

# **Selling Virtual Currency in Digital Games: Implications on Gameplay and Social Welfare<sup>1</sup>**

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## **Abstract**

Despite the fast growing popularity of in-game purchases of virtual currency (to obtain game enhancing virtual items) in digital games, there is very limited formal research that studies this new business model and its impacts on players' gameplay behavior and social welfare. This paper builds upon the classic labor-leisure model to capture players' dual experience of enjoying leisure and earning virtual currency when playing digital games. We examine the impact of selling virtual currency on players' gameplay behavior and social welfare, relating players' demand for in-game virtual currency to three key game characteristics – virtual wealth satiation point, rate of generating virtual currency, and rate of generating leisure. Our results suggest that the game provider can increase her profits by reducing the virtual currency's price and increasing the number of available virtual items. We find that selling virtual currency reduces the playing times for the majority of players, which helps alleviate the risk of excessive gaming. Selling virtual currency also leads to a larger player base and more people enjoy the many benefits from playing digital games. Finally, we demonstrate that selling virtual currency could lead to a “win-win-win” situation for the game provider, players, and society.

**Keywords:** Digital games, free-to-play games, virtual currency, in-game purchase.

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## Introduction

Digital games, including PC and console games, massively multiplayer online (MMO) games, social games, and mobile games, have experienced remarkable growth in recent years, generating a revenue of \$20.5 billion in the U.S. market alone in 2013 (Ewalt 2013). According to the Entertainment Software Association (ESA), 59% of Americans play video games and 51% of U.S. households own at least one device dedicated to game playing (ESA 2014). On average, U.S. gamers of age 13 or older spent 6.3 hours a week playing video games during 2013 (Nielsen 2014). Among all types of digital games, social games played on mobile devices have seen a significant growth and have now become the most played game type, accounting for 30% of all game play times. The gaming industry is ever-changing with constant introductions of creative game designs and exciting new generations of hardware. Digital games are now played by a highly diversified player base with an average age of 31, 48% being female (ESA 2014).

The business model of the gaming industry is also transforming rapidly. Traditionally, game makers, like other software producers, relied on either a recurring subscription fee and/or an upfront fixed fee to generate revenue (Sundararajan 2004; Zhang and Seidmann 2010). In the *subscription fee model*, players typically pay a monthly subscription fee to play the game. In the *fixed fee model*, the firm charges players an upfront lump sum fee for the game. For example, most console games offered in retail stores like GameStop are sold at a fixed price. Most paid mobile games, such as Minecraft (\$6.99) and Plants vs. Zombies (\$0.99), are also based on the fixed fee model. Some games charge both a subscription fee and a fixed fee. For example, the leading MMO game, Blizzard's World of Warcraft (WoW), charges players both a fixed fee of \$19.99 and a subscription fee of approximately \$13.99 per month.

With players' increasing comfort with and interest in spending real money to purchase virtual items, in-game microtransactions are becoming a standard feature in digital games. The gaming industry has seen a strong growth of in-game microtransactions, e.g., using real money to

purchase virtual items, as a new revenue source. A growing number of digital games are offering in-game virtual currency<sup>2</sup> microtransactions. The proliferation of microtransactions has made the *free-to-play business model* more attractive to game makers. In this free-to-play model, the game is offered to players for free and the game makers generate revenue through in-game microtransactions. There are other ways for monetizing free software applications, such as advertising (Lin et al. 2012; Chen and Stallaert 2014), feature-limited and time-limited freemiums (Lee and Tan 2013; Niculescu and Wu 2014). In this paper we focus on monetization through in-game virtual currency purchases. Several top-ranked MMOs such as League of Legends and Team Fortress 2 are also free-to-play games. Many game makers, especially the startups, are adapting to the free-to-play model (Chapman 2014). By the end of 2013, free-to-play games have gained a player base six times larger than the pay-to-play games in the U.S. For these fast-growing mobile games, only 27% of gamers paid for the games in the U.S. market.

The revenues generated by in-game microtransactions in the free-to-play models are not insignificant. For example, the top revenue-generating mobile game, *Candy Crush Saga*, brought an annual revenue of \$1.9 billion and a profit of \$568 million to its maker King Digital, which recently went public with a valuation of \$7.6 billion (Tassi 2013; Solomon 2014). King offers the game to players for free and makes money from selling virtual items, such as boosters, extra moves, or extra lives, to a small proportion of its players to enhance their gaming experience. Today, in-game virtual currency sales have become the second-largest source of revenue, only second to retail sales, and are still growing (Jenkins 2014). According to SuperData Research, a market intelligence firm on digital games, among the players who do pay for content, the average spend is \$40 per player and the worldwide market for virtual goods is predicted to exceed \$20 billion by 2015 (SuperData 2013).

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<sup>2</sup> In-game virtual currency studied in this paper is different from digital currency such as Bitcoin, which is a software-based online payment system serving as an alternative to traditional online payment systems such as credit cards.

Virtual currencies in digital games enhance game experiences through activating power-ups or boosters, exchanging for vanity items, weapons, vehicles, and class unlocks, etc. Navigating the new free-to-play and pay-for-content models and pricing virtual currency optimally are challenging tasks. In-game microtransactions may take different forms. Some games offer in-game purchases of virtual currency, which can then be used to purchase virtual items. Other games directly offer in-game purchases of virtual items, i.e., directly pay real money for virtual items. In this paper, we consider the value of virtual items, measured by virtual currency, and then study the general problem of pricing virtual currency. For example, in some games such as Grand Theft Auto, Clash of Clans, and Legend of Zelda, there are clear goals that must be completed to earn virtual currency that can be used to purchase virtual items. These goals vary across different games such as racing in new locations (Grand Theft Auto), raiding villages (Clash of Clans), and smashing a certain number of pots (Legend of Zelda). For games designed for smartphones, virtual currency can be purchased through the carrier's market, such as iTunes for iPhone owners and the Play Store for Android users. For example, players pay \$4.99 for 10,000 coins in Plants vs. Zombies and \$1 for 10 gold bars in Candy Crush Saga.

Despite the fast growing popularity of in-game purchases of virtual currency, there is very limited formal research that studies this new business model and its social impacts. This paper aims to fill this gap by analyzing selling virtual currency in digital games and its impact on players' gameplay behavior and social welfare. Specifically, we address the following research questions: What are the key factors in the market of digital games that affect players' gameplay behavior? How do game characteristics and the game provider's pricing strategy of virtual currency influence players' in-game purchasing decisions? When the players are provided with the option of purchasing virtual currency, how would their gaming behavior change? How does selling virtual currency affect consumer surplus and social welfare?

In this paper, we build upon the classic labor-leisure model to capture the dual experience of enjoying leisure and earning virtual currency for game players. If the game developer does not sell virtual currency in her game, the players can only earn virtual currency through playing the game. If the game developer does sell virtual currency in her game, the players have two options to obtain virtual currency – either through playing the game or directly purchasing virtual currency. We identify three key game characteristics – virtual wealth satiation point, rate of generating virtual currency, and rate of generating leisure. Our findings suggest that decreasing the virtual currency price for games with more purchasable virtual items can increase the game provider’s profits because of a more elastic demand for virtual currency. In addition, we shed light on the impact of game characteristics on the optimal pricing strategy. For example, we find that game makers are advised to reduce the virtual currency price when players can gain virtual currency faster through playing the game, due to a stronger substitution effect between playing time and virtual currency purchases. In contrast, they should raise the virtual currency price when the game leisure activities are more intense due to a weaker substitution effect. Interestingly, we find that the introduction of selling virtual currency as a new revenue source not only benefits the game provider but also increases consumer surplus and social welfare, resulting in a “win-win-win” situation for the game provider, players, and society as a whole. While playing digital games can generate some benefits, e.g., generating fun and entertainment for all age groups, facilitating learning, fostering social connectivity, and deriving higher meaning and purpose (Trudeau 2010; McGonigal 2011; Guarini 2013), we also note that many people are concerned about detrimental effects of gaming such as health risks, aggression, and even addiction, especially for those players with excessive game playing (Yousafzai et al. 2014; Robertson 2014; Park 2014). Our findings show that selling virtual currency can actually shorten the players’ gameplay time, which implies that selling virtual currency can reduce some negative effects of digital games.

The paper proceeds as follows. In the next section, we review the relevant literature. In the following section, we propose a game-theoretical model to describe the market of digital games, which consists of a game provider and a mass of game players. We then analyze the game provider's pricing strategies for the free-to-play model and the game players' corresponding gaming behavior. Based on the analysis, we present findings regarding pricing strategies of in-game virtual currency and suggestions regarding game characteristics. Finally, we conclude with managerial insights and directions for future work.

### **Literature Review**

Our paper is mainly related to two research areas: digital games and purchase of virtual items. In this section, we review these two streams and emphasize our contributions to the existing literature.

