Search and Authentication in Online Matching Markets

Abstract

Compared to offline matching markets, online matchmaking firms improve search in the matchmaking process, but at the same time, increase the problem of authenticating the features and credentials of prospective matches. This paper examines the interplay between these two processes in online matchmaking, using game-theoretic models. We examine whether an online matchmaking firm should target a broad market of match seekers, or an exclusive group of high-value seekers, and how the firm can use a two-part pricing approach for search and authentication services. Our results provide valuable insights for online matchmaking firms regarding the tradeoffs between search and authentication services, and providing guidelines for the pricing and positioning of their services. For instance, we show that the complementarity of the firm's optimal pricing for search and authentication services can lead to the firm offering an authentication service as a loss leader, and that higher quality authentication services may not justify higher authentication fees. We also develop guidelines for the firm's optimal strategies for different market conditions.

Keywords: matching, search, authentication, online markets, pricing

1 Introduction

"My mama always said - Life was like a box of chocolates. You never know what you're gonna get."

-Forrest Gump

Matching is an age-old problem, whether between people or organizations. It involves two parties finding each other by searching within a possibly large population. A common example of matching is the search for a spouse; another is the search for potential business partners. In recent years, firms such as Ariba Networks and Match.com have created online matchmaking platforms that significantly enhanced the ability of match seekers to reach larger populations of potential matches. At the same time, going online to search for partners is not always a clear choice. In fact, a number of early online firms trying to create matching platforms, such as Vertical Markets and Open Markets, were unsuccessful. One reason for their failures was their inability to generate enough participation due to concerns about the risks associated with the online environment and the quality of candidate matches available. In other words, although online matchmaking firms significantly enhance the ability of match-seekers to *search*, the online environment also introduces an *authentication* problem that can be significant.

Search is the process of enabling transacting parties to find each other, and of enabling buyers to find the products and services that they seek. And authentication is the process of enabling transacting parties to assure themselves of the authenticity and quality of their counter-parties, and of enabling buyers to assure themselves of the quality of the offered products/services. Clearly, both these processes are essential to the viability of any market. To see this, consider a market where there are many buyers and sellers, so that search is very effectively supported, but the participants know nothing about each other, or about the products and services that are offered by the sellers. Most buyers would balk at transacting in such a setting. For example, the strength of the New York Stock Exchange as a stock market derives not only from the broad selection of listed stocks that it offers to investors, but also from the rigorous authentication mechanisms it uses to qualify listed companies, including tests of financial robustness and compliance with reporting standards.

Online merchants and market operators face significant choices about the levels of search and authentication services they provide. For instance, some online intermediaries focus primarily on the search process, pinning their success to the number of participants they can attract and engage.

The early days of the commercial Internet saw plenty of such businesses, ranging from America Online (AOL) and MySpace in the consumer sector to Free Markets, Vertical Markets and Open Markets in the business-to-business sector. However, very few of these intermediaries survived, not least because they could not generate and sustain participation by quality-conscious participants. Other ventures that have been more successful, such as Facebook, Linked-In and Amazon, have clearly invested significantly in incorporating authentication mechanisms that increase confidence among potential participants that they will be interacting with credible and bona fide counterparties (see Basu and Muylle (2003) for various examples).

Matching markets introduce an additional challenge in that a matchmaking process involves two active match-seekers, both of whom have to evaluate the value of a potential match. Furthermore, a match is successful only if both match-seekers accept the match.[1] This is different from one-sided markets for products and services, where sellers present their offerings, and then the buyer is the only decision maker in the purchase process.

In the familiar context of matching among individuals (perhaps for dating or marriage), potential partners find each other in the offline setting through mechanisms such as meetings and social events; however, once they meet, the challenges of authentication are relatively low. When online matchmaking intermediaries such as e-Harmony.com and Match.com are used, they significantly improve the search process. However, in an online matching market, authentication is more complicated since they may not have the opportunity to directly interact before they accept a match. And as the old adage goes "On the Internet, no one knows you are a dog".

In B2B settings, the online matchmaking firms can offer match-seeking firms the use of authentication mechanisms such as financial disclosures, letters of reference and third party authenticators such as the Better Business Bureau and Standard and Poors. In consumer-oriented settings such as marriage markets, the online matchmaking firm can offer individual match-seekers mechanisms such as detailed application forms and multimedia tools, supplemented by educational and financial credentials and even third-party authenticators (e.g., I Am Real™), to authenticate themselves.

However, authentication mechanisms and processes can be expensive. Furthermore, while some of the mechanisms used by the match-maker, such as requiring match-seekers to submit copies of

[1] Although matches can also involve three or more parties, for ease of exposition, we limit our discussion to bilateral matches only.

credentials (e.g., certificates, diplomas, letters of reference) may help, they are imperfect, since they can be forged/edited. Achieving robust ("perfect") authentication may be possible, but only at a very high cost, which the intermediary typically passes along to the match-seekers in the form of (additional) fees.

The purpose of this paper is to examine the tradeoffs between search and authentication in online matching markets. As part of this, we also examine the question of whether an online matching firm should target a broad and inclusive (open) market of match seekers, or an exclusive (closed) group of high-value seekers. Using simple game-theoretic models, we study the effectiveness of pricing as a mechanism for achieving effective market structure and performance. We show that online matching firms can use two-part pricing for search and authentication services to effectively serve a broad range of customers, even in the presence of the authentication problem caused by low-value entities misrepresenting themselves as high-value entities.

Our results provide valuable insights for owners and operators of online match-making intermediaries, ranging from consumer matching sites such as Match.com and eHarmony to online agents matching potential business partners (e.g., Vantage Agora), by helping them understand the tradeoffs between search and authentication capabilities and services, and providing guidelines for the pricing and positioning of their services. From a research standpoint, this work contributes to two streams of literature. First, it adds to the rich literature on the economics of matching, by examining the interplay between search and authentication. Second, it adds to the literature on online markets, by bringing in the context of matching markets and the effective management of the two key transaction processes of search and authentication.

2 Relevant Literature

Our work builds upon a broad range of literature, ranging from bilateral search markets to online search and authentication processes. The literature on online search has broadly focused on search algorithms (Brin and Page, 1998), mechanisms to influence or guide the search process through recommendations and its impact on search (Xiao and Benbasat, 2007), search costs and its impact on search process flow (Brynjolfsson et al., 2011), the role of e-commerce platforms in reducing search frictions and improving conversions (Moe and Fader, 2004).

The term 'matching' refers to the two-sided nature of interchange between the two active seekers of the bilateral market Adachi (2003). In bilateral search markets, search is successful when a mutually acceptable match is found from two disjoint sets of seekers (Roth and Sotomayor, 1992). In such bilateral search markets, the intermediary's reputation and therefore its future profits may suffer due to unsuccessful matches on the platform. The intermediary's concern in such a setting can be alleviated by investing in better search or matching technology.

The extant literature on matching in bilateral search markets can be broadly classified into two streams. One stream of literature pioneered by Gale and Shapley has focused on the design and analysis of matching mechanisms that lead to stable matches (Gale and Shapley, 1962; Shapley and Shubik, 1971), whereby the two active seekers that have been matched prefer each other over any other active seekers; the Gale Shapley algorithm is used in the National Residents Matching Program[2] to match medical students seeking residency in medical programs in the USA. While Gale and Shapley (1962) assumed that both sides in a two sided matching market has enough information to rank agents on the other side. Roth (1989) and Chakraborty et al. (2010) analyze the role of uncertainty over other agent's and one's own preferences respectively, and the role of an intermediary in a bilateral search market. One other type of market in which bilateral search occurs is a double auction market(McAfee, 1992; Fan et al., 2003). However, the focus of such markets is on the support of efficient valuation and price discovery, which is very different from the matching problem addressed in this paper.

The other stream of matching literature has focused on the role of an intermediary in bilateral search markets (Cosimano, 1996; Burdett and Coles, 1997). Chade (2001) and Smith (2006) proposed the notion of perfect segregation, a selective behavior of seekers where only those belonging to the same class are matched to each other. Damiano and Li (2007) use a static framework to show that the seekers' selective behavior occurs as a result of intermediary's revenue maximization. Damiano and Hao (2008) shows that the selective behavior is less efficient in a duopolistic outcome compared to the monopolistic outcome because the role of search fee to facilitate selective behavior is undermined by the need to survive price competition. The selectivity in seeker's behavior caused by pricing in the matching market with search frictions differs from the standard Bertrand models where undercutting a rival's price may increase market share and revenues. In price competition in

[2] www.nrmp.org

bilateral markets, overtaking in prices may lead to higher revenues due to the perception of higher quality agents in such markets. Mailath et al. (2013) focus on investments by seekers in the matching markets and study the tradeoff between differentiated search fees and inefficient investments under uniform search fee.

The literature on authentication in search markets has focused on engineering challenges to deal with authentication problems (Jain et al., 1999). Also, the problem of reliably transacting online, even among anonymous parties, has been examined in Ba (2001) and Kalvenes and Basu (2006), while the impact of various factors on buyer trust in e-Commerce has been studied in Koh et al. (2012). The need for better authentication in bilateral search markets is well recognized by firms and seekers that have raised issues such as the impact of fake profiles in online dating markets, effect of fake resumes and job posts on the efficacy of the job search platform,[3,4,5] etc. However, the literature on authentication in bilateral search markets with dual active seekers and its impact on firm's strategy and seeker behavior is sparse.

In this paper, we focus on two types of bilateral search markets with dual active seekers; *exclusive* markets where seekers find a match acceptable if it belongs to the same class or type, and *inclusive* markets where a seeker is willing to accept anyone in that market. We study the market structures and firm's pricing strategies under which the firm finds it profitable to be exclusive or inclusive. While the presence of the firm can significantly improve the search process, the inability to examine each other and verify credentials, an implicit characteristic of the direct market raises authentication challenges in the intermediated bilateral search market. We analyze the role of authentication on seekers' behavior and the firm's investment strategies in search and authentication.

3 Model Description

We consider a setting in which match-seekers (individuals or firms) seek matches with other match-seekers within a marketplace. The process of matching occurs through encounters between active match-seekers. We assume that entities become active seekers randomly, and become candidates for matches with other seekers who become aware of them. The probability μ $(0 \leqslant \mu \leqslant 1)$ of

[3]http://online.wsj.com/news/articles/SB10001424052748703834804576300973195520918
[4]http://www.huffingtonpost.com/2012/02/14/online-dating-scams_n_1263837.html
[5]http://nypost.com/2013/11/22/model-suing-match-com-for-1-5b-over-fraudulent-fake-profiles/

a seeker detecting the availability of a particular match candidate defines the search efficiency of the matching market. In other words, if the search mechanism were perfect, a seeker would be alerted to each new active candidate. When a seeker encounters a potential candidate, the match is considered successful only if it is mutually acceptable to both parties involved. For simplicity of exposition and analytical tractability, we consider a 2-period setting for the matching process such that a match-seeker is offered at most one possible match in each period. Such a model is sufficient to capture the temporal element associated with any search process wherein match-seekers can choose to wait for a better match if they are not satisfied with the match they are offered in any given period. To account for the fact that each match-seeker would like to find a match as soon as possible, the value of a match achieved in the second period is discounted by a factor δ ($0 \leqslant \delta \leqslant 1$) for any match-seeker. The discount factor δ can be thought of as the level of patience of the match seekers in the market.

We assume that there are two types of match-seekers in the market: a set of high-type (H-type) match-seekers and a set of low-type (L-type) match-seekers. Let α ($0 \leqslant \alpha \leqslant 1$) be the proportion of H-types in the market. Then, α is also the probability that a newly active seeker is an H-type. Consistent with the notion of a bilateral match, the value derived by a match-seeker from a match depends on the type of match-seeker they are matched with, and we assume that a match with an H-type is preferred by all match-seekers. More specifically, we assume that obtaining a match with an H-type gives a value of v_H to any match-seeker, while a match with an L-type provides a value of v_L, where $v_H > v_L$. This implies that when an H-type matches with another H-type (the probability of which is $\mu\alpha$), both parties receive a value of v_H. In contrast, when the same H-type is matched with an L-type (with probability $\mu(1-\alpha)$), the value that the H-type derives from the match is v_L while the L-type derives a value v_H.

Given that a match with an H-type is preferred by all match-seekers, there are two choices for match-seekers. First, a match-seeker can choose to be *exclusive* by accepting matches with H-types only. Since only a fraction of the population is H-type, the probability ($\mu\alpha$) of obtaining a match would be lower when the seeker is exclusive than when they are *inclusive* by accepting matches with any type (the probability of which is 1 in any period). So inclusivity improves the odds of finding an acceptable match, but reduces the expected value.[6] If a seeker does not find

[6]Note that the L-type can be exclusive only when the H-type is willing to accept matches from L-types.

an acceptable match in period 1, they move on to the second period where the same process is repeated. However, in this terminal period, they accept whatever match is available to them. We also assume that when match-seekers exit the market, they are replaced by match-seekers of the same type, thus keeping the distribution of types invariant over time (Chade and Smith, 2006).

The direct market described above is simple, but may be time-consuming and inefficient (i.e., $\mu < 1$). The search process can be improved by using an online firm such as a matchmaking service or broker. We assume that such a firm detects every new active seeker, thereby generating a candidate match in each period (i.e., $\mu = 1$). The firm may use Web technologies (e.g., Ariba Networks, Match.com, eHarmony) to achieve the higher search efficiency.

While an online intermediary improves the search process, the anonymity that comes with an online setting makes authentication of match-seekers more difficult than in a direct marketplace. In a direct marketplace, potential matches occur through a process of direct interaction that allows each party to examine the other. In contrast, in an intermediated market, it is possible for an L-type to misrepresent itself as an H-type, and thereby potentially get matched with an H-type even when the H-type is exclusive. Thus the intermediated market solves the search problem, but at the same time introduces an authentication problem. This tension between search and authentication is at the core of this work.

One way to address the authentication challenge in the intermediated market is to use some type of authentication service, such as a certificate authority or other similar third party trustees. Using such an authentication service, the firm may be able to determine the true type of a match-seeker. In the setting of our models, the authentication service is only used to determine whether a match-seeker is an H-type. If the firm chooses to offer this service, it charges an additional fee q for it.

The authentication service always correctly identifies an H-type. On the other hand, we assume that γ (where $0 \leqslant \gamma \leqslant 1$) denotes the probability that the authentication service correctly classifies an L-type.[7] Therefore, when authentication is imperfect (i.e., $\gamma < 1$), an L-type motivated to mimic an H-type may succeed in defeating the authentication process. We assume that the cost of the authentication service to the firm is a linear function of γ of the form $\kappa + c\gamma$ where κ and c represent

[7] While we make this assumption for simplicity, the framework that we have developed can incorporate both type-1 and type-2 errors of the authentication system.

the fixed and marginal costs respectively.

