EFFECTS OF COMPETITION AMONG INTERNET SERVICE PROVIDERS AND CONTENT PROVIDERS ON THE NET NEUTRALITY DEBATE

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ABSTRACT

Supporters of net neutrality have often argued that more competition among Internet service providers (ISPs) is beneficial for an open Internet and that the market power of the ISPs lies at the heart of the net neutrality debate. However, the joint effects of the competition among ISPs and among content providers have yet to be examined. We study the critical linkage between ISP competition and content provider competition, as well as its policy implications. We find that even under competitive pressure from a rival ISP, an ISP still has the incentive and the ability to enforce charging content providers for priority delivery of content. Upending the commonly held belief that content providers will always support the preservation of net neutrality, we find that under certain conditions, it is economically beneficial for the dominant content provider to reverse its stance on net neutrality. Our paper also makes an important contribution in extending the traditional two-dimensional spatial-competition literature.

Keywords: Net Neutrality, Internet Service Provider Competition, Content Provider Competition, Packet Discrimination, Social Welfare

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INTRODUCTION

The Federal Communications Commission (FCC)'s path to enforce net neutrality rules to preserve an open Internet has been complicated and controversial. In its 2010 Open Internet Order, the FCC proposed net neutrality rules consisting of four core principles: transparency, no blocking, no unreasonable discrimination, and reasonable network management (FCC 2010). These rules were later struck down by the U.S. Court of Appeals for the District of Columbia Circuit, which assessed that the FCC only has limited regulatory options for broadband as an information service (Nagesh and Sharma 2014). During a five-month period in 2014, to better inform its new rulemaking, the FCC solicited public comments on net neutrality issue and received a total of 3.7 million comments, which makes it the most commented-upon issue in the agency's history.

The recent FCC decision to adopt new net neutrality rules that would allow "commercially reasonable" traffic management has been criticized by several proponents of net neutrality (Snider and Yu 2014; Wyatt 2014). Proponents of net neutrality have long opined that a lack of effective competition in the local broadband market enables Internet service providers (ISPs) to act as gatekeepers of content, and thus to be in a position to charge content providers (CPs) for priority delivery of their data packets[1]. The appeals court made the same argument as part of its ruling: "...if end users could immediately respond to any given broadband provider's attempt to impose restrictions on edge providers by switching broadband providers, this gatekeeper power might well disappear... For example, a broadband provider like Comcast would be unable to threaten Netflix that it would slow Netflix traffic if

[1] This paid prioritization is also referred to as packet discrimination.

all Comcast subscribers would then immediately switch to a competing broadband provider." (U.S. Court of Appeals 2014)

Supporters of net neutrality have often argued that more competition among ISPs is beneficial for an open Internet (Glaser 2014; Dunbar 2014) and that the market power of the ISPs lies at the heart of the net neutrality debate (Mcmillan 2014; Winegarden 2014). They argue that competition among ISPs would prevent them from gaining the market power that currently allows them to charge consumers supra-competitive prices for Internet access services and to potentially charge content providers for preferential delivery services. In fact, there have been several calls to remove the barriers to a competitive broadband market (Szoka et al. 2013). Net neutrality, however, has not been a major concern for countries with more competitive ISP markets, such as Australia and South Korea.

In this paper, we investigate whether the presence of competition in local broadband markets would indeed prevent ISPs from charging online content providers for preferred delivery. Our analysis builds upon the extant research of Choi and Kim (2010) and Cheng et al. (2011), who examined the net neutrality issue for a monopoly ISP. The contribution of our research is to show that even under competitive pressure from a rival ISP, an ISP still has the incentive *and* the ability to enforce charging content providers for priority delivery of content.

The issue of competition among content providers has thus far taken a back seat in the net neutrality debate. Content providers have been among the strongest proponents of net neutrality (The Internet Association 2014). Recent developments, however, show some deviations from this stance. Manjoo (2014) observed that in contrast to the recent grassroots movements over the ongoing discussions regarding net neutrality regulation, the large Internet companies "have not joined online protests, or otherwise moved to mobilize their users in favor of new rules." Google has been questioned for its stance on net neutrality after its entry into the broadband market through Google Fiber (Singel 2013). Two important issues remain unaddressed: to wit, how competitive pressures drive content providers to pay for preferential delivery and how their choices affect the ISPs' incentives to manage their traffic.

We investigate the impact of content provider competition *in the presence of ISP competition*. One question that is of particular interest is whether sufficient market power of the content provider encourages it to abandon its support for net neutrality. We find that a dominating content provider may be *better off* without net neutrality given sufficient market power relative to that of the ISPs. We study the critical linkage between ISP competition and content provider competition, as well as its policy implications. Our findings suggest that the impact of net neutrality regulation on the incentives of content providers depends critically on the market power of the competitors both within the local ISP market and within the content market.

Apart from these policy prescriptions, this paper makes an important contribution in extending the traditional two-dimensional spatial-competition literature. The proposed model captures the relevant characteristics of data transmission in the net neutrality debate. More generally, the modeling framework can be used to analyze the competition between two sets of firms providing complementary products that are consumed together to constitute the total end-user experience (e.g. computer hardware platforms and the software on those platforms).

The paper is organized as follows: In the next section, we review the related literature and discuss the contributions of this paper in that context. We then propose a two-sided market model of Internet data transmission with both ISP competition and content provider competition. Following that, we analyze the outcomes under both net neutrality and packet discrimination regimes, with a focus on the impact of ISP competition and content provider competition. The paper concludes with theoretical, managerial, and policy implications.

LITERATURE REVIEW

In this section, we review the fast-growing literature of economic analysis of net neutrality. For a broader review of the network neutrality literature see Krämer et al. (2013). Studies in network interconnection (Armstrong 1998; Laffont et al. 2003; Tan et al. 2006; Chiang and Jhang-Li 2014) focus on the issues of interconnection settlements among backbone network providers. The issue of net neutrality, however, focuses on “last-mile” ISPs that provide Internet access services to their local consumers.

Existing models make different assumptions about the market structure of Internet data transmission within the last mile and find a variety of results on the key economic outcomes (such as content innovation and social welfare, etc.) in the net neutrality debate.

Hermalin and Katz (2007) find that net neutrality reduces the set of available content and thus leads to lower content innovation. Economides and Tåg (2012) conclude that there are more active content providers under net neutrality, when the value of an additional consumer to the content providers exceeds the value of an additional content provider to the consumers. Krämer and Wiewiorra (2012) model congestion-sensitive content providers with differing congestion-sensitivity distributions and find that content innovation is lower under net neutrality for less congestion-sensitive content providers, and higher for more congestion-sensitive content providers. Guo et al. (2012) find that abandoning the principle of net neutrality can hinder the ability of startups to compete against established rivals and thus reduce innovation at the edge.

The results are mixed when it comes to evaluating the impact of the potential net neutrality regulation on social welfare. Some papers (Cheng et al. 2011; Guo et al. 2012; Krämer and Wiewiorra 2012; Bourreau et al. 2014; Guo and Easley 2014) find that net neutrality results in lower social welfare compared to packet discrimination, while others (Hermalin and Katz 2007; Choi and Kim 2010; Economides and Hermalin 2012; Economides and Tåg 2012) obtain mixed welfare results. Most of the articles agree, however, that if packet discrimination were permitted, the ISP would be better off while content providers would be worse off, due to the ISP's added flexibility in network management.

From the perspective of modeling ISP competition, most extant models consider a monopoly ISP, though Hermalin and Katz (2007) and Economides and Tåg (2012) extend their models to consider the effects of ISP competition and in either case find results similar to the monopoly case. Bourreau et al. (2014) extend the model proposed in Krämer and Wiewiorra (2012) to allow ISP competition. They find that the packet discrimination regime results in higher infrastructure investment, more content innovation, and higher overall social welfare. The ISPs, however, may be worse off under the packet discrimination regime, due to intensified competition in the consumer market. Bykowsky and Sharkey (2014) study the welfare effects of the zero-price rule under various conditions of the ISPs' market power. They find that the zero-price rule is welfare enhancing if and only if the ISP's ability to establish competitive prices for the content providers exceeds its ability to establish such prices for consumers.

From the perspective of modeling content provider competition, most authors assume that content providers are the sole providers of their own content and do not compete for consumers. As a result, these models are unable to capture the competitive pressure on the content providers to pay for premium delivery service under a packet discrimination regime. Two exceptions are Choi and Kim (2010) and Cheng et al. (2011), where two content providers compete for consumers but the content delivery service is provided by a monopolist ISP.

To the best of our knowledge, the joint effects of the competition among content providers and among ISPs have yet to be examined. This paper fills the research gap by studying the linkage between the two markets. Given the different roles of content providers and network providers in the “last mile” Internet access market, it is important to investigate the impact of the competition among both these types of players on the outcomes. If one of the two markets is modeled as a monopoly, then that monopolist has greater market power to influence the outcomes of the other market. This is different from what is observed in reality, where the effects of competition in one market mediate the effects of competition in the other market.

This paper also makes a theoretical contribution to the literature of two-dimensional spatial competition with horizontal differentiation in both dimensions. Tabuchi (1994) considers a two-dimensional setting and finds that maximum differentiation arises along one dimension, while minimum differentiation arises along the other dimension. Irmen and Thisse (1998) and Ansari et al. (1998) extend this model to multiple horizontal dimensions and find that firms maximize differentiation on one dimension while minimizing differentiation along all others. von Ehrlich and Greiner (2013) adapt two-dimensional spatial-competition models to the context of media markets and analyze two media outlets providing both online and offline platforms. They find that maximum differentiation may occur in both dimensions.

This paper deviates from the traditional two-dimensional spatial-competition models in three ways. First, we analyze two interrelated markets with complementary products (the markets of Internet access services and digital content), as opposed to two product attributes in the same market, which has been considered in prior studies. Specifically, our two dimensions represent two different duopoly markets. Consequently, consumers choose among four product combinations. Second, we allow for different unit fit costs for the two different markets of Internet access services and digital content. In other words, the two markets differ in terms of competition intensity. Third, we consider the ISPs’ pricing decisions and CPs’ content delivery service decisions. As a result, the outcome of each product combination is jointly determined by firms from both the ISP market and the content market. These three features are key characteristics of the Internet data-transmission process, which is crucial to modeling the net neutrality issue.

MODEL

We consider two competing ISPs, C and D, providing Internet access services to a unit mass of consumers and content delivery services to two competing content providers (CPs) Y and G (as shown in Figure 1).

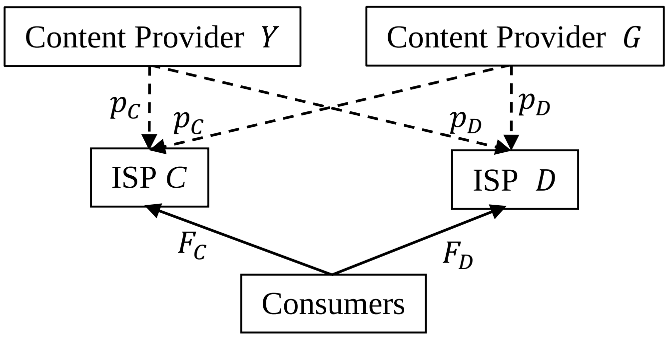


Figure 1: Market Structure

Figure 1 also demonstrates payments the ISPs receive from consumers and potentially from content providers. In the net neutrality regime, the ISPs charge consumers fixed fees F_C and F_D respectively for Internet access and this is their only revenue source. In the packet discrimination regime, in addition to fixed fees (F_C and F_D) from consumers, the ISPs may also charge the CPs usage-based fees p_C and p_D respectively for preferential delivery of their content. In other words, the ISPs have two revenue sources in the packet discrimination regime – Internet access fees from consumers and preferential delivery fees from content providers. Table 1 provides a list of all notations.

Table 1: List of Notations

Subscript a	ISP $a = C$ or D
Subscript b	CP $b = Y$ or G
Subscript i	Outcome $i = 1,2,3,4$ representing CPs' content delivery choices on ISP C
Subscript j	Outcome $j = 1,2,3,4$ representing CPs' content delivery choices on ISP D
F_{aij}	Fixed fee ISP $a = C$ or D changes consumers for Internet access service in outcome ij
p_{aij}	Usage-based fee ISP $a = C$ or D changes CPs for preferential delivery in outcome ij
V	Consumers' gross valuation for each ISP-CP combination
I_{ab}	Indicator function to represent whether CP b pays ISP a for preferential delivery
$u_{abij}(x, z)$	Utility of the ISP-CP combination ab for consumer (x, z) in outcome ij
N_{abij}	Market share of the ISP-CP combination ab in outcome ij
N_{aij}	Market share of ISP $a = C$ or D in outcome ij and $N_{aij} = N_{aYij} + N_{aGij}$
N_{bij}	Market share of CP $b = Y$ or G in outcome ij and $N_{bij} = N_{Cbij} + N_{Dbij}$
t	Unit fit cost for content
k	Unit fit cost for Internet access service
r_b	Revenue rate per packet of CP $b = Y$ or G
d	Consumers' unit delay cost (congestion cost)
w_{abij}	Expected waiting time (expected delay) for consumers who choose ISP a and CP b in outcome ij
λ	Poisson arrival rate of content requested by each consumer, expressed in packets per unit of time.
μ	Capacity that the ISPs provide to the consumers, expressed in packets per unit of time
π_{aij}	Profit of ISP $a = C$ or D
π_{bij}	Profit of CP $b = Y$ or G
CS_{ij}	Consumer surplus of outcome ij
SW_{ij}	Social welfare of outcome ij

We use a two-dimensional spatial-competition model to capture consumers' heterogeneous preferences for both content and Internet access service, and firm competition in these two markets. Specifically, consumers, who are characterized by their preference for content (x) and preference for Internet access service (z), are uniformly distributed on the unit square. Without loss of generality, we assume that the horizontal axis represents consumers' preference for content with (CP Y located at 0 and CP G located at 1); the vertical axis represents consumers' preference for Internet access service (ISP C located at 0 and ISP D located at 1). Consumers have four choices (in terms of the four ISP-CP combinations), represented by the four corners of the unit square.

Let V be consumers' gross valuation for each ISP-CP combination. We denote the unit fit cost for content and Internet access service as t and $k,$ respectively, and use the weighted box topology distance measure to calculate the fit cost in the consumers' utility functions. As shown in Figure 2, when choosing the ISP-CP combination of CY, the consumer located at (x, z) incurs a fit cost for content of tx and a fit cost for Internet access service of kz. Similarly, consumer (x, z)'s fit costs for the other three ISP-CP combinations can be calculated by multiplying the unit fit cost (t or k) and the corresponding (horizontal or vertical) distance between the consumer and the ISP-CP combination.

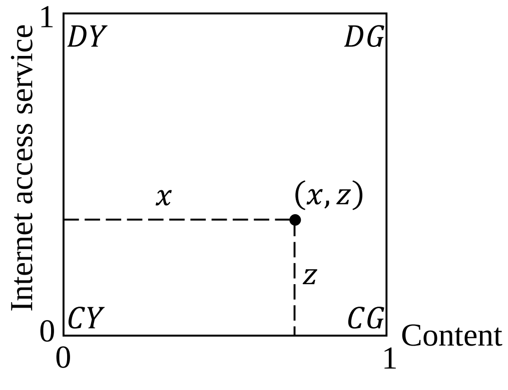


Figure 2: Consumer Choice

For ISP $a = C$ or D, CPs may decide whether to pay for preferential delivery of their content. The content delivery service choice of CP $b = Y$ or G can be represented by an indicator function I_{ab}, where $I_{ab} = 1$, if CP b pays ISP a for preferential delivery; $I_{ab} =$

0, otherwise. There are four outcomes for each ISP based on the two CPs' delivery service choices: outcome 1 (neither CP pays for preferential delivery), outcome 2 (only Y pays), outcome 3 (only G pays), outcome 4 (both CPs pay). Thus, there are 16 outcomes in all, based on the CPs' delivery service choices for the two ISPs, represented by outcome ij, with $i, j = 1,2,3,4$. Outcome ij is dictated by the CPs' delivery service choices for the two ISPs, i.e., $(I_{CYij}, I_{CGij}, I_{DYij}, I_{DGij})$. For example, in outcome 43, both CPs pay ISP C for preferential delivery (i.e., outcome 4 occurs on ISP C), but only G pays ISP D for preferential delivery (i.e., outcome 3 occurs on ISP D). In other words, $I_{CY43} = 1$, $I_{CG43} = 1$, $I_{DY43} = 0$, and $I_{DG43} = 1$.

Following prior work (Choi and Kim 2010; Cheng et al. 2011; Krämer and Wiewiorra 2012), we model the content delivery systems offered by the ISPs as M/M/1 queuing systems under net neutrality and two-class priority M/M/1 queuing systems under packet discrimination. In the packet discrimination regime, if only one CP pays for preferential delivery, then its data packets will be transmitted with higher priority compared to data packets from the other CP. However, if both CPs pay for preferential delivery, then all data packets will be treated the same.

