Selling or Leasing? Pricing Information Goods with Depreciation of Consumer Valuation

Yifan Dou
School of Management, Fudan University, Shanghai, 200433, China
yfdou@fudan.edu.cn

Yu Jeffrey Hu, D. J. Wu
Scheller College of Business, Georgia Institute of Technology, Atlanta, GA 30308, USA
{jeffrey.hu, dj.wu}@scheller.gatech.edu

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Should a monopolistic vendor adopt the selling model or the leasing model for information goods or services? We study this question in the context of value depreciation, using a two-period game-theoretic model. In particular, we model two types of value depreciation for information goods or services: vintage-depreciation and individual-depreciation. Vintage-depreciation assumes that a good or service loses some of its appeal to consumers as it becomes dated and this effect persists independent of usage. In contrast, individual-depreciation assumes that value depreciation happens only to a consumer who has consumed (or experienced) the good or service. We identify optimal regions (or conditions) of each pricing model: (1) for vintage-depreciation information goods, the leasing model dominates the selling model; in contrast, (2) for individual-depreciation information goods, the selling model dominates the leasing model when the magnitude of individual-depreciation exceeds a certain threshold; otherwise, leasing dominates selling. These findings are robust to several model extensions such as when considering network effects. We also discuss managerial implications.

Key words: pricing strategies; selling; leasing; vintage-depreciation; individual-depreciation; network effects; information goods or services

1. Introduction

To sell or to lease? Practitioners and academic researchers have been engaged in this debate for decades. With the rapid advancement of information technologies, especially the Internet, vendors of information goods and services are increasingly embracing the leasing model (thereafter "leasing" for short). Examples include video streaming services (e.g., Netflix, Apple iTunes), online storage services (e.g., Dropbox.com), software-as-a-service (e.g., Microsoft Office 365). Under the leasing model, users, instead of taking the perpetual ownership, rent information goods or services from the vendor and pay a periodic renting fee. In contrast, under the selling model (thereafter "selling" for short), users pay a lump-sum fee for the perpetual usage of information goods or services. The latter has been widely used by vendors of traditional information goods such as boxed software (e.g., Microsoft Windows, Adobe, Autodesk, and SAP), DVDs, hosted solutions (e.g., Rackspace), and mobile application marketplaces (e.g., paid apps in Google Play and Apple App Store).

In the academic literature, leasing is often believed to be more efficient in extracting consumer surplus *over time*. The pioneer work by Coase (1972) suggests that selling is suboptimal for a monopolistic vendor because under selling, consumers expect price markdown and therefore delay their purchase behavior. Leasing, on the other hand, can eliminate such strategic waiting behavior (Bond and Samuelson 1984).

Our paper reinvestigates this tradeoff by incorporating another important issue over the timeline – the depreciation of *consumer valuation*. While the extant literature has focused on quality depreciation on the product side, our work examines depreciation on the consumer-side. Consumer-side depreciation is ubiquitous in the market of information goods, such as books, CDs/DVDs, and video games. They share a common feature: the physical attributes of the product hardly depreciates but the consumption value to owners depreciates due to consumers' satiation (Ishihara and Ching 2012). In the market for video games, Shiller (2013) reports empirical evidence that consumers may tire quickly with playing. He finds that, for the average game, high valuation consumers reduce their valuation from \$80 in the 1st month of use to just a couple of dollars per month by the 6th month. Consumer-side depreciation is prevalent among information goods because information goods are often intangible (with no physical features for depreciation) and their consumption value are sensitive to consumer's experience over time (Shapiro and Varian 1999). While product-side depreciation is well documented in the durable goods literature, to the best of our knowledge, analytical modeling research on consumer-side deprecation is largely missing. Our paper aims to fill this gap.

We employ a two-period game-theoretic model to examine two types of consumer-side value depreciation in this paper: *vintage-depreciation* and *individual-depreciation*. Vintage-depreciation assumes that a good or service loses some of its appeal to consumers as it becomes dated and this effect persists independent of usage. For example, a dated version of a software is valued much less than a new release. In contrast, individual-depreciation (Hu 2005) assumes that value depreciation happens only to a consumer who has already consumed (or experienced) the good or service.

In an extension to our baseline model, network effects are considered as a unique feature of information goods (e.g., Farrell and Saloner 1986, Katz and Shapiro 1985). Many information goods, such as online games and chatting tools, are built upon user networks or product community in which the users' willingness-to-pay (WTP) are affected by the adoption of peers. Following the literature (e.g., Conner 1995), we model the network effects with an additive utility function which assumes that the information good has a standalone value as well as the ability to be extended through its user network. For example, in video games, core functionality includes features such as single game and media streaming. Users can also play with peers via the player network.

We identify optimal conditions of each pricing model. (1) for vintage-depreciation information goods, leasing dominates selling. This also holds true in the presence of network effects. (2) for individual-depreciation information goods, selling dominates leasing when the magnitude of individual-depreciation exceeds a certain threshold; otherwise leasing dominates selling, and (3) when both the magnitude of individual-depreciation is large and the degree of network effects is strong, selling and leasing are profit equivalent. These findings have immediate managerial implications for vendors of information goods and services. In particular, our results suggest that selling is still viable when individual-depreciation is significant and this insight is robust under network effects.

The rest of the paper is organized as follows. Section 2 reviews related literature. Section 3 provides a very simple example that intends to highlight the key idea and insight of our analytical model. Section 4 introduces our model assumptions. Section 5 establishes the baseline case of vintage-depreciation and Section 6 examines the case of individual-depreciation. Several model extensions, including network effects, are discussed respectively in Section 7. Section 8 discusses managerial implications and concludes.

2. Litearture Review

Academic debate of selling versus leasing of durable goods is rich, and can be traced back at least to the seminal work of Coase (1972). The key idea, as summarized by Bond and Samuelson (1984), is that a monopoly seller of a durable good is effectively unable to exercise its monopoly power. Once an initial stock of the good has been produced, the monopoly will find itself faced with a residual demand. Exploiting the residual demand by selling some additional quantity of the good, presumably at a lower price, allows the firm to earn additional profit. Therefore, the monopoly will produce until the competitive stock has been achieved. Rational consumers, on the other hand, will anticipate the price markdown and accordingly value the good only at the competitive price. As a result, the monopolist can thus earn no more profit than that of a competitive firm – this is well known in the literature as the Coase conjecture. To address this issue, Coase (1972) has suggested that leasing, rather than selling the product can improve the profit because leasing limits the market supply to the monopoly level, which in turn helps maintain the monopoly price.

The Coase conjecture has been analytically examined by the academic community using two typical approaches. The first approach is to directly formalize the stock-level decision considered by Coase (1972). In particular, the vendor of durable goods chooses the number of available stock at the beginning of each period (Swan 1970, Bulow 1982, Bond and Samuelson 1984, Gul et al. 1986, Suslow 1986, Bhaskaran and Gilbert 2009). Under stock-decision models, Coase conjecture is equivalent to the following: the monopoly seller's optimal stock level would converge to the competitive stock level at which the market price is equal to the marginal product cost. However, the (inverse) demand functions in these papers are aggregated at the market level, rather than at the individual level. Consequently, this approach does not and can not differentiate existing adopters who are subject to individual-depreciation from potential consumers at the individual level. To summarize this discussion, the stock-decision models are unable to capture consumer-side value depreciation, as we do in this paper.

It should be noted that, among papers using stock-decision models, there have been some researches looking at the issue of quality decay of physical products, e.g., Bond and Samuelson (1984), and Suslow (1986). In this work, two classic ways to characterize product depreciation for physical products are vintage-use type and one-hoss shay type (Desai and Purohit 1998). Under vintage-deprecation, as the durable good becomes dated, it loses some of its appeal to consumers and this effect persists independent of consumer usage or whether the good is used or not. In contrast, under one-hoss shay type, only some proportion of the durable goods depreciate fully but the remaining units do not depreciate. Again, neither of these two types of depreciation can differentiate existing adopters from potential adopters.

The second approach is the individual-utility-based model which focuses on determining the optimal cutoff prices from the distribution of consumer valuations. Such a model setting allows analysis of the consumers' choices at the individual (or group) level. In this stream of research, the early work by Stokey (1979) investigates durable goods pricing problem in a continuous time model with a wide range of utility functions. Similar to Coase's criticism to the selling model, she also finds that price discrimination over time periods is not optimal for the seller. A follow up work by Conlisk et al. (1984) revisits her utility-based model by considering new consumer arrivals in each period. They find that, with new consumer arrivals in each period, the selling model may be still optimal. Unfortunately, these two papers do not analytically solve the case of leasing. Bagnoli et al. (1989) also discover that selling might be optimal when consumer's base value is discrete. The utility-based model is also popular in marketing (e.g., Desai and Purohit 1998, Bhaskaran and Gilbert 2005), industry organization (e.g., Bensaid 1996), and economics of information systems (e.g., Chien and Chu 2008, Zhang and Seidmann 2010). Our work follows this stream of research with utility-based model and we use the standard two-period setting similar to Stokey (1979), Conlisk et al. (1984), and Desai and Purohit (1998). We contribute to this literature by considering individual-depreciation when selling or leasing information goods, which is novel.

Our work is also closely related to the burgeoning literature on software pricing. For example, Jain and Kannan (2002) examine the server cost structure and compare pricing schemes of information goods. In an auction setting with demand uncertainty, Bhargava and Sundaresan (2004) show that pay-as-you-go model is optimal when consumer valuation and demand realization is negatively correlated. Huang and Sundararajan (2005) compare on-demand and in-house computing from the cost perspective, and discuss the optimal transition path from in-house to on-demand computing. Choudhary (2007) endogenizes software upgrading decision and shows that the subscription model is always optimal. Dou et al. (2012), in the context of durable information goods, find that when consumers are fully strategic the license model dominates the subscription model. In our paper, the leasing (selling) model is a simplified form of the subscription (perpetual licensing) pricing in software industry. Therefore our results also provide insights to vendors of software products that exhibit characteristics of individual-depreciation.