As noted by many researchers (e.g., Webster and Martocchio 1993; Agarwal and Karahanna 2000; Finneran and Zhang 2005), there is limited research on digital games. Early work in this area focused on computer games. There exist some papers that investigate why people enjoy playing computer games. Malone (1981) investigated why playing computer games is fun and suggested a framework for a theory of intrinsically motivating instructions for computer games using three categories (challenge, fantasy, and curiosity). This framework could be used as a checklist for designing computer games. For example, Malone conjectured that adding a way of varying computer games' difficulty levels might improve players' satisfaction, which was confirmed by later studies (Malone and Lepper 1987; Fabricatore et al. 2002). In other words, they found that computer games can provide different levels of fun by varying difficulty levels of the games. Some other researchers studied how playing games can promote a state of heightened enjoyment. Epstein and Harackiewicz (1992) found that competition enhances task enjoyment for individuals with high achievement motivation compared to those with low achievement motivation. By extending Epstein

and Harackiewicz's (1992) findings, Tauer and Harackiewicz (1999) demonstrated that achievement orientation of positive and negative outcome feedback moderates the effects of competition. These findings on the sense of enjoyment associated with game activity have persuaded practitioners to use games for training. Venkatesh (1999) found that a more enjoyable experience with a game-based training program can more likely lead users to have favorable perceptions and Liu et al. (2013) identified that competition is the key element of game design that should be incorporated into organizational activity games such as employee training games.

Recently there has been a growing research interest in online digital games. Bartle (1996) identified four sources (Acting, Players, Interacting, and World) of a player's interest in a Multi-User Dungeon (MUD) and described that a player can see MUDs as games, pastimes, sports, or entertainments depending on the player's characteristics in terms of the four sources. Baek (2005) studied users' preferences by measuring willingness-to-pay for online games and found that human-to-human interactivity is the most influential attribute. Hsu and Lu (2004) applied the technology acceptance model (TAM) to explain why people play online games. By incorporating social influences and flow experiences, they found that social norms, attitude, and flow experience significantly affect intentions to play online games. Several papers have investigated motivations for multiplayer online games: Ryan et al. (2006) demonstrated that game enjoyment, autonomy, competence, and relatedness are important factors for intentions to play Massively Multiplayer Online (MMO) games; Yee (2006) showed that achievement, social, and immersion components are main reasons for playing Massively-Multiplayer Online Role-Playing Games (MMORPGs); and Chang et al. (2008) used the social identity theory and demonstrated that perceived enjoyment, reputation, and cohesion are significant factors for MMORPGs players' loyalty. To summarize, prior work has identified various factors that motivate players to play online digital games.

Finally, there are a few papers that explain why people purchase virtual items with real money. Castronova (2006) conducted a cost-benefit analysis for real-money trading (RMT) among

MMORPGs players and found that RMT among players generate negative externalities. He demonstrated that buyers and sellers engaged in the RMT transactions could benefit from RMT, while other players and the game companies bear the cost. Manninen and Kujanpää (2007) argued that virtual items in MMOGs have three values (Achievement, Social, and Immersion values) and demonstrated that the motivational items of play could be a potential profit source. Guo and Barnes (2007) developed a model to explain why people buy virtual items with a mixture of theories, including theory of planned behavior, technology acceptance model, trust theory, and unified theory of acceptance and use of technology. Hamari and Lehdonvirta (2010) proposed that practitioners need to place more emphasis on marketing virtual goods for MMO games because current marketing of virtual goods is short of their potential value.

To summarize, we believe we are the first to study the role of selling virtual currency in the free-to-play model in various aspects. Using our new utility function for playing digital games inspired by the classic labor-leisure model, we investigate the game provider's optimal pricing strategy of virtual currency, the players' optimal decisions on playing time and purchasing virtual currency, and social welfare implications.

## **Modeling Framework**

### ***Game Player's Utility Function***

We consider a game provider (hereafter called "provider") offering a digital game to a unit mass of game players (hereafter called "players") in the marketplace. We start our model setup by characterizing the players' utility function. When playing the game, we assume that players derive utility from two elements – virtual wealth (denoted by  $W$ ) and leisure (denoted by  $L$ ). Virtual wealth  $W$ , which is measured by the total amount of virtual currency a player owns, may take different forms in different games such as coins, gems, gold, etc. Virtual currency can be used to purchase virtual items that enhance the player's gaming experience, just as real currency can be used to purchase real items to enhance people's life experience. Leisure  $L$  in digital games also



resembles leisure in the real world. Players generate leisure through gaming activities such as winning a battle, advancing to next level, decorating a virtual place with more items, etc. By making such an analogy that parallels the virtual world in digital games and the real world, we are able to build on the classic labor-leisure model which characterizes a person’s utility from real currency and leisure in the real world to formulate a player’s utility function in the virtual gaming world.

In this paper, we specify a player’s gross utility  $V$ , which comes from two sources – virtual currency  $W$  and leisure  $L$ , as follows:

$$V = a_L L - \frac{L^2}{2} + a_W W.$$

Parameters  $a_L \geq 0$  and  $a_W \geq 0$  represent the relative values of leisure and virtual currency. This gross utility has a few properties that are usually assumed in the traditional labor and leisure models. As shown in Figure 1, indifferent curves in terms of players’ gross utility are downward sloping which captures the fact that a player prefers more of both virtual currency and leisure. In addition, indifferent curves are convex to the origin which captures the diminishing marginal rate of substitution between virtual currency and leisure. Parameter  $a_W$  determines the marginal rate of substitution. A higher  $a_W$  corresponds to a flatter indifference curve (i.e., a lower marginal rate of substitution), meaning that the players only require a small bribe of virtual currency to convince them to give up an additional unit of leisure.

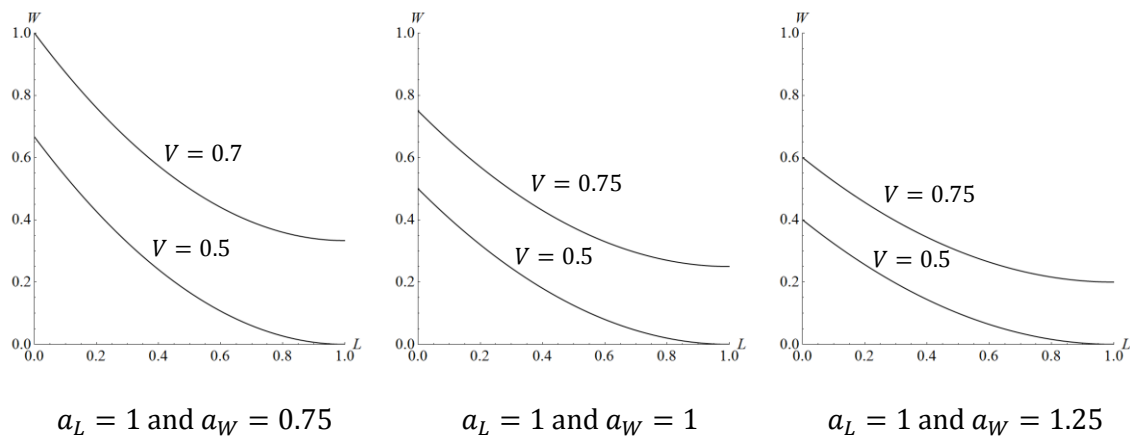


Figure 1: Indifference Curves for Players

In addition, the gross utility  $V$  captures the diminishing return of leisure  $L$ , i.e., the marginal benefit of leisure decreases as  $L$  increases. Furthermore, we use  $h$  to denote the satiation point of virtual wealth, and it is assumed that  $h$ , a game characteristic parameter, is higher for games with more abundant in-game virtual items. Therefore, for each player, there exists a satiation point  $(a_L, h)$  for  $(L, W)$  which he<sup>3</sup> aims to obtain to maximize his gross utility  $V$ . Virtual currency  $W$  and leisure  $L$  are obtained through players' time spent on activities in the digital game. A certain period of playing time generates both virtual currency  $W$  and leisure  $L$ . In other words, we cannot identify from a player's total playing time which fraction is dedicated to generating  $W$  only and which fraction is dedicated to generating  $L$  only. For example, after spending some playing time on winning a battle against virtual enemies, players often gain a certain amount of excitement and also a certain amount of virtual currency. This inseparability of  $L$  and  $W$  in the time dimension is our departing point from the traditional labor-leisure models where two utility components, wage from the time spent on labor activities and leisure from the time spent on hedonic activities, are separated in the time dimension.

To characterize this distinguishing feature of digital games, we denote the average rate of generating virtual currency (hereafter called "rate of virtual currency") and the average rate of generating leisure (hereafter called "rate of leisure") by  $k_W$  and  $k_L$  respectively. After spending time  $t$  on playing the game, players can gain  $k_W t$  amount of virtual currency and  $k_L t$  amount of leisure. Parameters  $k_W$  and  $k_L$  vary for different games. For example, a higher  $k_W$  indicates that it is faster for players to gain virtual currency through game playing, i.e., the game rewards the players more virtual currency per unit of playing time. Similarly, a higher  $k_L$  indicates that the game is more intense during gameplay, i.e., the game involves more leisure activities per unit of playing time. Different games have different  $k_W$  and  $k_L$ . Therefore,  $k_W$  and  $k_L$  are game characteristics. We further assume that  $h \geq \frac{a_L k_W}{k_L}$ . This assumption ensures that there are sufficient virtual items in the

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<sup>3</sup> In this paper, we refer to the provider as "she" and a player as "he."

game such that when the players achieve the bliss point on leisure activities, they still appreciate more virtual items.