We denote the expected waiting time (expected delay) for consumers that choose ISP a and CP b in outcome ij as w_{abij}. Let μ denote the capacity of the ISPs and λ denote the consumers' rate of requests for content. Table 2 presents the delays of the 16 outcomes under packet discrimination. Note that outcome 11, where neither CP pays for preferential delivery even though they have the option to do so, is essentially equivalent to the net neutrality regime.

Table 2: Delays under Packet Discrimination

Delays	Neither pays D	Y pays D	G pays D	Both pay D
Neither pays C	$w_{CY11} = w_{CG11} = w_{DY11} = w_{DG11} = \frac{1}{\mu - N_{CY11}\lambda}$	$w_{CY12} = w_{CG12} = \frac{1}{\mu - N_{C12}\lambda}$ $w_{DY12} = \frac{1}{\mu - N_{DY12}\lambda}$ $w_{DG12} = \frac{\mu}{(\mu - N_{DY12}\lambda)(\mu - N_{D12}\lambda)}$	$w_{CY13} = w_{CG13} = \frac{1}{\mu - N_{C13}\lambda}$ $w_{DY13} = \frac{\mu}{(\mu - N_{DG13}\lambda)(\mu - N_{D13}\lambda)}$ $w_{DG13} = \frac{1}{\mu - N_{DG13}\lambda}$	$w_{CY14} = w_{CG14} = w_{DY14} = w_{DG14} = \frac{1}{\mu - N_{CY14}\lambda}$
Y pays C	$w_{CY21} = \frac{1}{\mu - N_{CY21}\lambda}$ $w_{CG21} = \frac{\mu}{(\mu - N_{CY21}\lambda)(\mu - N_{C21}\lambda)}$ $w_{DY21} = w_{DG21} = \frac{1}{\mu - N_{D21}\lambda}$	$w_{CY22} = w_{DY22} = \frac{1}{\mu - N_{CY22}\lambda}$ $w_{CG22} = w_{DG22} = \frac{\mu}{(\mu - N_{CY22}\lambda)(\mu - N_{C22}\lambda)}$	$w_{CY23} = \frac{1}{\mu - N_{CY23}\lambda}$ $w_{CG23} = \frac{\mu}{(\mu - N_{CY23}\lambda)(\mu - N_{C23}\lambda)}$ $w_{DY23} = \frac{\mu}{(\mu - N_{DG23}\lambda)(\mu - N_{D23}\lambda)}$ $w_{DG23} = \frac{1}{\mu - N_{DG23}\lambda}$	$w_{CY24} = \frac{1}{\mu - N_{CY24}\lambda}$ $w_{CG24} = \frac{\mu}{(\mu - N_{CY24}\lambda)(\mu - N_{C24}\lambda)}$ $w_{DY24} = w_{DG24} = \frac{1}{\mu - N_{D24}\lambda}$
G pays C	$w_{CY31} = \frac{\mu}{(\mu - N_{CG31}\lambda)(\mu - N_{C31}\lambda)}$ $w_{CG31} = \frac{1}{\mu - N_{CG31}\lambda}$ $w_{DY31} = w_{DG31} = \frac{1}{\mu - N_{D31}\lambda}$	$w_{CY32} = \frac{\mu}{(\mu - N_{CG32}\lambda)(\mu - N_{C32}\lambda)}$ $w_{CG32} = \frac{1}{\mu - N_{CG32}\lambda}$ $w_{DY32} = \frac{1}{\mu - N_{DY32}\lambda}$ $w_{DG32} = \frac{\mu}{(\mu - N_{DY32}\lambda)(\mu - N_{D32}\lambda)}$	$w_{CY33} = w_{DY33} = \frac{\mu}{(\mu - N_{CG33}\lambda)(\mu - N_{C33}\lambda)}$ $w_{CG33} = w_{DG33} = \frac{1}{\mu - N_{CG33}\lambda}$	$w_{CY34} = \frac{\mu}{(\mu - N_{CG34}\lambda)(\mu - N_{C34}\lambda)}$ $w_{CG34} = \frac{1}{\mu - N_{CG34}\lambda}$ $w_{DY34} = w_{DG34} = \frac{1}{\mu - N_{D34}\lambda}$
Both pay C	$w_{CY41} = w_{CG41} = w_{DY41} = w_{DG41} = \frac{1}{\mu - N_{CY41}\lambda}$	$w_{CY42} = w_{CG42} = \frac{1}{\mu - N_{C42}\lambda}$ $w_{DY42} = \frac{1}{\mu - N_{DY42}\lambda}$ $w_{DG42} = \frac{\mu}{(\mu - N_{DY42}\lambda)(\mu - N_{D42}\lambda)}$	$w_{CY43} = w_{CG43} = \frac{1}{\mu - N_{C43}\lambda}$ $w_{DY43} = \frac{\mu}{(\mu - N_{DG43}\lambda)(\mu - N_{D43}\lambda)}$ $w_{DG43} = \frac{1}{\mu - N_{DG43}\lambda}$	$w_{CY44} = w_{CG44} = w_{DY44} = w_{DG44} = \frac{1}{\mu - N_{CY44}\lambda}$

In summary, the consumers' utility functions for the four ISP-CP combinations are:

$$u_{CYij}(x,z) = V - tx - kz - d\lambda w_{CYij} - F_{Cij}$$

$$u_{CGij}(x,z) = V - t(1-x) - kz - d\lambda w_{CGij} - F_{Cij}$$

$$u_{DYij}(x,z) = V - tx - k(1-z) - d\lambda w_{DYij} - F_{Dij}$$

$$u_{DGij}(x,z) = V - t(1-x) - k(1-z) - d\lambda w_{CYij} - F_{Dij}$$

where d represents the consumers' unit delay cost (congestion cost).

Each consumer compares the four ISP-CP combinations and chooses the option that yields the highest utility. We denote the market share of the ISP-CP combination ab in outcome ij as N_{abij}. For example, N_{DG43} represents the market share of DG when both CPs pay ISP C but only G pays ISP D.

CPs decide whether to pay ISP C, ISP D, or both. The profit of CP $b = Y$ or G is:

$$\pi_{bij} = \left(N_{Cbij} + N_{Dbij}\right)\lambda r_b - I_{Cbij} N_{Cbij} \lambda p_{Cij} - I_{Dbij} N_{Dbij} \lambda p_{Dij}$$

where r_b is the revenue rate per packet of CP b. The CPs' profit is their revenue generated from consumers served by both ISPs, net of their payments to the ISPs for preferential content delivery. Without loss of generality, we assume that $r_G \geq r_Y$, or, CP G is more effective in generating revenue from its customer base than is CP Y.

ISPs make their pricing decisions. ISP $a = C$ or D selects its prices F_{aij} and p_{aij} to maximize its profit π_{aij}. The decision problem of ISP a can be formulated as:

$$\max_{F_{aij}, p_{aij}} \pi_{aij} = \left(N_{aYij} + N_{aGij}\right) F_{aij} + \left(I_{aYij} N_{aYij} + I_{aGij} N_{aGij}\right)\lambda p_{aij}$$

$$\text{subject to } U_{ij}(x,z) = \max\left\{u_{CYij}(x,z), u_{CGij}(x,z), u_{DYij}(x,z), u_{DGij}(x,z)\right\} \geq 0$$

$$\pi_{Yij} \geq \pi_{Yi_1j_1}, \pi_{Yi_2j_2}, \pi_{Yi_3j_3}$$

$$\pi_{Gij} \geq \pi_{Gi_4j_4}, \pi_{Gi_5j_5}, \pi_{Gi_6j_6}.$$

The ISPs' profit consists of two components: payment from consumers for Internet access service and payment from CPs for preferential content delivery. The first constraint is consumers' participation constraint. Consumers choose the ISP-CP combination that yields the highest net utility, and consumers' participation constraint ensures that all consumers adopt the Internet access service. This full-market-coverage assumption is commonly made in the literature (Choi and Kim 2010; Guo et al. 2010; Cheng et al. 2011). The last two constraints are the CPs' incentive compatibility constraints. In response to the prices (F_{aij} and p_{aij}) set by the ISPs, the CPs choose whether to pay for preferential delivery and their choices of content delivery services determine the outcomes ij for both ISPs.

The CPs' incentive compatibility constraints ensure that neither CP has any incentive to deviate from outcome ij. In other words, given the other CP's delivery service choice, the profit for a CP in outcome ij needs to be higher than that with its alternative choices. Outcomes i_1j_1, i_2j_2, i_3j_3 represent Y's alternative choices and i_4j_4, i_5j_5, i_6j_6 represent G's alternative choices. For example, for outcome 43 (both CPs pay ISP C and only G pays ISP D) to be an equilibrium, there are three conditions for each CP's incentive compatibility constraint. Given that G pays both ISPs in outcome 43, Y has three options: (1) not pay either ISP (outcome 33); (2) pay both ISPs (outcome 44); or (3) not pay ISP C but pay ISP D (outcome 34). Similarly, Given Y's payment choice in outcome 43 (it pays C but not D), G also has three options: (1) not pay C but pay D (outcome 23); (2) pay C but not pay D (outcome 41); or (3) not pay either ISP (outcome 21). Formally, $i_1j_1, \cdots, i_6j_6$ are defined by the following CPs' decisions: $i_1j_1 \leftrightarrow \left(1 - I_{CYij}, I_{CGij}, I_{DYij}, I_{DGij}\right)$, $i_2j_2 \leftrightarrow \left(I_{CYij}, I_{CGij}, 1 - I_{DYij}, I_{DGij}\right)$, $i_3j_3 \leftrightarrow \left(1 - I_{CYij}, I_{CGij}, 1 - I_{DYij}, I_{DGij}\right)$, $i_4j_4 \leftrightarrow \left(I_{CYij}, 1 - I_{CGij}, I_{DYij}, I_{DGij}\right)$, $i_5j_5 \leftrightarrow \left(I_{CYij}, I_{CGij}, I_{DYij}, 1 - I_{DGij}\right)$, and $i_6j_6 \leftrightarrow \left(I_{CYij}, 1 - I_{CGij}, I_{DYij}, 1 - I_{DGij}\right)$. The details of the CPs' incentive compatibility constraints are presented in Table 3.

Table 3: CPs' Incentive Compatibility Constraints

IC constraints	Neither pays D	Y pays D	G pays D	Both pay D
Neither pays C	$\pi_{Y11} \geq \pi_{Y21}, \pi_{Y12}, \pi_{Y22}$ $\pi_{G11} \geq \pi_{G31}, \pi_{G13}, \pi_{G33}$	$\pi_{Y12} \geq \pi_{Y22}, \pi_{Y11}, \pi_{Y21}$ $\pi_{G12} \geq \pi_{G32}, \pi_{G14}, \pi_{G34}$	$\pi_{Y13} \geq \pi_{Y23}, \pi_{Y14}, \pi_{Y24}$ $\pi_{G13} \geq \pi_{G33}, \pi_{G11}, \pi_{G31}$	$\pi_{Y14} \geq \pi_{Y24}, \pi_{Y13}, \pi_{Y23}$ $\pi_{G14} \geq \pi_{G34}, \pi_{G12}, \pi_{G32}$
Y pays C	$\pi_{Y21} \geq \pi_{Y11}, \pi_{Y22}, \pi_{Y12}$ $\pi_{G21} \geq \pi_{G41}, \pi_{G23}, \pi_{G43}$	$\pi_{Y22} \geq \pi_{Y12}, \pi_{Y21}, \pi_{Y11}$ $\pi_{G22} \geq \pi_{G42}, \pi_{G24}, \pi_{G44}$	$\pi_{Y23} \geq \pi_{Y13}, \pi_{Y24}, \pi_{Y14}$ $\pi_{G23} \geq \pi_{G43}, \pi_{G21}, \pi_{G41}$	$\pi_{Y24} \geq \pi_{Y14}, \pi_{Y23}, \pi_{Y13}$ $\pi_{G24} \geq \pi_{G44}, \pi_{G22}, \pi_{G42}$
G pays C	$\pi_{Y31} \geq \pi_{Y41}, \pi_{Y32}, \pi_{Y42}$ $\pi_{G31} \geq \pi_{G11}, \pi_{G33}, \pi_{G13}$	$\pi_{Y32} \geq \pi_{Y42}, \pi_{Y31}, \pi_{Y41}$ $\pi_{G32} \geq \pi_{G12}, \pi_{G34}, \pi_{G14}$	$\pi_{Y33} \geq \pi_{Y43}, \pi_{Y34}, \pi_{Y44}$ $\pi_{G33} \geq \pi_{G13}, \pi_{G31}, \pi_{G11}$	$\pi_{Y34} \geq \pi_{Y44}, \pi_{Y33}, \pi_{Y43}$ $\pi_{G34} \geq \pi_{G14}, \pi_{G32}, \pi_{G12}$
Both pay C	$\pi_{Y41} \geq \pi_{Y31}, \pi_{Y42}, \pi_{Y32}$ $\pi_{G41} \geq \pi_{G21}, \pi_{G43}, \pi_{G23}$	$\pi_{Y42} \geq \pi_{Y32}, \pi_{Y41}, \pi_{Y31}$ $\pi_{G42} \geq \pi_{G22}, \pi_{G44}, \pi_{G24}$	$\pi_{Y43} \geq \pi_{Y33}, \pi_{Y44}, \pi_{Y34}$ $\pi_{G43} \geq \pi_{G23}, \pi_{G41}, \pi_{G21}$	$\pi_{Y44} \geq \pi_{Y34}, \pi_{Y43}, \pi_{Y33}$ $\pi_{G44} \geq \pi_{G24}, \pi_{G42}, \pi_{G22}$

The sequence of decisions is as follows: In stage 1, ISPs $a = C$ or D set the fixed fee F_a for end consumers and potentially a priority price of p_a per packet. In stage 2, CPs $b = Y$ or G decides whether to pay or not pay the ISPs C and D for preferential delivery of their content. In stage 3, consumers choose their preferred CP and ISP. In the following section, we solve for the symmetric equilibria in this game.

ANALYSIS OF NET NEUTRALITY AND PACKET DISCRIMINATION REGIMES

Consumers compare the four ISP-CP options and choose the option which yields the highest utility. In order to derive the market shares for each option, we need to calculate the six indifference curves based on the pairwise comparisons among them. Figure 3 demonstrates the various possibilities of how consumers are distributed among the four options. Solving for the exact market shares under each ISP-CP combination is difficult, since (1) there is a surfeit of parameter values; (2) we model the packet discrimination system as a priority queue, so the waiting times are a rational function of all the parameters; and (3) several of the variables are implicitly interrelated (for example, the waiting times are functions of consumer demand, which is dependent on the indifferent curves, that are in turn determined by the waiting times, and in general there is no closed-form solution for these implicit relations). Thus, the market shares under each ISP-CP option could look very different; see the dotted lines in Figure 3 as a possible example.

In order to solve this problem, we utilize the symmetry of the corresponding market shares when the content providers interchange their decisions on prioritizing their content delivery. For example, if the decisions of Y and G are simultaneously changed for ISPs C and D, the market shares under each ISP-CP option "flips". Depending on the shape of the "flipped" market shares, there are two types of flips – "horizontal" or "vertical". Interchanging the business decisions of the CPs (vertical and horizontal flips) induces what is called a $\mathbb{Z}_2 \times \mathbb{Z}_2$ (or the Klein 4-group) action in group theory. We use the symmetry associated with this group action to solve for the demand distributions. The details of this symmetry analysis using vertical and horizontal flips can be found in the appendix. The results of the consumer demand patterns are summarized in Lemma 1. All proofs are relegated to the appendix.

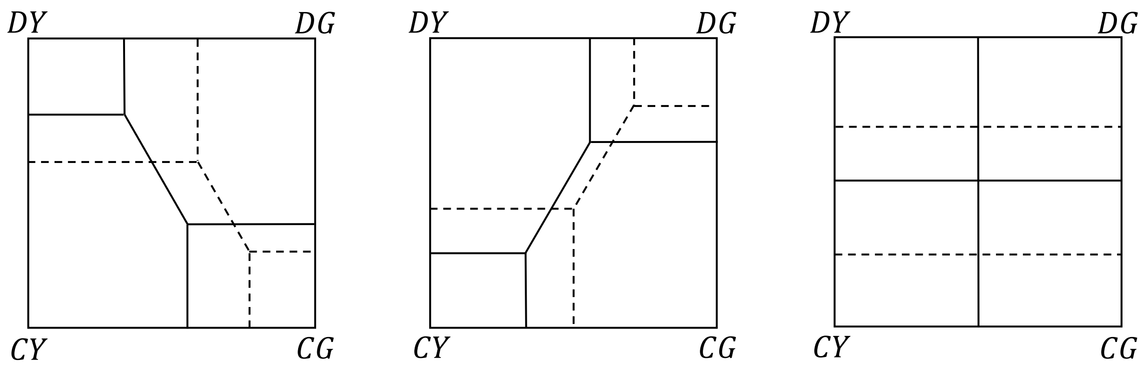


Figure 3: General Demand Distribution

Lemma 1 (Consumer Demand Patterns): *Depending on the ISPs' pricing decisions and the content providers' delivery service choices, there are 16 possible outcomes. These outcomes can be grouped into four classes (a through d) with similar consumer demand patterns.*

Figure 4 portrays the results in Lemma 1. For outcomes $ij = 11{,}14, 41{,}44$ in class a, all data packets receive equal priority and thus consumers incur the same congestion cost. As shown in Figure 4a, the four ISP-CP combinations split the consumer market equally, i.e., $N_{CYij} = N_{CGij} = N_{DYij} = N_{DGij} = 1/4$.

