To Share or Not to Share: Adjustment Dynamics in Sharing Markets

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Abstract

To aid in the description and estimation of the tremendous recent growth in the collaborative economy, we provide a model for the dynamics of sharing, subject to fixed costs and imperfect price formation. The sharing economy comprises a set of infinitely lived, heterogeneous suppliers, who take recurring decisions about entering or leaving the market. We provide a closed-form solution for the nonlinear evolution of the equilibrium in the sharing economy, typically resulting in an S-curve diffusion pattern. In general, the sharing economy can evolve in a nonmonotonic way, with downward adjustments followed either by a steady state or by a positive diffusion towards a steady state. The conversion costs produce a decision hysteresis for potential sharers. Unless these costs are so large that the economy rests in steady state (almost) immediately, the adjustment process does not converge in finite time. The study implies viability conditions for the sharing economy in the presence of an intermediary. The qualitative results are shown to be robust with respect to changes in the assumptions about price formation and hold even in the limiting case of a frictionless economy with almost immediate price updates.

JEL-Classification: C72, D43, D47, L13, L14, O18, R31.

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1 Introduction

The collaborative consumption of durable goods, such as cars, flats, power tools, and clothes, by strangers has become a noticeable part of the economy. As of December 2014, almost half (44%) of the U.S. adult population is familiar with the sharing economy, and close to one fifth (19%) of this population has been involved in a sharing transaction (PwC 2015). More than half (51%) of those familiar with the sharing economy could imagine themselves becoming a supplier over the coming two years, which foreshadows a substantial growth from the currently 23% of those individuals having acted as a supplier thus far (ibid.).[1] Already in 2011, *Time* magazine included "sharing" in their list of the 10 top ideas with the potential to change the world.[2] Since then, the sharing economy has been taking off. By 2014, more than 200 start-ups with an approximate two billion dollars worth of funding have been participating in the growing market for peer-to-peer sharing of physical assets (Teubner 2014). The business models in the various sharing markets include a vast range of personal assets, loans, parking spots, access to wifi-networks, creative projects, business ideas, or personal time.

For many suppliers of shared items, what may have started as a small income boost has turned into a significant source of revenue. *Forbes* magazine estimated that the sharing economy has generated \$3.5 billion dollars of revenue for people in 2013, with an annual growth exceeding 25%.[3] In a global survey, Nielsen (2014) observed that more than two-thirds (68%) of respondents share personal assets primarily for financial gain. This in turn suggests that economic logic is a major factor in the rationale for owners' decisions of whether to share assets with others rather than to keep them solely for their own personal use. This paper addresses the dynamic evolution of the sharing supply as a function of the demand for shared items and the owners' cost of converting personal assets to shared assets, and potentially vice versa. The latter costs create partial irreversibilities in the decisions, which in turn implies a certain stickiness or "decision inertia" in the sharing economy.

One of the main obstacles for sharing consists in gaining access to the marketplace, which includes systems for trust and matching. For this, usually a fixed "conversion cost" needs to be incurred by a prospective sharer. An asset such as a flat needs to be prepared for sharing transactions by removing personal components, creating an item description (including photos and text), listing with an intermediary, and possibly acquiring private insurance.[4] The existence of fixed costs usually implies an inaction region in the space of decision-critical parameters for the owner of an asset. Thus, before sharing becomes interesting to a particular owner, the "effective transaction price" needs to pass a threshold which is generally specific for this individual.[5] Conversely, for the individual to stop sharing, the effective transaction price in the sharing market needs to drop below another threshold which in general, due to the fixed cost involved in repossessing the item, lies below the earlier price threshold. This leads to a certain "hysteresis" in decision making, and thus to frictions in the sharing market.

[1] Based on survey data collected from more than 90,000 respondents, Vision Critical (2014) estimates that there are currently about 80 million sharers in the US (39%), 23 million sharers in the UK (52%), and 10 million sharers in Canada (41%).

[2] "10 Ideas That Will Change the World," *Time*, March 17, 2011.

[3] "AirBnB And The Unstoppable Rise Of The Share Economy," *Forbes,* February 11, 2013. The compound annual growth of 48.9% implied by the prospective increase over two years from 23% to 51% of individuals supplying shared goods mentioned earlier suggests that 25% annual growth may be a rather conservative estimate.