3. A Two-consumer Example

Before presenting our formal model, we use a simple example to demonstrate the impact of individual-depreciation. Consider a market consisting of only two consumers with initial valuation of $V_1 = 7$ and $V_2 = 3$ at the beginning of period 1, respectively. The lifecycle of the product is assumed to have two periods. In period 2, the consumer valuation will drop by 70% when depreciation applies. For simplicity, we assume the firm only consider leasing.

First consider the scenario of vintage-depreciation where both consumers' valuation is subject to depreciation in period 2 unconditionally. Thus the consumers' period-2 valuations are $7 \times 0.3 = 2.1$ and $3 \times 0.3 = 0.9$. The optimal leasing fee in period 2 is 2.1 because $0.9 \times 2 = 1.8 < 2.1 \times 1$. Similarly, it can be shown that in period 1, the optimal period 1 leasing fee is 7. The total profit under vintage deprecation is $7 + 2.1 = 9.1$, and only the consumer with valuation $V_1 = 7$ rents in each period.

Next consider the scenario of individual-depreciation where only the value of period-1 adopter depreciates. Intuitively, the vendor's profit should be no less than 9.1 because depreciation occurs to only one consumer. However, we show that the vendor becomes worse off.

In period 2, let's follow the previous case to assume only consumer V_1 adopted in period 1, then her valuation drops to 2.1 while consumer V_2 still has the valuation of 3. Then the optimal period-2 leasing fee becomes 2.1 with which the vendor can get $2.1 \times 2 = 4.2 > 3$ in period 2. Unfortunately, this is not a feasible strategy for the vendor because consumer V_1 would delay her adoption to period 2 and get a surplus of $7 - 2.1 = 4.9$, rather than adopting in both periods (the surplus is $7 - 7 + 0.3 \times 7 - 2.1 = 0$). As a result, it is optimal for the vendor to lower the leasing fee in period 1 to 2.1. The total profit is $2.1 \times 3 = 6.3 < 9.1$. Note that, the vendor's optimal strategy is to block sales in period 1, with a profit of $7 < 9.1$ (and only V_1 adopts in period 2). This example conveys the central idea of our paper: in the presence of individual-depreciation, consumers also have incentives to wait, even under leasing. This in turn hurts the vendor's profit. Surprisingly, selling can address such customer waiting behavior better than leasing when the magnitude of individual-depreciation can not be ignored. In what follows, we capture this idea and insight via analytical models.

4. Model Assumptions

A monopolistic vendor offers an information good or service with a lifecycle of two periods. The vendor wishes to maximize the total profit over both periods. We assume the marginal production cost of the information good or service is zero (e.g., Shapiro and Varian 1999). We examine and compare two representative pricing models: selling and leasing. Under the selling model, consumers pay a lump-sum price p_i for the perpetual ownership of the information good; under the leasing model, consumers pay r_i in period i for a single-period usage ($i = \{1, 2\}$). For simplicity, we assume that there are no disposal or switching costs if period 1 adopters give up in period 2.

We assume a unit mass of heterogenous consumers with their type v uniformly distributed on $[-K, 1]$ in which $K \geq 0$. Therefore the density of consumer distribution is $\frac{1}{1+K}$ everywhere. Specifically, for consumers with $v \in [0, 1]$, their type v represents period-1 valuation on the information good, which is subject to depreciation in period 2. In contrast, consumers distributed on $[-K, 0)$ are "not interested" in the information good or service. Thus their valuation is equal to 0 in both periods. In the baseline model without network effects, we only need to consider consumers with $v \in [0, 1]$. Later on in Section 7.1 we will extend the baseline model to incorporate network

Notation	
v	consumer type, $v \sim \mathrm{U}[-K, 1]$ where $K = 0$ for the case without network effects;
$1-\theta$	the magnitude of depreciation $\theta \in [0,1]$;
v_i	marginal consumer type in period $i \in \{1,2\}$;
p_i	selling price in period $i \in \{1,2\}$;
r_i	leasing fee in period $i \in \{1,2\}$;
N_i	number of adopters in equilibrium in period $i \in \{1,2\}$;
$S(L)$	indicator for the selling (leasing) model;
$A(B)$	indicator for vintage-depreciation (individual-depreciation) information goods;
s	the strength of network effects, $s \in [0,1]$;
π	the vendor's overall profit;
π_2	the vendor's profit in period 2;
$U(\tilde{U})$	consumer period-1 utility functions without (with) network effects;
Ω	consumer's adoption status over two periods, $\Omega \in \{DD, OD, DO, OO\}$ where D stands for "adopting" and O for "not adopting".

Table 1 Table of notation

effects under which those consumers with type $v < 0$ may become "interested" when expecting network-based benefits from peer adopters.

Next we introduce the mechanisms of value depreciation specifically for consumers with non-negative valuation. For *vintage-depreciation* information goods, any consumer v's valuation will depreciate from v to θv in period 2; in contrast, for *individual-depreciation* information goods, consumer v's valuation will depreciate to θv only when she is a period-1 adopter. Otherwise, consumer v will stay the same as her initial valuation in period 2 (Hu 2005). Denote N_i as the number of adopters (either under leasing or selling) at the equilibrium in period i ($i \in \{1,2\}$).

We summarize our notation in Table 1. In what follows, for both vintage-depreciation information goods and individual-depreciation information goods, we compare and contrast the selling model vs. the leasing model.

5. Benchmark: Vintage-Depreciation

We start our analysis with vintage-depreciation information goods, where all consumers, including those who do not adopt in period 1, depreciate their valuation in period 2. As a result, in period 2 the consumer valuation is uniformly distributed on $[0, \theta]$ (with a greater density). The update in distribution of consumer valuation is illustrated below in Figure 1. We analyze selling vs. leasing in the following subsections.

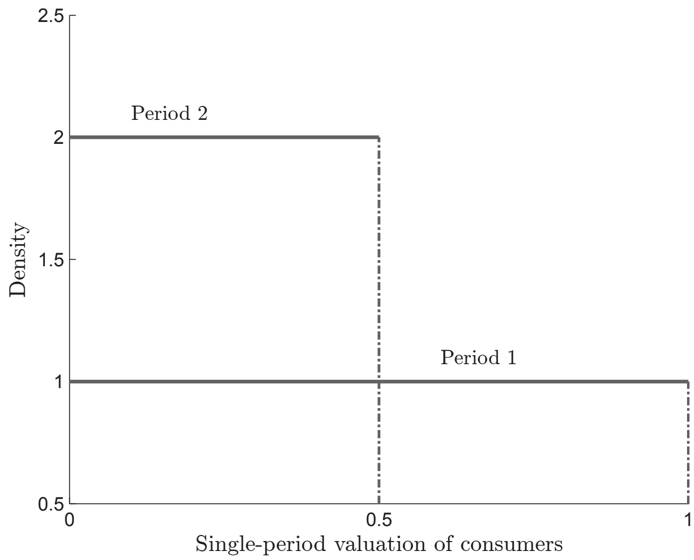

Figure 1 The distribution density function of consumers' single-period valuation ($\theta = 0.5, K = 0$)

5.1. The Selling Model

Under selling, the vendor announces p_i at the beginning of period $i \in \{1, 2\}$. Denote the marginal consumer type in period i as v_i ($i \in \{1, 2\}$). Marginal consumer type represents consumers who are indifferent between adopting and not adopting in each period. We solve the vendor's problem via backward induction.

At the beginning of period 2, the vendor only needs to consider potential consumers with initial type distributed on $[0, v_1]$ because period 1 adopters have purchased the information good in period 1. For any consumer $v \in [0, v_1]$, her period-2 valuation is θv. She will become a period-2 adopter when $\theta v \geq p_2$ holds. The population of paying consumers in period 2 is given by $v_1 - v_2$ in which v_2 is a function of period-2 price p_2 and determined by $\theta v_2 - p_2 = 0$. The vendor's period 2 problem is

$$\max_{v_2} \quad \pi_2(v_2|v_1) = (v_1 - v_2)\theta v_2.$$

Solving, we have $v_2^* = \frac{v_1}{2}$, $p_2^* = \frac{\theta v_1}{2}$, and the vendor's optimal period 2 profit is $\pi_2^*(v_1) = \frac{\theta v_1^2}{4}$.

Next we move to period 1. A type-v consumer 's utility function is denoted by $U_v(\Omega)$, where $\Omega \in \{DD, OD, DO, OO\}$ stands for the consumer v's adoption status in each period (D for "adopting", and O for "not adopting"). In period 1, all consumers face three options: purchasing in period 1 (i.e., adopting in both periods, denoted by $\Omega = DD$), delaying adoption to period 2 (i.e., $\Omega = OD$), and never adopting (i.e., $\Omega = OO$). The corresponding utility function $U_v(\Omega)$ is listed below (where superscripts S and A stand for selling and vintage-depreciation information goods, respectively)

$$\begin{aligned} &U_v^{S,A}(DD) = v + \theta v - p_1; \\ &U_v^{S,A}(OD) = \theta v - p_2; \\ &U_v^{S,A}(OO) = 0. \end{aligned} \tag{1}$$

Following the game theory literature (e.g., Fudenberg and Tirole 1991), we assume that, at the beginning of period 1, consumers can rationally expect $p_2^*(v_1)$ at the equilibrium (i.e., rational expectation equilibrium, REE). The marginal consumers (with type v_1) are indifferent between adopting in period 1 and delaying adoption to period 2. Then v_1 can be obtained by solving $U_{v_1}^{S,A}(DD) = U_{v_1}^{S,A}(ND)$, and the number of paying consumers is $N_1 = 1 - v_1$. Therefore, the vendor's problem is

$$\max_{v_1} \quad \pi^{S,A} = N_1(p_1(v_1)) \times p_1(v_1) + \pi_2^*(v_1) = (1 - v_1)p_1(v_1) + \pi_2^*(v_1), \tag{2}$$

where p_1 is a function of marginal consumer type v_1, which can be obtained by solving $U_{v_1}^{S,A}(DD) = U_{v_1}^{S,A}(OD)$.