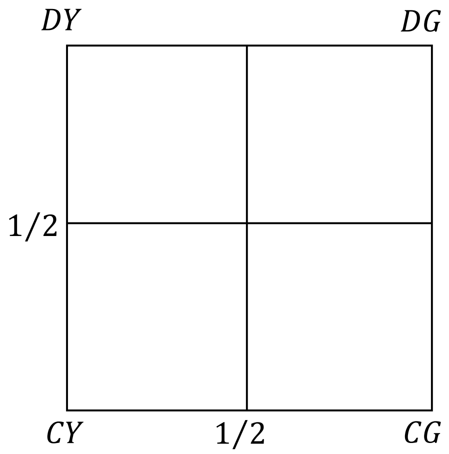


Figure 4a: Demand Distribution of Class a (outcomes 11, 14, 41, and 44)

For outcomes $ij = 22{,}33$ in class b, the CPs make the same delivery service choices with either ISP. As a result, ISPs C and D have the same market share, i.e., $N_{Cij} = N_{Dij} = 1/2$. Within each ISP, the paying CP gets more customers than the non-paying CP, i.e., $N_{CY22} = N_{DY22} = N_{CG33} = N_{DG33} \geq 1/4 \geq N_{CY33} = N_{DY33} = N_{CG22} = N_{DG22}$.

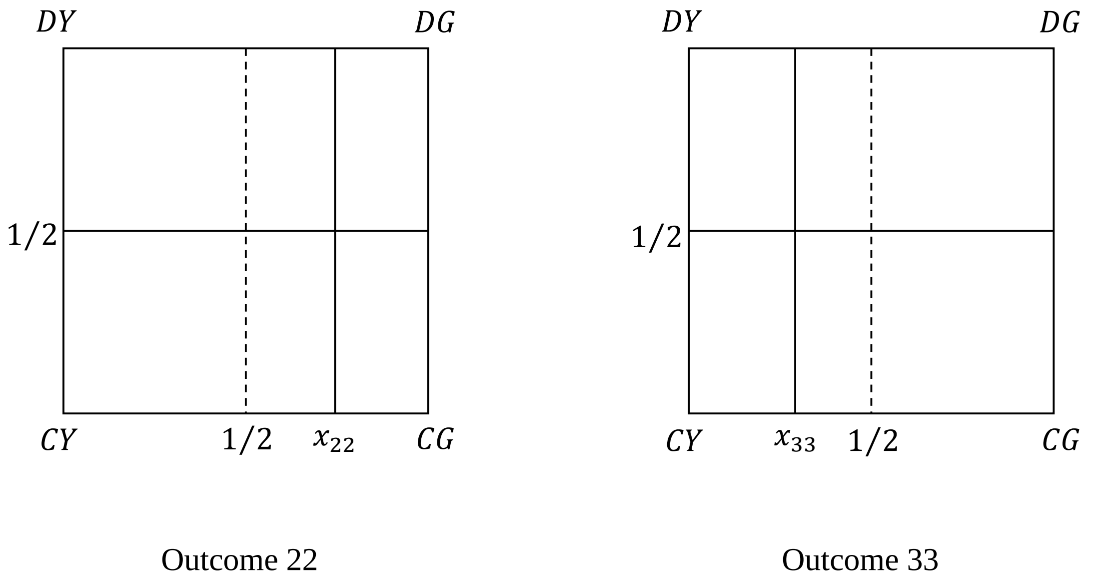


Figure 4b: Demand Distribution of Class b (outcomes 22 and 33)

For outcomes $ij = 23{,}32$ in class c, the CPs make the opposite delivery service choices with the two ISPs, leading to only one CP paying for the preferential delivery on each ISP. As a result, ISPs C and D have the same market share, i.e., $N_{Cij} = N_{Dij} = 1/2$. Within each ISP, the paying CP gets more customers than the non-paying CP, i.e., $N_{CY23} = N_{DG23} = N_{CG32} = N_{DY32} \geq 1/4 \geq N_{CY32} = N_{DG32} = N_{CG23} = N_{DY23}$.

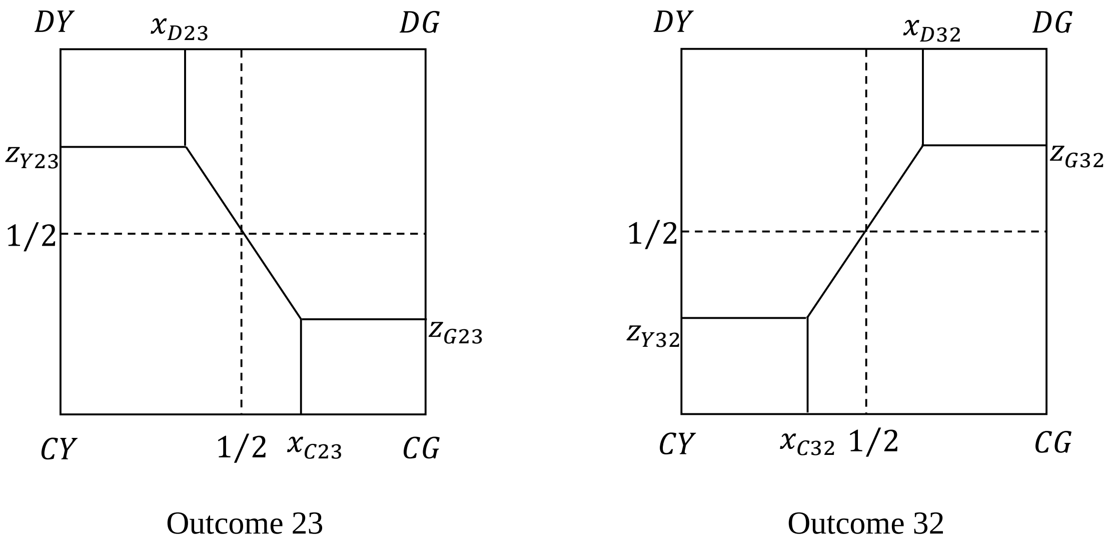
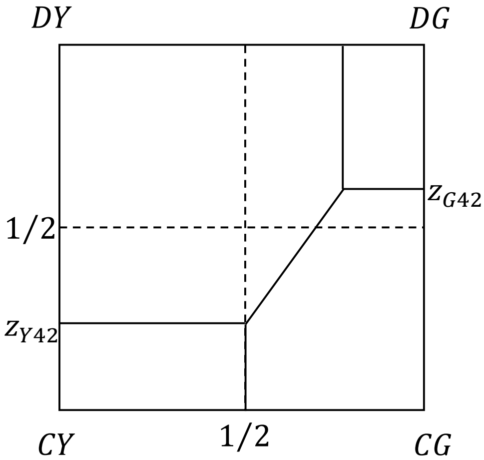
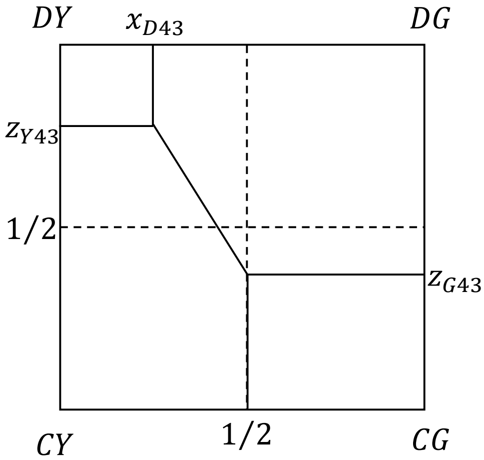


Figure 4c: Demand Distribution of Class c (outcomes 23 and 32)

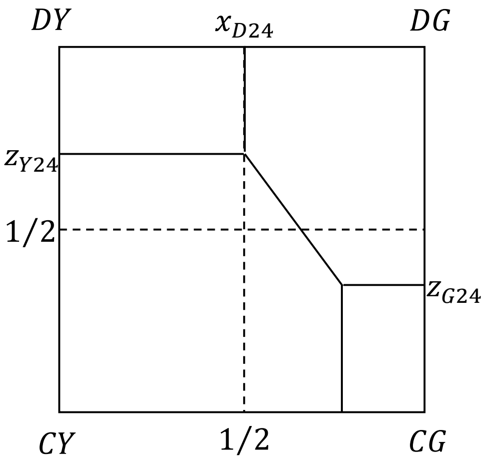
For outcomes $ij = 12,13,21,31,42,43,24,34$ in class d, $N_{Dij} \geq 1/2 \geq N_{Cij}$ for outcomes $ij = 12,13,42,43$ and $N_{Cij} \geq 1/2 \geq N_{Dij}$ for outcomes $ij = 21,31,24,34$. Outcomes in class d reveal particularly interesting demand patterns. For example, in outcome 43, although both CPs pay for preferential delivery on ISP C, Y gets fewer consumers than G from ISP C, i.e., $N_{CY43} > N_{CG43}$.



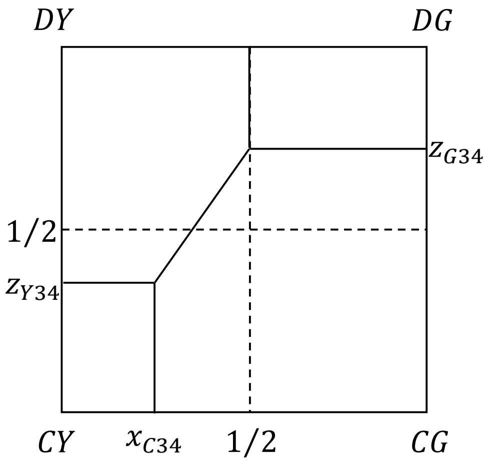
Outcomes 12 and 42



Outcomes 13 and 43



Outcomes 21 and 24



Outcomes 31 and 34

Figure 4d: Demand Distribution of Class d

(outcomes 12, 13, 21, 31, 42, 43, 24, and 34)

Lemma 2 (ISPs' Strategy): *Depending on market conditions, there are three possible symmetric equilibrium pricing strategies (i.e. the* F *and* p *choices) for the ISPs: (a) when* $r_G \geq max\{\alpha_1, \beta_1 r_Y, \alpha_2 + \beta_2 r_Y\}$*, the equilibrium is outcome 33, where only content provider G pays for priority delivery for its customers on both ISPs; (b) when* $\beta_3 r_Y \leq r_G < \alpha_1$ *and* $r_Y \leq \alpha_3$*, the equilibrium is outcome 43 (and equivalently, equilibrium 34), where content provider* G *pays for priority delivery for its customers on both ISPs, while Y pays for priority delivery for its customers with only one ISP*[2]*; and (c) otherwise, the equilibrium is outcome 44, where both content providers pay for priority delivery for their customers on both ISPs.*

We diagrammatically show the results of Lemma 2 in Figure 5a. The equilibrium results assuming $r_G \geq r_Y$ can be easily generalized to the case when $r_G < r_Y$. Figure 5b shows the equilibrium outcomes when the assumption of $r_G \geq r_Y$ is relaxed.

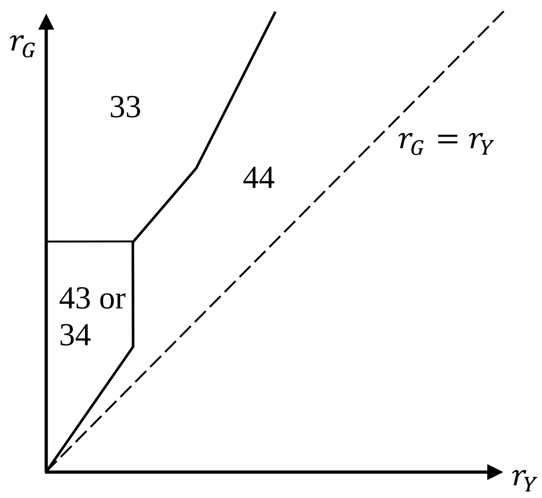


Figure 5a: Equilibrium outcomes when $r_G \geq r_Y$

[2] This equilibrium is equivalent to the one where G pays for priority delivery to one ISP while Y pays for priority delivery to both ISPs, in the sense that the profits of the content providers is the same in either case.

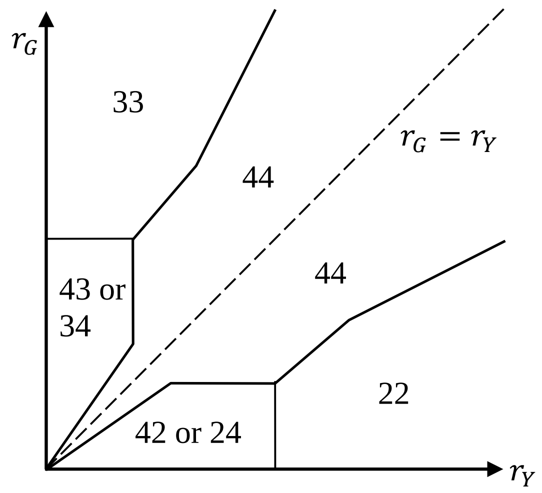


Figure 5b: Equilibrium outcomes when the assumption $r_G \geq r_Y$ is relaxed

Figure 5: Equilibrium Outcomes

In general, ISPs will charge higher prices to content providers when only one pays for priority delivery than when both content providers pay for priority delivery. Consumers, however, are charged less, indicating that as the revenue generation rate r_G increases, the relative contribution to ISPs' profit gradually switches from consumers to the content providers. This leads to our first proposition.

Proposition 1 (Competing ISPs still have an incentive to deviate from net neutrality): *The competing ISPs' profit is weakly higher under packet discrimination than under net neutrality, i.e.,* $\pi_C^{PD} = \pi_D^{PD} \geq \pi_C^{NN} = \pi_D^{NN}$.

Proposition 1 shows that the competing ISPs are always better off under packet discrimination even in the presence of ISP competition and thus have the incentive to deviate from net neutrality. In other words, when it comes to the net neutrality debate, ISPs will prefer abolishing net neutrality, even in the presence of ISP competition. This is a result that has been shown to be true when the ISP is a monopoly and it continues to hold with ISP competition. This finding is different from that of Bourreau et al. (2014), where the authors do not model the competition between content providers, and as a result, the effect of competition faced by the ISPs is exaggerated. Our analysis provides an explanation for the public stance of the Internet service providers in the ongoing net neutrality debate, as they continue to lobby for abolishing net neutrality. This result mirrors the results of Gans (2014), even though he does not explicitly model the effects of ISP competition.

Content providers, however, have supported the preservation of net neutrality. Would they continue to support net neutrality when there is competition between ISPs? Our next result (Proposition 2) shows that under certain conditions, it is economically beneficial for the dominant content provider to reverse its stance on net neutrality. In other words, some content providers might be better off when net neutrality is abolished in the presence of competition between ISPs. This is a crucial and, in some ways, a surprising result. Ever since the issue of net neutrality has been publicly debated, prominent content providers like Google, Yahoo!, Microsoft, Netflix and others have publicly supported net neutrality, and so far, the economic analyses have mirrored the public stances of the various parties in the debate: content providers are worse off when net neutrality is abolished, while the Internet service providers are better off. Proposition 2 shows that under ISP competition, the support for net neutrality from content providers – and more specifically, from the dominant content providers – might not be so forthcoming.