[4] Some intermediaries such as AirBnB offer basic insurance as part of the transaction, others such as HouseTrip do not. Acquiring personal insurance for the latter can be viewed as a capitalized version of the equivalent stream of insurance-related expenditures for the former (often folded into the commission structure).

[5] The *effective transaction price* is the product of posted price (minus commission) and relevant transaction probability.

In this paper, we construct a robust model to describe and analyze the diffusion of sharing in a possibly intermediated economy of heterogeneous agents, who face a stationary demand for shared items. While the demand assumption is somewhat conservative and simplifying, it allows us to focus on explaining sharing-related decisions in a dynamic setting. Interesting aspects of the analysis include the diffusion pattern for sharing, which under standard assumptions is comparable to logistic growth. The latter has been found to fit the diffusion of many consumer goods (Bass 1969; Bass et al. 1994). The findings in our infinite-horizon setting also characterize the law of motion for the economy, in closed form, including an invariance region of steady states. This allows for statements about how the diffusion of sharing depends on parameters such as an intermediary's commission rate, the price elasticity of demand, or the agents' conversion costs. It also implies conditions for the viability of sharing markets as well as characteristic patterns of evolution for the heterogeneous sharing supply.

1.1 Literature

The cultural transition from a mindset of owning property towards an openness for joint use (also termed "access-based consumption"), detected early on by the press (see, e.g., Levine 2009), has been described by Botsman and Rogers (2010), Ozanne and Ballantine (2010), Bardhi and Eckhardt (2012), Lamberton and Rose (2012) and Belk (2014), among others. Belk (2007, 2010) identified the motivation for the sharing of intangibles and tangibles from a psychological and sociological viewpoint, mainly in terms of altruism and a sense of community, including an extension of the self.[6]

In contrast to the abolition of private property, which falls under the contours of socialism (Shafarevich 1980, Ch. 6), modern sharing markets enable the temporary exchange of goods between owners and non-owners. As such, sharing allows for the "collaborative consumption" of durable goods (Felson and Spaeth 1978; Botsman and Rogers 2010), notwithstanding the imperfections and concomitant transaction costs in sharing markets. Apart from the standard problem of finding matching transaction partners in a market where it is difficult to inspect all possible counterparties, the main imperfections of sharing markets are related to informational asymmetries about the nature of an item to be shared (*ex ante*), the actions of the borrower once in possession of the item (*ad interim*), as well as the settlement of claims when the item is returned or turns out to be inoperable (*ex post*). The importance of trust and reputation in online markets was noted by Resnick and Zeckhauser (2002). Similarly, Jones and Leonard (2008) identified trust as one of the key enablers of success in peer-to-peer (P2P) markets and investigated the effect of a natural propensity to trust and of website quality on users' level of trust in sharing transactions. Nunes and Correia (2013) constructed a model for improving trust between users, based on online credibility sources and social-network quality metrics. Einav et al. (2015) provide a general discussion of the economics of P2P markets, including entry and professional sellers, leading to a market outcome with little or no surplus for all but the intermediary. Weber (2014) introduced an analytical model for sharing with intermediation, and found that in a setting with risk-neutral parties, intermediation can in fact solve the moral hazard problem; the latter can extract surplus up to the outside option of the sharers. Jiang and Tian (2015) provide an alternative model examining the effect of unresolved moral hazard on the transaction price; they also analyze the impact of sharing on manufacturer decisions such as quality. In this paper, we abstract from the standard transactional inefficiencies, and focus on the decisions of (rational) owners about whether or not to participate in the (possibly intermediated) sharing market.

[6] Kassan and Orsi (2012) give an overview of the legal foundations for the sharing economy.

The economic impact of sharing has been addressed based on empirical observations. In the context of room sharing, Zervas et al. (2014) found that sharing intermediaries such as AirBnB can have a negative effect on hotel revenues. Cervero et al. (2007) and Martin et al. (2010) observed that car sharing is associated with a significant decrease in car ownership, miles travelled, and gasoline consumption. Fraiberger and Sundararajan (2015) calibrate a stationary equilibrium model to data from the U.S. car market and observe significant consumption and surplus shifts as a result of car sharing. Weber (2015) provided a theoretical framework to analyze the impact of sharing on product sales of durable goods and showed that the impact is ambiguous, depending on the price of ownership. Weber (2016) showed that sharing markets tend to increase retail prices (even in the absence of agency effects), and that retailers or manufacturers prefer the presence of sharing markets when goods are relatively expensive to provide.