Solving the vendor's problem above yields

$$p_1^* = \frac{\theta}{2} + \frac{2}{4+\theta}, \; p_2^* = \frac{\theta(2+\theta)}{2(4+\theta)}, \; (\pi^{S,A})^* = \frac{(2+\theta)^2}{4(4+\theta)}. \tag{3}$$

The numbers of adopters under optimal selling prices are

$$N_1^* = \frac{2}{4+\theta},\ N_2^* = \frac{2+\theta}{2(4+\theta)} \tag{4}$$

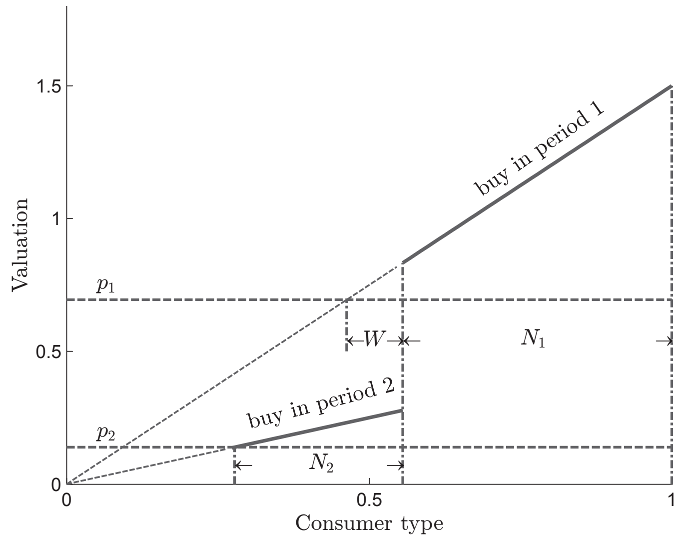

Figure 2 Consumers' valuation and adoption under optimal selling strategy and vintage-depreciation ($\theta = 0.5$)

Consumer adoptions under the optimal selling strategy are depicted by Figure 2. Consumers' valuation of adopting in period 1 (the curve of "buying in period 1") is always higher than that in period 2 (the curve of "buying in period 2") because the total benefit of buying in period 1 is $(1+\theta)v$ which is always greater than θv. In period 2, a rational vendor lowers the price ($p_2 < p_1$) to induce more purchases. Observing this future price markdown, there is a group of consumers (denoted by region W), even though they can afford p_1 (i.e., $(1+\theta)v \geq p_1$), delays their adoption to period 2 for a lower price p_2. For all period-1 adopters, originally they can afford a period-1 price of $(1+\theta v_1)$. However, the vendor has to lower the period-1 price to $p_1^* < (1+\theta v_1)$ to alleviate the waiting behavior. The profit loss incurred by consumer waiting is measured by $Loss^{S,A}$

$$Loss^{S,A} = N_1\left[(1+\theta)v_1 - p_1^*\right] = \frac{\theta(2+\theta)}{(4+\theta)^2}.$$

5.2. The Leasing Model

Under leasing, the vendor announces leasing fee r_i at the beginning of each period $i \in \{1, 2\}$. All consumers, including existing adopters in period 1, need to pay the leasing fee in period 2 if they opt to continue usage of the information good. Again we solve the vendor's problem via backward induction.

First consider period 2. Unlike selling, under leasing the vendor needs to take all consumers into consideration. The number of paying consumers in period 2 is $N_2 = 1 - v_2(r_2)$ where v_2 satisfies $\theta v_2 - r_2 = 0$. Period 2 profit is $\pi_2 = N_2 \times r_2$. Solving, we have $v_2^* = \frac{1}{2}$, $r_2^* = \frac{\theta}{2}$, and $\pi_2^* = \frac{\theta}{4}$.

At the beginning of period 1, in contrast to selling, under leasing consumers have the freedom to rent only in period 1. Therefore they have a total of four candidate strategies to consider.

$$
\begin{aligned}
U_v^{L,A}(DD) &= v - r_1 + \theta v - r_2; \\
U_v^{L,A}(DO) &= v - r_1; \\
U_v^{L,A}(OD) &= \theta v - r_2; \\
U_v^{L,A}(OO) &= 0.
\end{aligned}
$$

Under REE, the marginal consumer v_1 satisfies either $U_{v_1}^{L,A}(DD) = U_{v_1}^{L,A}(OD)$ (when there are new adopters in period 2) or $U_{v_1}^{L,A}(DO) = 0$ (when some consumers only rent in period 1). It is straightforward to see that these two equations are identical. Therefore marginal type v_1 is uniquely determined by $v_1 = r_1$ and $N_1 = 1 - v_1$. Therefore, the vendor's problem is

$$
\max_{v_1} \quad \pi^{L,A}(v_1) = N_1(r_1(v_1)) \times r_1(v_1) + \pi_2^* = (1 - v_1) \times v_1 + \pi_2^*.
$$

The optimal solution is $v_1^* = \frac{1}{2}$. Therefore $r_1^* = \frac{1}{2}$ and $(\pi^{L,A})^* = \frac{1+\theta}{4}$. The numbers of adopters in each period are $N_1^* = N_2^* = \frac{1}{2}$. Figure 3 below illustrates consumers' valuations and adoptions.

In each period, the marginal consumer's adoption decision only depends on their single-period valuation and the leasing fee (i.e., $v_1 = r_1$, $\theta v_2 = r_2$). It implies that consumer waiting does not exist under leasing (i.e., $Loss^{L,A} = 0$). The following Proposition 1 compares our results under vintage-depreciation in Section 5.1 and 5.2.

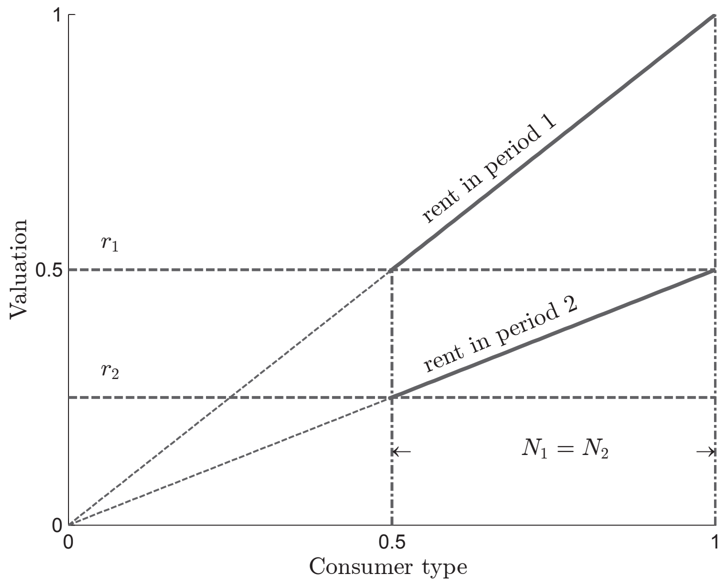

Figure 3 **Consumers' valuation and adoption under optimal leasing strategy and vintage-depreciation ($\theta = 0.5$)**

Proposition 1. *For vintage-depreciation information goods, leasing dominates selling.*

Proposition 1 follows immediately by comparing profits under leasing and selling.

$$(\pi^{L,A})^* - (\pi^{S,A})^* = \frac{1+\theta}{4} - \frac{(2+\theta)^2}{4(4+\theta)} = \frac{\theta}{4(4+\theta)} \geq 0.$$

Note that

$$\frac{Loss^{S,A}}{(\pi^{L,A})^* - (\pi^{S,A})^*} = \frac{2+\theta}{4+\theta} \geq \frac{1}{2},$$

which suggests that the majority of the profit gap is due to consumer waiting under selling. Proposition 1 extends Coase (1972) under a two-period model setting, it formalizes Coase's idea that leasing can effectively eliminates consumer waiting which causes the vendor's profit loss under selling. Proposition 1 provides a useful benchmark to the case of individual-depreciation information goods, the key focus of this paper, which we analyze in the next section.

6. Individual-Depreciation

For individual-depreciation information goods, only period 1 adopters depreciate their valuations in period 2 while valuations of other consumers do not depreciate. To compare with Figure 1, we visualize the density function of consumer valuation in period 2 in Figure 4.