Proposition 2 (CP G may be better off under packet discrimination): *When CP G is sufficiently dominant, its profit is higher under packet discrimination (corresponds to equilibrium 33) than that under net neutrality if the intensity of competition in the ISP market relative to that in the CP market is greater than a threshold. Formally,* $\pi_G^{PD} = \pi_{G33}^* > \pi_G^{NN}$ *if the ratio of* t/k *is higher than a threshold.*

Proposition 2 shows that under certain conditions, CP G might in fact do better than it could under net neutrality. When r_G and r_Y are similar and the corresponding equilibrium outcome is 44 (both CPs decide to pay the two ISPs), CP G is definitely worse off under packet discrimination than under net neutrality. As r_G becomes larger relative to r_Y, however, and the equilibrium shifts to outcome 33 (where only G prefers to pay the two ISPs), G's profit is at least as great as that under net neutrality. Specifically, the comparison result of G's profit depends on the relative magnitude of the intensity of competition in the ISP and CP markets. We can think of the unit fit costs t and k as the strengths of the consumer loyalties within the CP market and within the ISP market, respectively. A more differentiated market corresponds to a higher level of consumer loyalty. Thus, t and k can be interpreted as the reverse measures of the intensity of competition in the two markets, and the ratio of t/k measures the relative magnitude of the intensity of competition in the ISP market to that in the CP market. When the intensity of competition between the content providers is relatively low compared to that between the ISPs, such that t/k is greater than a threshold, the more efficient CP is actually better off under packet discrimination. In practice, it can be argued that the consumer loyalty in the CP market is relatively high compared to that in the ISP market, because digital content is more differentiated than Internet access service, which raises the possibility that this condition is likely to hold within markets for certain types of digital content.

This result is extremely significant. In the presence of ISP competition, a dominant content provider (in our case, G, with r_G being relatively large compared to r_Y) might no longer be concerned with preserving net neutrality. In fact, G might actually eke out a higher profit under packet discrimination, since its payments to the ISP for priority delivery might pay off in terms of the additional revenue garnered from consumers that have switched from a rival CP.

In the net neutrality debate thus far, content providers have generally been supportive of net neutrality. This stance has been vindicated in the literature (Choi and Kim 2010; Cheng et al. 2011), where it has been shown that content providers are never better off when ISPs deviate from net neutrality. Those results, however, have been derived under the assumption that there is no competition among ISPs. Indeed, a monopoly ISP can extract all the rent (and often more) from a content provider that gains market share by paying for priority delivery. But when there is competition among ISPs, while they do increase their profits by deviating from net neutrality, they cannot extract the entire surplus from the content provider that decides to pay for priority delivery of its content. The effect of competition moderates an ISP's ability to extract the surplus from the CPs, and in such situations, a dominant content provider can indeed be better off when ISPs deviate from net neutrality. In such cases (for example, in the mobile broadband market where there is effective ISP competition), we can expect the dominant content providers to be less supportive of the need for net neutrality.

In fact, such a shift in stance might already be underway (Manjoo 2014). He observed that "Large Internet businesses have written a few letters to regulators in support of the issue and have participated in the back-channel lobbying effort, but they have not joined online protests, or otherwise moved to mobilize their users in favor of new rules." He further goes on to speculate on the reason why the large Internet businesses have taken such a stance: "They may be too big to bother with an issue that primarily affects the smallest Internet companies" and that they "would escape relatively unscathed" by the paid prioritization. Our research shows that not only would some of these large Internet companies escape "relatively unscathed" from paid prioritization, they might actually *prosper* from such an arrangement.

Proposition 3 (CP Y is always worse off under packet discrimination): *CP Y's profit is lower under packet discrimination than under net neutrality, i.e.,* $\pi_Y^{NN} \geq \pi_Y^{PD}$.

Proposition 3 reminds us that although the dominant content provider may be better off under the packet discrimination regime, the economically less successful content provider, e.g., a startup in a market with an established market player, is always worse off. In such situations, packet discrimination, e.g., the option of a paid fast lane, can act as a disincentive to entry for newer entrants in a marketplace with well-established incumbents. From a policymaker's perspective, in the long term, this can have a debilitating effect on content innovation. The impact of net neutrality and packet discrimination on content innovation has been studied from different perspectives, such as the entry of content providers (Krämer and Wiewiorra 2012; Guo and Easley 2014), the investment of content providers (Choi and Kim 2010), and the profitability of content providers (Guo et al. 2012). Our analysis contributes to this discussion. It shows that even in the presence of ISP competition, we do not have a 'level playing field' since the dominant content provider can still marginalize a less efficient or newer rival, to the extent that it (the dominant provider) might be better off without net neutrality. So, in a way, the dominant CP can leverage the competition among ISPs to become even more dominant by taking advantage of the flexible traffic management options under a packet discrimination regime.

As Manjoo (2014) observed, recently it is the smaller Internet firms that have been most vocal in the net neutrality debate. Companies like Etsy, where consumers can shop directly from people around the world, "would not have been able to pay for priority access if broadband companies ever created a fast lane online." Its public policy director then went on to comment that "Delays of even fractions of a second result in dropped revenue for our users." Our result in Proposition 3 shows that this concern is justified because the smaller firms will certainly be negatively affected by paid prioritization.

Another way a small content provider can be in a disadvantageous position was recently illustrated by the experience of the online backup firm Backblaze. They observed that during the negotiations between Comcast and Netflix, some of their consumers, who were served by Comcast as their ISP, were having their Backblaze services throttled. After a thorough investigation (Backblaze 2014), Backblaze was left with only one conclusion: "...consumers and businesses…unrelated to Netflix were punished, and all this occurred without notice or explanation from Comcast."

Prior studies with a monopolist ISP and competing CPs (Choi and Kim 2010; Cheng et al. 2011) show that all content providers will be united in their stance in preserving net neutrality. With both ISP competition and CP competition, however, our findings suggest that under certain market conditions, it will be just the smaller content providers who will support net neutrality.

Next, we study the welfare effect under net neutrality and packet discrimination regimes. Consumer surplus is defined as $CS_{ij} = \int_0^1 \int_0^1 U_{ij}(x, z) dxdz$ and social welfare is defined as $SW_{ij} = \pi_{Cij} + \pi_{Dij} + \pi_{Yij} + \pi_{Gij} + CS_{ij}$.

Proposition 4 (Comparison of Social Welfare under Net Neutrality and Packet Discrimination): *Social welfare is weakly higher under packet discrimination than under net neutrality, i.e.,* $SW^{PD} \geq SW^{NN}$.

Proposition 4 indicates that packet discrimination with flexible network management options is welfare enhancing compared to the net neutrality regime. This result supports findings in prior studies (Cheng et al. 2011; Guo et al. 2012; Krämer and Wiewiorra 2012; Bourreau et al. 2014; Guo and Easley 2014).

CONCLUDING REMARKS

Theoretical Implications

We have proposed a modeling framework that captures the dynamics of two interrelated markets – that of Internet access service and that of digital content – providing complementary products. Duopolists compete for consumers in each market (ISPs C and D in the Internet access service market and CPs Y and G in the digital content market). Therefore consumers choose their preferred option among four product combinations (CY, CG, DY, and DG). Furthermore, user experiences are jointly determined by the ISPs' pricing decisions and the CPs' delivery service choices. We show that the interactions between the two markets and the relative market power of the agents in the two markets play a critical role in determining the equilibrium outcomes.

This modeling framework is not restricted to the Internet data transmission process and can be applied to a wide range of other contexts, where consumers derive their utility from a pair of complementary products. For example, in the ongoing hardware battle between competing hardware platforms (e.g. the Apple Macintosh versus the PC), a critical factor is software compatibility and functionality. While computer makers do not compete directly with software manufacturers, the competition between the computer hardware platforms nevertheless attenuates the competition between the software manufacturers, since consumers need to "consume" both the hardware and the software for their computing needs. When making their purchase decisions, users simultaneously consider the specifications of the device and the compatibility and ease-of-use of the corresponding software. The competitions between firms in both the computer hardware and software markets interact with each other and jointly determine the experiences of the end-users.

Managerial and Policy Implications

Our findings have important managerial implications. We find that net neutrality regulation (or conversely, the potential packet discrimination mechanisms) affects content providers differently. Moreover, this impact on the content providers' incentives and consequently content innovation critically depends on the market power of the content provider and the relative intensity of competition between the markets of Internet access service and digital content. In practice, content providers strive to improve their profit margin through lowering the cost of generating new content or licensing existing content. For example, instead of spending money indiscriminately on licensing content from other producers, content providers like Netflix are instead sifting through consumer viewing patterns to license (or greenlight for internal production) only those kinds of content that will probably be viewed extensively by their consumers. This trove of consumer information will become even more important over time, as it can be used to make increasingly accurate predictions, which in turn will help lower the cost of licensing content even further. In such scenarios, it is likely that the more efficient content provider would become increasingly dominant over time, which makes it more difficult for the less efficient content provider to compete. This is especially true in many online markets, where there is often a large gulf between the market leader and the next leading competitor. Our findings suggest that packet discrimination in the presence of ISP competition amplifies the competitive advantage of the more efficient content provider, even to the extent that the more efficient content provider is better off with paid prioritization compared to the outcome under the net neutrality regime.

Our findings also have important policy implications for net neutrality. Our results show that ISP competition may not substitute for net neutrality regulation, especially in the presence of content provider competition. Without net neutrality regulation, the competing ISPs still have the incentive to charge content providers for preferential delivery, and in the presence of content provider competition, they have the ability to induce content providers to pay for packet prioritization. Contrary to popular belief, we find that some advantaged content providers may benefit from paid prioritization because such arrangements further enforce their dominance in the content market. Paid prioritization, however, always hurts the disadvantaged content providers. In order to protect and encourage content innovation, the policy makers are advised to evaluate the specific market conditions (such as revenue generation ability of content providers and competition intensity) of individual markets of ISPs and CPs.

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ONLINE APPENDIX

Proof of Lemma 1

Consumers have four choices of ISP-CP combinations – CY, CG, DY, and DG. Consumer demands for these four choices can be derived by analyzing the indifference curves. There are six indifference curves based on the pairwise comparisons among the four ISP-CP combinations. For a given outcome ij, where $i, j = 1,2,3,4$, these six indifference curves can be characterized by four points x_{Cij}, x_{Dij}, z_{Yij}, and z_{Gij}: consumers located on $x = x_{Cij}$ are indifferent between CY and CG; consumers located on $x = x_{Dij}$ are indifferent between DY and DG; consumers located on $z = z_{Yij}$ are indifferent between CY and DY; consumers located on $z = z_{Gij}$ are indifferent between CG and DG; consumers located on the line that goes through points (x_{Cij}, z_{Yij}) and (x_{Dij}, z_{Gij}) are indifferent between CY and DG; and consumers located on the line that goes through points (x_{Cij}, z_{Gij}) and (x_{Dij}, z_{Yij}) are indifferent between CG and DY.

Comparing consumers' utility functions for the corresponding pairs of ISP-CP combinations yields $x_{Cij} = \frac{1}{2} + \frac{d\lambda(\mathrm{w}_{CGij} - \mathrm{w}_{CYij})}{2t}$, $x_{Dij} = \frac{1}{2} + \frac{d\lambda(\mathrm{w}_{DGij} - \mathrm{w}_{DYij})}{2t}$, $z_{Yij} = \frac{1}{2} + \frac{F_D - F_C}{2k} + \frac{d\lambda(\mathrm{w}_{DYij} - \mathrm{w}_{CYij})}{2k}$, and $z_{Gij} = \frac{1}{2} + \frac{F_D - F_C}{2k} + \frac{d\lambda(\mathrm{w}_{DGij} - \mathrm{w}_{CGij})}{2k}$. Considering symmetric equilibrium with $F_C = F_D$, we have $z_{Yij} = \frac{1}{2} + \frac{d\lambda(\mathrm{w}_{DYij} - \mathrm{w}_{CYij})}{2k}$ and $z_{Gij} = \frac{1}{2} + \frac{d\lambda(\mathrm{w}_{DGij} - \mathrm{w}_{CGij})}{2k}$. We observe that the sign $x_{Cij} - x_{Dij}$ is the same as the sign of $z_{Yij} - z_{Gij}$.

Each outcome ij is determined by the ISPs' pricing decisions and the corresponding content providers' delivery service choices. We use indicator functions I_{CYij}, I_{CGij}, I_{DYij} and I_{DGij}, which take values of 0 or 1, to represent whether content providers Y and G would pay for preferential delivery on ISPs C and D. To be consistent with the four ISP-CP combinations on the unit square, we denote outcome ij by the matrix $\begin{bmatrix} I_{DYij} & I_{DGij} \\ I_{CYij} & I_{CGij} \end{bmatrix}$. We introduce two types of actions (horizontal and vertical flips) to explore the connections among the 16 outcomes:

Horizontal Flip: Decisions of Y and G are simultaneously interchanged on ISPs C and D. Specifically, horizontal flip changes outcome ij dictated by $\begin{bmatrix} I_{DYij} & I_{DGij} \\ I_{CYij} & I_{CGij} \end{bmatrix}$ to outcome $i'j'$ dictated by $\begin{bmatrix} I_{DGij} & I_{DYij} \\ I_{CGij} & I_{CYij} \end{bmatrix}$, where $i' = \begin{cases} 1, & \text{if } i = 1 \\ 3, & \text{if } i = 2 \\ 2, & \text{if } i = 3 \\ 4, & \text{if } i = 4 \end{cases}$ and $j' = \begin{cases} 1, & \text{if } j = 1 \\ 3, & \text{if } j = 2 \\ 2, & \text{if } j = 3 \\ 4, & \text{if } j = 4 \end{cases}$.

Vertical Flip: Decisions of Y and G are simultaneously interchanged across ISPs C and D. Specifically, vertical flip changes outcome ij dictated by $\begin{bmatrix} I_{DYij} & I_{DGij} \\ I_{CYij} & I_{CGij} \end{bmatrix}$ to outcome ji dictated by $\begin{bmatrix} I_{CYij} & I_{CGij} \\ I_{DYij} & I_{DGij} \end{bmatrix}$.

Among the 16 outcomes, some outcomes permute amongst themselves when horizontal flip or vertical flip is applied and therefore can be grouped together into four invariant classes: (a) Outcomes 11, 14, 41, and 44; (b) Outcomes 22 and 33; (c) Outcomes 23 and 32; (d) Outcomes 12, 13, 21, 31, 42, 43, 24, and 34. In the following discussion, we give precise description of the changes to the indifferent customers when horizontal flip or vertical flip is applied to an outcome.

Horizontal Flip: The decisions of Y on the two ISPs are interchanged with the decisions of G in a given outcome. Horizontal flip changes outcome ij dictated by $\begin{bmatrix} I_{DYij} & I_{DGij} \\ I_{CYij} & I_{CGij} \end{bmatrix}$ to outcome $i'j'$ dictated by $\begin{bmatrix} I_{DGij} & I_{DYij} \\ I_{CGij} & I_{CYij} \end{bmatrix}$. That is we have $I_{CYi'j'} = I_{CGij}$, $I_{CGi'j'} = I_{CYij}$, $I_{DYi'j'} = I_{DGij}$, and $I_{DGi'j'} = I_{DYij}$. When the decisions in outcome ij are changed to $i'j'$, the decisions of Y on C and D and the decisions of G on C and D are interchanged. The queuing priorities are interchanged on ISPs C and D. This simultaneously interchanges the waiting times and market demand on C and D according to the new queuing priorities. We note that fees for all customers are equal so the redistribution is dependent solely on waiting times. Interchanging waiting times on ISPs C and D yields $w_{CYi'j'} = w_{CGij}$, $w_{CGi'j'} = w_{CYij}$, $w_{DYi'j'} = w_{DGij}$, and $w_{DGi'j'} = w_{DYij}$. This gives $x_{Cij} + x_{Ci'j'} = \frac{1}{2} + \frac{d\lambda(\mathrm{w}_{CGij} - \mathrm{w}_{CYij})}{2t} + \frac{1}{2} + \frac{d\lambda(\mathrm{w}_{CGi'j'} - \mathrm{w}_{CYi'j'})}{2t} = \frac{1}{2} + \frac{d\lambda(\mathrm{w}_{CGij} - \mathrm{w}_{CYij})}{2t} + \frac{1}{2} - \frac{d\lambda(\mathrm{w}_{CGij} - \mathrm{w}_{CYij})}{2t} = 1$, which implies $x_{Ci'j'} = 1 - x_{Cij}$. Similarly, we have $x_{Di'j'} = 1 - x_{Dij}$, $z_{Yij} = z_{Gi'j'}$, and $z_{Gij} = z_{Yi'j'}$. We note that the positions of these indifferent curves relative to the line of $x = \frac{1}{2}$ or $z = \frac{1}{2}$ remain the same according to the decisions of Y and G.