In such a setting, each match-seeker has to decide (1) whether to stay in the direct market or pay the search fee to use the firm, and if the latter option is chosen, (2) whether to pay the firm the additional fee for the authentication service. From the firm's perspective, the relevant decisions are (1) which market segments it should target, and (2) what prices it should set for its search and authentication services respectively.

We analyze the scope and pricing problems facing the matchmaking firm in three stages. We start by considering the case when the firm offers only search services. Next, when the firm can also offer an authentication service, we consider two alternatives. First, we examine a setting in which the authentication service is perfect (in correctly identifying match-seekers' types), and study the firm's optimal strategy and pricing of search and authentication services. We then consider the implications of the authentication service being imperfect, and once again study the firm's optimal strategy and pricing of search and authentication services.

3.1 Baseline Case: Direct Market

We start by considering the simple scenario in which match-seekers use a direct market (i.e., without any matching intermediary).

When the H-type prefers to be exclusive in the first period, the expected values that H- and L-types receive from an acceptable match are $\mu\alpha v_H$ and $(1-\alpha)\,\mu v_L$respectively. Given that the H-type accepts any match in the terminal period, we can denote the value functions of H- and L-types as

$$V_{eH}^{D} = \alpha\mu v_H + \delta\mu\,(1-\alpha\mu)\,(\alpha v_H + (1-\alpha)v_L) \quad (1)$$

$$V_{eL}^{D} = (1-\alpha)\,\mu v_L + \delta\mu\,(1-(1-\alpha)\,\mu)\,(\alpha v_H + (1-\alpha)v_L) \quad (2)$$

On the other hand, when the H-type is inclusive in the first period, the value functions of H- and L-types are,

$$V_{iH}^{D} = V_{iL}^{D} = (\mu + \delta\mu\,(1-\mu))\,(\alpha v_H + (1-\alpha)v_L) \quad (3)$$

		H-type	
		Direct	Online
L-type	Direct	V_H^D, V_L^D	V_H^{M1}, V_L^{M1}
	Online	V_H^{M3}, V_L^{M3}	V_H^{M2}, V_L^{M2}

Table 1: Payoffs for online market with search only

Comparing eq. 1 and 3 gives us the conditions under which an H-type prefers to be exclusive, as stated in the following proposition:

Proposition 1. *In a direct marketplace,*

i) There exists a threshold on $\eta = \frac{v_H}{v_L}$ *such that when* $\eta > \overline{\eta}$*, H-types prefer to be exclusive, while if* $\eta \leqslant \overline{\eta}$*, they prefer to be inclusive*

ii) $\overline{\eta}$ *is decreasing in* μ, α *and* δ*.*

Consistent with intuition, the H-type's decision to be exclusive or inclusive in the period 1 depends on the quality of the direct marketplace (α, η, μ). When the value derived by matching to an H-type relative to matching with an L-type increases beyond a threshold, the H-type prefers to be exclusive. Furthermore, as the probability of meeting with other match-seekers(μ) or the proportion of H-types (α) increases, the H-type prefers to be exclusive even for lower values of η. We see a similar effect when the match-seekers are also more patient (higher δ), since the perceived penalty for not finding a suitable match in the first period is lower when δ is higher.

3.2 Online Market Supporting Search Only

Now consider an online marketplace with a matchmaking firm that provides search services. While the search efficiency improves with the intervention of the online matchmaker, authentication issues are introduced due to the ability of the L-type to mimic the H-type. We start by assuming that the matchmaking firm offers only search services. Let p be the search fee that the firm charges to the match-seekers to use its services. Both H- and L-types have to choose between staying in the direct marketplace and employing the online match-making firm. The payoffs from the resulting 2x2 game are shown in Table 1.

We next describe each of the payoff functions represented in the table.

Case 1: When both types search in the direct market, their value functions would be as given in the previous section. So

$$V_k^D = \begin{cases} V_{ek}^D & \text{when } H\text{-type is exclusive} \\ \\ V_{ik}^D & o.w. \end{cases} \tag{4}$$

where $k \in \{H, L\}$.

Case 2: When only H-types are in the online marketplace, the value function for the H- and L-types from both periods are

$$V_H^{M1} = \alpha v_H + \delta (1-\alpha) \alpha v_H - p \tag{5}$$

$$V_L^{M1} = \mu (1-\alpha) v_L + \delta(1-\mu(1-\alpha))\mu (1-\alpha) v_L \tag{6}$$

Note that since the online market has only H-type seekers, they are implicitly exclusive. Similarly, since there are only L-types in the direct market, they are limited to matches amongst themselves.

Case 3: When both H- and L-types are online, the L-type has an incentive to mimic the H-type. In the absence of any authentication mechanism, the H-type has no means of identifying true H-types. As a result, being exclusive becomes infeasible and the value functions of both types are

$$V_H^{M2} = V_L^{M2} = \alpha v_H + (1-\alpha) v_L - p \tag{7}$$

Case 4: When only the L-type is in the online market, the value derived by the two types are

$$V_H^{M3} = \alpha \mu v_H + \delta (1-\alpha\mu) \mu \alpha v_H \tag{8}$$

$$V_L^{M3} = (1-\alpha) v_L + \delta\mu (1-(1-\alpha)) (1-\alpha) v_L - p \tag{9}$$

Given these four cases, the following lemma characterizes the equilibrium behavior of both types of match-seekers.

Lemma 1. *There exist thresholds $\overline{p}_M$ and $\underline{p}_M$ such that for any search fee p, the equilibrium behavior of the H- and L-type match-seekers will be as follows:*

i) When $p > \overline{p}_M$, both types search in the direct market, *ii) When $\underline{p}_M < p \leqslant \overline{p}_M$, the H-type searches in the online market, while the L-type searches in the direct market, and*

iii) When $p \leqslant \underline{p}_M$, both types search in the online market.

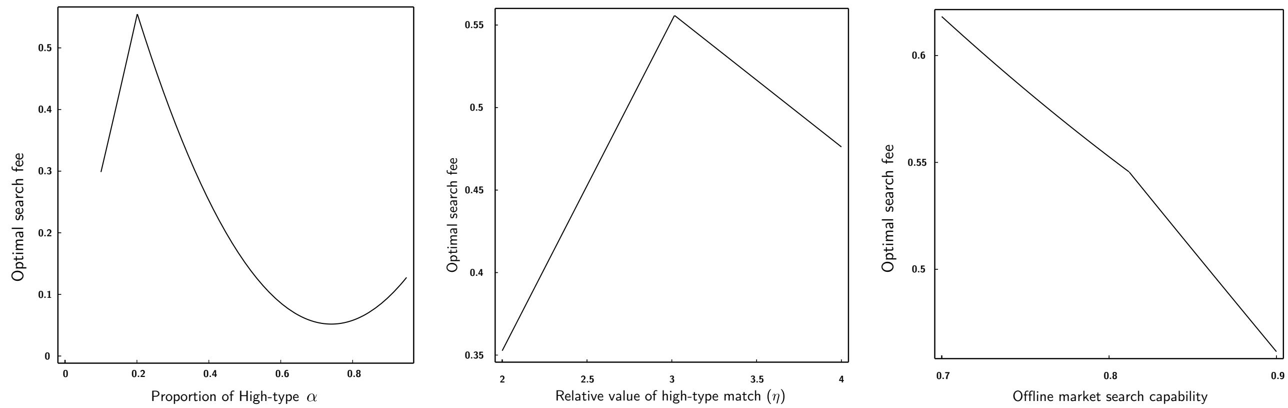
The above lemma shows that the behavior of the match seekers depends upon the search fees set by the firm, i.e., each type chooses to use the firm only if the search fee is low enough, consistent with intuition. An interesting observation is that the L-type never uses the online firm unless the H-type does so as well. The reason for this is that the search benefits of the online market are insufficient to motivate L-types to use the online market, unless there is the possibility of matching with an H-type as well (as in the direct market). Since there are no H-types in the online market in Case 4, it is never feasible.

Now that we have identified the match-seekers' behavior for a given search fee, we analyze the firm's equilibrium strategy. From the above discussion, we know that the firm can either offer its search capability to both types or cater to H-types alone. Should the firm charge a higher search fee and cater exclusively to the H-types (thereby losing the market share from L-type) or choose to cover the market with a lower price? The profit function of the firm when it chooses to serve the H-types only is $\pi^{M1} = \alpha p$ where $\underline{p}_M \leqslant p \leqslant \overline{p}_M$, and when it serves both types is $\pi^{M2} = p$ where $p \leqslant \underline{p}_M$.

Proposition 2. *It is never optimal for the firm to price such that only H-types search in the online market.*

The above proposition implies that the matchmaking firm cannot price the L-type out of the online market when it offers search service only. The reason for this is the interaction between the attractiveness or profitability of H-types and the incentive for L-types to mimic. When η is high, the firm may be motivated to focus on the H-type only. However, higher values of η also increase the incentives for L-types to use the online firm pretending to be H-types. As a result, the firm is better served by offering a search fee that attracts both types to the online market.

The maximum fee that H-types would be willing to pay to search in an online market with L-types is $V_H^{M2} - V_H^{M3}$. Similarly, the maximum fee that L-types would be willing to pay to search in an online market with H-types is $V_L^{M2} - V_L^{M1}$. In order to attract both types, the maximum search fee (which is also the optimal fee, p^*) that the firm can charge is the lower of these two



$\delta = 0.9; v_H = 3; v_L = 1; \mu = 0.8$ Effect of α | $\delta = 0.9; v_L = 1; \mu = 0.8; \alpha = 0.2$ Effect of η | $\delta = 0.9; v_H = 3; v_L = 1; \alpha = 0.2$ Effect of μ

Figure 1: Sensitivity analysis of optimal search fee under no-authentication

amounts, i.e. $p^* = \min\left[V_H^{M2} - V_H^{M3}, V_L^{M2} - V_L^{M1}\right]$.

We now examine how this optimal search fee changes with respect to market quality parameters η, α and μ.

Proposition 3. *The optimal search fee p^**

i) is non-monotonic in η, α; and

ii) is strictly decreasing in μ.

Given that the firm serves both types (Proposition2), the H-type's willingness to pay decreases and the L-type's willingness to pay increases, with an increase in η. This is driven by the absence of a mechanism to prevent the L-type from mimicking the H-type. Therefore, as η increases, the L-type finds the online market more attractive and in effect its willingness to pay increases. The increased presence of L-type in the market in turn reduces the H-type's willingness to pay resulting in the patterns shown in Figure 1. The effective search fee that the firm can charge is determined by the minimum of the willingness to pay of the two types. When η is low, the L-type's willingness to pay drives the firm's pricing decision. On the other hand, for high values of η, the decreasing willingness to pay of H-type becomes the driving factor resulting in the non-monotonic pattern shown in Figure 1.

For similar reasons, the market composition parameter α also has a non-monotonic effect on the optimal search fee. When α is low, the H-type's willingness to be inclusive makes the online market more attractive for the L-type. This results in the firm's optimal price being driven by the L-type's willingness to pay, which increases with α. The H-type's disutility from sharing the market with the L-type and its preference for exclusivity drives down its willingness to pay as the market quality improves.

However, as shown in Figure 1, when α is so high that the H-type represents a dominant proportion of the population, the optimal search fee could increase again. This is because the superior search benefits of the online market to the H-type under these conditions dominate the disutility due to the presence of (relatively few) L-types as a result of which the H-type's willingness to pay increases. Also, with regard to the effect of μ, since higher values of μ decrease the relative benefits of the online market, both types' willingness to pay for online search decreases as μ increases, as shown in Figure 1.

The above analysis demonstrates that the effects of market changes on the positioning and pricing strategies of an online matchmaking firm offering only search services are not always obvious. For instance, we show that it is never optimal for the firm to target just high-value match seekers for its services. Also, if the relative value of high-type matches (i.e., η) or the proportion of high-value seekers (i.e., α) increases, it is not always optimal for the firm to increase its search fees.

3.3 Online Market with Perfect Authentication

In this section, we examine the effects of the online firm offering a perfect authentication service (i.e. $\gamma = 1$) to counter the L-type's incentive to mimic the H-type in the online market. We assume that the firm charges an additional fee q for match seekers who use the authentication service in addition to the search fee p. Since $\gamma = 1$, the firm incurs a cost of c to authenticate an individual in addition to the fixed cost of κ. As before, each type has to make a decision to stay in the direct market or engage the online firm by paying the search fee. If they choose to go online, they also have to decide whether to use the authentication service and pay the additional fee, q. Thus, each type has three choices, and for any given search fee, p and authentication fee q, there are 9 possible outcomes. The payoffs for these different outcomes are as shown in the payoff matrix in Table 2.

The value functions of match-seekers when neither type chooses to pay for authentication are identical to the scenario in which the online firm offers search only (as discussed in Section 3.2). We now describe the value functions of match-seekers when at least one of the types uses the authentication services.

		H-type		
		Direct	Online	Online with authentication
L-type	Direct	V_H^D, V_L^D	V_H^{M1}, V_L^{M1}	V_H^{A1}, V_L^{A1}
	Online	V_H^{M3}, V_L^{M3}	V_H^{M2}, V_L^{M2}	V_H^{A2}, V_L^{A2}
	Online with authentication	V_H^{A3}, V_L^{A3}	V_H^{A4}, V_L^{A4}	V_H^{A5}, V_L^{A5}

Table 2: Payoffs for online markets with search and perfect authentication

Consider first the two scenarios in which H-types purchase authentication because they want to be exclusive[8]. First, when only H-types are in the online marketplace, their value function is

$$V_H^{A1} \quad = \quad \alpha v_H + \delta(1-\alpha)\alpha v_H - p - q \tag{10}$$

The corresponding value function of L-types in the direct market is

$$V_L^{A1} \quad = \quad (1-\alpha)\mu v_L + \delta(1-(1-\alpha)\mu)(1-\alpha)\mu \tag{11}$$

Second, when both H- and L-types are in the online market, the value function of the exclusive H-type is

$$V_H^{A2} \quad = \quad \alpha v_H + \delta(1-\alpha)(\alpha v_H + (1-\alpha)v_L) - p - q \tag{12}$$

The corresponding value function of the L-type in the online market is

$$V_L^{A2} \quad = \quad (1-\alpha)v_L + \delta(1-(1-\alpha))(\alpha v_H + (1-\alpha)v_L) - p \tag{13}$$

Now consider the two scenarios when only L-types purchase authentication. First, when only L-types are in the online market, the value function of the two types are

$$V_H^{A3} \quad = \quad \alpha\mu v_H + \delta\,(1-\alpha\mu)\,\mu\alpha v_H \tag{14}$$

$$V_L^{A3} \quad = \quad (1-\alpha)\,v_L + \delta\mu\,(1-(1-\alpha))\,(1-\alpha)\,v_L - p - q \tag{15}$$

[8]Note that authentication has no value when the H-type chooses to be inclusive.