### ***Game Provider's Problem Formulation***

When the provider does not sell virtual currency (the non-selling case), virtual currency can only be obtained through playing the game. Therefore, for the non-selling case, we have

$$W = k_W t \text{ and } L = k_L t.$$

However, when the provider does sell virtual currency (the selling case), the player has the option to purchase virtual currency from the provider in addition to obtaining it through gameplay. Let  $G$  be the amount of virtual currency that the player purchases from the provider, and for the selling case, we get

$$W = k_W t + G \text{ and } L = k_L t.$$

The cost of the player to gain  $W$  and  $L$  is two-fold. First, depending on their playing times, players incur an opportunity cost. For playing time  $t$ , the time cost is  $ct$ , where parameter  $c$  represents the unit opportunity cost for playing time. Second, suppose that the provider charges the virtual currency at price  $p$  per unit. For an amount of  $G$ , the purchasing cost is  $pG$ . As a result, we derive an individual player's net utility function as  $U = V - ct$  for the non-selling case and  $U = V - ct - pG$  for the selling case. Summarizing the above, in the non-selling case, a player chooses playing time  $t$  to maximize his net utility  $U$  as follows:

$$\begin{aligned} \max_t U &= V - ct, \\ \text{subject to: } &V = a_L L - \frac{L^2}{2} + a_W W, W = k_W t, L = k_L t, \\ &t \geq t_0 \text{ or } t = 0, \end{aligned}$$

where  $t_0$  represents the minimum playing time if the player decides to play the game. We assume  $t_0$  is sufficiently small such that if the players cannot achieve the bliss point on leisure activities through the minimum playing time  $t_0$ , i.e.,  $t_0 < a_L/k_L$ .

In the selling case, a player chooses playing time  $t$  and purchase an amount of virtual currency  $G$  to maximize his net utility  $U$  as follows:

$$\begin{aligned} \max_{t,G} U &= V - ct - pG, \\ \text{subject to: } V &= a_L L - \frac{L^2}{2} + a_W W, W = k_W t + G, L = k_L t, \\ t &\geq t_0 \text{ or } t = 0, \\ G &\geq 0. \end{aligned}$$

We assume that players are heterogeneous in terms of their time cost  $c$ , which is uniformly distributed on  $[0, \bar{c}]$ . The upper bound  $\bar{c}$  is assumed to be sufficiently large such that the market is not fully covered. In the selling case, let  $t(c)$  and  $G(c)$  denote the playing time and the amount of virtual currency chosen by a given player  $c$  in equilibrium. Then the provider's profit maximization problem is

$$\begin{aligned} \max_p \pi &= p \int_0^{\bar{c}} G(c) dc, \\ \text{subject to: } p &\geq 0. \end{aligned}$$

The provider chooses virtual currency price  $p$  to maximize her profit  $\pi$ .

In the non-selling case, the provider's profit is zero because the provider gives away the game for free. We do not include any other revenue source in the model to help isolate out the sole effect of selling virtual currency. Note that this non-selling case with zero profit for the provider only serves as a theoretical benchmark. All notations are summarized in Table 1.

Table 1. Table of Notations

<b>Game Characteristic Parameters</b>	
$k_W$	Rate of generating virtual currency
$k_L$	Rate of generating leisure
$h$	Satiation point of virtual wealth
<b>Other Parameters</b>	
$a_W$	Valuation of virtual currency
$a_L$	Valuation of leisure
$c$	Unit time cost
$t_0$	Minimum playing time if a player decides to play the game

<b>Decision Variables</b>	
<b>Player</b>	
$t$	Playing time
$G$	Amount of virtual currency a player purchases when the provider sells virtual currency
<b>Provider</b>	
$p$	Price per unit virtual currency
<b>Other Variables</b>	
$W$	Amount of virtual currency a player possesses
$L$	Amount of leisure a player obtains
$V$	Player's gross utility
$U$	Player's net utility
$\pi$	Provider's profit
$SW$	Social welfare

It is worth mentioning that our paper focuses on investigating the impact of selling the virtual currency compared to not selling. Therefore, we take game characteristic parameters  $h$ ,  $k_W$ , and  $k_L$  as exogenously given. We aim to investigate how selling virtual currency affects games with different characteristics, while not making game characteristic parameters decision variables as they are largely determined by factors outside the model such as the type of the game (role-playing games like Final Fantasy or puzzle games like Candy Crush).

## **Equilibrium Analysis**

### ***The Non-Selling Case: Free-to-Play Games with No Virtual Currency on Sale***

We start with analyzing the free-to-play model without selling virtual currency, the non-selling case, as the benchmark case. In the benchmark case, the players only choose playing time  $t$  to maximize their utility. Proposition 1 summarizes the optimal choice for the players. The proofs of all propositions and corollaries are relegated to the appendix.

### **Proposition 1 (Player's strategy for free-to-play games with no virtual currency on sale)**

*When playing free-to-play games with no virtual currency sales, players can be divided into four*

segments: players in  $0 \leq c < c_1^{BM} = a_W k_W$  choose  $t_1^{BM} = \frac{h}{k_W}$ ; players in  $c_1^{BM} \leq c < c_2^{BM} = a_L k_L + a_W k_W - k_L^2 t_0$  choose  $t_2^{BM} = \frac{a_L k_L + a_W k_W - c}{k_L^2}$ ; players in  $c_2^{BM} \leq c < c_3^{BM} = a_L k_L + a_W k_W - \frac{k_L^2 t_0}{2}$  choose  $t_3^{BM} = t_0$ ; and players in  $c \geq c_3^{BM}$  choose  $t = 0$ .

As mentioned before, the provider's profit in the non-selling case is zero and this case only serves as a theoretical benchmark.

### ***The Selling Case: Free-to-Play Games with Virtual Currency on Sale***

When the provider sells virtual currency, the timing of the game is as follows. In the first stage, the provider announces the virtual currency price  $p$ . In the second stage, each player decides whether to play the game; if so, he decides how long he wants to play the game and how much virtual currency to purchase. Note that a player may choose not to play the game at all, i.e.,  $t(c) = 0$  and  $G(c) = 0$ . Using backward induction, we first solve the player's optimal choices of  $t(c)$  and  $G(c)$  and then solve for the game provider's optimal pricing strategy in the next two subsections.

#### **Players' strategy for free-to-play games with virtual currency on sale**

In response to the provider's decision on virtual currency price  $p$ , the players choose their playing time  $t$  and their purchase amount of virtual currency  $G$ . Proposition 2 summarizes the optimal choices for the players for the case of  $p \leq a_W$  (note that if  $p > a_W$ , players will choose  $G = 0$  and players' choices of  $t$  depend on their time cost and the specific forms can be found in the appendix).

#### **Proposition 2 (Player's strategy for free-to-play games with virtual currency on sale)**

*Given the game provider's decision on virtual currency price  $p \leq a_W$ , players can be divided into four segments:*

1. *Pure Play: players in  $0 \leq c < c_1 = p k_W$  choose  $t_1 = \frac{h}{k_W}$  and  $G_1 = 0$ ;*

2. *Play and Purchase:* players in  $c_1 \leq c < c_2 = a_L k_L + p k_W - k_L^2 t_0$  choose  $t_2 = \frac{a_L k_L + p k_W - c}{k_L^2}$  and  $G_2 = h - \frac{k_W(a_L k_L + p k_W - c)}{k_L^2}$ ;
3. *Minimum Play and Purchase:* players in  $c_2 \leq c < c_3 = a_L k_L + p k_W - \frac{k_L^2 t_0}{2} + \frac{(a_W - p)h}{t_0}$  choose  $t_3 = t_0$  and  $G_3 = h - k_W t_0$ ; and
4. *Not Adopt:* players in  $c \geq c_3$  choose  $t = 0$  and  $G = 0$ .

Proposition 2 demonstrates how a player's optimal choices of playing time  $t$  and purchase amount of virtual currency  $G$  depend on individual-specific and game-specific factors, for example, the time cost  $c$ , the virtual currency price  $p$ , and the game characteristics such as virtual wealth satiation point  $h$ . When the virtual currency price is too high ( $p > a_W$ ), all the players in the market will not purchase any virtual currency, as the price does not justify the marginal benefit from one unit of virtual currency. Since this paper studies the provider's strategy on making profit from only selling virtual currency, our analysis focuses on the cases where  $p \leq a_W$  for both types of games. Based on their choices of  $(t, G)$ , players with heterogeneous time cost can be divided into four regions (shown in Figure 2): the "pure play" region for low time cost players ( $0 \leq c < c_1$ ), the "play and purchase" region for medium time cost players ( $c_1 \leq c < c_2$ ), the "minimum play and purchase" region for high time cost players ( $c_2 \leq c < c_3$ ), and the "not adopt" region for very high time cost players ( $c \geq c_3$ ).