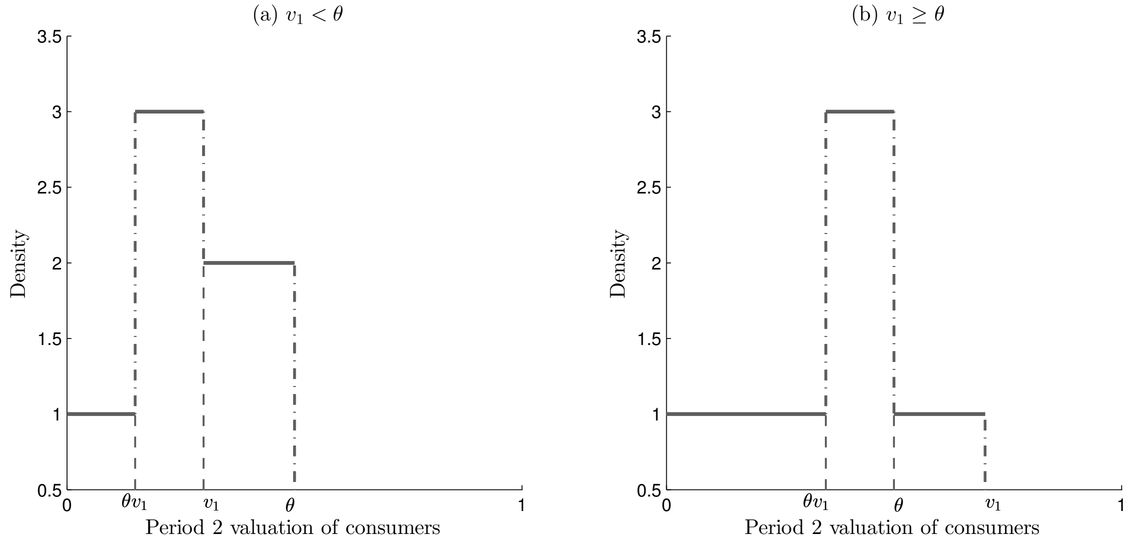

Figure 4 The distribution density function of consumers' single-period valuation ($\theta = 0.5$)

In Figure 4, in sharp contrasting to Figure 1, consumer type distribution is no longer uniform in period 2. Specifically, consumers with type $v \in [v_1, 1]$ (i.e., period 1 adopters) depreciate their valuation, while consumers with type $[0, v_1)$ do not. This leads to two cases: (a) $v_1 < \theta$ (see Figure 4a), and (b) $v_1 \geq \theta$ (see Figure 4b). Under case (a), the distribution of period 2 valuation is further segmented into three intervals: (a.1) interval $[0, \theta v_1)$ which consists of only period-1 non-adopters with density of 1, (a.2) interval $[\theta v_1, v_1)$ which consists of both period 1 adopters and non-adopters with density $1 + \frac{1}{\theta}$, and (a.3) interval $[v_1, 1]$ which consists of only period 1 adopters with density $\frac{1}{\theta}$. Under case (b) and similarly, the distribution of period 2 valuation is segmented into three intervals but with different densities: (b.1) interval $[0, \theta v_1)$ and $[\theta, v_1]$ which consist of only period 1 non-adopters with density 1 and (b.2) interval $[\theta v_1, \theta)$ which consists of both period 1 adopters and non-adopters with density $1 + \frac{1}{\theta}$.

Note well that, while θ is exogenous, v_1 is determined by the vendor's pricing strategy, which implies that the distribution density of consumer period-2 valuations can be manipulated by the vendor's period-1 pricing strategy. This connects our work with the literature on endogenous demand functions (e.g., Johnson and Myatt 2003, Johnson and Myatt 2006, Bhargava and Chen 2012). For individual-depreciation goods, upon observing θ, the vendor can strategically manipulate consumer valuations towards a certain distribution in period 2. For example, if the vendor charges a smaller price to induce more adoptions in period 1 (which leads to a smaller v_1), then period 2 distribution converges to a pattern as illustrated in Figure 4a where consumers are segmented into three intervals with different densities. Alternatively, if the vendor maintains a relatively small number of adoptions in period 1, then period 2 distribution converges to a pattern as illustrated in Figure 4b with a single bump in the center.

6.1. The Selling Model

Under selling, we denote selling prices as p_i $(i \in \{1,2\})$ for period i and solve the vendor's problem via backward induction.

At the beginning of period 2, the vendor only needs to consider consumers on $[0, v_1]$. The number of paying consumers in period 2 is $v_1 - v_2$ in which v_2 satisfies $v_2 - p_2 = 0$. Note that v_2 is not affected by θ as new adopters in period 2 have never purchased the product in period 1. The vendor's problem in period 2 is

$$\max_{v_2} \quad \pi_2 = (v_1 - v_2) \times v_2.$$

Solving, we have $v_2^* = \frac{v_1}{2}$, $p_2^* = \frac{v_1}{2}$, and $\pi_2^* = \frac{v_1^2}{4}$. Next consider period 1. At the beginning of period 1, consumer v faces three options with the following corresponding valuations:

$$\begin{aligned}
U_v^{S,B}(DD) &= v - p_1 + \theta v;\\
U_v^{S,B}(OD) &= v - p_2;\\
U_v^{S,B}(OO) &= 0.
\end{aligned}$$

where superscript B represents the case of individual-depreciation information goods. Note that existing adopters need not to pay p_2 for period 2 adoption. Further, period 1 non-adopters do not depreciate their evaluation in period 2. Thus, the number of adopters is $N_1 = 1 - v_1$ where v_1 satisfies $U_{v_1}^{S,B}(DD) = U_{v_1}^{S,B}(OD)$. The vendor's problem is

$$\max_{v_1} \quad \pi^{S,B}(v_1) = (1 - v_1)p_1(v_1) + \pi_2^* = (1 - v_1)p_1(v_1) + \pi_2^*(v_1), \tag{5}$$

Note that p_1 can be written as a function of v_1 by solving $U_{v_1}^{S,B}(DD) = U_{v_1}^{S,B}(v_1|OD)$. Solving the vendor's problem, we obtain the following Lemma 1.

Lemma 1. *For individual-depreciation information goods and under the selling model,* $\forall\, \theta \in [0,1]$, *the optimal price strategies* (p_1^*, p_2^*) *are*

$$p_1^* = \frac{(1+2\theta)^2}{2(1+4\theta)},\ p_2^* = \frac{1+2\theta}{2(1+4\theta)}.$$

The optimal profit is

$$(\pi^{S,B})^* = \frac{(1+2\theta)^2}{4(1+4\theta)}.$$

The numbers of adopters in each period are

$$N_1^* = \frac{2\theta}{(1+4\theta)},\ N_2^* = \frac{1+6\theta}{2(1+4\theta)}.$$

We illustrate consumer valuation and their adoptions in Figure 5. The parameter setting is the same as in Figure 2. It can be observed that there exists also a fraction of strategic consumers (also denoted by W) who would delay their purchase to period 2. The profit loss due to such waiting behavior is measured by $Loss^{S,B}$

$$Loss^{S,B} = N_1^* \times [(1+\theta)v_1 - p_1^*] = \frac{2\theta^2(1+2\theta)}{(1+4\theta)^2}.$$

Note that this loss is even greater than that under vintage-depreciation since

$$\frac{Loss^{S,B}}{Loss^{S,A}} = \frac{(4+\theta)^2(1+2\theta)}{(2+\theta)(4+\theta)^2} \geq 1,\ \text{for all } \theta \in [0,1].$$

For individual-depreciation information goods, consumers have even stronger incentives to wait until period 2 for a lower price since their period 2 valuation does not depreciate.

6.2. The Leasing Model

Finally, we examine the leasing model under individual-depreciation. The vendor announces a single-period leasing fee r_i at the beginning of each period $i = \{1,2\}$. Denote by v_i the marginal consumer type in period i ($i \in \{1,2\}$). As illustrated in Figure 4, there are two cases to consider here: (a) $\theta \geq v_1$ and (b) $\theta < v_1$. Note that the distribution of consumer type in period 2 is no longer a single uniform continuum but with intervals of different densities (see Figure 4).

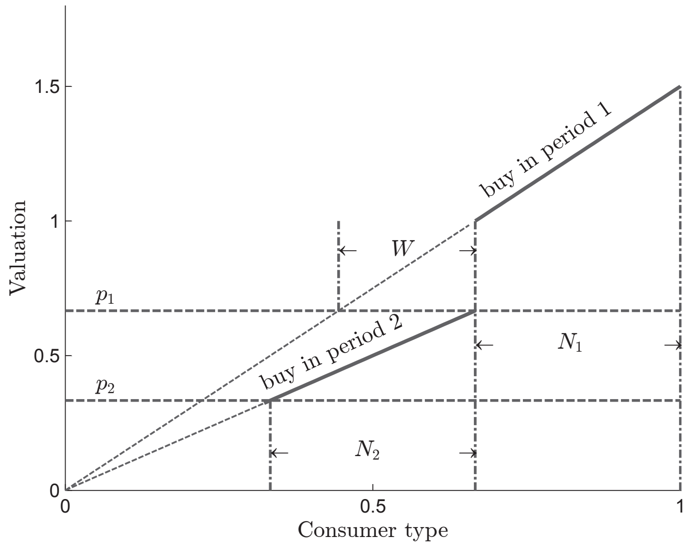

Figure 5 Consumers' valuation and adoption under optimal leasing strategy and vintage-depreciation ($\theta = 0.5$)

Consequently, the vendor's period-2 problem is non-trivial because the marginal consumer type v_2 can be located in any interval, as illustrated in Figure 4.

For case (a) in Figure 4, period-2 demand $N_2(r_2)$ depends on the location of marginal consumer type v_2

$$N_2(r_2|v_1 < \theta) = \begin{cases} 1 - r_2, & r_2 \in [0, \theta v_1); \\ \frac{1}{\theta}(\theta - v_1) + (1 + \frac{1}{\theta})(v_1 - r_2), & r_2 \in [\theta v_1, v_1); \\ \theta - r_2, & r_2 \in [v_1, \theta]. \end{cases}$$

Similarly, for case (b) in Figure 4, $N_2(r_2)$ is given as

$$N_2(r_2|v_1 \geq \theta) = \begin{cases} 1 - r_2, & r_2 \in \{[0, \theta v_1) \bigcup [\theta, v_1]\}; \\ v_1 - \theta + (1 + \frac{1}{\theta})(\theta - r_2), & r_2 \in [\theta v_1, v_1). \end{cases}$$

The vendor's period 2 problem is

$$\max_{v_2} \quad \pi_2 = N_2(v_1, r_2) \times r_2,$$
$$s.t. \quad r_2 = \begin{cases} \theta v_2, \ v_2 \geq v_1; \\ v_2, \quad v_2 < v_1. \end{cases}$$

At the beginning of period 1, consumers' utility functions for all candidate strategies are

$$\begin{aligned}
U_v^{L,B}(DD) &= v - r_1 + \theta v - r_2;\\
U_v^{L,B}(DO) &= v - r_1;\\
U_v^{L,B}(OD) &= v - r_2;\\
U_v^{L,B}(OO) &= 0.
\end{aligned}$$

The vendor's period 1 problem is:

$$\max_{v_1} \quad \pi^{L,B} = N_1(v_1) \times r_1 + \pi_2^*(v_1), \tag{6}$$

where r_1 can be written as a function of v_1. Solving, we have the following Lemma 2.