Second, when both types are in the online market, the value functions of the two types are

$$V_H^{A4} = \alpha v_H + (1-\alpha)\, v_L - p \tag{16}$$

$$V_L^{A4} = \alpha v_H + (1-\alpha)\, v_L - p - q \tag{17}$$

Finally, consider the scenario in which both types search in the online marketplace and choose to pay for the authentication service. The value functions of the two types are then

$$V_H^{A5} = \alpha v_H + \delta(1-\alpha)(\alpha v_H + (1-\alpha)v_L) - p - q \tag{18}$$

$$V_L^{A5} = (1-\alpha)v_L + \delta(1-(1-\alpha))(\alpha v_H + (1-\alpha)v_L) - p - q \tag{19}$$

The following lemma characterizes the equilibrium behavior of both types.

Lemma 2. *There exist thresholds $\underline{p}_A$,$\overline{p}_A$ and q_A such that for any search fee p and authentication fee q, the equilibrium behavior of the H- and L-types will be as follows:*
i) When $p > \overline{p}_A$, both H- and L-types search in the direct market only,
ii) When $\underline{p}_A < p \leqslant \overline{p}_A$, only H-types search in the online market,
iii) When $p \leqslant \underline{p}_A$ and $q > q_A$, both H- and L-types search in the online market and neither purchase authentication
iv) When $p \leqslant \underline{p}_A$ and $q < q_A$, both H- and L-types search in the online market, and the H-types purchase the authentication service.

Lemma 2 indicates that there are only 4 pure-strategy equilibria under perfect authentication. This is because of the following: since perfect authentication reveals the true type of a match seeker, L-types will not purchase authentication. Furthermore, H-types will find authentication useful only when they want to be exclusive and the L-types are also in the online market. Thus in addition to the equilibria characterized in Lemma 1, we find that there exists an equilibrium in which both types are in the online market, with only the H-types purchasing authentication, provided the authentication fee is sufficiently low.

Having identified the equilibrium behavior of the match-seekers, we now turn our attention to the firm's pricing strategy when the firm offers authentication services. In formulating this strategy, the firm has to balance the following considerations: (i) the fixed and variable costs of offering authentication services, (ii) the profits from offering authentication services to the H-types, and (iii) the benefits from including the L-types in the online marketplace.

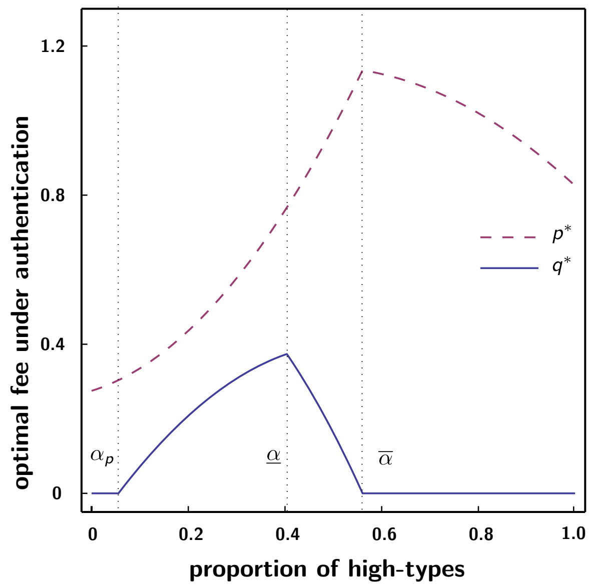
Specifically, we examine how the pricing strategy is impacted by the market composition. We know from Section 3.2, that it is not optimal for the firm to price the L-type out of the online market. If the H-type does not purchase authentication, the firm does not offer authentication services. This results in the same situation that was analyzed in Section 3.2. If however, the H-type purchases authentication, the firm's profit function will be $\pi_A = p + \alpha(q - c) - \kappa$ where $p \leqslant \underline{p}_A$ and $q \leqslant q_A$. It turns out that the optimal search and authentication fees depend upon the market composition (α) as characterized below.

Lemma 3. *There exists thresholds α_p,$\bar{\alpha}$ and $\underline{\alpha}$ on α such that the optimal search fee p^* and optimal authentication fee q^* are*

Case	***Search fee*** (p^*)	***Authentication fee*** (q^*)
$\alpha < \alpha_p$	$\alpha^2\delta v_H + (1-\alpha)(1-\mu)\cdot(1-\delta\mu+\alpha\delta(1+\mu))v_L$	$\epsilon \approx 0$
$\alpha < \underline{\alpha}$	$\alpha^2\delta v_H + (1-\alpha)(1-\mu)\cdot(1-\delta\mu+\alpha\delta(1+\mu))v_L$	$(1-\alpha)(\alpha\delta v_H - (1-\delta(1-\alpha))v_L)$
$\underline{\alpha} < \alpha < \overline{\alpha}$	$\alpha^2\delta v_H + (1-\alpha)(1-\mu)\cdot(1-\delta\mu+\alpha\delta(1+\mu))v_L$	$\alpha v_H\left(\alpha\delta\mu^2 - 2\alpha\delta - (\delta+1)\mu + \delta + 1\right) + v_L(1-\alpha)\left(\delta\left(-(1-\alpha)\mu^2 - 2\alpha + \mu + 1\right) + \mu - 1\right)$
$\alpha > \overline{\alpha}$	$(1-\alpha)^2\delta v_L + \alpha(1-\mu)\cdot(1+\delta(1-\alpha(1+\mu)))v_H$	$\epsilon \approx 0$

The optimal fees charged by the firm are influenced by two factors. The search fee is driven by the need to attract L-types to the online market while the authentication fee is driven by the incentive to encourage H-types to purchase authentication. When α is low, H-types who want to be exclusive derive a high value from the authentication service and are willing to pay for it. However the perceived need of authentication is low when the market is largely composed of H-types (high α). Indeed, when $\alpha > \overline{\alpha}$, the firm finds it optimal to fully subsidize authentication service ($q^* \approx 0$).

It is also useful to examine how the optimal fees change with various market parameters. Similar to our finding in Section 3.2, the search and authentication fees are non-monotonic in α. From Lemma 3 and as shown in Figure 2, there are three optimal strategies for the market covering firm.



$\delta = 0.9; v_H = 3; v_L = 1; \mu = 0.5$

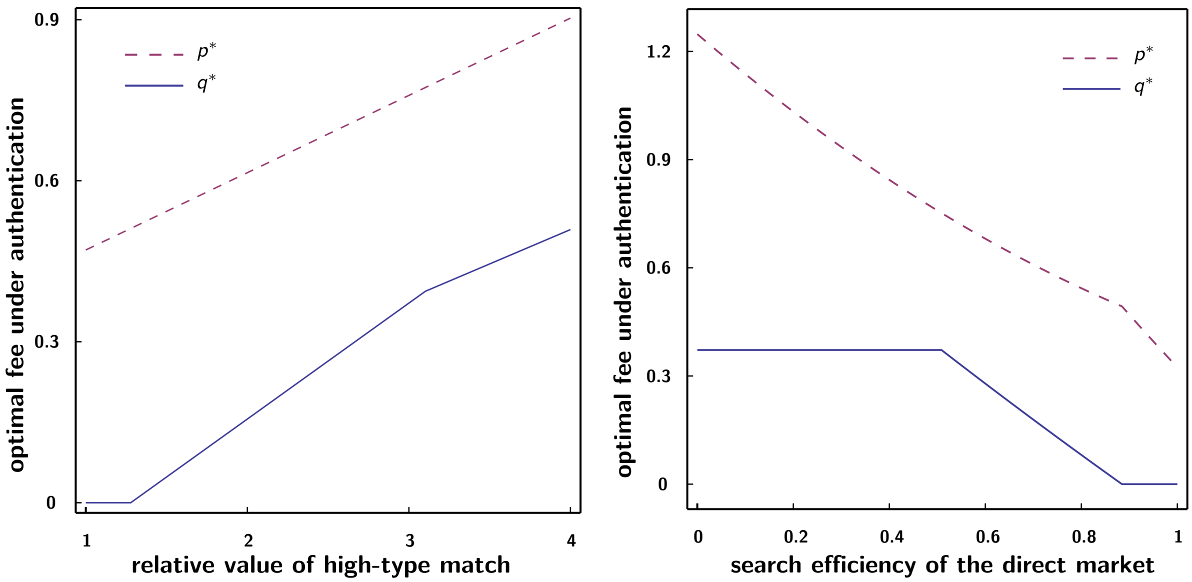
Figure 2: Effect of α on the optimal authentication fee

When $\alpha < \underline{\alpha}$, as α increases, the increasing attractiveness of the market to the L-types makes authentication service more valuable for the exclusive H-types, who are therefore willing to pay a higher authentication fee. When $\underline{\alpha} < \alpha < \overline{\alpha}$, the online market continues to be attractive to the L-type, thereby enabling the firm to continue to charge a higher search fee. However, it is also constrained by the total amount it can charge both types and as a result the optimal strategy of the market covering firm is to subsidize the authentication fee. Finally, in markets where $\alpha > \overline{\alpha}$, the threat of H-types searching in the direct marketplace is high enough to force the firm to reduce the search fee and fully subsidize authentication service in order to keep the H-types in the online marketplace.

With respect to the other market parameters η, the relative value of H-types and μ, the offline market efficiency, Figure 3 show that the optimal search and authentication fees are monotonic increasing in η and decreasing in μ respectively.

It is interesting to note that the authentication service is profitable in itself for the online firm only when the proportion of H-types is neither too high nor too low, as implied by the following proposition.

Proposition 4. *There exists thresholds on α and c such that $q^* > c$ (i.e. the authentication service is itself profitable) if $\alpha' < \alpha < \alpha''$ and $c < c'$. For other values of c and α, the authentication service*



$\delta = 0.9; v_L = 1; \mu = 0.5; \alpha = 0.4$
a. Effect of η

$\delta = 0.9; v_H = 3; v_L = 1; \alpha = 0.4$
b. Effect of μ

Figure 3: Effect of η, μ on the optimal authentication fee

will be a loss leader.

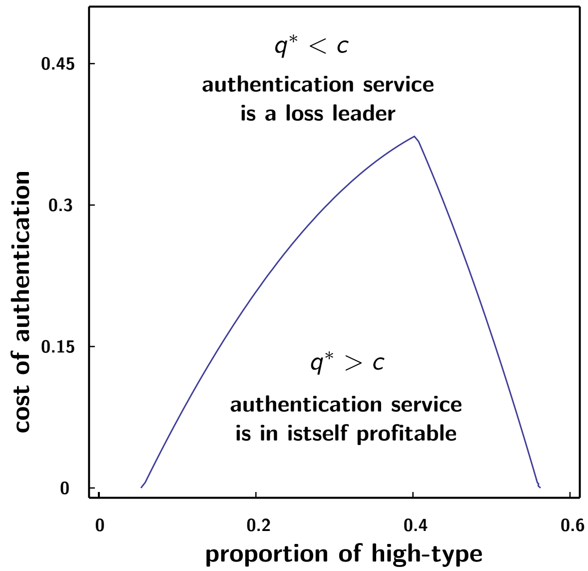
Proposition 4 implies that the authentication service can benefit the firm in two ways. First, authentication service can be a direct source of profits for the firm when $q^* > c$. This is shown in Figure 4 where for intermediate ranges of α, the firm is able to profit from authentication service when the cost is low enough. And second, the authentication service can be used to attract business for the firm even when it is not profitable by itself, i.e. as a loss-leader when $q^* < c$. This is because the availability of authentication service motivates the H-types enables them to be exclusive in the online market, which in turn allows the firm to charge higher search fees.

So far, we have analyzed the situation where the online firm offers both search and authentication services. However, this may not always be optimal for the firm. We next analyze the market conditions under which the firm would choose to offer perfect authentication services.

Proposition 5. *i) When $c = 0$, there exists threshold on α such that if $\alpha > \alpha_x$, perfect authentication is optimal. No authentication is optimal otherwise.*

ii) When $c > 0$, there exists thresholds on α and c such that when $c > c_y$ and $\alpha > \alpha_y$, no-authentication is the optimal strategy for the firm.

iii) When $\alpha < \overline{\alpha}$, there exists a threshold on c such that if $c > c_x$, no authentication is the optimal



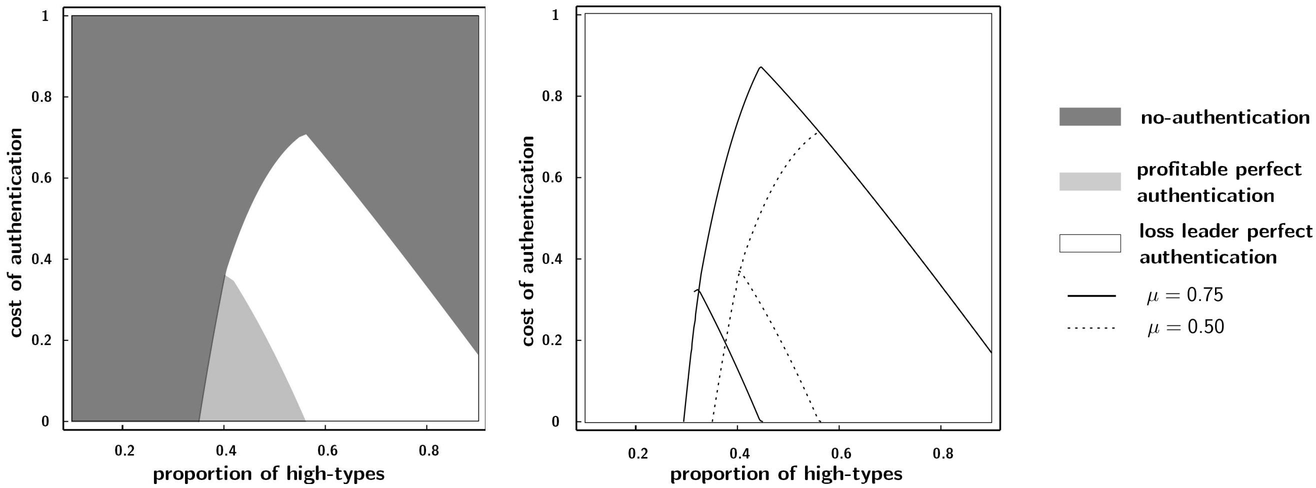
$\delta = 0.9; v_H = 3; v_L = 1; \mu = 0.5$

Figure 4: Profitability of authentication service.

strategy for the firm.

iv) α_x *is decreasing in* μ*, and* c_x *is increasing in* μ *i.e. region in which perfect authentication is optimal becomes larger when* μ *increases.*

The above proposition characterizes how the optimal strategy of the online firm depends on market composition. When there are relatively few H-types in the market (low values of α), H-types are better off being inclusive in the online market and the match-making firm prefers to cover the market by attracting the L-types as well. In such a market, authentication services are not needed. Interestingly, no-authentication is also the optimal strategy when α is high. This is because with relatively few L-types in the market, H-types are more likely to be matched with other H-types and thus unlikely to purchase authentication. However, for intermediate values of α, it becomes profitable for the firm to offer the authentication service to the H-type. Under these conditions, the efficiency of online search is sufficient to attract the L-types to the online market, while the H-types derive additional value from the authentication service that allows them to be exclusive in their search. This is illustrated in part (a) of Figure 5. Furthermore, as part (b) of Figure 5 shows, when the search efficiency in the direct market (μ) is higher, the firm finds it useful to offer authentication services for even lower values of α.



