$\begin{aligned} \frac{\partial \gamma^*}{\partial q_S} &= \frac{v_H \phi}{2\kappa} > 0 \\ \frac{\partial \gamma^*}{\partial v_H} &= \frac{q_S \phi}{2\kappa} > 0 \\ \frac{\partial \gamma^*}{\partial \phi} &= \frac{q_S v_H}{2\kappa} > 0 \end{aligned}$$

A.3 Proof of Proposition 3

Differentiating eq. 3.10 w.r.t. v_H and v_L, we can see that

$$\begin{aligned}
\frac{\partial q_{4P}^*}{\partial v_H} &= 2\kappa \Big(64\left(1-\alpha\right)^2 c^3\kappa^3 + 16\left(1-\alpha\right)c^2\kappa^2 v_H\phi^2\left(\alpha v_H + 2v_L\right) \\
&\quad \frac{+4\left(3-\alpha\right)\alpha c\kappa v_H^4\phi^4 + \alpha v_H^5\phi^6\left(\alpha v_H + 2v_L\right)\Big)}{\left(16\left(1-\alpha\right)c^2\kappa^2 + 8\alpha c\kappa v_H^2\phi^2 - \alpha v_H^4\phi^4\right)^2} > 0 \\
\frac{\partial q_{4P}^*}{\partial v_L} &= \frac{2\kappa v_H^2\phi^2}{\left(16\left(1-\alpha\right)c^2\kappa^2 + 8\alpha c\kappa v_H^2\phi^2 - \alpha v_H^4\phi^4\right)} > 0
\end{aligned}$$

Similarly, differentiating eq. 3.11 w.r.t. v_L, we can see that

$$\frac{\partial q_{4S}^*}{\partial v_L} = \frac{8c\kappa^2}{\left(16\left(1-\alpha\right)c^2\kappa^2 + 8\alpha c\kappa v_H^2\phi^2 - \alpha v_H^4\phi^4\right)} > 0$$

In addition, differentiating eq. 3.11 w.r.t. v_H, we can see that

$$\frac{\partial q_{4S}^*}{\partial v_H} = \frac{2\alpha\kappa\left(16c^2\kappa^2 v_H\phi^2\left(\left(3-\alpha\right)v_H - 4v_L\right) - 64\left(1-\alpha\right)c^3\kappa^3 - 4c\kappa v_H^3\phi^4\left(\alpha v_H - 4v_L\right) + \alpha v_H^6\phi^6\right)}{\left(16\left(1-\alpha\right)c^2\kappa^2 + 8\alpha c\kappa v_H^2\phi^2 - \alpha v_H^4\phi^4\right)^2}$$

Since the denominator is positive, the sign of the expression depends on the numerator $-64(1-\alpha)c^3\kappa^3 - 64c^2\kappa^2 v_H v_L\phi^2 - 16(\alpha-3)c^2\kappa^2 v_H^2\phi^2 + 16c\kappa v_H^3 v_L\phi^4 - 4\alpha c\kappa v_H^4\phi^4 + \alpha v_H^6\phi^6$. Let us look at the coefficients associated with the powers of v_H. Since the coefficient associated with v_H^6 is positive, the expression will be positive when v_H is very high. Similarly, the constant term (independent of v_H) and coefficients of lower powers of v_H are negative, the expression would be negative so long as v_H is sufficiently low.

A.4 Proof of Proposition 4

Differentiating eq. 3.10 and 3.11 w.r.t. α, we can see that

$$\begin{aligned}
\frac{\partial q_{4P}^*}{\partial \alpha} &= -\frac{2\kappa v_H^2\phi^2\left(4c\kappa - v_H^2\phi^2\right)\left(4c\kappa\left(v_H - v_L\right) + v_H^2 v_L\phi^2\right)}{\left(16\left(1-\alpha\right)c^2\kappa^2 + 8\alpha c\kappa v_H^2\phi^2 - \alpha v_H^4\phi^4\right)^2} \\
\frac{\partial q_{4S}^*}{\partial \alpha} &= -\frac{8c\kappa^2\left(4c\kappa - v_H^2\phi^2\right)\left(4c\kappa(v_H - v_L) + v_H^2 v_L\phi^2\right)}{\left(16\left(1-\alpha\right)c^2\kappa^2 + 8\alpha c\kappa v_H^2\phi^2 - \alpha v_H^4\phi^4\right)^2}
\end{aligned}$$

Both these expressions are negative if $\kappa > \dfrac{v_H^2\phi^2}{4c}$ and positive otherwise.