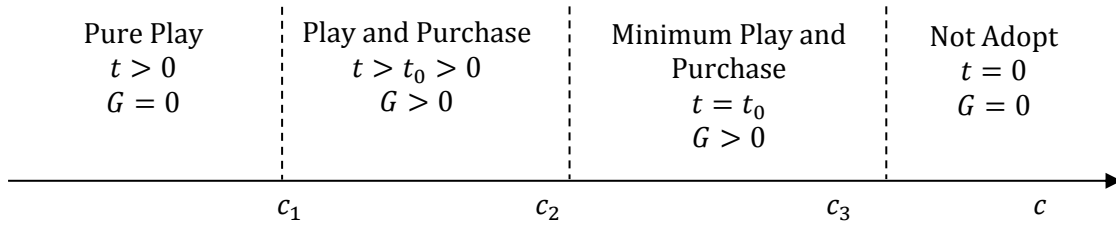


Figure 2: Player Segments

It is intuitive that when the player's time cost is low, the player will only spend time to gain the amount of virtual currency necessary for him to enhance his overall gaming experience and not

purchase any virtual currency using real money ( $G = 0$ ). One extreme case is that a player with zero time cost ( $c = 0$ ) will gain all the necessary virtual currency and also explore all the built-in game activities by purely playing, as playing incurs no cost for him. As the time cost  $c$  increases into the “play and purchase” region, players will find it is worthwhile spending money to purchase a certain amount of virtual currency to satisfy his needs of virtual wealth instead of gaining them all by purely playing. The high time cost outweighs the price of virtual currency, leading the player to bear the cost of buying virtual currency to achieve his needs of virtual wealth rather than to bear the cost of spending time to achieve it. In the meantime, the playing time  $t_2$  also decreases as a consequence of higher time cost. When the time cost  $c$  further increases into the “minimum play and purchase” region, the playing time  $t_3$  decreases to the minimum playing time and players heavily rely on purchasing the virtual currency to compensate for his high time cost. Eventually when the time cost  $c$  surpasses the threshold  $c_3$ , players stop adopting the game.

Now we summarize the substitution effect between playing time  $t$  and purchase amount of virtual currency  $G$  below:

**Corollary 1 (Substitution effect between  $t$  and  $G$ )**

*In the play and purchase region, players' choice of playing time  $t$  increases in the virtual currency price  $p$  and decreases in their time cost  $c$ , i.e.,  $\frac{\partial t_2}{\partial p} > 0$  and  $\frac{\partial t_2}{\partial c} < 0$ ; players' choice of the purchase amount of virtual currency  $G$  decreases in the virtual currency price  $p$  and increases in time cost  $c$ , i.e.,  $\frac{\partial G_2}{\partial p} < 0$  and  $\frac{\partial G_2}{\partial c} > 0$ .*

Corollary 1 reveals the intuitive yet non-trivial substitution effect between the playing time  $t$  and the purchase amount of virtual currency  $G$ . Intuitively, when virtual currency becomes more expensive, the players would purchase less virtual currency and spend more time on playing to gain the necessary amount of virtual currency. When the player's time cost  $c$  becomes higher, the player would spend less time playing the game and buy more virtual currency. Corollary 1 shows our



model results are consistent with these intuitive insights. Results in Corollary 1 are also non-trivial. In our model, two goods directly make up players' utility: leisure activities  $L$  and virtual wealth  $W$ . Since the players' playing time cannot be separated along these two dimensions, we consider  $(L, W)$  as being generated from  $(t, G)$ . Based on the generating function of  $(L, W)$  over  $(t, G)$ , a player's utility eventually comes from consuming two other goods: playing time  $t$  at the price of time cost  $c$  and purchased virtual currency  $G$  at the price of  $p$ . The positive cross-price elasticities ( $\partial t / \partial p > 0$  and  $\partial G / \partial c > 0$ ) demonstrate that playing time  $t$  and purchase amount of virtual currency  $G$  are substitutes in our model.

Next, we investigate the impact of rate of virtual currency on the players' optimal decisions.

**Corollary 2 (Impact of rate of virtual currency on players' strategy)**

*In the play and purchase region, players' choice of playing time  $t$  increases in rate of virtual currency  $k_W$ , i.e.,  $\frac{\partial t_2}{\partial k_W} > 0$ ; players' choice of purchase amount  $G$  decreases in  $k_W$ , i.e.,  $\frac{\partial G_2}{\partial k_W} < 0$ .*

Corollary 2 shows that for a game with higher rate of earning virtual currency  $k_W$ , the players tend to purchase less virtual currency  $G$  and in the meantime spend more time playing the game. This is because for a game with higher rate of earning virtual currency  $k_W$ , the player gains more  $W$  per unit time, which enhances the gaming experience and subsequently generates more utility per unit playing time. This augmentation of the marginal benefit of playing the game leads to longer playing time. For example, in Grant Theft Auto 5 (GTA5), players can rob small convenience stores for 1,000 virtual dollars in about 5 minutes, or armored trucks for 4,000 virtual dollars over a longer period of time, all of which can be summarized by an approximate average rate of virtual currency  $k_W$  of about 200 virtual dollars per minute. If the amount of virtual dollars players can earn increase to 300, then Corollary 2 suggests that players will play longer but purchase less virtual currency.

We now show that the rate of leisure has both positive and negative contributions to marginal benefit of players' playing time.

### Corollary 3 (Impact of rate of leisure on players' strategy)

In the play and purchase region, players' choice of playing time  $t$  first increases and then decreases in the rate of leisure  $k_L$ , i.e.,  $\frac{\partial t_2}{\partial k_L} > 0$  when  $k_L < \frac{2(c-pk_W)}{a_L}$  and  $\frac{\partial t_2}{\partial k_L} < 0$  when  $k_L > \frac{2(c-pk_W)}{a_L}$ ; players' choice of purchase amount  $G$  first decreases and then increases in the rate of leisure  $k_L$ , i.e.,  $\frac{\partial G_2}{\partial k_L} < 0$  when  $k_L < \frac{2(c-pk_W)}{a_L}$  and  $\frac{\partial G_2}{\partial k_L} > 0$  when  $k_L > \frac{2(c-pk_W)}{a_L}$ .

Corollary 3 indicates a non-monotonic relationship between  $t$  and  $k_L$ , which can be attributed to two countervailing effects of  $k_L$  on the marginal benefit of playing time  $t$ . First, the *intensifying effect* of  $k_L$  represents the positive contribution to marginal benefit of  $t$ : for a game with a higher  $k_L$ , the players would experience more game activities per unit playing time. Second, the *exhausting effect* of  $k_L$  represents the negative contribution to marginal benefit of  $t$ : for a game with a higher  $k_L$ , the players would grow more exhausted of the game per unit playing time as the novelty of previous stimuli wears off. For example, in Plants vs. Zombies, a popular strategy game, when more types of plants and/or zombies are introduced within one episode ( $k_L$  increases), the game becomes more intense (the intensifying effect) while in the meantime it also becomes easier for the players to explore most of the activities he wants to have in the game (the exhausting effect). The exhausting effect causes a drawdown to the marginal benefit of any further playing time and the higher *ex ante* playing time  $t$  is the greater is the drawdown. When  $k_L$  is low, relative to time cost  $c$ , i.e.,  $k_L < \frac{2(c-pk_W)}{a_L}$ , the intensifying effect dominates the exhausting effect since the relatively high time cost  $c$  renders the playing time  $t$  relatively low such that the exhausting effect has not taken off yet, i.e., the player is far from fully exploring the game as he has just played relatively little due to his relatively high time cost. Therefore, he will play longer in a game where he can experience more activities per unit time due to the intensifying effect. When  $k_L$  increases in the "high" range, compared to time cost  $c$ , i.e.,  $k_L > \frac{2(c-pk_W)}{a_L}$ , the exhausting effect dominates the intensifying effect

as the relatively low time cost renders a relatively high playing time  $t$ , making the exhausting effect very strong in comparison to the intensifying effect. Since  $t$  and  $G$  are substitutes in the game, the purchase amount of virtual currency  $G$  follows the opposite dynamics, i.e., spending more time playing the game earns more virtual currency and thus players purchase less virtual currency.

Besides the impact on the individual player's playing time  $t$  and virtual currency purchase amount  $G$ , the game characteristic parameters also affect how many individual players will play the game, i.e., the market size.

**Corollary 4 (Impact of virtual currency price and game characteristics on market size)**

*The market size decreases in virtual currency price  $p$ , i.e.,  $\frac{\partial c_3}{\partial p} < 0$ ; it increases in the satiation point of the virtual wealth  $h$ , i.e.,  $\frac{\partial c_3}{\partial h} > 0$ ; it increases in the rate of virtual currency  $k_W$ , i.e.,  $\frac{\partial c_3}{\partial k_W} > 0$ ; and it increases in the rate of leisure  $k_L$ , i.e.,  $\frac{\partial c_3}{\partial k_L} > 0$ .*

Corollary 4 shows that the market size, i.e., the number of players who play the game, shrinks when the virtual currency price increases while the market size expands when (i) there are more virtual items available in the game, (ii) the game has a higher rate of earning virtual currency through playing, and (iii) the game has a higher rate of earning leisure utility through playing. To explain the result regarding the virtual currency price  $p$ , we find that in our model the players on the verge of playing and not playing are the ones who incur high time cost relative to the value derived from the game. Those players will not adopt the game because their time cost is so high that playing results in a negative overall utility. If the virtual currency price decreases, they can derive more positive utility from purchasing virtual currency, which eventually turns the overall utility positive, i.e., they will choose to play the game.