$v_H = 3; v_L = 1; \delta = 0.9$

a. Optimal firm strategy b. Effect of μ on optimal strategy

Figure 5: Optimal firm strategy when perfect authentication is available.

3.4 Online Market with Imperfect Authentication

In order to get further insights into the interplay of search and authentication and the impact of varying authentication service quality, we now consider a setting in which the firm cannot always correctly identify the L-type match-seeker (i.e., $\gamma < 1$).

As stated earlier, the fixed cost of the authentication service is κ while the variable cost of authenticating each match-seeker is $c\gamma$. However, the key difference from the previous section is that while an H-type match-seeker is always detected correctly, an L-type match-seeker has a chance of mimicking the H-type and can pass itself on as a H-type with probability $1-\gamma$. In other words, when the authentication service classifies a match-seeker as being an H-type it provides a signal h and similarly, if it classifies the match-seeker as an L-type it provides a signal l. The probabilities of correct detection of the H- and L- types are $P\left(\omega = l \mid L\right) = \gamma$ and $P\left(\omega = h \mid H\right) = 1$ respectively. Therefore, the probabilities with which the authentication service provides the signals h and l are $P\left(\omega = h\right) = \alpha + \left(1-\alpha\right)\left(1-\gamma\right)$ and $P\left(\omega = l\right) = \gamma\left(1-\alpha\right)$ respectively. Furthermore, the probability that a match-seeker detected as an H-type is indeed an H-type is $P\left(H \middle| \omega = h\right) = \frac{\alpha}{\alpha+(1-\alpha)(1-\gamma)}$ and the probability that an L-type is falsely detected as an H-type is$P\left(L \mid \omega = l\right) = \frac{(1-\alpha)(1-\gamma)}{\alpha+(1-\alpha)(1-\gamma)}$. We make the reasonable assumption that only those match-seekers who are classified as H-types by the authentication service ($\omega = h$) will reveal that information to the market.

We start by analyzing the match-seekers' behavior regarding their use of search and authenti-

		H-type		
		Direct	Online	Online with authentication
L-type	Direct	V_H^D, V_L^D	V_H^{M1}, V_L^{M1}	V_H^{I1}, V_L^{I1}
	Online	V_H^{M3}, V_L^{M3}	V_H^{M2}, V_L^{M2}	V_H^{I2}, V_L^{I2}
	Online with authentication	V_H^{I3}, V_L^{I3}	V_H^{I4}, V_L^{I4}	V_H^{I5}, V_L^{I5}

Table 3: Payoffs for online markets with search and perfect authentication

cation services in the online market. As in the perfect authentication setting analyzed previously, each type has three choices, and for any given search fee p and authentication fee q, there are 9 possible scenarios. The payoffs for these different scenarios are as shown in the payoff matrix in Table 3.

In this payoff matrix, the value functions of match-seekers in the first two rows are identical to what we have developed in the previous sections. However, unlike under perfect authentication, the L-type can now choose to use the authentication service to mimic the H-type resulting in the payoffs corresponding to the last row in Table 3. We next discuss these value functions.

When the H-type searches in the direct marketplace and the L-type searches in the online market place and pays for authentication service as well, the value functions of the two types are $V_H^{I3} = V_H^{A3}$and $V_L^{I3} = V_L^{A3}$ respectively.

When both types are in the online market with only L-types purchasing authentication, an L-type will be classified as an H-type ($\omega = h$) with probability $1 - \gamma$. However, since H-types do not purchase authentication, the signal only serves to indicate that the match-seeker is an L-type. The value function of the H-type will depend on whether it wants to match with those L-types who are "identified" through the authentication process.[9] If H-types choose not to match with L-types receiving an h signal, its value function is

$$\begin{aligned} V_H^{I4a} &= \alpha v_H + \delta(1 - (1-\alpha)\gamma - \alpha)(\alpha v_H + (1-\alpha)v_L) + (1-\alpha)\gamma v_L - p \\ V_L^{I4a} &= \delta\left(2(\alpha-1)\gamma^2 + (2-3\alpha)\gamma + \alpha - 1\right)(\alpha v_H - \alpha v_L + v_L) \\ &\quad +(\gamma-1)(2(\alpha-1)\gamma v_L - \alpha v_H) - q - p \end{aligned}$$

[9]Those L-types who were correctly identified as L-types will by definition not reveal their type and hence cannot be distinguished from H-types.

On the other hand, if the H-type chooses to match also with L-types receiving an h signal, its value function is

$$\begin{aligned} V_H^{I4b} &= \alpha v_H + (1-\alpha)v_L - p \\ V_L^{I4b} &= \alpha v_H + (1-\alpha)v_L - p - q \end{aligned}$$

It follows that

$$\begin{aligned} V_H^{I4} &= \max\left[V_H^{I4a}, V_H^{I4b}\right] \\ V_L^{I4} &= \begin{cases} V_L^{I4a} & \text{if} \quad V_H^{I4a} > V_H^{I4b} \\ V_L^{I4b} & \text{if} \quad V_H^{I4a} \leqslant V_H^{I4b} \end{cases} \end{aligned}$$

Details on how these value functions are obtained are provided in the appendix.

When both types search in the online marketplace and the H-type purchases the authentication service, there are three types of matches possible involving match-seekers identified as H-type by the authentication system: (i) match between true H-types, (ii) match between a true H-type and an L-type falsely detected as an H-type, and (iii) match between two L-types who are both falsely detected as $H-$types.

The expected value of a match with a seeker authenticated as an H-type is given by $P\left(H|\ \omega = h\right) v_H + P\left(L|\ \omega = h\right) v_L$. With probability $P\left(\omega = l \mid L\right)$, an L-type identified correctly by the authentication service derives an expected value of $P\left(\omega = l\right) v_L$. So the value functions of each type are as follows:

$$\begin{aligned} V_H^{I5} &= P\left(\omega = h\right)\left(P\left(H|\ \omega = h\right) v_H + P\left(L|\ \omega = h\right) v_L\right) + \delta\left(P\left(\omega = l\right)\right)\left(\alpha v_H + (1-\alpha)v_L\right) - p - q \\ &= \alpha v_H + (\alpha-1)(v_L((\alpha-1)\gamma\delta + \gamma - 1) - \alpha\gamma\delta v_H) - q - p \qquad (20) \\ V_L^{I5} &= P\left(\omega = h \mid L\right) V_{4H} + P\left(\omega = l \mid L\right) P\left(\omega = l\right) v_L + \delta P\left(\omega = h\right)\left(\alpha v_H + (1-\alpha)v_L\right) - p - q \\ &= \alpha v_H(\gamma\delta(2(\alpha-1)\gamma - \alpha + 2) - \gamma + 1) \\ &\quad -(\alpha-1)v_L(\gamma\delta(2(\alpha-1)\gamma - \alpha + 2) + 2(\gamma-1)\gamma + 1) - q - p \qquad (21) \end{aligned}$$

Lemma 4. *There exist thresholds $\overline{p}$, $\underline{q}$ and $\overline{q}$ such that for any search fee p and authentication fee q, the equilibrium behavior of the H- and L-type match-seekers will be as follows:*

i) When $p > \overline{p}_I$, both H- and L-types search in the direct market only,

ii) When $\underline{p}_I < p \leqslant \overline{p}_I$, the H-type searches in the online market, while the L-type searches in the direct market,

iii) When $p \leqslant \underline{p}_I$ and $q > \overline{q}$, both types search in the online market and neither purchase authentication,

iv) When $p \leqslant \underline{p}_I$ and $\underline{q} < q < \overline{q}$, both types search in the online market and only the H-type uses the authentication service, and

v) When $p \leqslant \underline{p}_I$ and $q < \underline{q}$, both types search in the online market and use the authentication service.

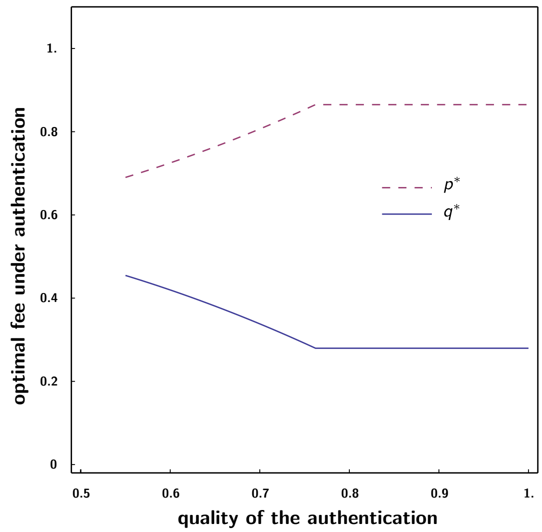
When the authentication fee, q is sufficiently high ($q > \underline{q}$), the equilibrium response of match seekers is identical to those described in Lemma 2 in the previous section. However, when $q < \underline{q}$, we have an additional scenario where both the $H-$and L-types engage the online firm and pay for both search and authentication services. The L-types purchase authentication in the hope that they would be able to successfully mimic the H-types. This raises an interesting strategic question for the firm. Should the firm price its services such that only the H-types purchase authentication or should it try to cover the market by offering a low price for authentication services? We address this question by examining the two cases: first, when authentication is targeted to H-types only and second, when the authentication service is targeted to both types.

Case 1: Authentication targeted to only H-type:

The profits of the firm when it chooses to target the authentication service to the H-type alone, are $\pi_{I1} = p + \alpha (q - \gamma c) - \kappa$ subject to the constraints that $p \leqslant \overline{p}$ and $\underline{q} \leqslant q \leqslant \overline{q}$. The analysis of this case is the same as when the firm offers perfect authentication and only H-types purchase it. However the authentication fee now has to be high enough to discourage L-types from purchasing the authentication service. Let p^* and q^* be the optimal search and authentication fees respectively.

Proposition 6. *When only H-types purchase the authentication service,*

i) p^* is non-decreasing in γ and,



$\delta = 0.9; v_H = 3; v_L = 1; \alpha = 0.45; \mu = 0.5$

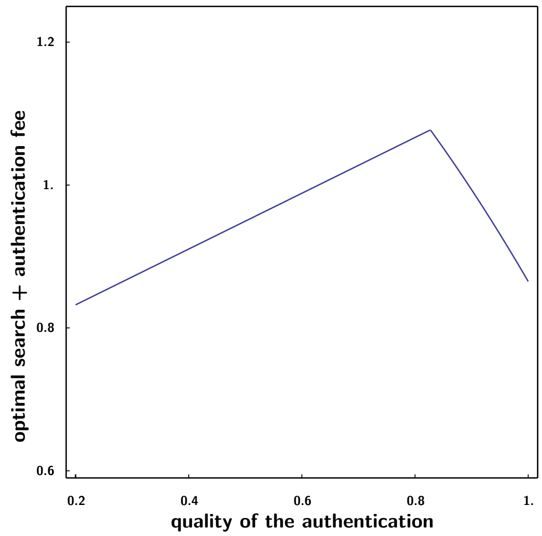
Figure 6: Effect of γ when only H-types buy authentication

ii) q^* is non-increasing in γ.

The above proposition characterizes how the quality of the authentication service γ impacts the fees charged by the firm. When γ is relatively low, the L-types are motivated to use the authentication service since there is a high probability that they will be classified as H-types. Therefore the firm has to maintain a high authentication fee to deter the L-type from using the authentication service. For higher values of γ, the reliability of the authentication service increasingly deters L-types from using it; so the firm no longer has to charge as high an authentication fee. This pattern in the authentication fee, shown in Figure 6, in turn affects the search fee that the firm can charge to both types. Thus, for low values of γ, the firm is forced to charge a lower search fee, so that the total fee paid by the H-types is not prohibitive. And at higher levels of γ, the firm can charge a higher search fee, as shown in Figure 6. This is interesting, in that while the firm can benefit from higher quality of its authentication service, this does not imply that it should charge a higher authentication fee.

Case 2: Authentication targeted to both types:

In this scenario, since both types of match seekers are purchasing authentication as well as search services, the firm can offer one price that will cover both services. As a result, we restrict our focus to the total price that the firm would offer. The total price $t = p + q$ that the firm can charge



$\delta = 0.9; v_H = 3; v_L = 1; \alpha = 0.45; \mu = 0.5$

Figure 7: Effect of γ when both types buy authentication

in this market is given as $p + q < t = \min[t_1, t_2]$ where, $t_1 = V_H^{I3} - V_H^{I5}$ and $t_2 = V_L^{I3} - V_L^{I1}$. Note that t_1 and t_2 are the total prices that ensure that both types find it worthwhile to purchase authentication and search services instead of staying in the direct market. The profit of the firm can now be stated as $\pi^{I5} = t - \gamma c - \kappa$.

Proposition 7. *When both types purchase authentication, the total fee (sum of authentication and search fees), t^*, is increasing in γ if $\gamma < \gamma_1$ and decreasing otherwise.*

While determining the total price for its services, the firm has to consider the preferences of both types. In this regard, the firm needs to understand the impact of γ on its decision to cover the market with both search and authentication services, rather than covering the market with search service and offering authentication as a premium service to H-types only. When γ is low, L-types are motivated to buy the authentication service, and pay a high price for it. However, H-types are less motivated to use the service, and thus their willingness to pay is the constraining factor on the price. As γ increases, the price increases, due to the increasing attractiveness of the service to the H-types. Beyond a point however, the demotivating effect of the higher reliability of the authentication service on the L-types makes the willingness to pay of the L-types the constraining factor, leading to the non-monotonic effect of γ on the total price charged by the firm as shown in Figure 7.