Lemma 2. *For individual-depreciation information goods and under the leasing model, $\forall\, \theta \in [0,1]$, the optimal leasing fees (r_1^*, r_2^*) are*

$$(r_1^*, r_2^*) = \begin{cases} (\emptyset, \frac{1}{2}), & \theta \in [0, \frac{1}{3}); \\ (\frac{\theta}{1+\theta}, \frac{\theta}{1+\theta}), & \theta \in [\frac{1}{3}, 1]; \end{cases} \tag{7}$$

The empty set $\emptyset$ indicates that the optimal strategy is to block leasing in period 1. The optimal profit $(\pi^{L,B})^*$ is

$$(\pi^{L,B})^* = \begin{cases} \frac{1}{4}, & \theta \in [0, \frac{1}{3}); \\ \frac{\theta}{1+\theta}, & \theta \in [\frac{1}{3}, 1]. \end{cases} \tag{8}$$

The numbers of adopters in each period (N_1^*, N_2^*) are

$$(N_1^*, N_2^*) = \begin{cases} (0, \frac{1}{2}), & \theta \in [0, \frac{1}{3}); \\ (\frac{\theta}{1+\theta}, \frac{1}{1+\theta}), & \theta \in [\frac{1}{3}, 1]. \end{cases} \tag{9}$$

Lemma 2 implies that, if the information goods were offered in both periods, a fixed rental fee is optimal, i.e., $r_1^* = r_2^*$. Comparing Lemma 2 with Lemma 1 gives the following insights for pricing information goods with individual-depreciation.

Proposition 2. *For individual-depreciation information goods, selling dominates leasing when $\theta \in [0, \frac{1}{2})$ otherwise when $\theta \in [\frac{1}{2}, 1]$ leasing dominates selling.*

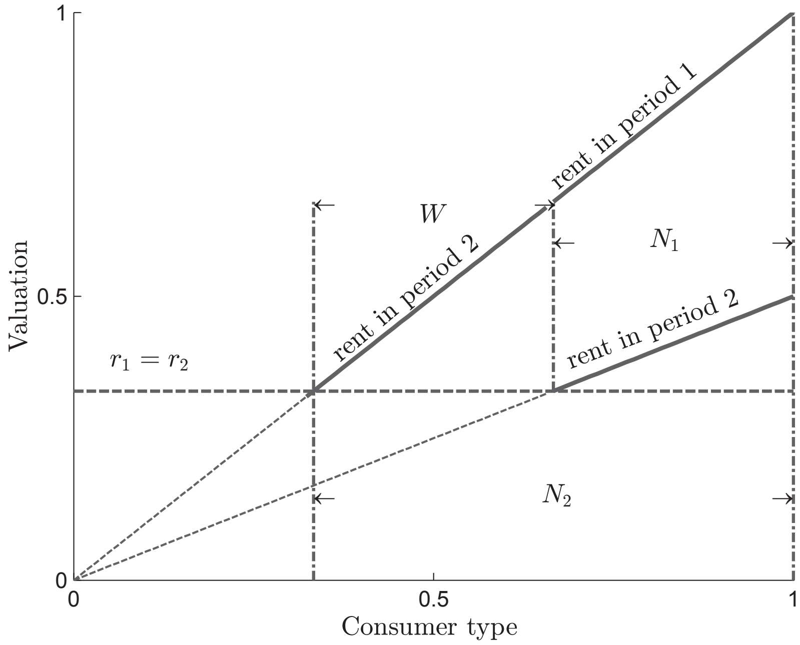

Figure 6 **Consumers' valuation and adoption under optimal leasing strategy and individual-depreciation (**$\theta = 0.5$**,** $K = 0$**)**

The insight from Proposition 2 is that, for information goods with individual-depreciation, selling dominates leasing when the magnitude (i.e., $1-\theta$) of individual-depreciation is large. The driving factor behind is consumers' waiting behavior. At REE, we show that consumers might choose to wait under leasing (see region W in Figure 6). In fact, when $\theta \geq \frac{1}{3}$, the profit loss due to waiting is

$$Loss^{L,B} = N_1^* \times [(1+\theta)v_1 - r_1^*] = \frac{\theta}{(1+\theta)^2}.$$

Comparing $Loss^{L,B}$ with $Loss^{S,B}$ gives

$$\frac{Loss^{L,B}}{Loss^{S,B}} = \frac{(1+4\theta)^2}{(1+\theta)^2(1+2\theta)} \geq 1 \text{ for all } \theta \in [0,1],$$

which implies that, under leasing, the loss due to consumer waiting is even worse than that loss under selling. Put it differently, selling is better in mitigating such consumer waiting behavior than leasing. We next demonstrate the robustness of these findings via several model extensions.

7. Extensions

In this section, we extend our baseline model in three dimensions. Section 7.1 incorporates network effects; Section 7.2 considers a membership discount under leasing where period 1 adopters enjoy a price discount if they choose to continue leasing in period 2; Section 7.3 discusses the hybrid-pricing model in our two-period setting when the vendor offers both selling and leasing simultaneously.

7.1. Network effects

It is well documented in the literature that network effects are profound in markets for information goods (e.g., Katz and Shapiro 1986, Katz and Shapiro 1992, Shapiro and Varian 1999), among others. A natural extension of our baseline model is to examine if our main results in proposition 2 hold under network effects.

Following the literature on network effects, we consider a number of potential users with initial value $v < 0$ distributed on $v \in [-K, 0)$ where K is large enough such that the vendor can never fully cover the entire market. The utility functions with network effects are denoted by $\tilde{U}$. In the presence of network effects, it may become possible that $\tilde{U}_v > 0$ even when $v < 0$, in which case the consumer's adoption is purely driven by adoptions of their peers. This setup is standard in the literature (e.g., Katz and Shapiro 1985, Conner 1995, Jing 2007). For vintage-depreciation information goods in section 5.1, we modify corresponding utility functions in equation 1 as follows

$$\begin{aligned} \tilde{U}_v^{S,A}(DD) &= v + sN_1 - p_1 + \theta(v + sN_2)^+; \\ \tilde{U}_v^{S,A}(OD) &= \theta(v + sN_2)^+ - p_2; \\ \tilde{U}_v^{S,A}(OO) &= 0, \end{aligned} \tag{10}$$

where s denotes the strength of network effects and $(v + sN_2)^+ = \max\{v + sN_2, 0\}$. The interpretation of this formulation is that depreciation only occurs when consumers have nonnegative utility for adoption. Utility functions in equation (10) generalize those in equation (1), and it is straightforward to see that they are identical when $s = 0$. We assume $s \in [0, 1]$ to maintain a reasonable strength of network effects. Similarly, we can adjust other utility functions accordingly.

Proposition 3. *For vintage-depreciation information goods with network effects, leasing dominates selling.*

Proposition 3 entails that the presence of network effects does not change the dominance of leasing over selling for vintage-depreciation information goods. Interestingly, profits under both selling and leasing models are affected by network effects, but in different ways. Under selling, as the strength of network effects increases, consumers have less incentives to delay adoption to period 2 because waiting becomes less attractive. Under leasing and in contrast, it can be shown that optimal leasing fees r_1 and r_2 are not affected by network effects, but the number of adopters increases as the strength of network effects increases.

Next we examine the case of individual-depreciation with network effects, and Proposition 4 summarizes our results.

Proposition 4. *For individual-depreciation information goods with network effects, if K is large enough such that $K \geq 3 + 2\sqrt{3}$,*

- *for $\theta \in (\frac{1+K-s}{2+2K-s}, 1]$, leasing dominates selling;*
- *for $\theta \in (\frac{s}{2+2K-s}, \frac{1+K-s}{2+2K-s}]$, selling dominates leasing;*
- *for $\theta \in [0, \frac{s}{2+2K-s}]$, it is optimal to block sales in period 1, making selling and leasing equivalent.*

We illustrate Proposition 3 via Figure 7. Consistent to our previous analysis in subsection 6.2, leasing also induces waiting under individual-depreciation. Consequently, when θ is relatively small (i.e., the magnitude of individual-deprecation $(1-\theta)$ is large), selling dominates leasing. In the extreme case when $\theta \to 0$, under either individual- or vintage-depreciation it is optimal for the vendor to shut down period 1 sales and only offer the information good in period 2, in which case selling and leasing are equivalent. Note also that when there does not exist network effects, i.e., when $s = 0$, Proposition 4 reduces to Proposition 2.

Comparing Proposition 4 to Proposition 3 offers the following insights for pricing information goods with depreciations and network effects. For vintage-depreciation information goods, leasing dominates selling. In contrast, optimal pricing scheme for individual-depreciation information goods largely depends on the magnitude of depreciation, and selling weakly dominates leasing when magnitude of depreciation is large. As illustrated in Figure 7, our findings in our baseline model are robust when considering network effects.