Corollary 4 also shows the impacts of game characteristics on market size. The marginal player will gain a higher utility when there are more virtual items available in the game (a higher

$h$ ), turning the utility from negative to positive, resulting in an increase in market size. Similarly, in a game with a higher rate of virtual currency  $k_W$  or a higher rate of leisure  $k_L$ , players will derive a higher utility per unit playing time, which provides a greater compensation for the high time cost marginal player on the verge of playing or not playing the game.

### **Provider's pricing strategy of virtual currency**

Anticipating the players' responses, the provider decides the virtual currency price  $p$  to maximize her profit. Proposition 3 summarizes the equilibrium pricing strategy and the equilibrium profit for the provider.

#### **Proposition 3 (Provider's pricing strategy of virtual currency)**

*Given the game characteristics, the provider's equilibrium choice of the virtual currency price*

$$p = \frac{a_W}{2} + \frac{t_0[hk_L(2a_L - k_L t_0) - a_L^2 k_W]}{4h(h - k_W t_0)} \quad \text{and} \quad \text{the equilibrium provider's profit}$$

$$\pi = \frac{[2a_W h(h - k_W t_0) + t_0(hk_L(2a_L - k_L t_0) - a_L^2 k_W)]^2}{16ht_0(h - k_W t_0)}.$$

Proposition 3 presents the provider's equilibrium choice of the virtual currency price  $p$  and its resulting profit. We next examine the dynamics of virtual currency price with respect to the game characteristics.

#### **Corollary 5 (Impact of game characteristics on virtual currency price)**

*The equilibrium virtual currency price  $p$  decreases in  $h$ , i.e.,  $\frac{\partial p}{\partial h} < 0$ ; decreases in rate of virtual currency  $k_W$ , i.e.,  $\frac{\partial p}{\partial k_W} < 0$ ; and increases in rate of leisure  $k_L$ , i.e.,  $\frac{\partial p}{\partial k_L} > 0$ .*

Corollary 5 shows that for games with more virtual items (a higher  $h$ ), the provider should charge a lower virtual currency price  $p$  to maximize her profit. According to Corollary 4, for a game with a higher  $h$ , individual demand on  $G$  is higher and the market size is larger. Both of these two effects

result in a higher price elasticity of demand on virtual currency purchases, leading to a lower equilibrium virtual currency price  $p$ . Therefore, the provider should charge a lower price for virtual currency. Corollary 5 also shows that for a game with a higher rate of virtual currency  $k_W$ , the provider should charge a lower virtual currency price  $p$ . From a theoretical perspective, for a game with a higher  $k_W$ , the overall demand of virtual currency becomes more elastic by having a greater market size joined by high time cost players. Their utilities are mostly derived from purchasing virtual currency, which makes them very sensitive to the virtual currency price (based on Corollary 4). Therefore, for a game with higher  $k_W$ , it will have a lower equilibrium virtual currency price. Following a similar approach, we can argue that for a game with higher  $k_L$ , the overall demand of virtual currency becomes more inelastic and consequently yields a higher equilibrium virtual currency price. This result is a consistent reflection of the substitution effect between  $t$  and  $G$ . How “close” a substitute playing time  $t$  is to the amount of virtual currency purchased  $G$  is influenced by the rate of virtual currency  $k_W$  and rate of leisure  $k_L$ . A higher  $k_W$  or lower  $k_L$  makes playing time  $t$  a “closer” substitute and, therefore, makes the demand of virtual currency more elastic, leading to a lower equilibrium price  $p$ . Next, we explore the impacts of various game characteristics on the provider’s profit.

**Corollary 6 (Impact of game characteristics on provider’s profit)**

*The game provider’s optimal profit  $\pi$  increases in  $h$ , i.e.,  $\frac{\partial \pi}{\partial h} > 0$ ; decreases in  $k_W$ , i.e.,  $\frac{\partial \pi}{\partial k_W} < 0$ ; and increases in  $k_L$ , i.e.,  $\frac{\partial \pi}{\partial k_L} > 0$ .*

First, the effect of  $h$  on the total profit is not difficult to comprehend. For a given virtual currency price  $p$ , players’ demand for virtual currency is higher for a game with a higher  $h$  because of a larger market size as well as a higher individual purchase amount of  $G$ . Therefore, the provider can set a

higher virtual currency price  $p$  for a game with a higher  $h$  to capitalize such improvements and generate a higher profit.

Next, the comparative statics analysis of the provider's optimal profit with respect to  $k_W$  demonstrates that in a game with a higher rate of gaining virtual currency  $k_W$ , the market size may be larger because it can entice more high time cost players to play (based on Corollary 4). However, there are fewer "play and purchase" players since some players will benefit more from playing the game with the higher  $k_W$  and become "pure play" players (not purchase virtual currency). In addition, given the substitutable goods of  $t$  and  $G$ , the individual purchase amount is also lower in a game with a higher  $k_W$ . Therefore, higher  $k_W$  leads to a lower provider's profit.

Finally, the comparative statics analysis of the provider's optimal profit with respect to  $k_L$  shows that when the provider's profit comes from selling virtual currency, a game with more intensive leisure activities could generate higher profit. From Corollary 3, we know that depending on whether the intensifying effect dominates the exhausting effect or not, a higher  $k_L$  could either lead to a higher playing time  $t$ , which accumulates a higher amount of virtual currency, or a lower playing time  $t$ , which accumulates a lower amount of virtual currency. Hence, the impact of a higher  $k_L$  on the players' demand on purchase amount of virtual currency seems to be uncertain. However, after the equilibrium price  $p$  increases in response to a higher  $k_L$  (Corollary 2), it reduces the size of the players where the intensifying effect dominates the exhausting effect (from Corollary 3, we can see that such size is determined by  $c > a_L k_L / 2 + p k_W$ ). Therefore, a higher  $k_L$  still provides an overall expansion of market demand for virtual currency, leading to a higher provider profit.

### **Impacts of selling virtual currency: Comparison between the non-selling case and the selling case**

In this section, we study the impacts of selling virtual currency on gameplay, consumer surplus, and social welfare by comparing the selling virtual currency case to the benchmark non-selling case.

Consumer surplus is the aggregate surplus of all participating consumers, i.e.,  $CS = \int_0^{c_1} U\left(t = \frac{h}{k_w}, G = 0\right) dc + \int_{c_1}^{c_2} U\left(t = \frac{a_L k_L + p k_W - c}{k_L^2}, G = h - \frac{k_W(a_L k_L + p k_W - c)}{k_L^2}\right) dc + \int_{c_2}^{c_3} U(t = t_0, G = h - k_w t_0) dc$ . Social welfare is the sum of consumer surplus and the provider's profit, i.e.,  $SW = CS + \pi$ . Now we present the impact of selling virtual currency on consumer surplus and social welfare.

**Proposition 4 (Social welfare of free-to-play games with virtual currency)**

*Selling virtual currency is welfare enhancing, i.e.,  $SW \geq SW^{BM}$ . Consumer surplus also increases, i.e.,  $CS \geq CS^{BM}$ .*

Proposition 4 presents an important finding of our paper: society as a whole will be better off if the monopolistic provider starts to sell in-game virtual currency, which we call the “welfare enhancing effect.” It is intuitive that the provider's profit increases by selling virtual currency. The provider can exploit the players' need for virtual currency at an appropriate price level (Proposition 3) and hence gain more profit. This result has been supported by the fact that many top games in the Apple App store are loaded with options for players to buy virtual currency using real money. More interesting finding lies in the players' side. Proposition 4 shows that consumer surplus also increases when the provider offers virtual currency for sale. This is because although the players have to pay real money to purchase virtual currency, they also benefit through saving their time that would otherwise have been spent to obtain that virtual currency. The overall effect of purchasing virtual currency is positive because the reduced time cost outweighs the price they pay, even as the price is set by a profit-maximizing monopolistic game provider. In summary, selling virtual currency results in a “win-win-win” situation, which benefits the provider, players, and society as a whole.

Nonetheless, such benefit is not equally applied to every player on the market. Figure 3 illustrates the comparison of the playing time and the net utility between the selling case and the

non-selling case for each individual player. It shows that for the players with low time cost, the playing time and net utility will remain the same regardless whether the provider offers the virtual currency for sale or not. For the players with moderate time cost, offering virtual currency for sale will reduce their playing time and improve their net utility. For the players with high time cost, selling the virtual currency will induce more of them choose to play the game, compared to the non-selling case in which they would not even adopt. This is because for high time cost players, purchasing the virtual currency at the equilibrium level brings extra utility from virtual items and reduces the extra playing time, which eventually makes playing the game a choice with positive utility.