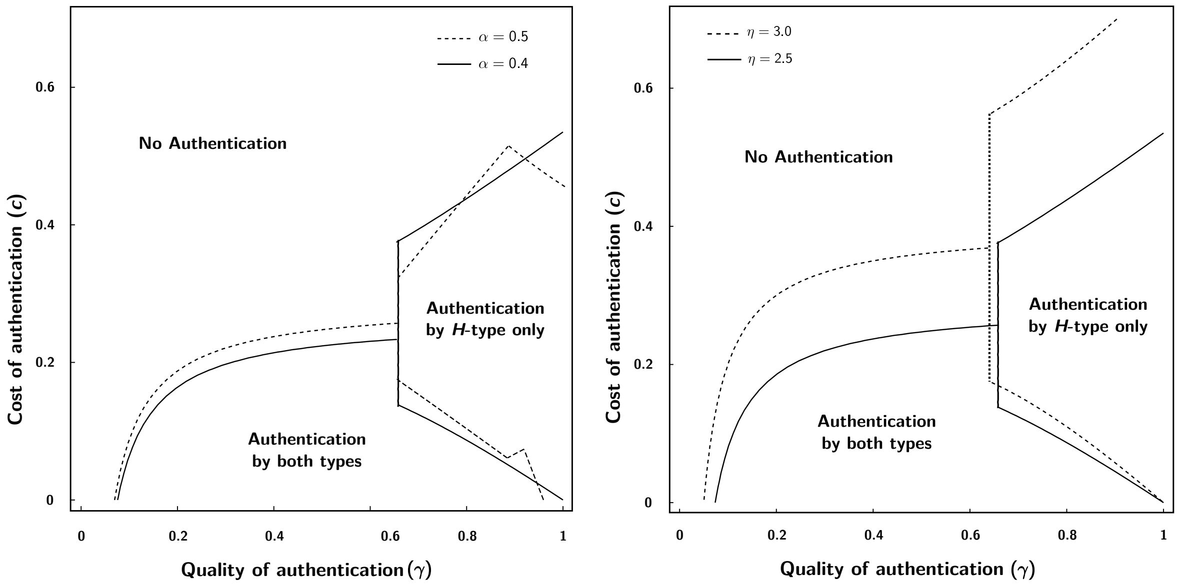
We next examine the optimal strategy for the online firm, i.e., the conditions under which it is optimal for the firm to target both types. We start by considering the simple case where the authentication service is costless (i.e., $c = 0$).

Proposition 8. *When $c = 0$, the firm's optimal strategy can be characterized as follows:*
i) The firm does not offer the authentication service if $\gamma < \underline{\gamma}$
ii) The firm targets the authentication service to both high and low types if $\underline{\gamma} < \gamma < \overline{\gamma}$
iii) The firm targets the authentication service to only the H-types if $\gamma \geqslant \overline{\gamma}$.

Due to the bilateral matching in this market, while the authentication service will not be offered when it is very poor at detecting L-types ($\gamma < \underline{\gamma}$), for intermediate levels of γ, the firm finds it optimal to cover the market with both search and authentication services. And when the authentication technology is high enough (above a threshold), the L-types may not find it attractive enough since the probability of correct detection of L-type posing as H-type increases.

Clearly, the firm would offer an authentication service only if it is not prohibitively expensive. In other words, there would exist a threshold $\overline{c}$ such that the firm would not offer the authentication service if $c \geqslant \overline{c}$. In addition, as shown above, the firm's strategy for targeting its authentication service also depends upon the quality of the service (γ). Figure 8 illustrates the collective effect of the cost and quality of the authentication service on the firm's optimal strategy.

When γ is very low, it is not worthwhile for the firm to offer the authentication service at all. For one thing, the service has little value for the H-types, and consequently, since the H-types do not use it, it has little value for L-types either. As γ increases and becomes increasingly attractive to both types, the firm can cover the market with the service, as long as the cost is reasonable. In this region, the quality of the authentication service is low enough for L-types to be willing to use it in the hope of successfully mimicking H-types, and at the same time, is high enough that H-types find it useful to be successfully exclusive, at least some of the time. Finally, as γ becomes relatively high, the authentication service is no longer attractive to L-types hoping to beat the service, and as a result the H-types become the target segment for the service. It is interesting to note that at very high levels of γ, it may be attractive for the firm to target the authentication service to H-types even at higher cost to keep them in the online market. This is because, as shown in Figure 6, the firm can charge a higher search fee from both types in this situation, even if it operates the



$\delta = 0.9; v_L = 1; \mu = 0.8; \kappa = 0.02$

a. Effect of α b. Effect of η

Figure 8: Optimal strategy under imperfect authentication

authentication service as a loss leader.

Figure 8 also shows the effect of the market characteristics, as reflected in α, the proportion of H-types and η, the relative value of the H-types, on the firm's strategy with respect to its authentication service. As shown in Figure 8(a), for low values of γ, an increase in α results in an increase in the cost threshold below which the firm should target its authentication service to both types. This is because both types find the service more useful when the proportion of H-types is higher. In contrast, a similar increase in α when γ is high, could result in a lower cost threshold below which authentication is targeted to both types. This is because most of the match-seekers are H-types, who therefore see less value in the authentication service, and at the same time, L-types are less motivated to use the higher-quality authentication service. Under these conditions, the firm is then motivated to try to keep the H-types online by offering them authentication services at lower prices (as a loss-leader), relying increasingly on the revenues from search fees from both types.

Figure 8(b) shows that when the relative value of the H-types is increased, it becomes more attractive for the firm to offer its authentication service. This is consistent with intuition, since the value of authentication to (H-type) match-seekers is clearly greater when the value of achieving a match with a true H-type is higher.

4 Discussion and Conclusion

The above analysis provides valuable insights into a number of issues regarding online matching markets, which can be leveraged by owners and operators of such firms to effectively position and price their services.

Since online markets inherently present authentication challenges, we show that when the online firm offers only search services, it will be unable to limit its services to high-value match-seekers. Thus, without access to an authentication service, customers will be forced to be inclusive in their search, and the firm will be unable to prevent low-value match-seekers from participating in the market and mimicking H-types. From a practical standpoint, this suggests that an online matchmaking firm such as match.com or Vantage Agora cannot effectively create a premium service targeted at high-quality or high-net-worth customers, even by charging very high fees. And increasing the search fees beyond what the low-types are willing to pay would render the market infeasible for high-types as well.

When the firm is able to offer an authentication service in addition to its search services, we show that it can use the pricing of these services to effectively limit the market to exclusive high-value match seekers. However, we show that the quality of the authentication service has a significant and non-intuitive effect on both the adoption of the service by different types of match-seekers, as well as the prices that the firm charges for its services. In particular, it is not always ideal for the firm to try and build a perfect authentication service that is limited to high-value match seekers As we show in section 3.3, depending on the cost and quality of the authentication service, the optimal decision for the firm may be to target it to both low and high-value match-seekers. We also examine the effect of authentication on the pricing strategy of the firm. Due to the complementarity between pricing of search and authentication services, the firm might find it optimal to offer authentication services even when it is not profitable by itself i.e. authentication serves as a loss leader. In fact, this also leads to conditions under which the firm might find it optimal to reduce the authentication fee when the quality of authentication improves.

While this paper is a significant step in the economic analysis of online matchmaking firms, there are a number of interesting directions for future work in this area. To start with, there are a number of potentially interesting extensions of our model through relaxation of some of our assumptions. First, we have assumed that the individual match-seeker knows their type and value. However, in practice, match-seekers' *a priori* perceptions of themselves may be revised as they go through the search process and interact with candidate matches. In this process, the firm could also learn about each match-seeker's preferences, leading to potentially better matches. The effect of such learning on the part of both match-seekers and the online firm is an interesting area. Second, we assume that the patience level parameter δ is the same for all match-seekers. The effect of heterogeneity in match-seeker patience on both the matching and learning process may be worth exploring. Third, we have assumed that the only cost that high-value match-seekers perceive in using authentication is the authentication fee charged by the online firm. However, the authentication service may require divulgence of private information that may lead to privacy concerns for the match-seekers, which represents an additional cost(Chellappa and Shivendu, 2010).

There are some other questions related to online matching that are also interesting for future work. For instance, there is the question of whether the use of an online matching market adds social value, relative to the direct market. In this paper, we assume that the value of a match is always positive, and that the value derived by each match-seeker is a positive value based on the type of its counter-party. Thus, if we were to have a market in which it is feasible for all match-seekers to be matched in a single period, then the total value of the matching process would remain the same, regardless of the distribution of mixed (HL, LH) and pure (HH, LL) matches. In other words, from the social planner's perspective, the goal is to minimize the number of unmatched seekers. In a multi-period setting the situation is changed, in that improved search that avoids the need for a second (or more) period(s) does add value, so even in our setting, search has some social value, and adopting the online firm can lead to a better social welfare outcome. However, authentication does not add to the social welfare. In fact, by causing some matches to be rejected in the first period, it can actually lead to lower social welfare. In other words, allowing high-value seekers to be exclusive in the (initial) search imposes a social cost. It would be interesting to explore the social welfare implications of online authentication in a matching market further, including when mixed matches may actually be dysfunctional.

Yet another interesting question is whether authentication should be positioned as a qualification mechanism, or not. For instance, when an imperfect authentication service targeted at both types of seekers is the optimal strategy, how should it be positioned? Suggesting that the system is fallible to attract L-types may at the same time drive away H-types. One possibility is to de-emphasize the authentication service, and possibly even disguise it as a feature that enriches the matching platform.

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Appendix: Search and Authentication in Online Matching Markets

A Proofs

Proof of Proposition 1: Direct Marketplace

From equations 1 and 3, the H-type participants prefers to be exclusive in a direct marketplace if$V_{eH}^{D} \geqslant V_{iH}^{D}$and inclusive otherwise. Comparing V_{eH}^{D} and V_{iH}^{D}, we have that

$$V_{eH}^{D} - V_{iH}^{D} \quad = \quad (1-\alpha)\mu v_L \left(\alpha\delta\mu\eta - \left(1 - (1-\alpha)\,\delta\mu\right)\right)$$

It follows that the threshold $\overline{\eta}$, above (below) which h-type prefers to be strictly exclusive (inclusive) is given by ,

$$\overline{\eta} = \frac{1 - (1-\alpha)\,\delta\mu}{\alpha\delta\mu}$$

Further, this threshold decreases in α, δ and μ.

$$\begin{aligned}
\frac{\partial\overline{\eta}}{\partial\alpha} &= -\frac{1-\delta\mu}{\alpha^2\delta\mu} < 0 \\
\frac{\partial\overline{\eta}}{\partial\mu} &= -\frac{1}{\alpha\delta\mu^2} < 0 \\
\frac{\partial\overline{\eta}}{\partial\delta} &= -\frac{1}{\alpha\delta^2\mu} < 0
\end{aligned}$$

It should be noted that when the H-type is exclusive in the first period, this condition ensures that the L-type participant also prefers to be exclusive in period $1 (V_{1eL} \geqslant V_{1iL})$ rather than wait until the second period to be inclusive.

Proof of Lemma 1: Online Market Supporting Search Only: Match-seeker's Equilibrium Behavior

First note that when only L-type searches online while H-type searches in the direct marketplace, the search fee must satisfy the conditions $V_H^{M3} > V_H^{M2}$ and $V_L^{M3} > V_{iL}^{D}$. For this to be an equilibrium strategy, we will require that

$$\begin{aligned} p &\geqslant \alpha v_H + (1-\alpha) v_L - \alpha\mu v_H (1 + \delta(1-\alpha\mu)) \\ p &\leqslant (1-\alpha) v_L (\alpha\delta + (1-\mu)(1-\delta\mu)) - \alpha\mu v_H (\delta(1-\mu)+1) \end{aligned}$$

Case 1: $\eta > \overline{\eta}$: . Comparing the upper and lower bounds p_1 and p_2, we have that

$$p_1 - p_2 = \alpha v_H \left(1 - (1-\alpha)\delta\mu^2\right) + (1-\alpha) v_L (1 + \mu(\delta(1-\mu)) - \alpha\delta) > 0$$

when $0 < \alpha, \mu, \delta < 1$.

Both types of match-seekers search in the direct marketplace if $V_H^D > V_H^{M1}$ and $V_L^D > V_L^{M3}$. From equations (4, 5 and 6), this results in the following condition:

$$\begin{aligned} p \geqslant \overline{p}_M = & \max\left[\alpha v_H (1-\mu)(\delta(1-\alpha\mu-\alpha)+1) + \delta\mu v_L (1-\alpha)(1-\alpha\mu)\right., \\ & \left.(1-\alpha) v_L (\alpha\delta + (1-\mu)(1-\delta\mu)) - \alpha\mu v_H (\delta(1-\mu)+1)\right] \end{aligned}$$

When both types search in the online marketplace, their value functions must satisfy $V_H^{M2} > V_H^{M3}$ and $V_L^{M2} > V_L^{M1}$. Therefore the search fee p must satisfy the following condition:

$$\begin{aligned} p \leqslant \underline{p}_M & = \min\left[\alpha v_H \left(\alpha\delta\mu^2 - (\delta+1)\mu + 1\right) - \alpha v_L + v_L\right., \\ & \left.\alpha v_H + v_L (1-\alpha)\left(1 + (1-\alpha)\delta\mu^2 - (\delta+1)\mu\right)\right] \end{aligned}$$

H-type searches in the online market while L-type searches in the direct marketplace if $V_H^{M1} > V_H^D$ and $V_L^{M1} > V_L^{M2}$. From equations 4, 5 and 6, we have the following condition: $\underline{p}_M < \alpha v_H + (1-\alpha) v_L \left(1 + (1-\alpha)\delta\mu^2 - (1+\delta)\mu\right) < p < \alpha v_H (1-\mu)(\delta(1-\alpha\mu-\alpha)+1) - \delta\mu v_L (1-\alpha)(1-\alpha\mu) < \overline{p}_M$

Proof of Proposition 2: Online Market Supporting Search Only: Optimal Search Fee

As shown in Lemma 1, the participation behavior is such that either (i) only the H-type searches online or (ii) both types participate in online search.

Case (i): Only H-type searches online

Since for this case to be feasible, we need $p < \alpha v_H + \delta(1-\alpha)\alpha v_H - V_H^D$, the profits under this case are given by

$$\pi_{M2} \quad = \quad \alpha\left(\alpha v_H + \delta(1-\alpha)\alpha v_H - V_H^D\right)$$

where $V_{DH} = \mu v_L(\alpha\eta + \delta(1-\alpha\mu)((1-\alpha)+\alpha\eta))$ if $\eta > \overline{\eta}$ and $V_{DH} = v_L(\mu - \delta(\mu-1)\mu)(\alpha\eta + (1-\alpha))$ if $\eta \leqslant \overline{\eta}$. So the profits of the firm are

$$\pi_{M2} \quad = \quad \begin{cases} \pi_{M21} = \alpha\eta(1+(1-\alpha)\delta) - (\mu + \delta(1-\mu)\mu)(1+\alpha(\eta-1)) & \text{if } \eta > \overline{\eta} \\ \pi_{M22} = \alpha\eta(1-\mu)(1-\delta(\alpha\mu+\alpha-1)) + \delta\mu(1-\alpha)(1-\alpha\mu) & \text{if } \eta \leqslant \overline{\eta} \end{cases}$$

Case (ii) Both types search online.

Since $p < \min\left[\alpha v_H\left(\alpha\delta\mu^2 - (\delta+1)\mu + 1\right) - \alpha v_L + v_L, \alpha v_H + (\alpha-1)v_L\left((\alpha-1)\delta\mu^2 + (\delta+1)\mu - 1\right)\right]$, the profits under this scenario are also $\min[\pi_{M31,}\pi_{M32}]$ where$\pi_{M31} = \alpha v_H\left(\alpha\delta\mu^2 - (\delta+1)\mu + 1\right) - \alpha v_L + v_L$ and $\pi_{M32} = \alpha v_H + (\alpha-1)v_L\left((\alpha-1)\delta\mu^2 + (\delta+1)\mu - 1\right)$.