To the best of our knowledge, we present the first model for sharing decisions in a fully dynamic context with market equilibrium, and thus the "diffusion" of sharing. On the equilibrium path, our findings relate directly to the well-known product diffusion model by Bass (1969) which fits the diffusion patterns for a wide range of products (and ideas) exceptionally well.[7] That is, for a sufficiently small initial number of sharers, the development of the sharing market follows an S-shaped growth pattern. The finding is remarkable because it is obtained in the absence of the standard assumptions on the sales of products (usually in the form of a differential equation with quadratic right-hand side); it holds for virtually all nondegenerate parameter values. Diffusion models have been used to estimate the adoption of products (Mahajan et al. 2000), and more recently, also to describe the spread of innovations in social networks (Peres et al. 2010). Our model can be interpreted as a diffusion of sharing supply in an economy, describing the expansion path of a sharing market which features frictions and supply-demand imbalances.

Sharing decisions in a static context, as for example examined in the finite-horizon model by Weber (2015), are generally based on a comparison of willingness-to-accept and willingness-to-pay, which can be quite different and thus lead to a (normative) endowment effect. This naturally introduces a transaction inertia.[8] The effect vanishes when individuals are risk-neutral which is the assumption adopted for the dynamic model in this paper. However, as alluded to earlier, the decision about whether or not to participate in the sharing market carries a fixed cost, which in turn implies that for small expected gains it is better for individuals to remain in their *status quo*. The resulting inaction region and the optimality of threshold-based decision rules are standard features of stochastic dynamic programs with fixed costs (see, e.g., Stokey 2008).[9] Our model finally relates to the literature on equilibrium search in labor markets with frictions (Alvarez and Veracierto 2000; Alvarez and Shimer 2011). This stream of literature was pioneered by Lucas and Prescott (1974) with a discussion about the equilibrium rate of unemployment across various industries/occupations/locations when unemployed workers can engage in costly search. Based on a system of Bellman equations for a set of heterogeneous individuals and a transversal equilibrium condition, we obtain a set of equilibrium threshold rules, which for each agent implies a decision hysteresis (Dixit 1992). The price thresholds vary between

[7]The Bass diffusion model was included as one of the top 10 contributions over the first 50 years of *Management Science* in 2004 (Vol. 50, No. 12 Supplement). The embedded logistic growth curve fits data so well, that Marshall Fisher used this model to estimate the diffusion of his own top 10 paper in that same issue.

[8]As long as WTA (or WTP) is increasing in income, at the same income level WTA exceeds WTP, leading to a normative endowment effect (Weber 2010).

[9]Fixed costs distort the classical net-present-value (NPV) rule because the implied partial irreversibility of sharing decisions turn opportunities into real options: a strictly positive NPV is required to make sharing decisions attractive (Dixit and Pindyck 1994).

individuals and also depend on the aggregate of all individuals, whose decisions therefore exert an externality on each other.

1.2 Outline

The remainder of this paper is organized as follows. Sec. 2 introduces the model primitives, including the heterogeneous consumers and their payoffs, the demand, and the market mechanism. Sec. 3 characterizes the equilibrium behavior of agents in a sharing economy with adjustment costs and a (possibly persistent) supply-demand imbalance. We provide comparative statics and analyze the limiting behavior of the economy as a function of the model parameters. Sec. 4 examines the supply-side sharing decisions on the equilibrium path in more detail and provides a specific example to illustrate the findings. Sec. 5 concludes. Robustness issues are examined in App. A, the proofs of formal results are in App. B, and App. C summarizes the notation.

2 Model

2.1 Supply

Consider a continuum of agents, each indexed by his type $\theta \in \Theta = [0, 1]$, and each of whom owns a potentially shareable durable good. Without any loss of generality, the total number of agents is normalized to 1.[10] At time $t \in \{0, 1, 2, \ldots\}$, any given agent can decide whether to change the state of his durable good (e.g., a flat or a car) from a "keeping" state ($x_t = 1$) to a "sharing" state ($x_{t+1} = 0$) or vice-versa, from $x_t = 0$ to $x_{t+1} = 1$. The corresponding conversion cost, to go from state $x_t = i$ to $x_{t+1} = j$, is denoted by c_{ij}, where $c_{ii} = 0$ and $c_{ij} \geq 0$, for any $i, j \in \mathcal{X} = \{0, 1\}$. The conversion cost c_{10} measures the effort required to get a good ready to be shared, including cleaning, installation of sharing-specific features (e.g., robust furniture in a flat), and creating a listing for the item with a sharing intermediary. On the other hand, by expending c_{01} an agent can repossess the shared item by reintegrating it into his private possessions, de-listing it with the sharing intermediary, and so forth. The switching costs between subsequent states x_t and x_{t+1} in the binary state space $\mathcal{X}$ introduce frictions and partial irreversibilities in the agents' decisions. While for most shared goods c_{10} is likely to be significant in magnitude, the relative importance of c_{01} depends on the particularities of the goods. In some markets (e.g., car sharing) c_{01} may be small by comparison, whereas in others (e.g., clothes sharing) this cost tends to be non-negligible.