It is interesting to note the left-hand-side region in Figure 4 where selling and leasing are equivalent in profit. In this region, since the individual-depreciation is so extreme that the vendor would

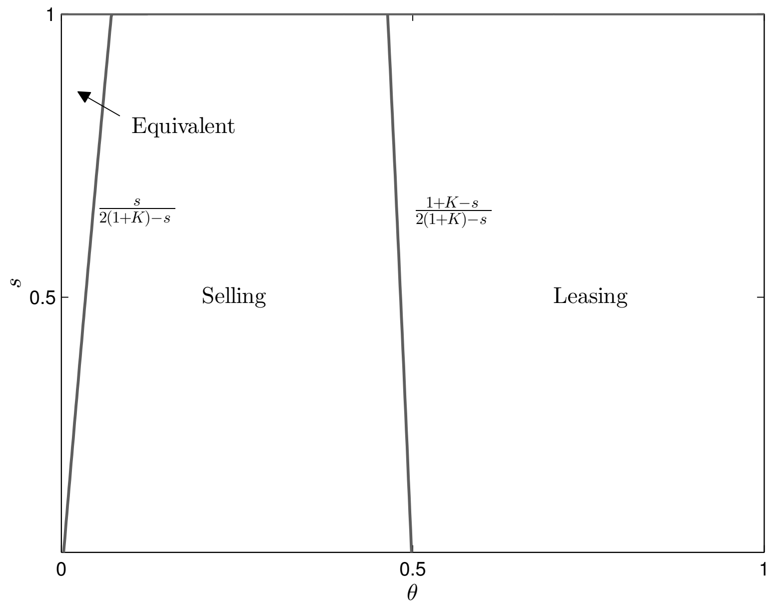

Figure 7 Optimal pricing strategies under individual-depreciation with network effects($K = 7$)

give up sales in period 1. Under a two-period setting, selling and leasing become equivalent thus obtaining the same profit. In business practice, this can be implemented by announcing a very high selling price (or leasing fee) in the early period or only offer to a tiny and negligible portion of customers in period 1.

7.2. Membership Discount

As Proposition 2 indicates, leasing is less profitable under individual-depreciation because consumers have more incentives to delay their adoption. To overcome this, the vendor can offer a discounted leasing fee $r_d \leq r_2$ in period 2 for existing adopters if they continue leasing. Note that such a strategy is sub-optimal for vintage-depreciation information goods as it does not further improve profit because there is no profit loss due to waiting.

For individual-depreciation information goods, this strategy entails the vendor to implement price discrimination between existing and new adopters in period 2. We have the following proposition.

Proposition 5. *For individual-depreciation information goods and under the leasing model, the vendor can further increase its profit with* $r_1^* = \frac{2\theta-\sqrt{\theta}}{4\theta-1}$ *and the optimal discounted leasing fee* $r_d^* = \theta r_1^*$

to only period-1 adopters when $\theta \in [\frac{1}{4}, \frac{1}{2})$. In this region only some existing adopters continue leasing in period 2. However, selling remains optimal for $\theta \in [0, \frac{1}{2})$.

Proposition 5 demonstrates robustness of our findings in baseline model but in another dimension. Membership discount is indeed profit improving under leasing when the magnitude of individual-depreciation is medium (i.e., $\theta \in [\frac{1}{4}, \frac{1}{2})$). However, its impact is limited when the magnitude of individual-depreciation is either very large (i.e., $\theta \in [0, \frac{1}{4}])$), or very small ($\theta \in [\frac{1}{2}, 1]$), for different reasons. In the former situation, when θ is extremely small, the vendor finds it optimal to close period 1, in which case membership discount does not apply. In the latter case, when θ is very large, existing adopters are willing to pay a period 2 leasing fee nearly as the same as period 1, membership discount is not needed and it does not further improve profit.

Surprisingly, even in the case when membership discount is profit improving (i.e., when $\theta \in [\frac{1}{4}, \frac{1}{2})$), the extra benefit due to membership discount is not big enough to beat selling. This is so because only a fraction of existing adopters find it attractive to continue leasing at a discounted period 2 leasing fee, while it helps the vendor to obtain a larger profit from high-value consumers, the down side of this strategy is that the new consumers are kept away by the high leasing fee r_2. As a matter of this trade-off, the value of membership discount is limited, and this strategy is dominated by selling.

7.3. The Hybrid Model

Finally, we briefly discuss the hybrid model where consumers have the freedom to choose between purchasing to use forever and/or renting for just a single period.

Proposition 6. *For both vintage- and individual- depreciation information goods, in a two-period setting, the hybrid model can not further increase the vendor's profit.*

As proved in the Appendix, for both types of depreciation, under REE, it is not possible to have both renting and buying consumers co-exist in either period. Under a two-period setting like ours, selling and leasing models are equivalent in period 2. Then it requires both selling and leasing adopters co-exist in period 1 if the hybrid model becomes optimal. Given that we focus on consumer-side depreciation, rather than product versioning, consumers have no incentives to choose differently on pricing schemes in period 1. In fact, if we assume a comparable setup of a two-period model and no production versioning (e.g., no product upgrading uncertainty over time), our findings in Proposition 6 are consistent with the literature (i.e., Zhang and Seidmann 2010). We leave it for future research to see whether such findings extend to a T-period setting ($T > 2$) and if so, under what conditions.

8. Conclusion

We re-examine the debate on selling vs. leasing information goods in the context of value depreciation, using a two-period game-theoretic model. Our model considers two types of consumer-side value depreciation for information goods or services: vintage-depreciation and individual-depreciation. We find that leasing dominates selling when the magnitude of individual-depreciation is small. This is consistent with observations from the industry, as attested by the case of Microsoft Office 365.[1] As yet another example, Wolfram Alpha, which is well known for its computing software solution Wolfram Mathematica, has also started to offer the software on a subscription basis since version 8.[2]

Contrary to the Coase conjecture, we find that selling dominates leasing when individual adopters heavily depreciate their valuation of the information goods, and this finding holds true in the presence of network effects. Our findings have immediate practical implications. For example, many mobile app games, such as “Angry Birds” and “Draw Something”, receive a lot of attention because of their novelty; but their novelty can wear off and their users may quickly get bored or distracted by similar products. In this case, selling could be more profitable than leasing. Amazon recently announced rental services to their digital book offerings (known as “kindle books”), but restricted renting to a very limited number of titles such as textbooks.[3] Our results suggest that it is suboptimal to use the leasing model for digital books with strong individual-depreciation (e.g., novels). Another interesting example comes from the music industry. Commonly mp3 files

[1] see http://www.microsoft.com/china/office365/buy.aspx

[2] http://www.wolfram.com/mathematica-home-edition/

[3] http://www.engadget.com/2013/01/18/Amazon-kindle-rentals/

are for selling model only due to its nature of individual-depreciation (e.g., iTunes). However, leading music streaming service website such as Spotify has been very successful with their periodic subscription pricing model. Spotify, as the streaming service provider for multiple music publishers, offer more than 20 million songs with weekly updates. Our model interpretation is that, Spotify's offering, as a package of songs, is much less sensitive to individual-depreciation and the leasing model should work. This is in contrast to iTunes' business model where each title is sold separately. For future research, it would be interesting to further extended our model to the multiple-period or continuous-time setting. Another fruitful avenue of future research would be further testing our model predictions empirically.

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Appendix

Proofs

Proof of Lemma 1. First consider the vendor's problem in period 2. At the beginning of period 2, period 1 adopters are type $v \in (v_1, 1]$. Under individual deprecation, the potential adopters for period 2 are on $[0, v_1]$ under the selling.

The marginal consumer type in period 2, v_2, must satisfy that $v_2 = p_2$. So period 2 profit is $p_2(v_1 - v_2)$. Solving period 2 profit maximization problem gives us $v_2^* = \frac{v_1}{2}$ and $\pi_2^* = \frac{v_1^2}{4}$.

Next consider the vendor's probem in period 1. The marginal consumers in period 1 are indifferent between (1) buying in period 1 and adopting for both periods, and (2) buying in period 2 and only adopting for one period. Thus marginal consumer type v_1 is given by $v_1 + \theta v_1 - p_1 = v_1 - p_2$. Under REE, all consumers can correctly expect $p_2 = \frac{v_1}{2}$. Therefore $p_1 = \frac{(1+2\theta)v_1}{2}$.

The vendor's profit function is $p_1(1 - v_1) + \pi_2^* = \frac{v_1[2(1+2\theta)-(1+4\theta)v_1]}{4}$, which is concave in v_1. The first order condition (FOC) gives the following interior solution $v_1^* = \frac{1+2\theta}{1+4\theta}$, which satisfies $v_1^* \in (0, 1)$, thus optimal. The corresponding optimal pricing strategics and profit can be obtained by inserting v_1^* back. $\square$

Proof of Lemma 2. First consider period 2. At the beginning of period 2, period 1 adopters types are distributed on $(v_1, 1]$. Under individual-depreciation, all consumers are potential adopters in period 2. However, those period 1 adopters have depreciated valuation toward the information good. There are three scenarios to consider in period 2.

Period 2, scenario 1: $v_1 = 1$;

Period 2, scenario 2: $v_1 < 1$, $v_1 < \theta$;

Period 2, scenario 3: $v_1 < 1$, $v_1 \geq \theta$.

Scenario 1 implies that there are no adopters in period 1. In this case, period 2 marginal consumer type v_2 satisfies that $v_2 = r_2$ and period 2 profit is $r_2(1 - v_2)$. It is straightforward to obtain that $v_2^* = \frac{1}{2}$ and $\pi_2^* = \pi^* = \frac{1}{4}$.

Under scenario 2, there are three pricing regions in period 2. (1) $v_2 \le \theta v_1$, (2) $v_2 \in (\theta v_1, v_1)$, and (3) $v_2 \in [v_1, \theta]$. We first rule out price region (2) which is infeasible via the following Lemma A1.