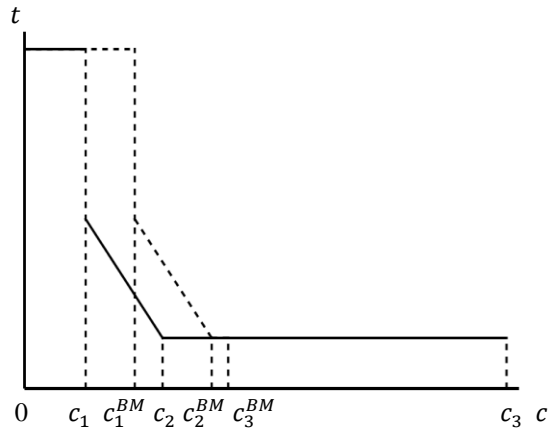


Figure 3a: Players' playing time

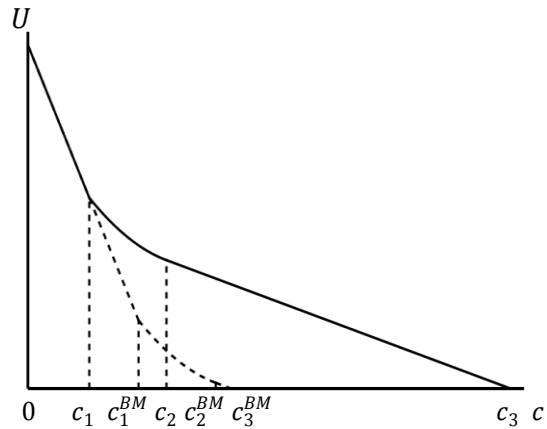


Figure 3b: Players' net utility

Notes: Solid lines correspond to the selling case and dashed lines correspond to the non-selling case.

Figure 3: Players' playing time and net utility

Now we further investigate the impact of three game characteristics on social welfare by deriving comparative statics.

### Corollary 7 (Impact of game characteristics on social welfare)

For both free-to-play games with and without selling virtual currency, social welfare levels  $SW$  and  $SW^{BM}$  increase in  $h$ , i.e.,  $\frac{\partial SW}{\partial h} > 0$  and  $\frac{\partial SW^{BM}}{\partial h} > 0$ ; increase in  $k_L$ , i.e.,  $\frac{\partial SW}{\partial k_L} > 0$  and  $\frac{\partial SW^{BM}}{\partial k_L} > 0$ ; and increase in  $k_W$ , i.e.,  $\frac{\partial SW}{\partial k_W} > 0$  and  $\frac{\partial SW^{BM}}{\partial k_W} > 0$ .



Corollary 7 reveals the welfare effect of the three key game characteristics – satiation point of virtual wealth  $h$ , rate of leisure  $k_L$ , and rate of virtual currency  $k_W$ . In both selling and non-selling cases, social welfare is higher for a game with more abundant virtual items, i.e., higher  $h$ . This is not difficult to understand since players will hardly find it harmful to include more available virtual items in the game, no matter whether they need to obtain them by earning the corresponding virtual currency through playing or purchasing. In addition, the provider would not refuse a higher abundance of virtual items either, since it would generate more sales of the virtual currency. However, we emphasize that Corollary 5 suggests that the provider should charge a lower virtual currency price accordingly.

Second, we observe that a similar situation happens to  $k_L$ . According to Corollary 6, we know that the provider's profit increases in  $k_L$ . Interestingly, we find that the consumer surplus also increases in  $k_L$ . This is because for a more intense game (higher  $k_L$ ), players gain more utility per unit time they play. Therefore, social welfare is increasing in  $k_L$ .

Next, we also find that social welfare increases in  $k_W$ . For a game with a higher  $k_W$ , the playing time required for obtaining a certain amount of virtual currency decreases, which saves the players' time cost that would otherwise have been spent to earn them through playing. Meanwhile, since a higher  $k_W$  also enhances the potential players' utility per unit playing time, it will cause more players to play (i.e., a larger market size), which also positively contributes to the social welfare. Note that the effect of higher  $k_W$  on the social welfare through players' purchase amount  $G$  is irrelevant since it is an internal transfer within the social welfare.

Finally, we note that one interesting finding lies in the dynamics of  $k_W$ . From Corollary 7, we know that, similar to  $h$  and  $k_L$ , social welfare is greater for games with higher  $k_W$  in both selling and non-selling cases. However, from Corollary 6, unlike  $h$  and  $k_L$  (both of which are positively correlated with the provider's profit),  $k_W$  is negatively correlated with the provider's profit. This

finding reveals a misalignment of interests between the provider and the social planner regarding the rate of virtual currency  $k_W$ , but not  $h$  and  $k_L$ . To determine its cause, first consider the game characteristic parameters  $h$  and  $k_L$ . In a game with higher  $h$ , players will find more virtual items they want to have so they want to buy more virtual currency. In a game with higher  $k_L$ , players will take less time to get exhausted about the game so that the amount of virtual currency earned through playing time is not sufficient enough to cover their needs. Hence, they also want to buy more virtual currency. In other words, both higher  $h$  and higher  $k_L$  will generate more purchase demand of virtual currency. However, although higher  $k_W$  will shorten the playing time as higher  $k_L$  does, it also diminishes players' incentive to purchase virtual currency as they can earn them through playing time more quickly. Consequently, the provider faces less purchase demand on virtual currency and not being able to keep the same amount of profit.

### **Concluding Remarks**

Selling virtual currency through in-game microtransactions has become an important revenue source for firms in the digital gaming industry. This paper aims to analyze the impact of this new business model on players' gaming behavior, the game providers' pricing strategy of virtual currency and their profits, and social welfare. We characterize a digital game by three key parameters – virtual wealth satiation point, rate of generating virtual currency, and rate of generating leisure. In our model, players vary in terms of their time cost of playing the game, and their gaming behavior is determined by their time cost, the virtual currency price, and game characteristics. We derive the provider's optimal pricing strategy of virtual currency, taking game characteristics as exogenously given. We analyze the impact of selling virtual currency on players' gameplay behavior and social welfare by comparing the selling case to the non-selling benchmark case.

Our findings have important managerial implications for the digital gaming industry. In order to realize the full potential and ensure long-term growth of in-game microtransactions, the creative and ingenious game designs should be carefully aligned with the business side of the game. Specifically, our results suggest that the provider's optimal pricing strategy of virtual currency critically depends on game characteristics. In general, it is beneficial for the provider to develop and introduce new virtual items. However, the virtual currency price should be reduced accordingly because a lower price will make the more abundant virtual items more attractive to high time cost players, which expands the market and eventually leads to a greater profit. Furthermore, the rate of generating virtual currency (how fast players gain virtual currency) and the rate of generating leisure (how intense game activities are) during gameplay have different impacts on the provider's pricing strategy of virtual currency. Specifically, when it is faster for the players to gain virtual currency, the players' playing time becomes a stronger substitute of virtual currency purchases, and thus, the players have lower incentive to purchase virtual currency. Therefore, the provider should set a lower price to avoid losing too much demand. On the other hand, when the leisure activities in the game become more intense, the players' playing time becomes a weaker substitute of virtual currency purchases, and thus, the players have a higher incentive to purchase virtual currency. Therefore, the provider should set a higher price to take advantage of the increased incentive to purchase virtual currency.

Our findings also provide important social implications. We find selling virtual currency has different impacts on different player segments. When the game developer introduces the option of purchasing virtual currency, the "extreme" players with extremely low time cost will refuse to purchase any virtual currency and will not reduce their playing time. In other words, selling virtual currency does not alter the gaming behavior of the extreme players. Selling virtual currency will entice the "heavy" players with relatively low time cost into the play and purchase segment, which consists of players who will purchase some virtual currency and play the game to earn more virtual

currency to obtain their desirable level of virtual currency. In addition, when the players are provided the option of purchasing virtual currency, there will be an increase in “light” players who will only play the game for a limited time but purchase virtual currency to optimize their gaming experience. In other words, selling virtual currency reduces the playing times for the majority of players, which helps alleviate the risk of excessive gaming. Selling virtual currency also leads to a larger player base and more people enjoy the many benefits from playing digital games. We conclude that offering in-game purchases of virtual currency as a new business model benefits society as a whole.

We also find that both the game provider and the social planner prefer games with more virtual items and higher rate of leisure. However, the preferences of the game provider and the social planner are at odds when it comes to the rate of virtual currency. The social planner prefers games with a higher rate of virtual currency, whereas the game provider prefers games with a lower rate of virtual currency. This misalignment of incentives between the game provider and the social planner may result in the production of games with an excessively low rate of virtual currency. Regulators in the gaming industry are advised to provide incentives to encourage game providers to increase rate of virtual currency in designing their games.

Finally, our work has several limitations. For example, in this paper, we only considered free-to-play business model. While this free-to-play model was selected to make the impact of selling virtual currency clear, we conjecture that even in the other business models our main findings and managerial insights may hold. Another limitation of our paper is that we do not model the trading of the virtual currency among the game players. Nowadays game players in several digital games, for example Second Life, can buy the virtual currency from not only the game provider but also other players. As our model focuses on the implications of the virtual currency on the game play and social welfare, we leave the study of the between-player trading for future research.