A.5 Proof of Proposition 5

Differentiating eq. 3.10 and 3.11 w.r.t. ϕ, we can see that

$$\begin{aligned}
\frac{\partial q_{4P}^{*}}{\partial \phi} &= \frac{4\kappa v_H^2\phi\left(-16\left(1-\alpha\right)c^2\kappa^2\left(\alpha v_H - v_L\right) + 8\left(1-\alpha\right)\alpha c\kappa v_H^3\phi^2 + \alpha v_H^4\phi^4\left(\alpha v_H + v_L\right)\right)}{\left(16\left(1-\alpha\right)c^2\kappa^2 + 8\alpha c\kappa v_H^2\phi^2 - \alpha v_H^4\phi^4\right)^2} \\
\frac{\partial q_{4S}^{*}}{\partial \phi} &= \frac{4\alpha\kappa v_H^2 y\left(16c^2\kappa^2\left(\alpha v_H + v_H - 2v_L\right) + 8c\kappa v_H^2\phi^2\left(v_L - \alpha v_H\right) + \alpha v_H^5\phi^4\right)}{\left(16\left(1-\alpha\right)c^2\kappa^2 + 8\alpha c\kappa v_H^2\phi^2 - \alpha v_H^4\phi^4\right)^2}
\end{aligned}$$

Examining q_{4S}^{*} in eq. 3.11we can see that $q_{4S}^{*} > 0$ only if $\kappa < \frac{\alpha v_H^3\phi^2}{4\alpha c v_H - 4c v_L}$. For these range of κ values, the above expressions are positive.

A.6 Proof of Proposition 6

1. Note that the difference between the optimal premium product quality levels with and without co-design is

$$q_{4P}^{*} - q_{1H}^{*} = \frac{v_H^2\phi^2\left(-4\alpha c\kappa v_H + \alpha v_H^3\phi^2 + 4c\kappa v_L\right)}{2c\left(16\left(1-\alpha\right)c^2\kappa^2 + 8\alpha c\kappa v_H^2\phi^2 - \alpha v_H^4\phi^4\right)}$$

It is easy to see that the expression is positive when $\alpha v_H < v_L$ which is the only region where product line is optimal when co-design is not an option (Proposition 1).

$$q_{4S}^{*} - q_{1L}^{*} = \frac{\alpha v_H^2\phi^2\left(4c\kappa\left(\alpha v_H + v_H - 2v_L\right) + v_H^2\phi^2\left(v_L - \alpha v_H\right)\right)}{2c\left(1-\alpha\right)\left(16\left(1-\alpha\right)c^2\kappa^2 + 8\alpha c\kappa v_H^2\phi^2 - \alpha v_H^4\phi^4\right)}$$

This expression is positive for the region in which a product line is optimal both with and without co-design option. As we show in Proposition 7, this is true as long as κ is below a certain threshold.

For the second part of the proposition, let us compare the optimal standard product quality (with co-design option) to the efficient quality of the low-end segment when there is no co-design option

$$q_{4S}^{*} - q_{1L}^{*} = \frac{\alpha\left(v_H^2\phi^2 - 4c\kappa\right)\left(4c\kappa(v_H - v_L) + v_H^2 v_L\phi^2\right)}{2c\left(16\left(1-\alpha\right)c^2\kappa^2 + 8\alpha c\kappa v_H^2\phi^2 - \alpha v_H^4\phi^4\right)}$$

This expression is positive when $\kappa < \dfrac{v_H^2\phi^2}{4c}$.

A.7 Proof of Proposition 7

Part 1: As detailed in the proof of Propsoition 5, $q_{4S}^* > 0$ when

$$\kappa < \bar{\kappa} = \frac{\alpha v_H^3 \phi^2}{4\alpha c v_H - 4 c v_L}$$

The product line strategy is optimal as long as q_{4S}^* is positive. When it is negative, the single product strategy becomes optimal. In addition, differentiating the threshold w.r.t α gives us

$$\frac{\partial \bar{\kappa}}{\partial \alpha} = -\frac{v_H^3 v_L \phi^2}{4c\left(v_L - \alpha v_H\right)^2} < 0$$

Part 2: Recall that when $\alpha > \alpha_1 = \dfrac{v_L}{v_H}$, single product was optimal when there was no co-design option. The threshold in the previous part can be rearranged as $\alpha > \alpha_2 = \dfrac{\kappa\left(4\alpha c v_H - 4 c v_L\right)}{v_H^3 \phi^2}$. Note that

$$\begin{aligned}\alpha_2 - \alpha_1 &= \frac{\kappa\left(4\alpha c v_H - 4 c v_L\right)}{v_H^3 \phi^2} - \frac{v_L}{v_H} \\ &= \frac{v_H v_L \phi^2}{4c\kappa - v_H^2 \phi^2} > 0\end{aligned}$$

So, the region of optimality for a product line increases when the co-design option is viable.