Vertical Flip: The decisions of Y and G on C are interchanged with their decisions on D in a given outcome. Vertical flip changes outcome ij dictated by $\begin{bmatrix} I_{DYij} & I_{DGij} \\ I_{CYij} & I_{CGij} \end{bmatrix}$ to outcome ji dictated by $\begin{bmatrix} I_{CYij} & I_{CGij} \\ I_{DYij} & I_{DGij} \end{bmatrix}$. That is we have $I_{CYji} = I_{DYij}$, $I_{CGji} = I_{DGij}$, $I_{DYji} = I_{CYij}$, and $I_{DGji} = I_{CGij}$. When the decisions in outcome ij are changed to ji, the decisions of Y and G on C are swapped with the decisions of Y and G on D. The queuing priorities are interchanged on ISPs C and D. This simultaneously interchanges the waiting times and market demand on ISPs C and D according to the new queuing priorities. We note that fees for all customers are equal so the redistribution is dependent solely on waiting times. Interchanging waiting times on ISPs C and D yields $w_{CYji} = w_{DYij}$, $w_{CGji} = w_{DGij}$, $w_{DYji} = w_{CYij}$, and $w_{DGji} = w_{CGij}$. This gives $x_{Cij} = \frac{1}{2} + \frac{d\lambda(\mathrm{w}_{CGij} - \mathrm{w}_{CYij})}{2k} = \frac{1}{2} + \frac{d\lambda(\mathrm{w}_{DGji} - \mathrm{w}_{DYji})}{2k} = x_{Dji}$. Similarly, $x_{Dij} = x_{Cji}$. We also have $z_{Yij} + z_{Yji} = \frac{1}{2} + \frac{d\lambda(\mathrm{w}_{DYij} - \mathrm{w}_{CYij})}{2t} + \frac{1}{2} + \frac{d\lambda(\mathrm{w}_{DYji} - \mathrm{w}_{CYji})}{2t} = \frac{1}{2} - \frac{d\lambda(\mathrm{w}_{DYji} - \mathrm{w}_{CYji})}{2t} + \frac{1}{2} + \frac{d\lambda(\mathrm{w}_{DYji} - \mathrm{w}_{CYji})}{2t} = 1$, which implies $z_{Yji} = 1 - z_{Yij}$. Similarly, $z_{Gji} = 1 - z_{Gij}$.

Next we apply the above results of horizontal and vertical flips to each of the classes (a) through (d) to characterize the demand distribution under each outcome.

(Class a) Outcomes 11, 14, 41, and 44

Under outcomes 11, 14, 41, and 44, all customers have equal queuing priorities. Therefore applying horizontal flip or vertical flip to these outcomes will not change the queuing priorities. Hence the indifferent customers remain unchanged when horizontal flip or vertical flip is applied.

From horizontal flip relations, we have $x_{C11} = 1 - x_{C11}$, $x_{D11} = 1 - x_{D11}$, $x_{C14} = 1 - x_{C14}$, $x_{D14} = 1 - x_{D14}$, $x_{C41} = 1 - x_{C41}$, $x_{D41} = 1 - x_{D41}$, $x_{C44} = 1 - x_{C44}$, and $x_{D44} = 1 - x_{D44}$. That is $x_{C11} = x_{C14} = x_{C41} = x_{C44} = \frac{1}{2}$ and $x_{D11} = x_{D14} = x_{D41} = x_{D44} = \frac{1}{2}$. From vertical flip relations, we have $z_{G11} = 1 - z_{G11}$, $z_{Y11} = 1 - z_{Y11}$, $z_{G14} = 1 - z_{G14}$, $z_{Y14} = 1 - z_{Y14}$, $z_{G41} = 1 - z_{G41}$, $z_{Y41} = 1 - z_{Y41}$, $z_{G44} = 1 - z_{G44}$, $z_{Y44} = 1 - z_{Y44}$. That is $z_{G11} = z_{G14} = z_{G41} = z_{G44} = \frac{1}{2}$ and $z_{Y11} = z_{Y14} = z_{Y41} = z_{Y44} = \frac{1}{2}$.

Therefore, as shown in Figure 4a, the market demand for CY, DY, CG, and DG are equal under outcomes 11, 14, 41, and 44. That is $N_{CY11} = N_{DY11} = N_{CG11} = N_{DG11} = \frac{1}{4}$, $N_{CY14} = N_{DY14} = N_{CG14} = N_{DG14} = \frac{1}{4}$, $N_{CY41} = N_{DY41} = N_{CG41} = N_{DG41} = \frac{1}{4}$, $N_{CY44} = N_{DY44} = N_{CG44} = N_{DG44} = \frac{1}{4}$.

(Class b) Outcomes 22 and 33

Under outcome 22, only Y pays for preferential delivery on both ISPs. Under outcome 33, only G pays for preferential delivery on both ISPs. Thus $w_{CG22} - w_{CY22} > 0$, $w_{DG22} - w_{DY22} > 0$, $w_{CG33} - w_{CY33} < 0$, and $w_{DG33} - w_{DY33} < 0$.

Vertical flip does not change the decisions of Y and G on C and D in outcomes 22 and 33. Therefore we have $x_{C22} = x_{D22} > \frac{1}{2}$, $z_{Y22} = 1 - z_{Y22} \implies z_{Y22} = \frac{1}{2}$, $z_{G22} = 1 - z_{G22} \implies$

$z_{G22} = \frac{1}{2}$, $x_{C33} = x_{D33} < \frac{1}{2}$, $z_{Y33} = 1 - z_{Y33} \implies z_{Y33} = \frac{1}{2}$, and $z_{G33} = 1 - z_{G33} \implies z_{G33} = \frac{1}{2}$.

Moreover, horizontal flip applied to outcome 22 gives outcome 33 and vice versa. This gives $x_{C22} = 1 - x_{C33} = x_{D22} = 1 - x_{D33}$. Thus, we simplify the notations to $x_{C22} = x_{D22} = x_{22}$ and $x_{C33} = x_{D33} = x_{33}$. Therefore, as shown in Figure 4b, the demands for CY, DY, CG, and DG in outcome 22 and 33 are related such that $N_{CY22} = N_{DY22} = N_{CG33} = N_{DG33} = \frac{1-x_{33}}{2}$ and $N_{CY33} = N_{DY33} = N_{CG22} = N_{DG22} = \frac{x_{33}}{2}$.

(Class c) Outcomes 23 and 32

Under outcome 23, only Y pays for preferential delivery on C and only G pays for preferential delivery on D. Under outcome 32, only G pays for preferential delivery on C and only Y pays for preferential delivery on D. Thus $w_{CG23} - w_{CY23} > 0$, $w_{DG23} - w_{DY23} < 0$, $w_{CG32} - w_{CY32} < 0$, and $w_{DG32} - w_{DY32} > 0$. Therefore we have $x_{C23} > \frac{1}{2} > x_{D23}$ and $x_{C32} < \frac{1}{2} < x_{D32}$. Since the sign $x_{Cij} - x_{Dij}$ is the same as the sign of $z_{Yij} - z_{Gij}$ for any outcome ij, we have $z_{Y23} > z_{G23}$, and $z_{Y32} < z_{G32}$.

Observe that both horizontal flip and vertical flip applied to outcome 23 gives outcome 32 and vice versa. Through the connection of horizontal flip, we have $x_{C23} = 1 - x_{C32}$, $x_{D23} = 1 - x_{D32}$, $z_{Y23} = z_{G32}$, and $z_{G23} = z_{Y32}$. Through the connection of Vertical flip, we have $x_{C23} = x_{D32}$, $x_{D23} = x_{C32}$, $z_{Y23} = 1 - z_{Y32}$, and $z_{G23} = 1 - z_{G32}$. Combining the two set of equalities gives $x_{D23} = 1 - x_{C23}$, $x_{D32} = 1 - x_{C32}$, $z_{G23} = 1 - z_{Y23}$, and $z_{G32} = 1 - z_{Y32}$. Since $z_{Y23} > z_{G23}$ and $z_{Y32} < z_{G32}$, the last set of equalities says that $z_{Y23} > \frac{1}{2} > z_{G23}$ and $z_{Y32} < \frac{1}{2} < z_{G32}$. This says that the indifferent customers x_{C23} and x_{D23} (as well as x_{C32} and x_{D32}) are symmetrically positioned on either side of $x = \frac{1}{2}$. Likewise, z_{Y23} and z_{G23} (as well as

z_{Y32} and z_{G32}) are symmetrically positioned on either side of $z = \frac{1}{2}$. Therefore the demands for CY, DY, CG, and DG in outcomes 23 and 32 are related such that $N_{CY23} = N_{DG23} = N_{CG32} = N_{DY32}$ and $N_{CG23} = N_{DY23} = N_{CY32} = N_{DG32}$.

(Class d) Outcomes 12, 13, 21, 31, 42, 43, 24, and 34

Based on CPs' delivery service choices in outcomes 12, 13, 21, 31, 42, 43, 24, and 34, we know that $w_{CG12} - w_{CY12} = w_{CG13} - w_{CY13} = 0$, $w_{DG21} - w_{DY21} = w_{DG31} - w_{DY31} = 0$, $w_{DG12} - w_{DY12} > 0$, $w_{DG13} - w_{DY13} < 0$, $w_{CG21} - w_{CY21} > 0$, and $w_{CG31} - w_{CY31} < 0$. Therefore we have $x_{C12} = x_{C13} = \frac{1}{2}$, $x_{D21} = x_{D31} = \frac{1}{2}$, $x_{D12} > \frac{1}{2} > x_{D13}$, and $x_{C21} > \frac{1}{2} > x_{C31}$. Since the sign $x_{Cij} - x_{Dij}$ is the same as the sign of $z_{Yij} - z_{Gij}$ for any outcome ij, we have $z_{Y12} < z_{G12}$ and $z_{Y21} > z_{G21}$. Likewise, we have $z_{Y13} > z_{G13}$ and $z_{Y31} < z_{G31}$.

Successive application of horizontal flip and vertical flip connect outcomes 12, 13, 21, and 31 as follows:

$$
\begin{array}{ccc}
\text{Outcome 12} & \xleftrightarrow{\textit{Horizontal Flip}} & \text{Outcome 13} \\
\textit{Vertical Flip} \updownarrow & & \updownarrow \textit{Vertical Flip} \\
\text{Outcome 21} & \xleftrightarrow{\textit{Horizontal Flip}} & \text{Outcome 31}
\end{array}
$$

Through horizontal flip, we have $x_{C13} = 1 - x_{C12} = \frac{1}{2}$, $x_{D13} = 1 - x_{D12} < \frac{1}{2}$, $z_{Y13} = z_{G12}$, $z_{G13} = z_{Y12}$, $x_{C31} = 1 - x_{C21} < \frac{1}{2}$, $x_{D31} = 1 - x_{D21} = \frac{1}{2}$, $z_{Y31} = z_{G21}$, and $z_{G31} = z_{Y21}$.

Through vertical flip, we have $x_{C21} = x_{D12} > \frac{1}{2}$, $x_{D21} = x_{C12} = \frac{1}{2}$, $z_{Y21} = 1 - z_{Y12}$, $z_{G21} = 1 - z_{G12}$, $x_{C31} = x_{D13} < \frac{1}{2}$, $x_{D31} = x_{C13} = \frac{1}{2}$, $z_{Y31} = 1 - z_{Y13}$, and $z_{G31} = 1 - z_{G13}$. Therefore the demand for CY, DY, CG, and DG in outcomes 12, 13, 21, and 31 are related such that $N_{DY12} =$

$N_{DG13} = N_{CG31} = N_{CY21}$, $N_{DG12} = N_{DY13} = N_{CY31} = N_{CG21}$, $N_{CG12} = N_{CY13} = N_{DY31} = N_{DG21}$, and $N_{CY12} = N_{CG13} = N_{DG31} = N_{DY21}$.

The demand analysis for outcomes 42, 43, 24, and 34 is the same as that in outcomes 12, 13, 21, and 31 since both CPs receive the same queuing priority when they both pay for preferential delivery. Therefore, the demand for CY, DY, CG, and DG in outcomes 42, 43, 24, and 34 are related such that $N_{DY42} = N_{DG43} = N_{CG34} = N_{CY24}$, $N_{DG42} = N_{DY43} = N_{CY34} = N_{CG24}$, $N_{CG42} = N_{CY43} = N_{DY34} = N_{DG24}$, and $N_{CY42} = N_{CG43} = N_{DG34} = N_{DY24}$.

If Y and G make identical decisions (1 or 4) on any ISP (C or D), consumers on that ISP will receive the same queuing priority. For example, under outcomes 13 and 43, indifferent customers of all four ISP-CP combinations are the same, which leads to identical demand distribution for CY, DY, CG, and DG. That is $N_{CY13} = N_{CY43}$, $N_{DY13} = N_{DY43}$, $N_{CG13} = N_{CG43}$, and $N_{DG13} = N_{DG43}$. By the same arguments above, we obtain the pairings with identical demand distribution for CY, DY, CG, and DG: outcomes 12 and 42, outcomes 21 and 24, and outcomes 31 and 34.

Summarizing the above analysis, we conclude that the 16 outcomes can be grouped into four classes invariant under horizontal and vertical flips with similar consumer demand patterns.

Proof of Lemma 2

We derive the ISPs' equilibrium pricing strategies and the corresponding equilibrium outcomes in the packet discrimination regime in the following steps: step 1, prove that any outcome involving only Y pays on an ISP cannot be an equilibrium; step 2, derive properties of the equilibrium fixed fee F; step 3, eliminate dominated outcomes; step 4, solve for the equilibrium fixed fee F and preferential delivery fee p for the candidate outcomes; step 5, compare the candidate outcomes and derive equilibrium outcomes.

Step 1: Prove that any outcome involving only Y pays on an ISP cannot be an equilibrium

In step 1, we show that there is no feasible p for any outcome involving only Y pays on an ISP. Therefore such outcomes (12, 21, 42, 24, 23, 32, and 22) cannot be an equilibrium. Since some outcomes are infeasible for similar reasons, we group them together.

<u>Outcomes 12 and 21</u>

Here we focus on showing that there is no feasible p for outcome 12, as the analysis for outcome 21 is similar. For outcomes 12 to be feasible, all the CPs' incentive compatibility constraints need to be satisfied: (1) $\pi_{Y12} - \pi_{Y22} \geq 0$; (2) $\pi_{Y12} - \pi_{Y11} \geq 0$; (3) $\pi_{Y12} - \pi_{Y21} \geq 0$; (4) $\pi_{G12} - \pi_{G31} \geq 0$; (5) $\pi_{G12} - \pi_{G14} \geq 0$; and (6) $\pi_{G12} - \pi_{G34} \geq 0$.

Inequality (2) is $-N_{DY12}p + (N_{CY12} - N_{CY11} - N_{DY11} + N_{DY12})r_Y \geq 0$. Since $N_{CY12} + N_{DY12} > \frac{1}{2}$ and $N_{CY11} + N_{DY11} = \frac{1}{2}$, inequality (2) can be reduced to $p \leq \frac{(N_{CY12}+N_{DY12}-1/2)r_Y}{N_{DY12}}$.

Inequality (5) is $N_{DG14}p + (N_{CG12} - N_{CG14} + N_{DG12} - N_{DG14})r_G \geq 0$. Since $N_{DG14} = \frac{1}{4}$, $N_{CG14} + N_{DG14} = \frac{1}{2}$, and $N_{CG12} + N_{DG12} < \frac{1}{2}$, inequality (5) can be reduced to $p \geq \frac{(1/2-N_{CG12}-N_{DG12})r_G}{1/4}$.

We know that $\frac{1}{2} - N_{CG12} - N_{DG12} = N_{CY12} + N_{DY12} - \frac{1}{2}$, $N_{DY12} > \frac{1}{4}$, and $r_G \geq r_Y$. Thus we have $\frac{(N_{CY12}+N_{DY12}-1/2)r_Y}{N_{DY12}} < \frac{(1/2-N_{CG12}-N_{DG12})r_G}{1/4}$. Therefore (2) and (5) are inconsistent. Hence there is no solution for p that satisfies all incentive compatibility constraints for outcome 12.

Outcomes 42 and 24

Outcomes 24 and 42 are infeasible for similar reasons. Outcome 24 is not feasible since the following incentive compatibility constraints are inconsistent: (1) $\pi_{Y24} - \pi_{Y13} \geq 0$ and (2) $\pi_{G24} - \pi_{G44} \geq 0$.

Inequality (1) can be reduced to $p \leq \frac{(N_{CY24}+N_{DY24}-N_{CY13}-N_{DY13})r_Y}{(N_{CY24}+N_{DY24})}$. Note that we have $N_{CY24} + N_{DY24} - N_{CY13} - N_{DY13} = N_{CY24} + N_{DY24} - \frac{1}{2} + \frac{1}{2} - N_{CY13} - N_{DY13}$. Since $N_{CY24} + N_{DY24} - \frac{1}{2} = \frac{1}{2} - N_{CY13} - N_{DY13}$, we have $N_{CY24} + N_{DY24} - N_{CY13} - N_{DY13} = 2\left(N_{CY24} + N_{DY24} - \frac{1}{2}\right) = 2\left(\frac{1}{2} - N_{CG24} - N_{DG24}\right)$. Thus inequality (1) can be simplified to $p \leq \frac{(1/2-N_{CG24}-N_{DG24})r_Y}{(N_{CY24}+N_{DY24})/2}$.