First let us consider the case when $\eta > \overline{\eta}$, Here $\pi_{M2} = \pi_{M21}$. Comparing π_{M2} and π_{M31}, we have that

$$\begin{aligned} \pi_{M2} - \pi_{M31} &= \alpha v_H\left(\alpha^2\delta\left(\mu^2-1\right) - \alpha(\delta+1)(\mu-1) - 1\right) \\ &\quad -(\alpha-1)v_L\left(\left(\alpha^2+\alpha-1\right)\delta\mu^2 + \mu(-\alpha\delta+\delta+1) - 1\right) \\ \frac{\partial\left(\pi_{M2} - \pi_{M31}\right)}{\partial v_H} &= \alpha(\alpha(\mu-1)(\delta(\alpha\mu+\alpha-1)-1)-1) < 0 \end{aligned}$$

Since $\pi_{M2} - \pi_{M31}$ is decreasing in v_H, the highest value occurs when $v_H = v_L$ where

$$\pi_{M2} - \pi_{M31}\big|_{v_H=v_L} \quad = \quad v_L\left(\alpha^3(-\delta) + \alpha^2(\delta-\mu+1) + \alpha\mu(2\delta(\mu-1)-1) - (\mu-1)(\delta\mu-1)\right) < 0$$

Now let us compare π_{M2} and π_{M32}, which gives us

$$\pi_{M2} - \pi_{M32} = (1-\alpha)\left(\alpha v_H\left(\delta\left(\alpha+\mu-\alpha\mu^2\right)+\mu-1\right) - v_L\left(1+\alpha\delta\mu\left(1-\alpha\mu\right)\right)\right)$$

Note that the co-efficient for v_L is negative, which implies that the expression is positive only if the co-efficient forv_H is positive i.e. $\left(\delta\left(\alpha+\mu-\alpha\mu^2\right)+\mu-1\right) > 0$ which occurs only when $\alpha > \frac{1-\delta\mu-\mu}{\delta(1-\mu^2)}$. For high-type only search to be feasible, we also require that $p_A = \alpha v_H + (1-\alpha)v_L\left(1+(1-\alpha)\delta\mu^2-(1+\delta)\mu\right) < p < \alpha v_H(1-\mu)(\delta(-\alpha\mu-\alpha+1)+1)-\delta\mu v_L(1-\alpha)(1-\alpha\mu) = p_B$. Comparing p_A and p_B, it can be see that

$$\begin{aligned} p_B - p_A &= \alpha v_H\left(\delta\left(1-\mu\right)\left(1-\alpha\mu-\alpha\right)-\mu\right) - v_L\left(1-\alpha\right)\left(\mu-\left(1-2\alpha\right)\delta\mu^2-1\right) \\ \frac{\partial\left(p_B-p_A\right)}{\partial v_H} &= \alpha\left(\delta\left(1-\mu\right)\left(1-\alpha\mu-\alpha\right)-\mu\right) \end{aligned}$$

Note that when $\alpha > \frac{\delta-\delta\mu-\mu}{\delta(1-\mu^2)}$, $\frac{\partial(p_B-p_A)}{\partial v_H} < 0$. So the maximum value of $p_B - p_A$ will occur when $v_H = v_L$. Also note that the co-efficient of v_H is negative when $\alpha > \frac{\delta-\delta\mu-\mu}{\delta(1-\mu^2)}$. Since $p_B - p_A$ can be positive only if v_H is sufficiently high, we only need to check the sign of the co-efficient of v_L at $\alpha = \frac{\delta-\delta\mu-\mu}{\delta(1-\mu^2)}$. It can be see that $\left(\mu-\left(1-2\alpha\right)\delta\mu^2-1\right)\Big|_{\alpha=\frac{\delta-\delta\mu-\mu}{\delta(1-\mu^2)}} = -\frac{\mu^2\left(\delta(\mu-1)^2-2\mu\right)}{\mu^2-1}+\mu-1 < 0$. Since $\frac{1-\delta\mu-\mu}{\delta(1-\mu^2)} > \frac{\delta-\delta\mu-\mu}{\delta(1-\mu^2)}$, when $\pi_{M2} = \pi_{M21} > \pi_{M32}$, $p_B < p_A$ which makes high-type only online search infeasible. So high-type only online search cannot be optimal when $\eta > \overline{\eta}$.

When $\eta < \overline{\eta}$, $\pi_{M2} = \pi_{22} < \pi_{21}$. So high-type only search cannot be optimal even when $\eta < \overline{\eta}$.

Proof of Proposition 3: Online Market Supporting Search Only: Sensitivity of optimal search fee with α and μ and η.

It is optimal for the firm to price its search service at p_M^* such that both types can search online in absence of authentication support (as shown in Proposition 2) where $p_M^* = \min\left[\pi_{M31}, \pi_{M32}\right]$

First note that

$$\frac{\partial\left(\pi_{M31}-\pi_{M32}\right)}{\partial\alpha} = \mu\left(\left(1+\delta\right)\left(v_H+v_L\right)-2\delta\mu\left(\alpha v_H+\left(1-\alpha\right)v_L\right)\right) > 0$$

Thus, the difference in search fees that the L-type and H-type are respectively willing to pay is increasing in α. Since the optimal search fee that the online intermediary charges is $p_M^* =$

$\min[\pi_{M31}, \pi_{M32}]$, when α is sufficiently high, the optimal price $p_M^* = \pi_{M32}$; otherwise it is $p_M^* = \pi_{M31}$. Let α_k be the threshold on α above which $p_M^* = \pi_{M32}$.

When $\alpha < \alpha_k$, we have that,

$$\frac{\partial p_M^*}{\partial \alpha} = v_H - v_L \left(1 - \mu \left(1 + \delta \left(1 - 2\mu \left(1 - \alpha\right)\right)\right)\right) > 0$$

Further, $\alpha > \alpha_k$, $p_M^* = \pi_{M32}$, and

$$\frac{\partial^2 \pi_{M32}}{\partial \alpha^2} = 2 v_H \delta \mu^2 > 0$$

implying that π_{M32} is convex in α. Additionally, we have that

$$\frac{\partial \pi_{M32}}{\partial \alpha} = v_H \left(2\alpha\delta\mu^2 - (\delta+1)\mu + 1\right) - v_L$$

Note that $\frac{\partial \pi_{M32}}{\partial \alpha} > 0$ if $\alpha > \alpha_m = \frac{v_H(\delta\mu+\mu-1)+v_L}{2\delta\mu^2 v_H}$ and $\frac{\partial \pi_{M32}}{\partial \alpha} > 0$ if $\alpha < \alpha_m$.

If follows that when $\alpha < \alpha_k$, p_M^* is increasing in α. When $\alpha_k < \alpha < \alpha_m$, p_M^* is decreasing in α and when $\alpha > \alpha_m$, p_M^* is increasing in α.

To determine the sensitivity w.r.t η, note that

$$\frac{\partial \left(\pi_{M31} - \pi_{M32}\right)}{\partial \eta} = \alpha\mu \left(1 + \delta - \alpha\delta\mu\right) > 0$$

In addition

$$\begin{aligned} \frac{\partial \left(\pi_{M31}\right)}{\partial \eta} &= \alpha > 0 \\ \frac{\partial \left(\pi_{M32}\right)}{\partial \eta} &= \alpha \left(\alpha\delta\mu^2 - (\delta+1)\mu + 1\right) \end{aligned}$$

Note that $\frac{\partial(\pi_{M32})}{\partial \eta} < 0$ if δ is sufficiently high. So $p_M^* = \min\left[\pi_{M31}, \pi_{M32}\right]$ is non-monotonic in η.

ii) For the sensitivity of p_M^* with respect to μ, note that

$$\begin{aligned}
\frac{\partial \pi_{M31}}{\partial \mu} &= -v_L\left(1-\alpha\right)\left(1+\delta\left(1-2\left(1-\alpha\right)\mu+1\right)\right)<0 \\
\frac{\partial \pi_{M32}}{\partial \mu} &= -v_H\alpha\left(1+\delta\left(1-2\alpha\mu\right)\right)<0
\end{aligned}$$

Since both π_{M31} and π_{M32} are decreasing in μ, $p_M^* = \min\left[\pi_{M31}, \pi_{M32}\right]$ should also be decreasing in μ.

Proof of Lemma 2: Online Market with Perfect Authentication: Match-seeker's Equilibrium Behavior

First, it is easy to see that L-type purchasing authentication is a dominated strategy. Similarly, $V_H^{A1} > V_H^{M1}$; so the case in which H-type purchases authentication when L-type is in the direct market can also not be an equilibrium. From Lemma 1**,** we know that the case where L-type searches online, while H-type stays in the direct market is also not an equilibrium. So only the 4 cases given in the proposition are possible equilibria.

From Table 2, both types search in the direct market if $V_H^D > \max\left[V_H^{M1}, V_H^{A1}\right] = V_H^{M1}$ and $V_L^D > V_L^{M3}$. From equations (4, 5 , 6 and 10), this results in the following condition:

$$\begin{aligned}
p \geqslant \overline{p}_A &= \max\left[\alpha v_H(1-\mu)(\delta(-\alpha\mu-\alpha+1)+1)-\delta\mu v_L(1-\alpha)(1-\alpha\mu)\right., \\
&\qquad \left.(1-\alpha)\,v_L\left(\alpha\delta+(1-\mu)\left(1-\delta\mu\right)\right)-\alpha\mu v_H\left(\delta(1-\mu)+1\right)\right]
\end{aligned}$$

When only H-type searches online and L-type searches in the direct marketplace, $V_H^{M1} > V_H^D$and $V_L^{M1} > V_L^{M2}$. Consistent with Lemma **1**, this can be possible only if $\underline{p}_A < p < \overline{p}_A$. When both types search in the online market and neither purchase authentication, we require $V_H^{M2} > \max\left[V_H^{A2}, V_H^{M3}\right]$and $V_L^{M2} > V_L^{M1}$. Comparing these value functions, it can be seen that the search fee p and authentication fee q should be,

$$\begin{aligned}
p &< \underline{p}_A = \min\left[\alpha(1-\mu(1+\delta\left(1-\alpha\mu\right)v_H+\left(1-\alpha\right)v_L,\right. \\
&\qquad \left.\alpha v_H+(1-\alpha)(1-(1+\delta)\mu+(1-\alpha)\delta\mu^2)v_L\right] \\
q &> q_A = (1-\alpha)(\alpha\delta v_H-(1-\delta\left(1-\alpha\right))v_L)
\end{aligned}$$

When both types search online and only the H-type purchases authentication service, $V_H^{A2} > \max\left[V_H^{M2}, V_H^{M3}\right]$and $V_L^{A2} > V_L^{A1}$. This implies that the search fee p and authentication fee q are such that

$$
\begin{aligned}
p &< \alpha^2\delta v_H + (1-\alpha)(1-\mu)(1-\delta\mu+\alpha\delta(1+\mu))v_L \\
q &< q_A < \min\left[(1-\alpha)\left(\alpha\delta v_H - (1-\delta(1-\alpha))\, v_L\right),\right. \\
&\quad \left.(1-\alpha)^2\delta v_L + \alpha(1-\mu)(1+\delta(1-\alpha(1+\mu)))\, v_H - p\right]
\end{aligned}
$$

Proof of Lemma 3: Online Market with Perfect Authentication: Optimal Search and Authentication Fee

From the proof of Lemma 2, we know that for this case to be an equilibrium, we require $V_H^{A2} > V_H^{M2}$, $V_H^{A2} > V_H^{M3}$ and $V_L^{A2} > V_L^{A1}$. This gives us the following constraints:

$$
\begin{aligned}
q &\geqslant q_M = (1-\alpha)\left(\alpha\delta v_H - v_L\left(1-(1-\alpha)\delta\right)\right) \\
p &\geqslant p_M = \alpha^2\delta v_H + v_L(1-\alpha)(1-\mu)\left(\alpha\delta(\mu+1) - \delta\mu + 1\right) \\
p+q &\geqslant t_M = (1-\alpha)^2\delta v_L + \alpha v_H(1-\mu)\left(1+\delta(1-\alpha(1+\mu))\right)
\end{aligned}
$$

First note that this case is feasible only if $p_M, t_M \geqslant 0$. Differentiating each of these w.r.t. v_H, we have

$$
\begin{aligned}
\frac{\partial p_M}{\partial v_H} &= \delta\alpha^2 > 0 \\
\frac{\partial t_M}{\partial v_H} &= \alpha(1-\mu)(1+\delta(1-\alpha-\mu\alpha)) > 0
\end{aligned}
$$

implying that that p_M and t_M is increasing in v_H . So v_H is sufficiently high i.e.

$$
\begin{aligned}
v_H \geqslant v_{HM} = \max&\left[\frac{v_L(1-\delta(1-\alpha))}{\delta\alpha},\right. \\
&\left.-\frac{v_L(1-\alpha)(1-\mu)(1+\delta\alpha-\delta\mu(1-\alpha))}{\delta\alpha^2}\right]
\end{aligned}
$$

we will have $p_M, t_M \geqslant 0$ and this case is feasible. The profits of the firm are $\pi_M = p + \alpha(q-c)$ where $p, q \geqslant 0$ and satisfies the above constraints. Given that the profit function is increasing in

p, q and $0 < \alpha < 1$, the optimal fees has to be one of the following:

If $q_M \leqslant 0$, then $p^* = p_M$ and $q^* = \epsilon \approx 0$

If $p_M + q_M < t_M$, then $p^* = p_M$ and $q^* = q_M$.

If $p_M < t_M < p_M + q_M$, then $p^* = p_M$and $q^* = t_M - p_M$.

If $t_M < p_M$, then $p^* = t_M$and $q^* = \epsilon \approx 0$.

Now let us examine which of these cases occur for different values of α. First note that

$$\begin{aligned} \frac{\partial^2 (p_M - t_M)}{\partial \alpha^2} &= 2(v_H - v_L)\delta\left(2 - \mu^2\right) > 0 \\ \frac{\partial^2 (p_M + q_M - t_M)}{\partial \alpha^2} &= 2(v_H - v_L)\delta\left(1 - \mu^2\right) > 0 \end{aligned}$$

This implies that both $p_M + q_M - t_M$ and $p_M - t_M$ are convex in α. In addition

$$p_M + q_M - t_M|_{\alpha=0} = -\mu v_L (1 + \delta(1 - \mu)) < 0$$

So in the region $v_M \geqslant v_{HM}$, $p_M + q_M - t_M = 0$ and $p_M - t_M = 0$ can only have one positive root each. Let $\overline{\alpha}$ be the positive root for $p_M + q_M - t_M = 0$ and let $\underline{\alpha}$ be the positive root for $p_M - t_M = 0$. Since in the region $v_M \geqslant v_{HM}$, $q_M > 0$, we also have that $\overline{\alpha} > \underline{\alpha}$. Finally, note that $q_M < 0$ if $\alpha < \frac{(1-\delta)v_L}{\delta(v_H - v_L)} = \alpha_p$. It follows that

If $\alpha < \alpha_p$, then $q_M < 0$ which implies that $p^* = p_M$ and $q^* = 0$.