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## Appendix

### *Proof of Proposition 1*

When playing free-to-play games with no virtual currency sales, players' optimal choice of  $t$  can be derived by solving a constrained optimization problem. We follow the standard procedure of the Lagrangian method. Based on the characteristics of the utility function, we divide the entire optimization problem into the following sub problems:

$$\text{i) } \max_t U = a_L L - \frac{L^2}{2} + a_W W - ct, \text{ subject to } L \leq a_L, W \leq h, \text{ and } t \geq t_0,$$

$$\text{ii) } \max_t U = \frac{a_L^2}{2} + a_W W - ct, \text{ subject to } L > a_L, W \leq h, \text{ and } t \geq t_0,$$

$$\text{iii) } \max_t U = a_L L - \frac{L^2}{2} + a_W h - ct, \text{ subject to } L \leq a_L, W > h, \text{ and } t \geq t_0, \text{ and}$$

$$\text{iv) } \max_t U = \frac{a_L^2}{2} + a_W h - ct, \text{ subject to } L > a_L, W > h, \text{ and } t \geq t_0,$$

where  $W = k_W t$ ,  $L = k_L t$ . Solving each sub optimization problem, from i) to iv), yields the optimal  $t$  for the corresponding sub problem given its own functional form of the utility function. The overall problem's optimal  $t$  can be derived based on a cross comparison among the optimal  $t$  of the sub problems, i.e., finding the one that yields the highest utility value across the sub problems for any given parameter values.

To illustrate how the Lagrange method is executed in our analysis, we take the problem (i) with the constraint  $W \leq h$  being binding as an example. We sketch part of its derivation as follows.

First we construct the Lagrange function  $\mathcal{L}$

$$\mathcal{L} = a_L(k_L t) - \frac{(k_L t)^2}{2} + a_W(k_W t) - ct + \lambda[h - (k_W t)],$$

where  $\lambda$  is the LaGrange multiplier. Because this is a maximization problem, we require  $\lambda > 0$ . We

solve the first order conditions  $\left\{ \frac{\partial \mathcal{L}}{\partial t} = 0, \frac{\partial \mathcal{L}}{\partial \lambda} = 0 \right\}$  and obtain  $t^* = \frac{h}{k_W}$  and  $\lambda = \frac{a_L k_L k_W + a_W k_W^2 - h k_L^2 - c k_W}{k_W^2}$ .

Since we require  $\lambda > 0$ , the first necessary condition for  $t^*$  to be optimal is  $c < \frac{a_L k_L k_W + a_W k_W^2 - h k_L^2}{k_W}$ .

Then we do this for the remaining constraints one by one. After finding all the constrained optimal

solutions and their necessary conditions using  $> 0$ , we verify their second order condition and then conduct a cross comparison among the optimal solutions that has common range regarding  $c$  to find the overall best choice given any  $c$ . In this way, we obtain Proposition 1.

### ***Proof of Proposition 2***

Given the game provider's decision on virtual currency price  $p$ , players' optimal choice of  $(t, G)$  can be derived by solving a constrained optimization problem. We follow the standard procedure of the Lagrangian method. Based on the characteristics of the utility function, we divide the entire optimization problem into the following sub problems:

$$\text{i) } \max_{t,G} U = a_L L - \frac{L^2}{2} + a_W W - ct - pG, \text{ subject to } L \leq a_L, W \leq h, t \geq t_0 \text{ and } G \geq 0,$$

$$\text{ii) } \max_{t,G} U = \frac{a_L^2}{2} + a_W W - ct - pG, \text{ subject to } L > a_L, W \leq h, t \geq t_0 \text{ and } G \geq 0,$$

$$\text{iii) } \max_{t,G} U = a_L L - \frac{L^2}{2} + a_W h - ct - pG, \text{ subject to } L \leq a_L, W > h, t \geq t_0 \text{ and } G \geq 0, \text{ and}$$

$$\text{iv) } \max_{t,G} U = \frac{a_L^2}{2} + a_W h - ct - pG, \text{ subject to } L > a_L, W > h, t \geq t_0 \text{ and } G \geq 0,$$

where  $W = k_W t + G$ ,  $L = k_L t$ . Solving each sub optimization problem, from i) to iv), yields the optimal  $(t, G)$  for the corresponding sub problem given its own functional form of the utility function. The overall problem's optimal  $(t, G)$  can be derived based on a cross comparison among the optimal  $(t, G)$  of the sub problems, i.e., finding the one that yields the highest utility value across the sub problems for any given parameter values.

To illustrate how the Lagrange method is executed in our analysis, we take the problem (i) with the constraint  $W \leq h$  being binding as an example. We sketch part of its derivation as follows. First we construct the Lagrange function  $\mathcal{L}$

$$\mathcal{L} = a_L(k_L t) - \frac{(k_L t)^2}{2} + a_W(k_W t + G) - ct - pG + \lambda[h - (k_W t + G)]$$

where  $\lambda$  is the LaGrange multiplier. Because this is a maximization problem, we require  $\lambda > 0$ . We solve the first order conditions  $\left\{ \frac{\partial \mathcal{L}}{\partial t} = 0, \frac{\partial \mathcal{L}}{\partial G} = 0, \frac{\partial \mathcal{L}}{\partial \lambda} = 0 \right\}$  and obtain  $t^* = \frac{a_L k_L + p k_W - c}{k_L^2}$ ,  $G^* = h -$

$\frac{k_W(a_L k_L + p k_W - c)}{k_L^2}$  and  $\lambda = a_W - p$ . Since we require  $\lambda > 0$ , the first necessary condition for  $(t^*, G^*)$  to be optimal is  $p < a_W$ . Then we do this for the remaining constraints one by one. Substitute  $(t^*, G^*)$  into the constraint  $t \geq t_0$  and it yields the second necessary condition  $c \leq (a_L - k_L t_0)k_L + p k_W$ . The constraint  $L \leq a_L$  yields the third necessary condition  $c \geq p k_W$ . The constraint  $G \geq 0$  yields the fourth necessary condition  $c \geq k_W p + a_L k_L - \frac{h k_L^2}{k_W}$ . The last constraint,  $W \leq h$ , is automatically satisfied as we treated it as binding. Find the common range of  $c$  that satisfies all the previous necessary conditions and verify the second order condition. Then we have when  $h \geq \frac{a_L k_W}{k_L}$ ,  $p \leq a_W$  and  $p k_W \leq c \leq (a_L - k_L t_0)k_L + p k_W$ , the optimal choice is  $t^* = \frac{a_L k_L + p k_W - c}{k_L^2}$  and  $G^* = h - \frac{k_W(a_L k_L + p k_W - c)}{k_L^2}$ .

We repeat the previous steps for problem (i) with the other single constraints being binding, which are  $L \leq a_L$ ,  $t \geq t_0$ , and  $G \geq 0$ , plus other combinations of constraints being binding, which are  $\{L \leq a_L, G \geq 0\}$ ,  $\{L \leq a_L, W \leq h\}$ ,  $\{W \leq h, G \geq 0\}$ ,  $\{W \leq h, t \geq t_0\}$ , and  $\{t \geq t_0, G \geq 0\}$ . Each round we generate a candidate of  $(t^*, G^*)$  and the corresponding range on  $c, h$ , and  $p$ . Eventually we have the following results. When  $h \geq a_L k_W / k_L$ ,

- a) If  $0 < p \leq a_W$ , when  $0 \leq c \leq p k_W$  we have  $t^* = \frac{a_L}{k_L}$  and  $G^* = \frac{h k_L - a_L k_W}{k_L}$ ; when  $k_W p \leq c \leq a_L k_L + p k_W - k_L^2 t_0$  we have  $t^* = \frac{a_L k_L + p k_W - c}{k_L^2}$  and  $G^* = h - \frac{k_W(a_L k_L + p k_W - c)}{k_L^2}$ ; and when  $a_L k_L + p k_W - k_L^2 t_0 \leq c \leq a_L k_L + b h - \frac{k_L^2 t_0}{2} - \frac{p(h - k_W t_0)}{t_0}$  we have  $t^* = t_0$  and  $G^* = h - k_W t_0$ .
- b) If  $p > a_W$ , when  $0 \leq c \leq a_W k_W$ , we have  $t^* = \frac{a_L}{k_L}$  and  $G^* = 0$ ; when  $a_W k_W < c \leq a_L k_L + a_W k_W - k_L^2 t_0$  we have  $t^* = \frac{a_L k_L + a_W k_W - c}{k_L^2}$  and  $G^* = 0$ ; and when  $a_L k_L + a_W k_W - k_L^2 t_0 < c \leq a_L k_L + a_W k_W - \frac{k_L^2 t_0}{2}$  we have  $t^* = t_0$  and  $G^* = 0$ .

After exhausting all the combinations of constraints being binding for problem (i), we repeat the steps for (ii) to (iv). Eventually we have a number of  $(t^*, G^*)$  each of which corresponds

to its own feasible range. Next, we integrate all those feasible ranges through cross comparing  $(t^*, G^*)$  based on which one generate the highest utility for the consumer. Finally, we find the best  $(t^*, G^*)$  for any given parameter values, leading to Proposition 2. It is worth noting that since all the cases under  $p > a_W$  (case b) give  $G^* = 0$ , they are not presented in Proposition 2.