Inequality (2) can be reduced to $p \geq \frac{(1/2-N_{CG24}-N_{DG24})r_G}{(1/2-N_{DG24})}$. Note that we have $N_{CY24} + N_{DY24} + N_{CG24} + N_{DG24} = 1$. But $N_{CG24} < N_{DG24}$. Thus we have $\frac{N_{CY24}+N_{DY24}}{2} > \frac{1}{2} - N_{DG24}$. Therefore $p \geq \frac{(1/2-N_{CG24}-N_{DG24})r_G}{(1/2-N_{DG24})} > \frac{(1/2-N_{CG24}-N_{DG24})r_G}{(N_{CY24}+N_{DY24})/2}$.

In addition, we know $r_G \geq r_Y$. Thus we have $\frac{(1/2-N_{CG24}-N_{DG24})r_G}{(1/2-N_{DG24})} > \frac{(1/2-N_{CG24}-N_{DG24})r_G}{(N_{CY24}+N_{DY24})/2} \geq \frac{(1/2-N_{CG24}-N_{DG24})r_Y}{(N_{CY24}+N_{DY24})/2}$. Therefore inequalities (1) and (2) are inconsistent. Hence outcome 24 is infeasible.

Outcomes 23 and 32

Outcomes 23 and 32 are infeasible for similar reasons. Outcome 23 is feasible provided $\pi_{Y23} - \pi_{Y14} \geq 0$, i.e., $\left(N_{CG23} - \frac{1}{4}\right)p + \left(\frac{1}{2} - N_{CG32} - N_{CG23}\right)r_Y \geq 0$. Note that $N_{CG23} + N_{CY23} = \frac{1}{2}$ and $N_{CY23} = N_{CG32}$. Thus we have $N_{CG23} + N_{CG32} = \frac{1}{2}$. This gives $\left(N_{CG23} - \frac{1}{4}\right)p \geq 0$. Since $N_{CG23} <$

$\frac{1}{4}$, we have $p \leq\ 0$. Hence there is no positive solution for p and therefore outcome 23 is not feasible.

<u>Outcome 22</u>

Outcome 22 is not feasible since the following incentive compatibility constraints are inconsistent: (1) $\pi_{Y22} - \pi_{Y11} \geq 0$ and (2) $\pi_{G22} - \pi_{G44} \geq 0$.

Inequality (1) is $\left(N_{CG22} - N_{CG33} - \frac{1}{2}\right)p + \ (N_{CG33} - N_{CG22})r_Y\ \geq\ 0$. Note that $N_{CG22} + N_{CY22} = \frac{1}{2}$. Thus we have $\left(N_{CG22} - N_{CG33} - \frac{1}{2}\right) = -N_{CG33} - N_{CY22} = -2N_{CG33} < 0$. Therefore inequality (1) can be reduced to $p \leq\ \left(\frac{1}{2} - \frac{N_{CG22}}{2N_{CG33}}\right)r_Y$. Inequality (2) can be reduced to $p\ \geq\ (1 - 4N_{CG22})r_G$.

Recall that $N_{CG22} = N_{CY33}$, $N_{CY22} = N_{CG33}$, and $N_{CY33} + N_{CG33} = \frac{1}{2}$. Thus inequality (1) may be re-written as $p \leq \left(1 - \frac{1}{4N_{CG33}}\right)r_Y$ and inequality (2) may be re-written as $p \geq 4N_{CG33}\left(1 - \frac{1}{4N_{CG33}}\right)r_G$. Since $r_Y \leq r_G$ and $4N_{CG33} > 1$, inequality (1) implies that $p \leq \left(1 - \frac{1}{4N_{CG33}}\right)r_Y < r_G$ but inequality (2) implies that $p \geq 4N_{CG33}\left(1 - \frac{1}{4N_{CG33}}\right)r_G > r_G$. Therefore inequalities (1) and (2) are inconsistent and there is no feasible p for outcome 22.

In summary, outcomes 12, 21, 42, 24, 23, 32, and 22 cannot be an equilibrium.

Step 2: Derive properties of the equilibrium fixed fee F

In step 2, we derive properties of the equilibrium fixed fee F. Here we first discuss some properties for all 16 outcomes and thus the subscript ij is omitted in this discussion. Under the assumption of full market coverage, the profit maximizing fixed fee F is such that the consumers of all four ISP-CP combinations (CY, DY, CG, and DG) with the lowest net utility will get zero net utility.

We now define the global utility function $U(x,z)$ for the entire market $[0,1] \times [0,1]$. First recall the definition of the demand distribution of each ISP-CP combinations characterized by the utility functions.

$$R_{CY} = \{(x,z) \in [0,1] \times [0,1];\ u_{CY}(x,z) \geq \max\{u_{CG}(x,z), u_{DY}(x,z), u_{DG}(x,z)\}\}$$

$$R_{DY} = \{(x,z) \in [0,1] \times [0,1];\ u_{DY}(x,z) \geq \max\{u_{DG}(x,z), u_{CY}(x,z), u_{CG}(x,z)\}\}$$

$$R_{CG} = \{(x,z) \in [0,1] \times [0,1];\ u_{CG}(x,z) \geq \max\{u_{CY}(x,z), u_{DG}(x,z), u_{DY}(x,z)\}\}$$

$$R_{DG} = \{(x,z) \in [0,1] \times [0,1];\ u_{DG}(x,z) \geq \max\{u_{DY}(x,z), u_{CG}(x,z), u_{CY}(x,z)\}\}$$

Note that each of the following inequalities reduces to regions on $[0,1] \times [0,1]$ dictated by the indifference customers between mutual pairs of ISP-CP combinations:

$$u_{CG}(x,z) - u_{CY}(x,z) \geq 0 \Leftrightarrow x \geq x_C$$

$$u_{DG}(x,z) - u_{DY}(x,z) \geq 0 \Leftrightarrow x \geq x_D$$

$$u_{DY}(x,z) - u_{CY}(x,z) \geq 0 \Leftrightarrow z \geq z_Y$$

$$u_{DG}(x,z) - u_{CG}(x,z) \geq 0 \Leftrightarrow z \geq z_G$$

$$u_{DG}(x,z) - u_{CY}(x,z) \geq 0 \Leftrightarrow z \geq L_-(x)$$

$$u_{DY}(x,z) - u_{CG}(x,z) \geq 0 \Leftrightarrow z \geq L_+(x)$$

Then the demand distributions can be written in terms of the indifference customers as follows:

$$R_{CY} = \{(x,z) \in [0,1] \times [0,1];\ x \leq x_C, z \leq z_Y, z \leq L_-(x)\}$$

$$R_{DY} = \{(x,z) \in [0,1] \times [0,1];\ x \leq x_D, z \geq z_Y, z \geq L_+(x)\}$$

$$R_{CG} = \{(x,z) \in [0,1] \times [0,1];\ x \geq x_C, z \leq z_G, z \leq L_+(x)\}$$

$$R_{DG} = \{(x,z) \in [0,1] \times [0,1];\ x \geq x_D, z \geq z_G, z \geq L_-(x)\}$$

Define the global utility function $U(x, z)$ over the entire market $[0,1] \times [0,1]$:

$$U(x, z) = \begin{cases} u_{CY}(x, z), & \text{if } (x, z) \in R_{CY} \\ u_{DY}(x, z), & \text{if } (x, z) \in R_{DY} \\ u_{CG}(x, z), & \text{if } (x, z) \in R_{CG} \\ u_{DG}(x, z), & \text{if } (x, z) \in R_{DG} \end{cases}$$

By definition of the demand regions R_{CY}, R_{DY}, R_{CG}, and R_{DG}, the global utility function gives the maximal utility value for the consumer (x, z) according to its choice of ISP-CP combination. We also note that $U(x, z)$ is a continuous function over the set $[0,1] \times [0,1]$. Indeed, first note that the functions $u_{CY}(x, z)$, $u_{DY}(x, z)$, $u_{CG}(x, z)$, and $u_{DG}(x, z)$ are linear functions in (x, z) and thus are all continuous. Since $U(x, z)$ is piecewise defined over demand regions R_{CY}, R_{DY}, R_{CG}, and R_{DG}, we only need to check that $U(x, z)$ is continuous at each point on the boundaries between mutual pairs of the demand regions R_{CY}, R_{DY}, R_{CG}, and R_{DG}. We check each boundary:

- Between R_{CY} and R_{DY}, the boundary is along the line $z = z_Y$ on which $u_{CY} = u_{DY}$.
- Between R_{CY} and R_{CG}, the boundary is along the line $x = x_C$ on which $u_{CY} = u_{CG}$.
- Between R_{CY} and R_{DG}, the boundary is along the line $z = L_-(x)$ on which $u_{CY} = u_{DG}$.
- Between R_{DG} and R_{CG}, the boundary is along the line $z = z_G$ on which $u_{DG} = u_{CG}$.
- Between R_{DG} and R_{DY}, the boundary is along the line $x = x_D$ on which $u_{DG} = u_{DY}$.
- Between R_{DY} and R_{CG}, the boundary is along the line $z = L_+(x)$ on which $u_{DY} = u_{CG}$.

Since corresponding utility functions all matches along the boundaries between mutual pairs of the demand regions R_{CY}, R_{DY}, R_{CG}, and R_{DG}, the global utility function $U(x, z)$ is continuous over the entire set $[0,1] \times [0,1]$.

The global utility function $U(x, z)$ is a continuous function over the closed and bounded set $[0,1] \times [0,1]$. Therefore $U(x, z)$ attains its maximum and minimum at some points in the set $[0,1] \times [0,1]$. Under the assumption of full market coverage, the optimal fixed fees the ISPs charge consumers are such that the minimum of $U(x,z)$ equal to zero. In other words, the optimal fixed fee is the maximum fee such that all consumers get nonnegative utility.

Since $U(x,z)$ is piecewise defined by linear functions, it has no critical points in the interior of each demand regions R_{CY}, R_{DY}, R_{CG}, and R_{DG}. Therefore we only need to analyze the value of $U(x,z)$ along each mutual boundaries to capture the minimum of $U(x,z)$. Before we analyze the boundaries between R_{CY}, R_{DY}, R_{CG}, and R_{DG}, we recall that the demand distributions split into the three geometric types (i) $x_C = x_D$ and $z_Y = z_G$; (ii) $x_C < x_D$ and $z_Y < z_G$; and (iii) $x_C > x_D$ and $z_Y > z_G$.

The feasible outcomes 11, 14, 41, 44, and 33 are of type (i), where the demand regions are all rectangular in shape. The feasible outcomes 31 and 34 are of type (ii), which have exactly two rectangles, and two pentagonal regions sharing a boundary along $z = L_+(x)$. And finally, the feasible outcomes 13 and 43 are of type (iii), which have exactly two rectangles, and two pentagonal regions sharing a boundary along $z = L_-(x)$.

We organize the analysis into two cases (A): $x_C \leq x_D$and $z_Y \leq z_G$ and (B): $x_C \geq x_D$ and $z_Y \geq z_G$. Obviously, Cases (A) and (B) overlaps in those of type (i) here the diagonal boundary on $z = L_+(x)$ or $z = L_-(x)$ collapses to the point of intersection of these lines.

Case (A): $\boldsymbol{x_C \leq x_D}$ and $\boldsymbol{z_Y \leq z_G}$

There are five boundaries including a segment on $z = L_+(x)$.

(A1) Boundary between R_{CY} and R_{DY}. This boundary is along the horizontal line $z = z_Y$ and is the line segment joining $(0, z_Y)$ and the point (x_C, z_Y). Since $u_{CY} = u_{DY}$ on this boundary, along the boundary we may write for $0 \leq x \leq x_C$, $U(x, z_Y) = u_{CY}(x, z_Y) = V - tx - kz_Y - d\lambda w_{CY} - F_C$, or $U(x, z_Y) = u_{DY}(x, z_Y) = V - tx - k(1 - z_Y) - d\lambda w_{DY} - F_D$. In either formula, we see that on this boundary $U(x, z)$ is a decreasing function of x. Therefore $U(x, z)$ minimizes at (x_C, z_Y) on the boundary between R_{CY} and R_{DY}.

(A2) Boundary between R_{DG} and R_{CG}. This boundary is along the horizontal line $z = z_G$ and is the line segment joining (x_D, z_G) and the point $(1, z_G)$. Since $u_{DG} = u_{CG}$ on this boundary, along the boundary we may write for $x_D \leq x \leq 1$, $U(x, z_G) = u_{CG}(x, z_G) = V - t(1 - x) - kz_G - d\lambda w_{CG} - F_C$, or $U(x, z_G) = u_{DG}(x, z_G) = V - t(1 - x) - k(1 - z_G) - d\lambda w_{DG} - F_D$. In either formula, we see that on this boundary $U(x, z)$ is a increasing function of x. Therefore $U(x, z)$ minimizes at (x_D, z_G) on the boundary between R_{DG} and R_{CG}.

(A3) Boundary between R_{CY} and R_{CG}. This boundary is along the vertical line $x = x_C$ and is the line segment joining $(x_C, 0)$ and the point (x_C, z_Y). Since $u_{CY} = u_{CG}$ on this boundary, along the boundary we may write for $0 \leq z \leq z_Y$, $U(x_C, z) = u_{CY}(x_C, z) = V - tx_C - kz - d\lambda w_{CY} - F_C$, or $U(x_C, z) = u_{CG}(x_C, z) = V - t(1 - x_C) - kz - d\lambda w_{CG} - F_C$. In either formula, we see that on this boundary $U(x, z)$ is a decreasing function of z. Therefore $U(x, z)$ minimizes at (x_C, z_Y) on the boundary between R_{CY} and R_{CG}.

(A4) Boundary between R_{DG} and R_{DY}. This boundary is along the vertical line $x = x_D$ and is the line segment joining (x_D, z_G) and the point $(x_D, 1)$. Since $u_{CY} = u_{CG}$ on this boundary, along the boundary we may write for $z_G \leq z \leq 1$, $U(x_D, z) = u_{DY}(x_D, z) = V - tx_D - k(1 - z) - d\lambda w_{DY} - F_D$, or $U(x_D, z) = u_{DG}(x_D, z) = V - t(1 - x_D) - k(1 - z) - d\lambda w_{DG} - F_D$. In either formula, we see that on this boundary $U(x, z)$ is a increasing function of z. Therefore $U(x, z)$ minimizes at (x_D, z_G) on the boundary between R_{DG} and R_{DY}.

(A5) Boundary between R_{CG} and R_{DY}. This boundary is along the line $z = L_+(x)$ and is the line segment joining (x_C, z_Y) and the point (x_D, z_G). We parameterize the directed line segment as follows: For $0 \leq s \leq 1$, $x = (1 - s)x_C + sx_D$ and $z = (1 - s)z_Y + sz_G$. On this boundary the utility function U is a function of the parameter s. Since $u_{CG} = u_{DY}$ on this boundary, along the boundary we may write for $0 \leq s \leq 1$, $U(s) = u_{DY}((1-s)x_C + sx_D, (1-s)z_Y + sz_G) = V - t[(1-s)x_C + sx_D] - k[1-(1-s)z_Y - sz_G] - d\lambda w_{DY} - F_D = V + st(x_C - x_D) - tx_C - k(1-z_Y) + sk(z_G - z_Y) - d\lambda w_{DY} - F_D$, or $U(s) = u_{CG}((1-s)x_C + sx_D, (1-s)z_Y + sz_G) = V(\lambda) - t[1-(1-s)x_C - sx_D] - k[(1-s)z_Y + sz_G] - d\lambda w_{CG} - F_C = V(\lambda) - t(1-x_C) + st(x_D - x_C) - kz_Y + sk(z_Y - z_G) - d\lambda w_{CG} - F_C$. If $x_D = x_C$ and $z_Y = z_G$ then $U(s)$ is a constant not dependent on s. However, in general we note that the slope of $z = L_+(x)$ is given by $\frac{t}{k} = \frac{z_G - z_Y}{x_D - x_C}$, i.e., $k(z_G - z_Y) = t(x_D - x_C)$. Thus the values of $U(s)$ reduces to the constant: $U(s) = V - tx_C - k(1-z_Y) - d\lambda w_{DY} - F_D$ or $U(s) = V - t(1 - x_C) - kz_Y - d\lambda w_{CG} - F_C$. From the analysis above, we could see that $U(x,z)$ minimizes on the points along the boundary on the line $z = L_+(x)$. In particular, $U(x,z)$ minimizes at (x_C, z_Y) or (x_D, z_G) with the same value.

Case (B): $x_C \geq x_D$ and $z_Y \geq z_G$

There are five boundaries including a segment on $z = L_-(x)$.

(B1) Boundary between R_{CY} and R_{DY}. This boundary is along the horizontal line $z = z_Y$ and is the line segment joining $(0, z_Y)$ and the point (x_D, z_Y). Since $u_{CY} = u_{DY}$ on this boundary, along the boundary we may write for $0 \leq x \leq x_D$, $U(x, z_Y) = u_{CY}(x, z_Y) = V - tx - kz_Y - d\lambda w_{CY} - F_C$, or $U(x, z_Y) = u_{DY}(x, z_Y) = V - tx - k(1 - z_Y) - d\lambda w_{DY} - F_D$. In either formula, we see that on this boundary $U(x,z)$ is a decreasing function of x. Therefore $U(x,z)$ minimizes at (x_D, z_Y) on the boundary between R_{CY} and R_{DY}.