If $\alpha_p < \alpha < \underline{\alpha}$, then $p_M + q_M < t_M$ which implies that $p^* = p_M$ and $q^* = q_M$.

If $\underline{\alpha} < \alpha < \overline{\alpha}$, then $p_M < t_M < p_M + q_M$ and so $p^* = p_M$and $q^* = t_M - p_M$.

If $\alpha > \overline{\alpha}$, then $t_M < p_M$, and so $p^* = t_M$and $q^* = \epsilon \approx 0$.

Proof of Proposition 4

First, when $\alpha < \alpha_p$, $q^* = 0$ implying that $q^* < c\ \forall c > 0$. Second, note that

$$\begin{aligned} \left.\frac{\partial q^*}{\partial \alpha}\right|_{\alpha > \overline{\alpha}} &= (1 - 2\alpha)\delta v_H + v_L(2(\alpha - 1)\delta + 1) \\ \left.\frac{\partial^2 q^*}{\partial \alpha^2}\right|_{\alpha > \overline{\alpha}} &= -2(v_H - v_L)\delta < 0 \end{aligned}$$

From the above, it can seen that q^* is concave in α and decreasing in α when α is sufficiently high. Combined with the fact that $q^* \approx 0$ when $\alpha < \alpha_p$ and $\alpha > \overline{\alpha}$, it follows that $q^* < c$ both for very high values of α and for very low values of α.

In addition, from Lemma 3, since $q_M > t_M - p_M$ when $\underline{\alpha} < \alpha < \overline{\alpha}$, there should exist two thresholds α' and α''such that authentication generates positive profits only if $\alpha' < \alpha < \alpha''$ and would be a loss leader otherwise.

Proof of Proposition 5: Online Market with Perfect Authentication: Optimal Firm Strategy

i) When α is sufficiently low ($\alpha < \min(\underline{\alpha}, \alpha_k)$), where α_k is as defined in the proof of Proposition 3, and $c = 0$, the profits under no-authentication is π_{M32} (from the proof of Proposition 3) and that under perfect authentication is p_M which we denote as π_{P1}. First, note that

$$\begin{aligned} \frac{\partial^2 (\pi_{P1} - \pi_{M32})}{\partial \alpha^2} &= 2\delta (v_H - v_L) > 0 \\ \pi_{M32} - \pi_{P1}\big|_{\alpha=0} &= 0 \end{aligned}$$

Let α_1 be the positive root for $\pi_{P1} - \pi_{M32} = 0$. From the above, $\pi_{M32} < \pi_{P1}$ when $\alpha > \alpha_1$. So there exists a threshold on α such that when α is above that threshold, perfect authentication is optimal.

ii) When $\alpha > \overline{\alpha}$, the profits under perfect authentication are $t_M - c\alpha$, which we denote as π_{P2} and profits under no-authentication is π_{M31}

$$\frac{\partial^2 (\pi_{P2} - \pi_{M31})}{\partial \alpha^2} = -2\delta (v_H - v_L) < 0$$

In addition, we know that (from proof of proposition 2 and 3) that π_{M31} is convex in α (and hence increasing in α when α is sufficiently high). Also, $t_M - \pi_{M31} = 0$ when $\alpha = 1$. Together these imply that when c and α are sufficiently high, $\pi_{P2} < \pi_{M31} = \pi_M$.

iii) Consider the region $\alpha_p < \alpha < \underline{\alpha}$. The profits under perfect authentication are $p_M + \alpha (q_M - c)$

which we denote by π_{P3}. The profits for no-authentication would be $\min\left[\pi_{M31}, \pi_{M32}\right]$. Let us compare π_{P3} with π_{M32}

$$\pi_{P3} - \pi_{M32}\big|_{c=0} \quad = \quad (1-\alpha)\, v_L\left((2-\alpha)\,\delta - 1\right) - v_H\left(\alpha^2\delta - 2\alpha\delta + 1\right) < 0$$

So in the region $\alpha < \min\left[\underline{\alpha}, \alpha_k\right]$, no-authentication is optimal even for $c = 0$. Now let us compare $\pi_{P3}$with π_{M31}. Since $\pi_{M31}$is convex in α, it is decreasing in α when α is sufficiently low. In addition

$$\pi_{P3} - \pi_{M31} \quad = \quad -\mu v_L(\delta(1-\mu) + 1) < 0$$

Also,

$$\frac{\partial^2\left(p_M - \pi_{M31}\right)}{\partial\alpha^2} \quad = \quad 2\delta\left(1-\mu^2\right)\left(v_H - v_L\right) > 0$$

Since $\pi_3 = p_M + \alpha\left(q_M - c\right)$,there would exist a threshold on c above which $\pi_{M31} > \pi_{P3}$. Also, $\pi_{P3} > \pi_{M31}$ only if α is sufficiently high.

iv) Differentiating $\left(\pi_{P3} - \pi_{M31}\right)$ w.r.t to μ, we have

$$\begin{aligned}
\frac{\partial\left(\pi_{P3} - \pi_{M31}\right)}{\partial\mu} &= \alpha v_H(-2\alpha\delta\mu + \delta + 1) + (\alpha - 1)v_L(2(\alpha-1)\delta\mu + \delta + 1) \\
&= \alpha\left(1+\delta\right)v_H - \left(1-\alpha\right)\left(1+\delta\right)v_L + 2\mu\left((1-\alpha)^2\delta v_L - \alpha^2\delta v_H\right)
\end{aligned}$$

The expression is positive when α is sufficiently high i.e. $\alpha > \frac{v_H}{v_H + v_L}$. Since $\pi_{P3} - \pi_{M31}$ is decreasing in c, this implies that the threshold on c above which $\pi_{P3} < \pi_{M31}$ would be increasing in μ if $\alpha > \frac{v_H}{v_H + v_L}$.

Similarly, we also know (from above) that $\pi_{P3} - \pi_{31}$ is increasing at the point at which $\pi_{P3} - \pi_{31} = 0$. Again, since $\partial_\mu\left(\pi_{P3} - \pi_{31}\right) > 0$ in this region, the threshold beyond which $\pi_{P3} - \pi_{M31} > 0$ would be decreasing in μ.

Now let us compare profits under no-authentication to profits under perfect authentication when $\underline{\alpha} < \alpha < \overline{\alpha}$. Here the profits under perfect authentication are $p_M + \alpha\left(t_M - p_M - c\right)$ which we denote by π_{P4}. We only need to compare π_{M31} to π_{P4}. This is because $\pi_{M32} > \pi_{P3} > \pi_{P4}$ and because $t_M - p_M < q_M$ when $\underline{\alpha} < \alpha < \overline{\alpha}$.

Comparing we have

$$\begin{aligned}\pi_{P4} - \pi_{M31}\big|_{c=0} &= -(1-\alpha)\left(\alpha v_H\left(\alpha\delta\left(\mu^2-2\right)-(\delta+1)\mu+1\right)+\right.\\ &\left.\left(v_L\left(\alpha^2(-\delta)\left(\mu^2-2\right)+\alpha\left(\delta\left(2\mu^2-\mu-2\right)-\mu+1\right)+\mu(\delta(-\mu)+\delta+1)\right)\right)\right)\end{aligned}$$

Note that $\pi_{P4} - \pi_{M31} = (1-\alpha)\, A(\alpha)$ where

$$\begin{aligned}A(\alpha) &= -\alpha v_H\left(\alpha\delta\left(\mu^2-2\right)-(\delta+1)\mu+1\right)\\ &\quad -v_L\left(\alpha^2(-\delta)\left(\mu^2-2\right)+\alpha\left(\delta\left(2\mu^2-\mu-2\right)-\mu+1\right)+\mu(\delta(-\mu)+\delta+1)\right)\end{aligned}$$

Also

$$\frac{\partial^2 A(\alpha)}{\partial\alpha^2} = 2\left(v_H - v_L\right)\left(2-\mu^2\right) > 0$$

So, again for sufficiently high α, $\pi_{P4} > \pi_{M31}$ for $c = 0$ and when c sufficiently high, we will again have $\pi_4 < \pi_{M31}$. In addition

$$\begin{aligned}\frac{\partial\left(\pi_{P4} - \pi_{M31}\right)}{\partial\mu} &= (1-\alpha)\left(\alpha v_H\left(1+\delta-2\alpha\delta\mu\right)+v_L\left(1-\alpha\right)\left(1+\delta+2\alpha\delta\mu-2\delta\mu\right)\right)\\ &= (1-\alpha)\left((1+\delta)\left(\alpha v_H-(1-\alpha)\,v_L\right)+2\mu\left((1-\alpha)^2\,\delta v_L-\alpha^2\delta v_H\right)\right)\end{aligned}$$

Again, this expression is positive when α is sufficiently high i.e. $\alpha > \frac{v_H}{v_H+v_L}$. This combined with the fact that $\pi_{P4} - \pi_{M31}$ is decreasing in c, implies that the threshold on c above which $\pi_4 < \pi_{M31}$ would be increasing in μ.

In the same way as above, we know that $\pi_{P4} - \pi_{32}$ is increasing at the point at which $\pi_{P4} - \pi_{31} = 0$. Again, since $\partial_\mu\left(\pi_{P4} - \pi_{32}\right) > 0$ in this region, the threshold beyond which $\pi_{P3} - \pi_{M31} > 0$ would be decreasing in μ.

Proof of Lemma 4: Online Market with Imperfect Authentication: Match-seeker's Equilibrium Behavior

Proofs for part i), ii) and iii) are same as Lemma 2. When both types search online and only the H-type purchases authentication service, $V_H^{I2} > \max\left[V_H^{M2}, V_H^{M3}\right]$and $V_L^{I2} > \max\left[V_L^{I1}, V_L^{I5}\right]$. This implies that the search fee p and authentication fee q are such that,

$$
\begin{aligned}
p &< \alpha^2\delta v_H + (1-\alpha)(1-\mu)(1-\delta\mu+\alpha\delta(1+\mu))v_L \leqslant \overline{p} \\
q &< q_A < \min\left[(1-\alpha)(\alpha\delta v_H - (1-\delta(1-\alpha))v_L),\right. \\
&\quad \left.(1-\alpha)^2\delta v_L + \alpha(1-\mu)(1+\delta(1-\alpha(1+\mu)))v_H - p\right] \leqslant \overline{q} \\
q &> (1-\gamma)(\alpha v_H(1+2\gamma\delta-\alpha(2\gamma\delta+\delta)) \\
&\quad -(1-\alpha)v_L(2\gamma(-(1-\alpha)\delta+1)+\alpha\delta)) \geqslant \overline{q}
\end{aligned}
$$

This implies that the above case is an equilibrium only $p < \overline{p}$ and $\underline{q} < q < \overline{q}$.

For both types to purchase authentication, we need the following conditions to be true: $V_H^{I5} > \max\left[V_H^{I4}, V_H^{I3}\right]$ and $V_L^{I4} > \max\left[V_L^{I1}, V_L^{I2}\right]$. The first of these conditions leads to the following constraints:

$$
\begin{aligned}
q &< \min\left[(1-\alpha)(1-2\gamma)(v_L(1-(1-\alpha)\delta)-\alpha\delta v_H),\right. \\
&\quad \gamma(1-\alpha)(v_L((1-\alpha)\delta-1)+\alpha\delta v_H), \\
&\quad \left.(1-\gamma)(\alpha v_H(-\alpha(2\gamma\delta+\delta)+2\gamma\delta+1)-v_L(1-\alpha)(2\gamma(-(1-\alpha)\delta+1))+\alpha\delta))\right] \leqslant \underline{q} \\
p &< \min\left[(1-\alpha)v_L(1-\alpha\gamma\delta+\gamma\delta-\gamma)+\alpha v_H\left((1-\alpha)\gamma\delta+\alpha\delta\mu^2-\delta\mu-\mu+1\right)-q,\right. \\
&\quad \alpha v_H(\gamma\delta(2-2(1-\alpha)\gamma-\alpha)-\gamma+1)-q \\
&\quad \left.+(1-\alpha)v_L\left(2\gamma^2(1-(1-\alpha)\delta)-\gamma(2-(2-\alpha)\delta)-\mu(1+\delta-(1-\alpha)\delta\mu)+1\right)\right] \leqslant \overline{p}
\end{aligned}
$$

This implies that for the above case to be an equilibrium, $p < \overline{p}$ and $q < \underline{q}$.

Proof of Proposition 6: Imperfect Authentication: H-type Only

From the proof of Lemma 4, we know that for the case where only H-type buys authentication to be an equilibrium requires $V_H^{I2} > V_H^{M2}$, $V_H^{I2} > V_H^{M3}$, $V_L^{I2} > V_L^{I1}$ and $V_L^{I2} > V_L^{I5}$. This gives us the following constraints:

$$
\begin{aligned}
q_M \quad \geqslant q \geqslant \quad & q_{M1} = (1-\gamma)\left(\alpha v_H\left(1+2\gamma\delta-\alpha\left(2\gamma\delta+\delta\right)\right)\right. \\
& \left.-(1-\alpha)\,v_L\left(2\gamma\left(-(1-\alpha)\,\delta+1\right)+\alpha\delta\right)\right) \\
p \quad \geqslant \quad & p_M \\
p+q \quad \geqslant \quad & t_M
\end{aligned}
$$

The profits of the firm are $\pi_{I1} = p + \alpha\left(q - c\gamma\right)$ where $p, q \geqslant 0$ and satisfies the above constraints. As under perfect authentication, since the profit function is increasing in p, q and $0 < \alpha < 1$, the optimal fees has to be one of the following:

1. If $p_M + q_M < t_M$, then $p^* = p_M$ and $q^* = q_M$.
2. If $p_M + q_{M1} < t_M < p_M + q_M$, then $p^* = p_M$and $q^* = t_M - p_M$.
3. If $t_M < p_M + q_{M1}$, then $p^* = t_M - q_{M1}$and $q^* = q_{M1}$.

From proof of Lemma 3, we know the following:

1. If $\alpha_p < \alpha < \underline{\alpha}$, then $p_M + q_M < t_M$ which implies that $p^* = p_M$ and $q^* = q_M$.
2. If $\underline{\alpha} < \alpha < \overline{\alpha}$, then $p_M < t_M < p_M + q_M$ and so $p^* = p_M$ and $q^* = t_M - p_M$.

First let us compare q_M and q_{M1} which gives us that:

$$
\begin{aligned}
q_M - q_{M1} \quad = \quad & \alpha v_H\left(\gamma\left(\left(-2\alpha\gamma+\alpha+2\gamma-2\right)+1\right)\right)+\delta-1) \\
& -(1-\alpha)\,v_L\left(\gamma\delta\left(-2\left(1-\alpha\right)\gamma-\alpha+2\right)-2\left(1-\gamma\right)\gamma-\delta+1\right)
\end{aligned}
$$

The above expression is negative when $\gamma < \frac{1}{2}$ which implies that $q_M < q_{M1}$ which contradicts the constraint above and this case is infeasible.