Lemma A1. *For individual-depreciation information goods employing the leasing model, given any rental price pair* (r_1, r_2)*, if there are simultaneous period-1-only and period-2-only adopters, then* (r_1, r_2) *does not constitute a REE.*

We prove Lemma A1 by contradiction. Assume that consumers correctly anticipate the rental price pair $\{r_1, r_2\}$ and period-1-only and period-2-only adopters co-exist in the market. The surplus for period-1-only adopters v is $v - r_1$. Similarly, the surplus for period-2-only adopters v is $v - r_2$. Rational consumers will rent only in the period with smaller rental price unless $r_1 = r_2$ (in which case there is no decision for the vendor to make in period 2). This contradicts the above co-existence assumption.

Lemma A1 implies that only price region (1) and (3) are feasible for the above scenario 2. In price region (1), the optimal period 2 rental price is obtained by maximizing period 2 profit $r_2(1 - v_2)$ s.t. $r_2 < \theta v_1$ and $r_2 = v_2$. The optimal solutions are

$$r_2^* = \begin{cases} \frac{1}{2}, & v_1 \ge \frac{1}{2\theta}; \\ \theta v_1, & v_1 < \frac{1}{2\theta}. \end{cases}, \quad \pi_2^* = \begin{cases} \frac{1}{4}, & v_1 \ge \frac{1}{2\theta}; \\ \theta v_1(1 - \theta v_1), & v_1 < \frac{1}{2\theta}. \end{cases} \tag{A.1}$$

Similarly, the optimal rental price r_2^* and the corresponding profit π_2^* in price region (3) are

$$r_2^* = \begin{cases} v_1, & v_1 \ge \frac{\theta}{2}; \\ \frac{\theta}{2}, & v_1 < \frac{\theta}{2}. \end{cases}, \quad \pi_2^* = \begin{cases} v_1(1 - \frac{v_1}{\theta}), & v_1 \ge \frac{\theta}{2}; \\ \frac{\theta}{4}, & v_1 < \frac{\theta}{2}. \end{cases}$$

It can be verified that under scenario 3, there is only 1 feasible price region $v_2 \le \theta v_1$. Therefore the solution is identical to equation (A.1). Therefore we have a total of 5 candidate strategies from period 1. We need to combine them for optimal period 2 strategies under all possible value of v_1.

Period 2, candidate strategy 1: $v_1 = 1$, $r_2 = \frac{1}{2}$ and $\pi_2 = \frac{1}{4}$;

Period 2, candidate strategy 2: $v_1 \in [\frac{1}{2\theta}, 1)$, $r_2 = \frac{1}{2}$ and $\pi_2 = \frac{1}{4}$;

Period 2, candidate strategy 3: $v_1 < \frac{1}{2\theta}$, $r_2 = \theta v_1$ and $\pi_2 = \theta v_1(1-\theta v_1)$;

Period 2, candidate strategy 4: $v_1 < \frac{\theta}{2}$, $r_2 = \frac{\theta}{2}$ and $\pi_2 = \frac{\theta}{4}$;

Period 2, candidate strategy 5: $v_1 \in [\frac{\theta}{2}, \theta]$, $r_2 = v_1$ and $\pi_2 = v_1(1-\frac{v_1}{\theta})$.

Combining all 5 candidate strategies gives the following period-2 optimal strategies on different regions of v_1 and θ.

Region 1: $v_1 = 1$, $r_2 = \frac{1}{2}$ and $\pi_2 = \frac{1}{4}$;

Region 2: $v_1 \in [\frac{1}{2\theta}, 1]$ and $\theta \in (\frac{1}{2}, 1)$, $r_2 = \frac{1}{2}$ and $\pi_2 = \frac{1}{4}$;

Region 3: $v_1 \in [\frac{\theta}{1+\theta+\theta^2}, 1]$ and $\theta \in (0, \frac{1}{2})$, $r_2 = \theta v_1$ and $\pi_2 = \theta v_1(1-\theta v_1)$;

Region 4: $v_1 \in [\frac{\theta}{1+\theta+\theta^2}, \frac{1}{2\theta}]$ and $\theta \in [\frac{1}{2}, \frac{\sqrt{5}-1}{2}]$, $r_2 = \theta v_1$ and $\pi_2 = \theta v_1(1-\theta v_1)$;

Region 5: $v_1 \in [\frac{1-\sqrt{1-\theta}}{2\theta}, \frac{1}{2\theta}]$ and $\theta \in [\frac{\sqrt{5}-1}{2}, 1]$, $r_2 = \theta v_1$ and $\pi_2 = \theta v_1(1-\theta v_1)$;

Region 6: $v_1 \in [0, \frac{\theta}{2}]$ and $\theta \in [0, \frac{\sqrt{5}-1}{2}]$, $r_2 = \frac{\theta}{2}$ and $\pi_2 = \frac{\theta}{4}$;

Region 7: $v_1 \in [\frac{1-\sqrt{1-\theta}}{2\theta}, \frac{1}{2\theta}]$ and $\theta \in [\frac{\sqrt{5}-1}{2}, 1]$, $r_2 = \frac{\theta}{2}$ and $\pi_2 = \frac{\theta}{4}$;

Region 8: $v_1 \in [\frac{\theta}{2}, \frac{\theta}{1+\theta+\theta^2}]$ and $\theta \in [0, \frac{\sqrt{5}-1}{2}]$, $r_2 = v_1$ and $\pi_2 = v_1(1-\frac{v_1}{\theta})$.

Next consider period 1. In period 1, we go through region 1 to 8 above to solve v_1^*. There are a total of 6 candidate strategies for period 1.

Period 1, candidate strategy 1: $v_1 = 1$ and $\pi = \frac{1}{4}$;

Period 1, candidate strategy 2: $\theta \in [\frac{1}{2}, 1]$, $v_1 = \frac{1}{2\theta}$ and $\pi = \frac{3}{4} - \frac{1}{4\theta}$;

Period 1, candidate strategy 3: $v_1 = \frac{1}{1+\theta}$ and $\pi = \frac{\theta}{1+\theta}$;

Period 1, candidate strategy 4: $\theta \in [0, \frac{\sqrt{5}-1}{2}]$, $v_1 = \frac{\theta}{2}$ and $\pi = \frac{\theta(3-\theta)}{4}$;

Period 1, candidate strategy 5: $\theta \in [\frac{\sqrt{5}-1}{2}, 1]$, $v_1 = \frac{1-\sqrt{1-\theta}}{2\theta}$ and $\pi = \frac{\theta^3+2[(1-\theta)\sqrt{1-\theta}-1]+3\theta}{4\theta^2}$;

Period 1, candidate strategy 6: $\theta \in [0, \frac{\sqrt{5}-1}{2}]$, $v_1 = \frac{\theta}{1+\theta+\theta^2}$ and $\pi = \frac{\theta(1+\theta+2\theta^2)}{(1+\theta+\theta^2)^2}$.

Optimal solution in Lemma 2 can be obtained by comparing all candidate strategies in period 1. □

Proof of Proposition 3. We first show the optimal profit under selling and leasing, respectively. Proposition 3 can be obtained by comparing their profits.

3.1. Vintage-depreciation: Selling under network effects

First consider period 2. The marginal consumer type v_2 satisfies that $p_2 = \theta(v_2 + s\frac{1-v_2}{1+K})$. Period 2 profit is $\frac{p_2(v_1-v_2)}{1+K} = \frac{\theta(v_1-v_2)(s+(1+K-s)v_2-2)}{(1+K)^2}$ which is always concave in v_2. The interior optimal solution is $v_2^* = \frac{1}{2}\left[v_1 - \frac{s}{1+K-s}\right]$. p_2^* is nonnegative when $v_1 \geq -\frac{s}{1+K-s}$. Therefore, for $v_1 \geq -\frac{s}{1+K-s}$, $p_2^* = \frac{\theta(s+(1+K-s)v_1)}{2(1+K)}$ and $\pi_2^* = \frac{\theta(s+(1+K-s)v_1)^2}{4(1+K)^2(1+K-s)}$. Otherwise $v_2^* = v_1$ which implies that the information good should not be offered in period 2.

Next consider period 1. The marginal consumer v_1 is indifferent between buying in period 1 and buying in period 2, which implies that

$$v_1 + sN_1 - p_1 + \theta(v_1 + sN_2) = \theta(v_1 + sN_2) - p_2.$$

This leads to $p_1 = \frac{(2+\theta)[s+(1+K-s)v_1]}{2(1+K)}$ in equilibrium. Inserting p_1 back to the profit function $p_1(1 - v_1) + \pi_2^*$ gives

$$\pi = \frac{2s(2+\theta)(1+2K) - s^2(4+\theta)}{4(1+K)^2(1+K-s)} - \frac{2(1+K-s)v_1(2s(4+\theta) - 2(1+K)(2+\theta))}{4(1+K)^2(1+K-s)} - \frac{(1+K-s)(4+\theta)v_1^2}{4(1+K)^2},$$

which is concave in v_1. The interior solution gives

$$v_1^* = 1 - \frac{2(1+K)}{(1+K-s)(4+\theta)}.$$

This interior solution is valid as $v_1^* > -\frac{r}{1+K-r}$ always hold. The optimal profit is $\frac{(2+\theta)^2}{4(1+K-s)(4+\theta)}$.

3.2. Vintage-depreciation: Leasing under network effects

First consider period 2. The marginal consumer v_2 satisfies that $r_2 = \theta\left(v_2 + \frac{s(1-v_2)}{1+K}\right)$. Inserting this back into the profit function $\frac{r_2(1-v_2)}{1+K}$ gives

$$\pi_2 = \frac{\theta(1-v_2)[s + (1+K-s)v_2]}{(1+K)^2},$$

which is concave in v_2. The interior solution is $v_2^* = \frac{1+K-2s}{2(1+K-s)}$ and the optimal period 2 profit is $\pi_2^* = \frac{\theta}{4(1+K-s)}$.

Next consider period 1. In period 1, the marginal consumer type v_1 satisfies $v_1 + sN_1 - r_1 = 0$, which implies that consumers are purely myopic. Using an argument similar to that in period 2, it can be obtained that $v_1^* = v_2^* = \frac{1+K-2s}{2(1+K-s)}$. The profit is $\frac{1+\theta}{4(1+K-s)}$.