(B2) Boundary between R_{DG} and R_{CG}. This boundary is along the horizontal line $z = z_G$ and is the line segment joining (x_C, z_G) and the point $(1, z_G)$. Since $u_{DG} = u_{CG}$ on this boundary, along the boundary we may write for $x_C \le x \le 1$, $U(x, z_G) = u_{CG}(x, z_G) = V - t(1 - x) - kz_G - d\lambda w_{CG} - F_C$, or $U(x, z_G) = u_{DG}(x, z_G) = V - t(1 - x) - k(1 - z_G) - d\lambda w_{DG} - F_D$. In either formula, we see that on this boundary $U(x, z)$ is a increasing function of x. Therefore $U(x, z)$ minimizes at (x_C, z_G) on the boundary between R_{DG} and R_{CG}.

(B3) Boundary between R_{CY} and R_{CG}. This boundary is along the vertical line $x = x_C$ and is the line segment joining $(x_C, 0)$ and the point (x_C, z_G). Since $u_{CY} = u_{CG}$ on this boundary, along the boundary we may write for $0 \le z \le z_G$, $U(x_C, z) = u_{CY}(x_C, z) = V - tx_C - kz - d\lambda w_{CY} - F_C$, or $U(x_C, z) = u_{CG}(x_C, z) = V - t(1 - x_C) - kz - d\lambda w_{CG} - F_C$. In either formula, we see that on this boundary $U(x, z)$ is a decreasing function of z. Therefore $U(x, z)$ minimizes at (x_C, z_G) on the boundary between R_{CY} and R_{CG}.

(B4) Boundary between R_{DG} and R_{DY}. This boundary is along the vertical line $x = x_D$ and is the line segment joining (x_D, z_Y) and the point $(x_D, 1)$. Since $u_{CY} = u_{CG}$ on this boundary, along the boundary we may write for $z_Y \le z \le 1$, $U(x_D, z) = u_{DY}(x_D, z) = V - tx_D - k(1 - z) - d\lambda w_{DY} - F_D$, or $U(x_D, z) = u_{DG}(x_D, z) = V - t(1 - x_D) - k(1 - z) - d\lambda w_{DG} - F_D$. In either formula, we see that on this boundary $U(x, z)$ is a increasing function of z. Therefore $U(x, z)$ minimizes at (x_D, z_Y) on the boundary between R_{DG} and R_{DY}.

(B5) Boundary between R_{CY} and R_{DG}. This boundary is along the line $z = L_{-}(x)$ and is the line segment joining (x_D, z_Y) and the point (x_C, z_G). We parameterize the directed line segment as follows: For $0 \le s \le 1$, $x = (1 - s)x_D + sx_C$ and $z = (1 - s)z_Y + sz_G$. On this boundary the utility function U is a function of the parameter s. Since $u_{CY} = u_{DG}$ on this boundary, along the boundary we may write for $0 \le s \le 1$, $U(s) = u_{DG}((1 - s)x_D + sx_C, (1 - s)z_Y + sz_G) = V - t[1 - (1 - s)x_D - sx_C] - k[1 - (1 - s)z_Y - sz_G] - d\lambda w_{DG} - F_D = V + st(x_C - x_D) - t(1 - x_D) - k(1 - z_Y) + sk(z_G - z_Y) - d\lambda w_{DG} - F_D$, or $U(s) = u_{CY}((1 -$

$s)x_D + sx_C, (1-s)z_Y + sz_G) = V - t[(1-s)x_D + sx_C] - k[(1-s)z_Y + sz_G] - d\lambda w_{CY} - F_C = V - tx_D + st(x_D - x_C) - kz_Y + sk(z_Y - z_G) - d\lambda w_{CY} - F_C$. If $x_D = x_C$ and $z_Y = z_G$ then $U(s)$ is a constant not dependent on s. However, in general we note that the slope of $z = L_-(x)$ is given by: $-\frac{t}{k} = \frac{z_G - z_Y}{x_C - x_D}$, i.e., $k(z_G - z_Y) = t(x_D - x_C)$. Thus the values of $U(s)$ reduces to the constant: $U(s) = V - t(1-x_D) - k(1-z_Y) - d\lambda w_{DG} - F_D$ or $U(s) = V - tx_D - kz_Y - d\lambda w_{CY} - F_C$. From the analysis above, we could see that $U(x,z)$ minimizes on the points along the boundary on the line $z = L_-(x)$. In particular, $U(x,z)$ minimizes at (x_D, z_Y) or (x_C, z_G) with the same value.

<u>Maximum Fees for Case A:</u> The maximum fees occur when the minimum of the global utility function is zero. Therefore from the formulas in (A5), the maximum fees are given by: $V - tx_C - k(1-z_Y) - d\lambda w_{DY} - F_D = 0$ and $V - t(1-x_C) - kz_Y - d\lambda w_{CG} - F_C = 0$. This gives the maximum fees: $F_C = V - t\left(1 - \frac{x_C + x_D}{2}\right) - k\left(\frac{z_Y + z_G}{2}\right) - d\lambda w_{CG}$ and $F_D = V - t\left(\frac{x_C + x_D}{2}\right) - k\left(1 - \frac{z_Y + z_G}{2}\right) - d\lambda w_{DY}$.

<u>Maximum Fees for Case B:</u> The maximum fees occur when the minimum of the global utility function is zero. Therefore from the formulas in (B5), the maximum fees are given by: $V - t(1-x_D) - k(1-z_Y) - d\lambda w_{DG} - F_D = 0$ and $V - tx_D - kz_Y - d\lambda w_{CY} - F_C = 0$. This gives the maximum fees: $F_C = V - t\left(\frac{x_C + x_D}{2}\right) - k\left(\frac{z_Y + z_G}{2}\right) - d\lambda w_{CY}$ and $F_D = V - t\left(1 - \frac{x_C + x_D}{2}\right) - k\left(1 - \frac{z_Y + z_G}{2}\right) - d\lambda w_{DG}$.

We next solve for the optimal fixed fee for feasible outcomes in symmetric equilibrium when $F_C = F_D = F$.

<u>Optimal F for outcomes 11, 14, 41, and 44:</u> All waiting times are the same and $x_C = x_D = z_Y = z_G = \frac{1}{2}$. Using the formulas for maximum fees above, we get $F_{11} = F_{14} = F_{41} = F_{44} = V - \frac{t}{2} - \frac{k}{2} - \frac{d\lambda}{\mu - \lambda/2}$.

<u>Optimal F for outcome 33:</u> In this outcome, $z_{Y33} = z_{G33} = \frac{1}{2}$ and $x_{D33} = x_{C33} < \frac{1}{2}$. We have four formulas for F which must be consistent. We verify that those in Case A and Case B both reduces to the following formula for F: $F_{33} = V - t(1 - x_{D33}) - \frac{k}{2} - \frac{2d\lambda}{2\mu - (1 - x_{D33})\lambda}$.

<u>Optimal F for outcomes 13 and 43:</u> These outcomes have the same demand distributions and so the same indifferent customers and waiting times. We use the formulas for Case B for these outcomes: $F_{43} = F_{13} = V - t(1 - x_{D43}) - k(1 - z_{Y43}) - \frac{d\lambda}{\mu - N_{DG43}\lambda}$.

<u>Optimal F for outcomes 31 and 34:</u> These outcomes have the same demand distributions and so the same indifferent customers and waiting times. We use the formulas for Case A for these outcomes: $F_{34} = F_{31} = V - t(1 - x_{C34}) - kz_{Y34} - \frac{d\lambda}{\mu - N_{CG34}\lambda}$.

Step 3: Eliminate dominated outcomes

From Step 1, we know that any outcome involving only Y pays on an ISP cannot be an equilibrium. Therefore, outcomes 12, 21, 22, 23, 24, 32, and 42 can be eliminated from the equilibrium analysis as shown in the following table.

		Decisions of Y and G on ISP D			
		1: Neither pays D	2: Y pays D	3: G pays D	4: both pay D
Decisions of Y and G on ISP C	1: Neither pays C	(π_{C11}, π_{D11})	$\cancel{(\pi_{C12}, \pi_{D12})}$	(π_{C13}, π_{D13})	(π_{C14}, π_{D14})
	2: Y pays C	$\cancel{(\pi_{C21}, \pi_{D21})}$	$\cancel{(\pi_{C22}, \pi_{D22})}$	$\cancel{(\pi_{C23}, \pi_{D23})}$	$\cancel{(\pi_{C24}, \pi_{D24})}$
	3: G pays C	(π_{C31}, π_{D11})	$\cancel{(\pi_{C32}, \pi_{D32})}$	(π_{C33}, π_{D33})	(π_{C34}, π_{D34})
	4: both pay C	(π_{C41}, π_{D41})	$\cancel{(\pi_{C42}, \pi_{42})}$	(π_{C43}, π_{D43})	(π_{C44}, π_{D44})

Next, we further eliminate other dominated outcomes by comparing CPs' profits. Recall that: $\pi_{C11} = \pi_{D11} = \frac{F_{11}}{2}$; $\pi_{C14} = \frac{F_{14}}{2}$ and $\pi_{D14} = \frac{F_{14}}{2} + \frac{\lambda p_{14}}{2}$; $\pi_{C41} = \frac{F_{41}}{2} + \frac{\lambda p_{41}}{2}$ and $\pi_{D41} = \frac{F_{41}}{2}$; and $\pi_{C44} = \pi_{D44} = \frac{F_{44}}{2} + \frac{\lambda p_{44}}{2}$. For outcomes 11, 14, 41, and 44, we have $F_{11} = F_{14} = F_{41} = F_{44}$ and equal demand distributions amongst all ISP-CP combinations. Comparing pairs of these outcomes yields $\pi_{D11} < \pi_{D14}$, $\pi_{D41} < \pi_{D44}$, and $\pi_{C14} < \pi_{C44}$. Therefore, outcomes 11, 41, and 14 are dominated and can be eliminated from the equilibrium analysis as shown in the following table.

<table>
<tr><td colspan="2" rowspan="2"></td><td colspan="4">Decisions of Y and G on ISP D</td></tr>
<tr><td>1: Neither pays D</td><td>2: Y pays D</td><td>3: G pays D</td><td>4: both pay D</td></tr>
<tr><td rowspan="4">Decisions of Y and G on ISP C</td><td>1: Neither pays C</td><td>$\cancel{(\pi_{C11}, \pi_{D11})}$</td><td>$\cancel{(\pi_{C12}, \pi_{D12})}$</td><td>$(\pi_{C13}, \pi_{D13})$</td><td>$\cancel{(\pi_{C14}, \pi_{D14})}$</td></tr>
<tr><td>2: Y pays C</td><td>$\cancel{(\pi_{C21}, \pi_{D21})}$</td><td>$\cancel{(\pi_{C22}, \pi_{D22})}$</td><td>$\cancel{(\pi_{C23}, \pi_{D23})}$</td><td>$\cancel{(\pi_{C24}, \pi_{D24})}$</td></tr>
<tr><td>3: G pays C</td><td>(π_{C31}, π_{D11})</td><td>$\cancel{(\pi_{C32}, \pi_{D32})}$</td><td>$(\pi_{C33}, \pi_{D33})$</td><td>$(\pi_{C34}, \pi_{D34})$</td></tr>
<tr><td>4: both pay C</td><td>$\cancel{(\pi_{C41}, \pi_{D41})}$</td><td>$\cancel{(\pi_{C42}, \pi_{42})}$</td><td>$(\pi_{C43}, \pi_{D43})$</td><td>$(\pi_{C44}, \pi_{D44})$</td></tr>
</table>

Next, we compare outcomes 13, 31, 43, and 34. We know $F_{13} = F_{31} = F_{43} = F_{34}$, and the following demand distributions amongst all ISP-CP combinations: $N_{CY13} = N_{DY31} = N_{CY43} = N_{DY34}$, $N_{DY13} = N_{CY31} = N_{DY43} = N_{CY34}$, $N_{CG13} = N_{DG31} = N_{CG43} = N_{DG34}$, and $N_{DG13} = N_{CG31} = N_{DG43} = N_{CG34}$. Recall that $\pi_{C13} = (N_{CY13} + N_{CG13})F_{13}$, $\pi_{D13} = (N_{DY13} + N_{DG13})F_{13} + \lambda p_{13} N_{DG13}$, $\pi_{C31} = (N_{CY31} + N_{CG31})F_{31} + \lambda p_{31} N_{CG31}$, $\pi_{D31} = (N_{DY31} + N_{DG31})F_{31}$, $\pi_{C43} = (N_{CY43} + N_{CG43})(F_{43} + \lambda p_{43})$, $\pi_{D43} = (N_{DY43} + N_{DG43})F_{43} + \lambda p_{43} N_{DG43}$, $\pi_{C34} = (N_{CY34} + N_{CG34})F_{34} + \lambda p_{34} N_{CG34}$, and $\pi_{D34} = (N_{DY34} + N_{DG34})(F_{34} + \lambda p_{34})$. Comparing pairs of these outcomes yields $\pi_{C13} < \pi_{C43}$ and $\pi_{D31} < \pi_{D34}$. Therefore, outcomes 13 and 31 are dominated and can be eliminated from the equilibrium analysis as shown in the following table.

		Decisions of Y and G on ISP D			
		1: Neither pays D	2: Y pays D	3: G pays D	4: both pay D
Decisions of Y and G on ISP C	1: Neither pays C	$\cancel{(\pi_{C11}, \pi_{D11})}$	$\cancel{(\pi_{C12}, \pi_{D12})}$	$\cancel{(\pi_{C13}, \pi_{D13})}$	$\cancel{(\pi_{C14}, \pi_{D14})}$
	2: Y pays C	$\cancel{(\pi_{C21}, \pi_{D21})}$	$\cancel{(\pi_{C22}, \pi_{D22})}$	$\cancel{(\pi_{C23}, \pi_{D23})}$	$\cancel{(\pi_{C24}, \pi_{D24})}$
	3: G pays C	$\cancel{(\pi_{C31}, \pi_{D11})}$	$\cancel{(\pi_{C32}, \pi_{D32})}$	(π_{C33}, π_{D33})	(π_{C34}, π_{D34})
	4: both pay C	$\cancel{(\pi_{C41}, \pi_{D41})}$	$\cancel{(\pi_{C42}, \pi_{42})}$	(π_{C43}, π_{D43})	(π_{C44}, π_{D44})

Therefore, after eliminating all the dominated outcomes, we are left with outcomes 33, 34, 43, and 44 as the only four candidate equilibria.

Step 4: Solve for the equilibrium fixed fee F and preferential delivery fee p for the candidate outcomes

In step 4, we solve for the equilibrium fixed fee F and preferential delivery fee p for the candidate outcomes one by one. Among the four candidate equilibria, outcome 43 and outcome 34 are symmetric. Thus, we focus on outcomes 44, 43, and 33 in this analysis.

<u>Outcome 44</u>

The preferential delivery fee p for outcome 44 is determined by the following two CPs' incentive compatibility constraints: $\pi_{Y44} \geq \pi_{Y43}$ yields $p_{44} \leq \frac{(1/2 - N_{DY43} - N_{CY43}) r_Y}{1/2 - N_{CY43}}$; $\pi_{Y44} \geq \pi_{Y33}$ yields $p_{44} \leq \frac{(1/2 - N_{CY33} - N_{DY33}) r_Y}{1/2}$.

Therefore, $p_{44}^* = H_{44} r_Y$, where $H_{44} = \min\left\{\frac{1/2 - N_{DY43} - N_{CY43}}{1/2 - N_{CY43}}, \frac{1/2 - N_{CY33} - N_{DY33}}{1/2}\right\}$. In addition, we know from the results in Step 2 that $F_{44}^* = V - \frac{t}{2} - \frac{k}{2} - \frac{d\lambda}{\mu - \lambda/2}$.

Outcome 43

The preferential delivery fee p for outcome 43 is determined by the following three CPs' incentive compatibility constraints: $\pi_{Y43} \geq \pi_{Y33}$ yields $p_{43} \leq \frac{(N_{CY43}+N_{DY43}-N_{CY33}-N_{DY33})r_Y}{N_{CY43}}$; $\pi_{Y43} \geq \pi_{Y44}$ yields $p_{43} \geq \frac{(1/2-N_{CY43}-N_{DY43})r_Y}{1/2-N_{CY43}}$; $\pi_{G43} \geq \pi_{G41}$ yields $p_{43} \leq \frac{(N_{CG43}+N_{DG43}-1/2)r_G}{N_{CG43}+N_{DG43}-1/4}$.