Now let us look at $p_M + q_{M1} - t_M$:

$$\begin{aligned} p_M + q_{M1} - t_M &= \alpha v_H \left(\delta\left(\alpha + \mu - 1 - \alpha\mu^2\right) + \mu - 2\left(1-\alpha\right)\gamma^2\delta - \gamma\left(1 - \left(2-\alpha\right)\delta\right)\right) \\ &\quad + \left(1-\alpha\right) v_L \left(2\gamma^2\left(1 - \left(1-\alpha\right)\delta\right) - \gamma\left(2 - \left(2-\alpha\right)\delta\right)\right. \\ &\quad \left. + \delta\mu\left(-\alpha\mu + \mu - 1\right) - \left(1-\alpha\right)\delta - \mu + 1\right) \end{aligned}$$

In addition,

$$\frac{\partial^2\left(p_M + q_{M1} - t_M\right)}{\partial\mu^2} = 2\delta\left(1 - \gamma\left(1 - 2\gamma\right) - \mu^2\right)\left(v_H - v_L\right)$$

This expression is positive when $\gamma \geqslant \frac{1}{2}$ implying that $\left(p_M + q_{M1} - t_M\right)$ is convex in the region where H-type only authentication is feasible. Also note that

$$p_M + q_{M1} - t_M\big|_{\alpha=0} = v_L\left(-2\gamma\left(1-\gamma\right)\left(1-\delta\right) + \left(1-\mu\right)\left(1-\delta\mu\right) - \delta\right) < 0$$

This in conjunction with the fact that $q_M > q_{M1}$ implies that when $\alpha > \overline{\alpha}$, $t_M < p_M + q_{M1}$ and the optimal fees will be $p^* = t_M - q_{M1}$ and $q^* = q_{M1}$.

Given above, the optimal authentication fees $q^* = \max\left[\min\left[q_M, t_M - p_M\right], q_{M1}\right]$. Both q_M and $t_M - p_M$ are independent of γ. We also have

$$\begin{aligned} \frac{\partial q_{M1}}{\partial\gamma} &= \alpha v_H\left(\delta\left(4\alpha\gamma - \alpha - 4\gamma + 2\right) - 1\right) \\ &\quad + v_L\left(1-\alpha\right)\left(4\gamma\left(1 - \left(1-\alpha\right)\delta\right) - \alpha\delta + 2\delta - 2\right) \end{aligned}$$

This expression is negative when $q_M > q_{M1}$ implying $\partial_\gamma q_{M1} \leqslant 0$.

Proof of Proposition 7: Imperfect Authentication: Both types

From the proof of Lemma 4, we know that for this case to be an equilibrium, we require $V_H^{I5} > V_H^{I4}$, $V_H^{I5} > V_H^{I1}$, $V_L^{I5} > V_L^{I1}$ and $V_L^{I5} > V_L^{I1}$. This gives us the following constraints:

$$
\begin{aligned}
q &\leqslant q_{M1} = (1-\gamma)\left(\alpha v_H\left(1+2\gamma\delta-\alpha\left(2\gamma\delta+\delta\right)\right)\right. \\
&\quad \left.-\left(1-\alpha\right)v_L\left(2\gamma\left(-\left(1-\alpha\right)\delta+1\right)+\alpha\delta\right)\right) \\
q &\leqslant q_{M2} = (1-\alpha)\left(1-2\gamma\right)\left(v_L\left(1-\left(1-\alpha\right)\delta\right)-\alpha\delta v_H\right) \\
p+q &\geqslant t_{M1} = (1-\alpha)v_L\left(1-\alpha\gamma\delta+\gamma\delta-\gamma\right) \\
&\quad +\alpha v_H\left(\left(1-\alpha\right)\gamma\delta+\alpha\delta\mu^2-\delta\mu-\mu+1\right) \\
p+q &\geqslant t_{M2} = \alpha v_H\left(1-2\left(1-\alpha\right)\gamma^2\delta-\gamma\left(1-(2-\alpha)\delta\right)\right) \\
&\quad +v_L\left(1-\alpha\right)\left(2\gamma^2\left(1-\left(1-\alpha\right)\delta\right)-\gamma\left(2-\left(2-\alpha\right)\delta\right)+\mu^2\delta\left(1-\alpha\right)-\left(\delta+1\right)\mu+1\right)
\end{aligned}
$$

Let the sum of the authentication and search fee $(p+q)$ be t. The profits of the firm are then $\pi_{I2} = t - c - \kappa$. So the optimal total fee t^* has to be $\min\left[t_{M1}, t_{M2}\right]$. Differentiating t_{M1} and t_{M2} w.r.t γ, we obtain

$$
\frac{\partial t_{M1}}{\partial \gamma} = (1-\alpha)\left(\alpha\delta v_H + (1-\alpha)\delta v_L - v_L\right)
$$

Note that in order for authentication to be considered, the threshold η_1 above which H-type is exclusive (ref: Proposition 1) has to be such that $\eta > \eta_1|_{\mu=1}$. In the region $\eta > \eta_1|_{\mu=1}$, the above expression is positive implying that t_{M1} is increasing in γ. In addition,

$$
\begin{aligned}
\frac{\partial t_{M2}}{\partial \gamma} &= \alpha v_H\left(\delta\left(4\alpha\gamma-\alpha-4\gamma+2\right)-1\right)+v_L\left(1-\alpha\right)\left(4\gamma\left(1-\left(1-\alpha\right)\delta\right)-\alpha\delta+2\delta-2\right) \\
\frac{\partial^2 t_{M2}}{\partial \gamma^2} &= -4\left(1-\alpha\right)\left(\alpha\delta v_H - v_L\left(1-\left(1-\alpha\right)\delta\right)\right)
\end{aligned}
$$

Note that $\partial^2_{\mu^2}\left(t_{M2}\right) < 0$ when $\eta > \eta_1$ implying that t_{M2} is concave in this range. So for sufficiently high γ, t_{M2} is decreasing in γ. It follows that t_{M2} is non-monotonic in γ.

Proof of Proposition 8: Online Market with Imperfect Authentication

From the proof of Proposition 6, we know that when γ is sufficiently low, H-type only authentication is infeasible. So for low γ, we can restrict our comparison to no-authentication and authentication by both types. Also note that both t_{M1} and t_{M2} are increasing in γ when γ is sufficiently low. When $c = 0$, no-authentication can be optimal only if $\min\left[\pi_{M31}, \pi_{M32}\right] > \min\left[t_{M1}, t_{M2}\right] - \kappa$. Comparing these values at $\gamma = 0$, we have

$$\begin{aligned} t_{M1}|_{\gamma=0} &= \alpha v_H \left(\alpha\delta\mu^2 - (\delta+1)\mu + 1\right) + (1-\alpha) v_L = \pi_{M31} \\ t_{M2}|_{\gamma=0} &= \alpha v_H + (1-\alpha) v_L \left(1 + (1-\alpha)\delta\mu^2 - (\delta+1)\mu\right) = \pi_{M32} \end{aligned}$$

So $\pi_{I2} = \min\left[t_{M1}, t_{M2}\right] - \kappa < \min\left[\pi_{M31}, \pi_{M32}\right]$ when $\kappa > 0, \gamma = 0$. This implies that when $\gamma = 0$, no authentication is optimal. Since t_{M1} and t_{M2} are increasing in γ, there should exist a threshold above which authentication by both types leads to higher profits than no-authentication and below which, no-authentication is optimal.

Now let us evaluate what happens when γ is sufficiently high. Here, we only need to compare authentication by both types to authentication by H-type only. Recall that $q^* = \max\left[q_{M1}, \min\left[q_{M2}, t_M - p_M\right]\right]$.

Let us assume that $\kappa = 0$. When $q^* = q_{M1}$, the profits of the firm are $t_M + (1-\alpha) q_{M1}$ which we denote by π_{IH1}. Comparing it with t_{M1} we have

$$\pi_{IH1} - t_{M1} = (1-\alpha)(1-\gamma)\left(\alpha v_H(1-(\alpha-1)(2\gamma+1)\delta) + v_L\left(2(\alpha-1)\gamma((\alpha-1)\delta+1) + (\alpha-1)^2\delta - 1\right)\right)$$

Also, note that

$$\frac{\partial^2\left(\pi_{IH1} - t_{M1}\right)}{\partial\gamma^2} = 4(1-\alpha)^2\left(v_L\left(1-(1-\alpha)\delta\right) - \alpha\delta v_H\right) < 0$$

So $(\pi_{IH1} - t_{M1})$ is concave in γ. This implies that when γ is sufficiently high, $(\pi_{IH1} - t_{M1})$ is decreasing in γ. At $\gamma = 1$, $\pi_{IH1} - t_{M1} = 0$. So there should exist a threshold on γ above which $\pi_{IH1} > t_{M1}$.

Now let us compare t_{M1} and t_{M2}. Since t_{M1} is linear in γ and t_{M2} is concave in γ,

$$\begin{aligned} t_{M1} - t_{M2}|_{\gamma=1} &= \alpha v_H \left(\delta \left(\alpha \left(\mu^2 - 2\right) - \mu + 1\right) - \mu + 1\right) \\ &\quad + (1-\alpha) v_L \left(\delta \left(\alpha \left(\mu^2 - 2\right) - \mu^2 + \mu + 1\right) + \mu - 1\right) \end{aligned}$$

Suppose the above expression is positive; then $t_{M1} > t_{M2}$ if γ is sufficiently high. So the optimal total fee (and profits) would be t_{M2}.

Now let us compare t_{M2} to $p_M + \alpha (t_M - q_M)$. We have that

$$\begin{aligned} p_M + \alpha (t_M - q_M) - t_{M2}|_{\gamma=1} &= \alpha \left(\alpha v_H \left(\delta \left(\alpha \left(\mu^2 - 2\right) - \mu + 1\right) - \mu + 1\right)\right. \\ &\quad \left. + (1-\alpha) v_L \left(\delta \left(\alpha \left(\mu^2 - 2\right) - \mu^2 + \mu + 1\right) + \mu - 1\right)\right) \\ &= \alpha (t_{M1} - t_{M2}) > 0 \end{aligned}$$

So for sufficiently high γ, $t_{M2} < p_M + \alpha (t_M - q_M) - t_{M2}$.

Suppose $t_{M1} - t_{M2}|_{\gamma=1} < 0$. Then

$$\begin{aligned} p_M + \alpha (t_M - q_M) - t_{M1}|_{\gamma=1} &= -(1-\alpha) \left(\alpha v_H \left(\delta \left(\alpha \left(\mu^2 - 2\right) - \mu + 1\right) - \mu + 1\right)\right. \\ &\quad \left. + (1-\alpha) v_L \left(\delta \left(\alpha \left(\mu^2 - 2\right) - \mu^2 + \mu + 1\right) + \mu - 1\right)\right) \\ &= (1-\alpha) (t_{M2} - t_{M1}) > 0 \end{aligned}$$

Thus, again for sufficiently high values of γ, $t_{M1} < p_M + \alpha (t_M - q_M)$.

Finally, let us compare t_{M2} with the case in which $q^* = q_{M1}$. Here, profits under high-type only authentication are $p_M + \alpha q_M$. Recall that t_{M2} is convex in γ. In addition

$$\partial_\gamma (t_{M2})|_{\gamma=1} = \alpha v_H ((3\alpha - 2)\delta - 1) + (1-\alpha) v_L ((3\alpha - 2)\delta + 2) < 0$$

So t_{M2} is decreasing in γ when $\gamma = 1$. Also note that

$$p_M + \alpha q_M - t_{M2}|_{\gamma=1} = \alpha (1-\alpha) (\alpha \delta v_H - v_L (1 - (1-\alpha)\delta)) > 0$$

These two in conjunction imply that when γ is sufficiently high, $p_M + \alpha q_M > t_{M2}$.

It follows that for sufficiently high values of γ, the optimal strategy will be to target authentication to H-types only.

B Derivation Of Value Functions Under Imperfect Authentication

In this appendix, we derive the value function for H- and L-types when only the L-type purchases authentication. When only L-types purchase authentication, with probability γ, an L-type would be correctly identified and these L-types have no incentive to reveal their signal. In the first period, the expected number of seekers with no h-signal are α of H-types and $(1-\alpha)\gamma$ of L-types. So the value from the first period is $\alpha v_H + (1-\alpha)\gamma v_L$. The probability of no match in period 1 is $1-(\alpha+(1-\alpha)\gamma)$ and the value from a second-period match is $\alpha v_H + (1-\alpha)v_L$. Putting it all together, the value of H-types when they choose to accept matches from only those without an h-signal would be

$$V_{Ha}^{I4} = \alpha v_H + (1-\alpha)\gamma v_L + \delta(1-(1-\alpha)\gamma-\alpha)(\alpha v_H + (1-\alpha)v_L) - p$$

For the L-types, first note that when they are correctly identified, they are able to obtain matches from H-types. In this case the expected value is $\alpha v_H + (1-\alpha)\gamma v_L$. Again, with probability $1-(\alpha+(1-\alpha)\gamma)$, they do not find a match in period 1 and move to period 2 in which they receive a value of $\delta(\alpha v_H + (1-\alpha)v_L)$.

If L-types receive an h-signal, they are able to match only with other L-types who also received an h-signal in period 1. In this case, the expected value from a match would be $(1-\alpha)(1-\gamma)v_L$. Again, with probability $1-(1-\alpha)(1-\gamma)$, they do not find a match in period 1 and move to period 2 in which they receive a value of $\alpha v_H + (1-\alpha)v_L$.

So the expected value from period 1 after taking into consideration the probability of the authentication correctly identifying the L-types is

$$
\begin{aligned}
V_{La}^{I4} &= \gamma\left(\alpha v_H + (1-\alpha)\gamma v_L + \delta\left(1-(\alpha+(1-\alpha)\gamma)\right)\right)\left(\alpha v_H + (1-\alpha)v_L\right) \\
&\quad +(1-\gamma)\left(\left(1-\alpha\right)\left(1-\gamma\right)v_L + \delta\left(1-\left(1-\alpha\right)\left(1-\gamma\right)\right)\left(\alpha v_H + (1-\alpha)v_L\right)\right) - p - q \\
&= \delta\left(2(\alpha-1)\gamma^2 + (2-3\alpha)\gamma + \alpha - 1\right)\left(\alpha v_H - \alpha v_L + v_L\right) \\
&\quad +(\gamma-1)(2(\alpha-1)\gamma v_L - \alpha v_H) - p - q
\end{aligned}
$$

If the H-types accept matches from everybody, then the value for both types is as before when H-types were inclusive

$$
\begin{aligned}
V_{Hb}^{I4} &= \alpha v_H + (1-\alpha)v_L - p \\
V_{Lb}^{I4} &= \alpha v_H + (1-\alpha)v_L - p - q
\end{aligned}
$$