3.3. Vintage-depreciation: The comparison

Leasing dominates selling because $\frac{1+\theta}{4(1+K-s)} > \frac{(2+\theta)^2}{4(1+K-s)(4+\theta)}$. $\square$

Proof of Proposition 4. This proof is similar to the above proofs of Lemma 1 and 2 and it contains discussions over several regions in both periods. Due to page limit, we provide a sketch of proof here and a more detailed one is available from the authors upon request.

4.1. Individual-depreciation: Selling

In period 2, it can be shown that when $v_1 \geq -\frac{s}{1+K-s}$, there exists a unique interior solution such that $v_2 = \frac{1}{2}\left(v_1 - \frac{s}{1+K-s}\right)$ and the optimal period 2 profit is $\frac{[s+(1+K-s)v_1]^2}{4(1+K)^2(1+K-s)}$.

In period 1, marginal consumer type v_1 satisfies $v_1 + sN_1 - p_1 + \theta(v_1 + sN_2) = v_1 + sN_2 - p_2$. Inserting N_2 and p_2 back gives

$$p_1 = \frac{(s(1-v_1) + v_1(1+K))(1+K-s\theta - 2(s-(1+K)\theta))}{2(1+K)(1+K-s)}.$$

The profit function is concave in v_1 when $s < \frac{(1+K)(1+4\theta)}{3+2\theta}$. There are two cases to consider:

Case 1: $s \geq \frac{(1+K)(1+4\theta)}{3+2\theta}$, profit function is convex in v_1. $v_1^* = 1$ and $pi^* = \frac{1}{4(1+K-s)}$;

Case 2: $s < \frac{(1+K)(1+4\theta)}{3+2\theta}$, profit function is concave in v_1. The interior solution is $v_1^* = 1 - \frac{1+K}{2}\left(\frac{1}{1+K-s} - \frac{1}{1+K-3s+4\theta+4K\theta-2s\theta}\right)$ which is no greater than 1 when $s \leq \frac{2\theta(1+K)}{1+\theta}$.

Combining case 1 and 2 gives the optimal solution: when $s < \frac{2\theta(1+K)}{1+\theta}$, the interior solution in case 2 is valid and the optimal profit is $\frac{((1+K)(1+2\theta)-s(2+\theta))^2}{4(1+K-s)^2((1+K)(1+4\theta)-s(3+2\theta))}$; otherwise when $s \geq \frac{2\theta(1+K)}{1+\theta}$, $v_1^* = 1$ and $\pi^* = \frac{1}{4(1+K-s)}$.

4.2. Individual-depreciation: Leasing

Similar to the proof of Lemma 2, we consider multiple period 2 scenarios and obtain the following candidate strategies:

Period 2, candidate strategy 1: $v_1 = 1$, $v_2 = \frac{1+K-2s}{2(1+K-s)}$ and $\pi_2 = \frac{1}{4(1+K-s)}$;

Period 2, candidate strategy 2: $v_1 \geq \frac{1+K-s(1+\theta)}{2(1+K-s)\theta}$, $v_2 = \frac{1+K-s}{2(1+K-s)}$, and $\pi_2 = \frac{1}{4(1+K-s)}$;

Period 2, candidate strategy 3: $v_1 < \frac{1+K-s(1+\theta)}{2(1+K-s)\theta}$, $v_2 = \frac{(1+K)\theta v_1 - s(1-\theta)}{1+K-s(1-\theta)}$, and $\pi_2 = \frac{\theta[s+(1+K-s)v_1](1-\theta v_1)}{(1+K-s(1-\theta))^2}$;

Period 2, candidate strategy 4: $v_1 \geq \frac{\theta}{2} - \frac{s}{2(1+K-s)}$, $v_2 = \frac{s(1-\theta)+v_1(1+K)}{s(1-\theta)+\theta(1+K)}$, and $\pi_2 = \frac{\theta[s+(1+K-s)v_1](1-\theta v_1)}{(1+K-s(1-\theta))^2}$;

Period 2, candidate strategy 5: $v_1 < \frac{\theta}{2} - \frac{s}{2(1+K-s)}$, $v_2 = \frac{1+K-2s}{2(1+K-s)}$, and $\pi_2 = \frac{\theta}{4(1+K-s)}$.

Combing all these candidate strategies produce the optimal period 2 solution which contains a total of 7 regions (details are omitted). In period 1, we search through all these 7 regions for an optimal solution that satisfies REE. If K is sufficiently large such that $K \geq 3+2\sqrt{3}$, the optimal period 1 solution contains the following three optimal solutions, each of which covers a certain parameter space (subregions) of $\{\theta, s\}$.

Region 1: For subregion 1-1 and 1-2, $\pi^*_{region1} = \frac{(2(1+K)\theta - s(1-\theta))^2}{4(1+K-s)(s^2(1-\theta)^2+(1+K)^2)\theta(1+\theta)-(1+K)s(1-\theta^2)}$;

Region 2: $\pi^*_{region2} = \frac{1}{4(1+K-s)}$;

Region 3 (all other regions):

$\pi^*_{region3} = \frac{(1+K-2s)(1+K-s(1-\theta))(s(1-\theta)+\theta(1+K))(1-2s(1-\theta)^2+2s^2(1-\theta)^2+\theta(1+2\theta)+K^2(1+\theta(1+2\theta))+2K(1-s(1-\theta)^2+\theta(1+2\theta)))}{(1+K)^2((1+K-s)^2+((1+K)^2-2s^2)\theta+(1+K-s)^2\theta^2)^2}$,

where subregion 1-1 is defined by $s \geq \frac{(1+K)(1-2\theta)}{1-\theta}$; subregion 1-2 is defined by $s < \frac{(1+K)(1-2\theta)}{1-\theta}$, $\theta \geq \frac{1}{3}$, and $\pi^*_{region1} > \pi^*_{region3}$; Region 2 is defined by $s < \frac{(1+K)(1-2\theta)}{1-\theta}$, $\theta < \frac{1}{3}$, and $\pi^*_{region2} > \pi^*_{region3}$.

4.3. Individual-depreciation: Comparison

For region 1-1, $s \geq \frac{(1+K)(1-2\theta)}{1-\theta}$, leasing dominates selling;

For $s \geq \frac{2\theta(1+K)}{1+\theta}$, it can be shown that these two pricing models converge with $v_1 = 1$ and $\pi^* = \frac{1}{4(1+K-s)}$;

For all other regions, it can be shown that selling dominates leasing. $\square$.

Proof of Proposition 5. This proof is similar to the proof of Lemma 2. We only provide a sketch here due to space constraint. In period 2, when membership discount is allowed, it can be verified that Lemma A1 still holds; However, the vendor now has a greater flexibility in capturing those period 1 adopters. Period 2 optimal solutions are given below.

Period 2, region 1: $\theta < \frac{1}{2}$, $v_1 \in [0, \frac{2\theta-\sqrt{\theta}}{4\theta-1})$, $\pi^*_2 = \frac{\theta}{4}$;

Period 2, region 2: $\theta < \frac{1}{2}$, $v_1 \in [\frac{2\theta-\sqrt{\theta}}{4\theta-1}, 1]$, $\pi^*_2 = \theta v_1(1-v_1) + \frac{v_1^2}{4}$;

Period 2, region 3: $\theta \geq \frac{1}{2}$, $v_1 \in [0, \frac{1-\sqrt{1-\theta}}{2\theta})$, $\pi^*_2 = \frac{\theta}{4}$;

Period 2, region 4: $\theta \geq \frac{1}{2}$, $v_1 \in [\frac{1-\sqrt{1-\theta}}{2\theta}, \frac{1}{2\theta})$, $\pi^*_2 = \theta v_1(1-\theta v_1)$;

Period 2, region 5: $\theta \geq \frac{1}{2}$, $v_1 \in [\frac{1}{2\theta}, 1]$, $\pi_2^* = \frac{1}{4}$.

In each region we solve for v_1^* that satisfies REE. Specifically, we are interested in the region with an optimal solution different from those solutions in Lemma 2. We find a new strategy $v_1 = \frac{2\theta - \sqrt{\theta}}{4\theta - 1}$ which is optimal when $\theta \in [\frac{1}{4}, \frac{1}{2})$ with $\pi^* = \frac{1}{4}\left(1 + \theta - \frac{1}{(1-\sqrt{\theta})^2}\right)$. However, this new strategy is dominated by selling. □

Proof of Proposition 6.

Prove by contradiction. In a two-period setting, if a hybrid pricing model dominates selling or leasing, it indicates that either in period 1 or 2 there are renting and buying consumers concurrently. First consider period 2. In period 2, potential consumers will always choose the lower price (renting or buying). Thus $p_2 = r_2$ holds, thus these two price models are equivalent. The hybrid pricing model does not further improve profit in period 2.

Next consider period 1, there are two cases: (1) vintage-depreciation and (2) individual-depreciation goods. For vintage-depreciation goods, if there are concurrent renting and buying consumers in period 1, it should be noted that some of those renting consumers have to pay again in period 2 (at a presumably lower price), but this violates REE because these consumers should just buy in period 1. Thus, there will be no one renting in period 1. The hybrid pricing model doest not improve profit. This contradicts to the above assumption.

For individual-depreciation goods, consider those renting consumers in period 1. If they rent again in period 2, under REE they should choose to buy in period 1 (otherwise no one would buy in period 1, which is also a contradiction as the hybrid pricing model doest not further improve profit). Alternatively, they may only rent in period 1, this implies that period 1 leasing fee is smaller than period 2 leasing fee. However, in this case no one would rent in period 2, which is also a contradiction as the hybrid pricing model doest not further improve profit.