### ***Proof of Corollary 1***

Based on the results from Proposition 2, we derive the first derivative of  $t_2$  with respect to  $p$  and  $c$ :

$$\frac{\partial t_2}{\partial p} = \frac{k_W}{k_L^2} > 0 \text{ and } \frac{\partial t_2}{\partial c} = -\frac{1}{k_L^2} < 0. \text{ Next we derive the first derivative of } G_2 \text{ with respect to } p \text{ and } c:$$

$$\frac{\partial G_2}{\partial p} = -\frac{k_W}{k_L^2} < 0 \text{ and } \frac{\partial G_2}{\partial c} = \frac{1}{k_L^2} > 0.$$

### ***Proof of Corollary 2***

Based on the results from Proposition 2, we derive the first derivative of  $t_2$  and  $G_2$  with respect to

$$k_W: \frac{\partial t_2}{\partial k_W} = \frac{p}{k_L^2} > 0 \text{ and } \frac{\partial G_2}{\partial k_W} = -\frac{p}{k_L^2} < 0.$$

### ***Proof of Corollary 3***

Based on the results from Proposition 2, we derive the first derivative of  $t_2$  with respect to  $k_L$ :

$$\frac{\partial t_2}{\partial k_L} = \frac{2(c-pk_W)-a_L k_L}{k_L^3}. \text{ Next we derive the first derivative of } G_2 \text{ with respect to } k_L :$$

$$\frac{\partial G_2}{\partial k_L} = \frac{k_W[a_L k_L - 2(c-pk_W)]}{k_L^3}. \text{ Solving } \frac{\partial t_2}{\partial k_L} < 0, \frac{\partial t_2}{\partial k_L} > 0, \frac{\partial G_2}{\partial k_L} < 0, \text{ and } \frac{\partial G_2}{\partial k_L} > 0 \text{ yields the results of Corollary}$$

3.

### ***Proof of Corollary 4***

Based on the results from Proposition 2, we derive the first derivative of  $c_3$  with respect to  $p, h, k_W,$

$$\text{and } k_L: \frac{\partial c_3}{\partial p} = k_W - \frac{h}{t_0} < 0, \frac{\partial c_3}{\partial h} = \frac{a_W - p}{t_0} > 0, \frac{\partial c_3}{\partial k_W} = p > 0, \text{ and } \frac{\partial c_3}{\partial k_L} = a_L - k_L t_0 > 0 \text{ for a sufficiently}$$

small  $t_0$ .

### **Proof of Proposition 3**

The provider chooses  $p$  to maximize  $\pi = p \left( \int_{c_1}^{c_2} G_2 dc + \int_{c_2}^{c_3} G_3 dc \right) = \left( \frac{p}{2t_0} \right) [2h(a_W - p)(h - k_W t_0) + t_0(hk_L(2a_L - k_L t_0) - a_L^2 k_W)]$ , which is a concave quadratic function of  $p$ . The optimal profit is achieved at the critical point  $p = \frac{a_W}{2} + \frac{t_0[hk_L(2a_L - k_L t_0) - a_L^2 k_W]}{4h(h - k_W t_0)}$ . Substituting the optimal  $p$  back into  $\pi$  yields the equilibrium provider's profit  $\pi = \frac{[2a_W h(h - k_W t_0) + t_0(hk_L(2a_L - k_L t_0) - a_L^2 k_W)]^2}{16ht_0(h - k_W t_0)}$ .

### **Proof of Corollary 5**

Based on the results in Proposition 3, we derive the first derivative of  $p$  with respect to  $h$ ,  $k_W$ , and  $k_L$ :  $\frac{\partial p}{\partial h} = -\frac{t_0(hk_L - a_L k_W)[2a_L h - t_0(hk_L + a_L k_W)]}{4h^2(h - k_W t_0)^2} < 0$ ,  $\frac{\partial p}{\partial k_W} = -\frac{t_0(a_L - k_L t_0)^2}{4(h - k_W t_0)^2} < 0$ , and  $\frac{\partial p}{\partial k_L} = \frac{t_0(a_L - k_L t_0)}{2(h - k_W t_0)} > 0$  for a sufficiently small  $t_0$ .

### **Proof of Corollary 6**

Based on the results in Proposition 3, we derive the first derivative of  $\pi$  with respect to  $h$ ,  $k_W$  and  $k_L$ :  $\frac{\partial \pi}{\partial h} = \left[ \frac{2ha_W(h - k_W t_0)(2h - k_W t_0) + k_W t_0(2a_L^2 h - a_L t_0(2hk_L + a_L k_W) + hk_L^2 t_0^2)}{16t_0 h^2 (h - k_W t_0)^2} \right] [2ha_W(h - k_W t_0) + t_0(hk_L(2a_L - k_L t_0) - a_L^2 k_W)] > 0$ ,  $\frac{\partial \pi}{\partial k_W} = -\left[ \frac{2ha_W(h - k_W t_0) + t_0(hk_L(2a_L - k_L t_0) - a_L^2 k_W)}{16h(h - k_W t_0)^2} \right] [2h(a_L^2 + a_W h) - t_0(2h(a_L k_L + a_W k_W) + a_L^2 k_W) + hk_L^2 t_0^2] < 0$ , and  $\frac{\partial \pi}{\partial k_L} = \frac{(a_L - k_L t_0)[2ha_W(h - k_W t_0) + t_0(hk_L(2a_L - k_L t_0) - a_L^2 k_W)]}{4(h - k_W t_0)} > 0$  for a sufficiently small  $t_0$ .

### **Proof of Proposition 4**

We first derive consumer surplus and social welfare in both the selling and non-selling cases. In the

$$\text{selling case, } CS = \int_0^{c_1} U\left(t = \frac{h}{k_W}, G = 0\right) dc + \int_{c_1}^{c_2} U\left(t = \frac{a_L k_L + p k_W - c}{k_L^2}, G = h - \frac{k_W(a_L k_L + p k_W - c)}{k_L^2}\right) dc + \int_{c_2}^{c_3} U(t = t_0, G = h - k_W t_0) dc = \frac{a_W^2 h^2}{8t_0} - \frac{k_L^4 t_0^3}{24} - \frac{3a_L^4 k_W^2 t_0}{32h(h - k_W t_0)} + \frac{a_L^3 k_L(4h + 5k_W t_0)}{24(h - k_W t_0)} + \frac{3a_L^2 k_L^2 t_0(2h + k_W t_0)}{16(h - k_W t_0)}$$

$\frac{3hk_L^3t_0^2(4a_L-k_Lt_0)}{32(h-k_Wt_0)} + \frac{a_W[h(2a_Lk_L+3a_Wk_W-k_L^2t_0)+3a_L^2k_W]}{8}$  and  $SW = CS + \pi$ . In the non-selling case,

$$SW^{BM} = CS^{BM} = \int_0^{c_1^{BM}} U\left(t = \frac{h}{k_W}\right) dc + \int_{c_1^{BM}}^{c_2^{BM}} U\left(t = \frac{a_Lk_L+a_Wk_W-c}{k_L^2}\right) dc + \int_{c_2^{BM}}^{c_3^{BM}} U(t = t_0) dc =$$

$\frac{4a_L^2(a_Lk_L+3a_Wk_W)+12a_W^2hk_W-k_L^4t_0^3}{24}$ . Comparing  $CS$  and  $SW$  to  $CS^{BM}$  and  $SW^{BM}$ , we get  $CS \geq CS^{BM}$  and

$SW \geq SW^{BM}$  for a sufficiently small  $t_0$ .

### **Proof of Corollary 7**

Based on the results in Proposition 4, we derive the first derivative of  $SW$  and  $SW^{BM}$  with respect

to  $h$ ,  $k_L$  and  $k_W$ :

$$\frac{\partial SW}{\partial h} = \frac{3a_Wk_L(2a_L-k_Lt_0)}{8} + \frac{a_W^2(6h-k_Wt_0)}{8t_0} - \frac{k_Wt_0[a_L(2hk_L-a_Lk_W)-hk_L^2t_0][2a_L^2h-a_Lt_0(2hk_L+a_Lk_W)+hk_L^2t_0]}{32h^2(h-k_Wt_0)^2} > 0,$$

$$\frac{\partial SW}{\partial k_L} = (a_L - k_Lt_0) \left[ \frac{18a_Wh+a_L^2+4k_L^2t_0^2}{24} + \frac{3a_L^2h-2a_Lk_Lt_0(h+2k_Wt_0)+3hk_L^2t_0^2}{24(h-k_Wt_0)} \right] > 0, \text{ and}$$

$$\frac{\partial SW}{\partial k_W} = \frac{[2h(a_L^2+a_Wh)-t_0(2ha_Lk_L+2ha_Wk_W+a_L^2k_W)+hk_L^2t_0][2ha_W(h-k_Wt_0)+t_0(2ha_Lk_L-a_L^2k_W-hk_L^2t_0)]}{32h(h-k_Wt_0)^2} > 0 \text{ for a}$$

sufficiently small  $t_0$ . Next we derive the first derivative of  $SW^{BM}$  with respect to  $h$ ,  $k_L$ , and  $k_W$ :

$$\frac{\partial SW^{BM}}{\partial h} = \frac{a_W^2k_W}{2} > 0, \frac{\partial SW^{BM}}{\partial k_L} = \frac{a_L^3-k_L^3t_0^3}{6} > 0 \text{ for a sufficiently small } t_0, \text{ and } \frac{\partial SW^{BM}}{\partial k_W} = \frac{a_W(a_L^2+a_Wh)}{2} > 0.$$