The type $\theta \in \Theta$ of a given agent characterizes the probability with which he will need the item at any given time t.[11] More specifically, it describes the distribution of the agent's need state $\tilde{s}_t$, with realizations s_t in $\mathcal{S} = \{L, H\}$, that can be either "low need" ($s_t = L$) or "high need" ($s_t = H$), in the sense that

$$P(\tilde{s}_t = H) = \theta.$$

The type distribution is assumed to be uniform and is described by the cumulative distribution function $F : \Theta \to [0, 1]$, where $F(\theta) \equiv \theta$. Agents are risk-neutral, and at each time t they all

[10] Accordingly, model predictions about, say, the number of agents participating in the sharing market are to be interpreted as fractions of the total number of agents; the latter is assumed to be stationary for simplicity.

[11] An agent's type θ can also be interpreted as the marginal value for an increase in the utility difference Δ between the high-need state and the low-need state.

take individual binary adjustment decisions

$$a_t \in \mathcal{A} = \{0, 1\},$$

where the values of 0 and 1 correspond to sharing and keeping, respectively. This reflects the fact that in the presence of a sharing market, owners currently using their items (i.e., "keepers") rationally decide in each time period whether it is beneficial to get their items ready to be offered on the sharing market. Similarly, owners currently present on the sharing market (i.e., "sharers") take a decision about possibly converting their assets back to personal use. Adjustments in the usage between sharing and keeping are costly, and the best decision for any given agent in each period depends on his beliefs about all agents' current and future actions.[12]

An agent's per-period payoff $u(s, x)$ depends on both the current need state $s \in \mathcal{S}$ and the item availability (sharing state) $x \in \mathcal{X}$. For simplicity, we assume that in the low-need state the agent's utility vanishes, no matter if the item is being shared or not, i.e., $u(L, 0) = u(L, 1) = 0$. In the high-need state, the agent experiences a disutility from not having the item at his disposal, and we introduce u_0 and $u_1 = u_0 + \Delta$ such that [13]

$$u_0 = u(H, 0) \leq 0 < u(H, 1) = u_1.$$

2.2 Demand

At any given time, the demand for the durable good consists of individuals who do not own the item and find themselves in a high-need state. In the tradition of Mussa and Rosen (1978), a potential renter is assumed to have a marginal utility $\mu \in [0, 1]$ for using a durable item of quality $\gamma > 0$ (see also Jiang and Tian 2015; Weber 2014). For simplicity, we assume that marginal utilities are distributed uniformly on $[0, 1]$. If a potential renter decides not to participate in the sharing market, he resorts to his second-best (outside) option, the payoff of which is normalized to 0. Thus, a non-owner is willing to rent the item if $\mu\gamma - p \geq 0$, so that the demand on the sharing market becomes $n = \max\{0, 1 - p/\gamma\}$. Equivalently, the (inverse) demand for shared items describes the market price,

$$p(n) = \gamma \cdot (1 - n), \tag{1}$$

as a function of $n \in [0, 1]$, where γ also parametrizes the elasticity of demand.[14] We concentrate on the interesting case where u_1 is sufficiently large, so

$$0 = p(1) < p(0) = \gamma \leq u_1 \leq \Delta. \quad (= u_1 - u_0) \tag{2}$$

This means that if all owners were to offer their goods, for the sharing market to clear (in a steady state), the price would have to vanish, i.e., there is enough potential supply to satisfy demand in the long run, and rationing could therefore not be persistent. The utility of an owner in a high-need state (u_1) is larger than the gross benefit (γ) a renter with maximum marginal utility $\mu = 1$ can obtain. This is plausible because any contract between owner and non-owner

[12] As is standard in dynamic market models, we assume that expectations about the future are rational (Muth 1961).