A.8 Proof of Proposition 8

For the separating equilibrium to be possible, the low-type firm should find it optimal to not mimic the high-type firm's quality levels. If the low-type firm does not mimic the high-type firm's quality, it is easy to see that the optimal quality levels of both products would be given by eq. 3.10 and 3.11 where we substitute ϕ_l for ϕ to adjust for the capability level. This implies that the profits of low-type firm when it does not mimic the high-type firm's quality levels will be $\pi_{dl1}\left(q_P^*\left(\phi_l\right), q_S^*\left(\phi_l\right)\right)$.

Now consider the high-type firm's profits as a function of the quality levels. In the absence of any informational asymmetry, the optimal quality levels of this firm would be given again by eq. 3.10 and 3.11 where we substitute ϕ_l for ϕ to adjust for the capability level.

Let us first see what happens when these quality levels are offered. By imitating these levels, the low-type firm's profits will be given by

$$\pi_{dl2}\left(q_{dhP},q_{dhS},p_P,p_S\right) \;=\; \alpha\left(p_P\left(\phi_l,\underline{\gamma}\right)-cq_{dhP}^2\right)+\left(1-\alpha\right)\left(p_S-cq_{dhS}^2\right)$$

Two things to note here: first, since customer is not able to distinguish between the two firms, the effort she invest would be $\underline{\gamma}$. Second, since the customer, after putting in the effort and co-designing the product, would realize the true capability of the low-type firm, and thus would consider only the extent to which her effort increased her willingness to pay while making her purchase decision. So the price for the co-designed product will be determined by the customer effort $\underline{\gamma}$, the revealed capability ϕ_l and the quality of the co-designed product. Hence, the profits of the low-type firm under this scenario will be $\pi_{dl2}\left(q_P^*\left(\phi_h\right),q_S^*\left(\phi_h\right)\right)$.

If $\pi_{dl2}\left(q_P^*\left(\phi_h\right),q_S^*\left(\phi_h\right)\right)<\pi_{dl1}\left(q_P^*\left(\phi_l\right),q_S^*\left(\phi_l\right)\right)$, then the high-type firm can offer quality levels $q_P^*\left(\phi_h\right),q_S^*\left(\phi_h\right)$ and be certain that the low-type firm will not imitate its quality levels. Thus the high-type firm's quality levels under the full information case would be sufficient for signaling its type to customers.

If, however, $\pi_{dl2}\left(q_P^*\left(\phi_h\right),q_S^*\left(\phi_h\right)\right)>\pi_{dl1}\left(q_P^*\left(\phi_l\right),q_S^*\left(\phi_l\right)\right)$, then the low-type firm will find it profitable to imitate the high-type firm's quality levels. To understand how the high-type firm should react to signal its type, first note that

$$\frac{\partial^2\pi_h}{\partial q_P\partial q_S} \;=\; \frac{\alpha v_H^2\phi_h^2}{2\kappa}>0$$

Second, the quality levels that maximize $\pi_{dl2}\left(q_{dhP},q_{dhS},p_P,p_S\right)$ is given by

$$\begin{aligned} q_{P'}^* &= \frac{2\kappa v_H\left(v_H\phi_l\left(\alpha v_H+v_L\right)\bar{\phi}+4\left(1-\alpha\right)c\kappa\right)}{16\left(1-\alpha\right)c^2\kappa^2+8\alpha c\kappa v_H^2\phi_l\bar{\phi}-\alpha v_H^4\phi_l^2\bar{\phi}^2} \\ q_{S'}^* &= \frac{2\kappa\left(\alpha v_H\left(4c\kappa-v_H^2\phi_l\bar{\phi}\right)-4c\kappa v_L\right)}{16\left(1-\alpha\right)c^2\kappa^2+8\alpha c\kappa v_H^2\phi_l\bar{\phi}-\alpha v_H^4\phi_l^2\bar{\phi}^2} \end{aligned}$$

It can be shown that $q_P^*\left(\phi_h\right)>q_{P'}^*$ and that $q_S^*\left(\phi_h\right)>q_{S'}^*$. Thus, π_{dl2} is decreasing in q_P and q_S $\forall$ values of $q_P>q_{P'}^*$ and $q_S>q_{S'}^*$. This in conjunction with the fact that $\dfrac{\partial^2\pi_h}{\partial q_P\partial q_S}>0$ implies that $q_{dhP}>q_P^*\left(\phi_h\right)$ and $q_{dhS}>q_S^*\left(\phi_h\right)$ under a separating equilibrium.