Thus, there exists a feasible p_{43} if and only if $\frac{(1/2-N_{CY43}-N_{DY43})r_Y}{1/2-N_{CY43}} \leq \min\left\{\frac{(N_{CY43}+N_{DY43}-N_{CY33}-N_{DY33})r_Y}{N_{CY43}}, \frac{(N_{CG43}+N_{DG43}-1/2)r_G}{N_{CG43}+N_{DG43}-1/4}\right\}$, which can be reduced to $r_G \geq \frac{(N_{CG43}+N_{DG43}-1/4)r_Y}{1/2-N_{CY43}}$ and $\frac{1/2-N_{CY43}-N_{DY43}}{1/2-N_{CY43}} \leq \frac{N_{CY43}+N_{DY43}-N_{CY33}-N_{DY33}}{N_{CY43}}$. When these feasible conditions hold, we obtain $p_{43}^* = \min\{H_{Y43}r_Y, H_{G43}r_G\}$, where $H_{Y43} = \frac{N_{CY43}+N_{DY43}-N_{CY33}-N_{DY33}}{N_{CY43}}$ and $H_{G43} = \frac{N_{CG43}+N_{DG43}-1/2}{N_{CG43}+N_{DG43}-1/4}$.

We know that in a symmetric equilibrium, $\pi_{C43} = \pi_{D43}$, i.e., $(N_{DY43}+N_{DG43})F_{43} + \lambda p_{43}N_{DG43} = (N_{CY43}+N_{CG43})(F_{43}+\lambda p_{43})$. Thus, $F_{43}^* = \frac{(N_{CY43}+N_{CG43}-N_{DG43})\lambda p_{43}^*}{N_{DY43}+N_{DG43}-N_{CY43}-N_{CG43}}$. Note that since $N_{CY43}+N_{CG43} > N_{DG43}$ and $F_{43}^* \geq 0$, we have $N_{DY43}+N_{DG43} > N_{CY43}+N_{CG43}$.

Outcome 33

The preferential delivery fee p for outcome 33 is determined by the following three CPs' incentive compatibility constraints: $\pi_{Y33} \geq \pi_{Y43}$ yields $p_{33} \geq \frac{(N_{CY43}+N_{DY43}-N_{CY33}-N_{DY33})r_Y}{N_{CY43}}$; $\pi_{Y33} \geq \pi_{Y44}$ yields $p_{33} \geq \frac{(1/2-N_{CY33}-N_{DY33})r_Y}{1/2}$; $\pi_{G33} \geq \pi_{G11}$ yields $p_{33} \leq \frac{(N_{CG33}+N_{DG33}-1/2)r_G}{N_{CG33}+N_{DG33}}$; $\pi_{G33} \geq \pi_{G13}$ yields $p_{33} \leq \frac{(N_{CG33}+N_{DG33}-N_{CG43}-N_{DG43})r_G}{N_{CG33}+N_{DG33}-N_{DG43}}$.

Let $L_{33} = \max\left\{\frac{N_{CY43}+N_{DY43}-N_{CY33}-N_{DY33}}{N_{CY43}}, \frac{1/2-N_{CY33}-N_{DY33}}{1/2}\right\}$ and $H_{33} = \min\left\{\frac{N_{CG33}+N_{DG33}-1/2}{N_{CG33}+N_{DG33}}, \frac{N_{CG33}+N_{DG33}-N_{CG43}-N_{DG43}}{N_{CG33}+N_{DG33}-N_{DG43}}\right\}$. Thus, there exists a feasible p_{33} if and only if $L_{33}r_Y \le H_{33}r_G$. Here we note that $L_{33} \ge \frac{1/2-N_{CY33}-N_{DY33}}{1/2}$ and $H_{33} \le \frac{N_{CG33}+N_{DG33}-1/2}{N_{CG33}+N_{DG33}}$. So we have $\frac{L_{33}}{H_{33}} \ge \frac{\frac{1/2-N_{CY33}-N_{DY33}}{1/2}}{\frac{N_{CG33}+N_{DG33}-1/2}{N_{CG33}+N_{DG33}}} = \frac{N_{CG33}+N_{DG33}}{1/2} > 1$. When these feasible conditions hold, we obtain $p^*_{33} = H_{33}r_G$. In addition, we know from the results in Step 2 that $F^*_{33} = V - t(1-x_{33}) - \frac{k}{2} - \frac{d\lambda}{\mu-(1-x_{D33})\lambda/2}$.

We note here that the solution of price p in outcomes 44, 43, and 33 form three non-overlapping intervals. Specifically, we have $p_{44} \le H_{44}r_Y \le \frac{(1/2-N_{CY43}-N_{DY43})r_Y}{1/2-N_{CY43}} \le p_{43} \le \min\{H_{Y43}r_Y, H_{G43}r_G\} \le L_{33}r_Y \le p_{33} \le H_{33}r_G$. The non-overlapping solution reflects the fact that incentive criteria for content providers in outcomes 44, 43, and 33 are mutually exclusive. Observe also that the endpoints of the non-overlapping intervals are given by constant multiples of the revenue rates r_Y and r_G.

Step 5: Compare the candidate outcomes and derive equilibrium outcomes

In step 5, we compare the ISPs' profits in outcomes 44, 43, and 33 to determine the equilibrium outcomes. Since ISPs C and D have the same profit level in a given outcome, we simplify the notations to $\pi_{C44} = \pi_{D44} = \pi_{44}$, $\pi_{C43} = \pi_{D43} = \pi_{43}$, and $\pi_{C33} = \pi_{D33} = \pi_{33}$.

Outcome 33 is the equilibrium provided all the following inequalities are satisfied: $L_{33}r_Y \le H_{33}r_G$, $\pi_{33} \ge \pi_{43}$, and $\pi_{33} \ge \pi_{44}$. These reduces to the following inequalities: $r_G \ge$

$\frac{L_{33} r_Y}{H_{33}} \equiv \beta_1 r_Y$, $r_G \geq \frac{(N_{CY43}+N_{CG43})p_{43}}{H_{33}N_{CG33}} + \frac{(N_{CY43}+N_{CG43})F_{43}-F_{33}/2}{\lambda H_{33}N_{CG33}} \equiv \alpha_1$, and $r_G \geq \frac{H_{44} r_Y}{2H_{33}N_{CG33}} + \frac{F_{44}-F_{33}}{2\lambda H_{33}N_{CG33}} \equiv \beta_2 r_Y + \alpha_2$.

Outcome 43 (or outcome 34) is the equilibrium provided all the following inequalities are satisfied: $r_G \geq \frac{(N_{CG43}+N_{DG43}-1/4)r_Y}{1/2-N_{CY43}} \equiv \beta_3 r_Y$, $\pi_{43} > \pi_{33}$, and $\pi_{43} \geq \pi_{44}$. These reduces to the following inequalities: $r_G \geq \beta_3 r_Y$, $r_G < \alpha_1$, and $r_Y \leq \frac{2(N_{CY43}+N_{CG43})p_{43}}{H_{44}} - \frac{F_{44}-2(N_{CY43}+N_{CG43})F_{43}}{\lambda H_{44}} \equiv \alpha_3$.

When the above market conditions are not satisfied, outcome 44 is the equilibrium. Summarizing the above analysis yields Lemma 2.

Proof of Proposition 1

Since the net neutrality regime is essentially equivalent to outcome 11, where neither CP pays for preferential delivery even though they have the option to do so. Based on the results from Lemma 2, we know that in the net neutrality regime, $\pi_C^{NN} = \pi_D^{NN} = \pi_{11}^* = \frac{F_{11}^*}{2}$. In addition, there are three possible equilibria in the packet discrimination regime, i.e., $\pi_C^{PD} = \pi_D^{PD} = \pi_{33}^* = \frac{F_{33}^*+\lambda p_{33}^*(1-x_{33})}{2}$, or $\pi_C^{PD} = \pi_D^{PD} = \pi_{43}^* = N_{C43}(F_{43}^* + \lambda p_{43}^*)$, or $\pi_C^{PD} = \pi_D^{PD} = \pi_{44}^* = \frac{F_{44}^*+\lambda p_{44}^*}{2}$. From the results in Step 3 in the proof of Lemma 2, we know $\pi_{44}^* \geq \pi_{11}^*$. Therefore, we get $\pi_C^{PD} = \pi_D^{PD} \geq \pi_{44}^* \geq \pi_{11}^* = \pi_C^{NN} = \pi_D^{NN}$.

Proof of Proposition 2

In the net neutrality regime, we know that $\pi_G^{NN} = \pi_{G11}^* = \frac{\lambda r_G}{2}$. In the packet discrimination regime, there are three possible equilibria – outcomes 33, 43, and 44. The corresponding profit for content provider G is: $\pi_{G33}^* = \lambda(N_{CG33} + N_{DG33})(r_G - p_{33}^*)$, $\pi_{G43}^* = \lambda(N_{CG43} + N_{DG43})(r_G - p_{43}^*)$, and $\pi_{G44}^* = \frac{\lambda(r_G - p_{44}^*)}{2}$. Next we focus on comparing π_{G33}^* and π_{G11}^*.

Recall that $p_{33}^* = H_{33} r_G$, where $H_{33} = \min\left\{\frac{N_{CG33}+N_{DG33}-1/2}{N_{CG33}+N_{DG33}}, \frac{N_{CG33}+N_{DG33}-N_{CG43}-N_{DG43}}{N_{CG33}+N_{DG33}-N_{DG43}}\right\}$. If $p_{33} = \frac{(N_{CG33}+N_{DG33}-1/2)r_G}{N_{CG33}+N_{DG33}}$, then $\pi_{G33}^* = \lambda(N_{CG33} + N_{DG33})\left(r_G - \frac{(N_{CG33}+N_{DG33}-1/2)r_G}{N_{CG33}+N_{DG33}}\right) = \frac{\lambda r_G}{2} = \pi_{G11}^*$. If $p_{33} = \frac{(N_{CG33}+N_{DG33}-N_{CG43}-N_{DG43})r_G}{N_{CG33}+N_{DG33}-N_{DG43}}$, then $\pi_{G33}^* = \lambda(N_{CG33} + N_{DG33})\left(r_G - \frac{(N_{CG33}+N_{DG33}-N_{CG43}-N_{DG43})r_G}{N_{CG33}+N_{DG33}-N_{DG43}}\right) = \frac{N_{CG43}(N_{CG33}+N_{DG33})\lambda r_G}{N_{CG33}+N_{DG33}-N_{DG43}} \geq \frac{\lambda r_G}{2} = \pi_{G11}^*$. Thus, CP G's profit in outcome 33 is higher than that in outcome 11 if and only if $\frac{N_{CG33}+N_{DG33}-N_{CG43}-N_{DG43}}{N_{CG33}+N_{DG33}-N_{DG43}} < \frac{N_{CG33}+N_{DG33}-1/2}{N_{CG33}+N_{DG33}}$, which can be simplified to $(N_{CG33} + N_{DG33})N_{CG43} > \frac{N_{CG33}+N_{DG33}-N_{DG43}}{2}$.

From the proof of Lemma 1, we know $N_{CG33} + N_{DG33} = 1 - x_{33}$, $N_{CG43} = \frac{z_{G43}}{2}$, and $\frac{1}{2} - x_{D43} = \frac{k}{t}(z_{Y43} - z_{G43})$. This gives: $N_{DG43} = (1 - x_{D43})(1 - z_{G43}) - \frac{1}{2}(z_{Y43} - z_{G43})\left(\frac{1}{2} - x_{D43}\right) = (1 - x_{D43})(1 - z_{G43}) - \frac{k}{2t}(z_{Y43} - z_{G43})^2$.

Substituting these equations into the $(N_{CG33} + N_{DG33})N_{CG43} > \frac{N_{CG33}+N_{DG33}-N_{DG43}}{2}$ yields $(1 - x_{33})\left(\frac{z_{G43}}{2}\right) > \frac{1}{2}\left((1 - x_{33}) - (1 - x_{D43})(1 - z_{G43}) + \frac{k}{2t}(z_{Y43} - z_{G43})^2\right)$. Rearranging this inequality gives $\frac{t}{k} > \frac{(z_{Y43}-z_{G43})^2}{2(x_{33}-x_{D43})(1-z_{G43})}$. Therefore, if the ratio of $\frac{t}{k}$ is higher than a threshold, $\pi_{G33}^* > \pi_{G11}^*$.

In general, comparisons of π_{G33}^*, π_{G44}^*, and π_{G11}^* show that CP G's profit may be lower, unchanged or higher in the packet discrimination regime than that in the net neutrality regime.

Specifically, it is lower under equilibrium 44, but is unchanged or higher under equilibrium 33, i.e., $\pi^*_{G33} \geq \pi^*_{G11} \geq \pi^*_{G44}$.

Proof of Proposition 3

In the net neutrality regime, we know that $\pi_Y^{NN} = \pi^*_{Y11} = \frac{\lambda r_Y}{2}$. In the packet discrimination regime, there are three possible equilibria – outcomes 33, 43, and 44. The corresponding profit for content provider Y is: $\pi^*_{Y33} = \lambda x_{33} r_Y$, $\pi^*_{Y43} = \lambda(N_{Y43} r_Y - N_{CY43} p^*_{43})$, and $\pi^*_{Y44} = \frac{\lambda(r_Y - p^*_{44})}{2}$. We compare Y's profit in the three possible equilibria in the packet discrimination regime to its profit in the net neutrality regime one by one. We first note that $\pi^*_{Y11} \geq \pi^*_{Y44}$. Furthermore, since $x_{33} \leq \frac{1}{2}$, we get that $\pi^*_{Y11} \geq \pi^*_{Y33}$. Lastly, since $p^*_{44} \leq p^*_{43}$, $\pi^*_{Y44} \geq (N_{CY43} + N_{DY43})\lambda r_Y - N_{CY43}\lambda p^*_{44} \geq (N_{CY43} + N_{DY43})\lambda r_Y - N_{CY43}\lambda p^*_{43} = \pi^*_{Y43}$.

Summarizing the above, we conclude that Y's profit is higher in the net neutrality regime than that in all three possible equilibria in the packet discrimination regime. Therefore, $\pi_Y^{NN} \geq \pi_Y^{PD}$.

Proof of Proposition 4

Substituting the equilibrium prices into the social welfare formula $SW_{ij} = \pi_{Cij} + \pi_{Dij} + \pi_{Yij} + \pi_{Gij} + \int_0^1 \int_0^1 U_{ij}(x, z) dx dz$, we get that, in the net neutrality regime, $SW^{NN} = SW_{11} = V - \frac{t+k}{4} - \frac{d\lambda}{\mu - \lambda/2} + \frac{\lambda(r_Y + r_G)}{2}$. In the packet discrimination regime, there are three possible equilibria – outcomes 33, 43, and 44. The corresponding social welfare is: $SW_{33} = V - t\left(\frac{1}{2} - x_{33}^2\right) - \frac{k}{4} - \frac{d\lambda}{\mu - (1 - x_{33})\lambda/2} + \lambda x_{33} r_Y + \lambda(1 - x_{33}) r_G$, $SW_{43} = F_{43} + \frac{\lambda(r_Y + r_G)}{2} + \frac{\lambda(r_G - r_Y)}{2}(2 - z_{Y43} - z_{G43})\left(\frac{1}{2} - \right.$

$x_{D43}\Big) + \lambda N_{Y43} r_Y + \lambda N_{G43} r_G + t\left(x_{D43} - \frac{1}{2}\right) + k\left(z_{G43} - \frac{1}{2}\right) + \frac{k}{2}\left(x_{D43} + \frac{1}{2}\right)(z_{Y43} - z_{G43}) +$

$\frac{t}{2}(z_{Y43} + z_{G43})\left(\frac{1}{2} - x_{D43}\right) + \frac{2t^2}{3k}\left(\frac{1}{8} - x_{D43}^3\right) + \frac{2k^2}{3t}(z_{Y43}^3 - z_{G43}^3) - \frac{t}{2k}(t + 2kz_{G43})\Big(\frac{1}{4} -$

$x_{D43}^2\Big) - \frac{k}{2t}(t + 2kz_{G43})(z_{Y43}^2 - z_{G43}^2)$, and $SW_{44} = V - \frac{t+k}{4} - \frac{d\lambda}{\mu - \lambda/2} + \frac{\lambda(r_Y + r_G)}{2}$.

We first note that $SW_{44} = SW_{11}$. Furthermore, since $x_{33} \leq \frac{1}{2}$, we get that $SW_{33} \geq SW_{11}$. Lastly, we compare SW_{43} and SW_{11}. Let $\Delta SW = SW_{43} - SW_{11}$. We can show that $\frac{\partial \Delta SW}{\partial \mu} \geq 0$ and $\Delta SW = 0$ at $\mu = \lambda$. Therefore, $SW_{43} \geq SW_{11}$.

Summarizing the above, we conclude that social welfare is weakly higher in all three possible equilibria in the packet discrimination regime than that in the net neutrality regime. Therefore, $SW^{PD} \geq SW^{NN}$.