[13] Essential for the results is only that the difference Δ between u_1 and u_0 is positive; apart from that, neither the specific values of the utilities nor their signs matter.

[14] The (price) elasticity of demand, $\varepsilon = p/(\gamma - p) = (1 - n)/n$, takes on all values in $(0, \infty)$ as p varies from $p(1) = 0$ to $p(0) = \gamma$. As is shown in App. B (see La. 12) the demand elasticity ε is decreasing in the (in)elasticity parameter γ. App. A discusses an extension of the model to nonlinear demand specifications.

would necessarily be incomplete and allocate all residual claims to the owner (Grossman and Hart 1986; Hart and Moore 1990). In addition, an owner can usually disable certain features of the durable good that is being shared, such as access to spare parts, accessories, or special functionality (e.g., a locked utility closet in shared house or administrator privileges for a shared personal computer). Another desirable consequence of the assumption $u_1 \geq \gamma$ is that this excludes the (economically) degenerate situation where *all* owners would be willing to share, irrespective of their need state and price movements in the sharing market.

2.3 Price Formation

As noted before, the prices in the sharing market are critical for the agents' incentives to share. However, it seems unreasonable to assume that the price discovery in such markets is efficient and instantaneous, even using a centralized exchange (see, e.g., Cohen et al. 1980).[15] As in any market, prices in the sharing economy are (at least somewhat) sticky: when at the beginning of time t the supply of shared goods is n_t, the corresponding market price adjusts at the end of that period, so

$$p_{t+1} = p(n_t), \tag{3}$$

for all $t \geq 0$, where the initial supply n_0 at time $t = 0$ is given. The unit delay in the price response allows for a temporary supply-demand imbalance, as can be observed in actual sharing markets.[16] Its main effect is that price discovery in the market is not immediate but takes place over some finite time horizon. Accordingly, the supply and demand in the sharing market can fluctuate. The robustness of all results with respect to changes in the price-adjustment time scale is examined in App. A.

For any time $t > 0$, a type-θ agent's current-period expected net payoff, given action a, sharing state x, and the price in the sharing market p, is

$$\bar{g}(a, x, p|\theta) = \begin{cases} p + \theta u_0, & \text{if } (a, x) = (0, 0), \\ p + \theta u_0 - c_{01}, & \text{if } (a, x) = (1, 0), \\ \theta u_1, & \text{if } (a, x) = (1, 1), \\ \theta u_1 - c_{10}, & \text{if } (a, x) = (0, 1). \end{cases} \tag{4}$$

Fig. 1 depicts an agent's decision process in form of a lattice, with payoffs that include the transition (conversion) costs between different sharing states and the *ex ante* expected per-period payoffs. All agents are assumed to behave optimally with respect to their objectives and the information available to them. Since the value of keeping the item for private use is increasing in an agent's type, at each time t there is a sharing threshold ϑ_t, below which low-θ agents participate in the sharing market. On the other hand, high-θ agents, above the sharing threshold, abstain from the market.[17] Hence, for any agent of type $\theta \in \Theta$, the sharing decision is

$$x_t = \xi(\theta, \vartheta_t) \triangleq \begin{cases} 1, & \text{if } \theta > \vartheta_t, \\ 0, & \text{otherwise}, \end{cases} \tag{5}$$

[15]To counter inefficiencies, sharing intermediaries may offer price-optimization tools as guidance. For example, using data-mining techniques AirBnB provides a utility to recommend a daily updated rental price as a function of the transaction probability. Furthermore, AirBnB routinely recommends that hosts check similar listings to gauge appropriate price levels.

[16]The Economist (2013) provides the example of AirBnB's renting about 40,000 out of 250,000 listed rooms in any given night, corresponding to a transaction probability of 16%.

[17]The optimality of a threshold rule is formally established by Prop. 2 below.

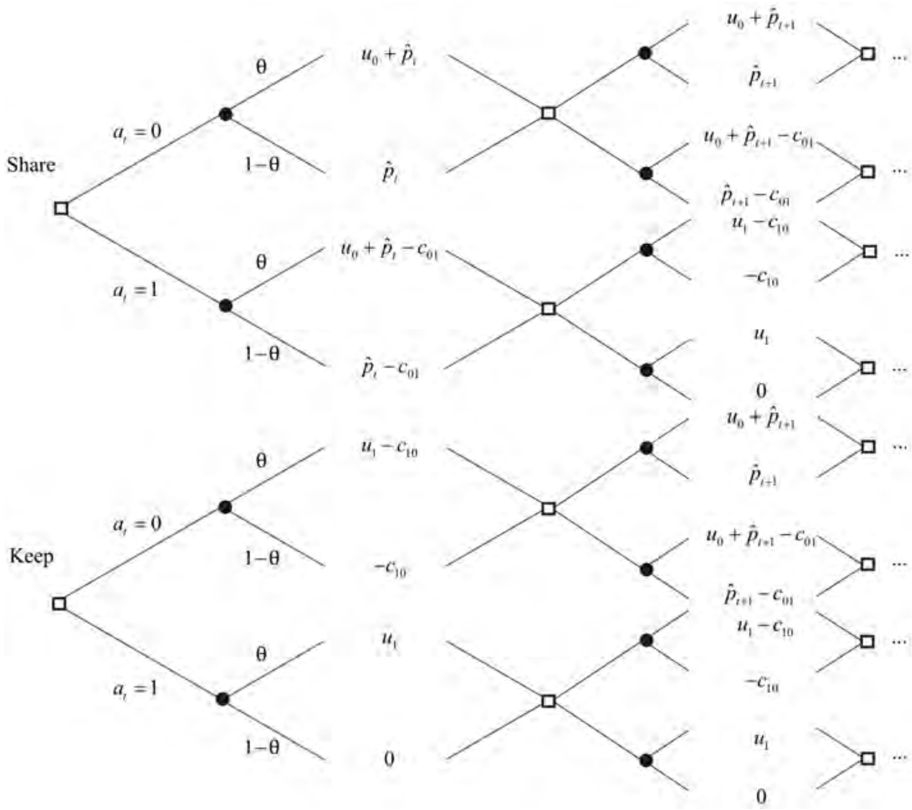

Figure 1: Decision process for a type-θ agent, viewed at time t.

where $\xi(\theta, \vartheta_t)$ describes the sharing state of type θ, given the highest type willing to share his item ϑ_t. Consequently, the supply in the sharing market (i.e., the total number of agents sharing the item) at time $t > 0$ is

$$n_t = \int_\Theta \mathbf{1}_{\{\xi(\theta,\vartheta_t)=0\}} d\theta = 1 - \int_\Theta \xi(\theta, \vartheta_t)\, d\theta = \vartheta_t, \tag{6}$$

where the initial size $n_0 = \vartheta_0 \in [0, 1]$ of the sharing supply is given.

3 Supply-Demand (Im)balance

A natural consequence of imperfections in the price-formation and supply-adjustment processes is that sharing markets may not always clear.[18] Given the price $p_t = p(n_{t-1})$, the demand at time t lags behind the supply for one period, and excess demand is

$$z_t \triangleq n_{t-1} - n_t. \tag{7}$$

If excess demand is positive ($z_t > 0$), then some potential users willing to rent an item on the sharing market at the current price are unable to find a seller and therefore have to take their outside option instead (at zero payoff). On the other hand, if excess demand is negative ($z_t < 0$), then there is in fact excess supply of shared items. Accordingly, the *transaction probability* for any given sharer is

$$q(\vartheta_t, \vartheta_{t-1}) \triangleq \min\{1, \vartheta_{t-1}/\vartheta_t\} \quad (= \min\{1, n_{t-1}/n_t\}). \tag{8}$$

[18]In contrast to Weber (2015), who uses Nash-bargaining for the price discovery in potentially unbalanced sharing markets, we use the market mechanism to determine the dynamic price in a rational-expectations equilibrium.

Moreover, let $\rho \in [0, 1]$ be a sharing intermediary's *commission rate*, corresponding to the captured fraction of the posted price p.[19] Therefore, instead of considering the transaction probability and pass-through transfer after deduction of the sharing intermediary's commission explicitly, one can restrict attention to the *effective transaction price*

$$\hat{p}(\vartheta_t, \vartheta_{t-1}) \triangleq (1-\rho)\, p(\vartheta_{t-1}) \cdot q(\vartheta_t, \vartheta_{t-1}). \tag{9}$$

Remark 1. The percentage charged by the intermediary varies across different platforms. For example AirBnB charges a 3% host service for every completed booking, while TaskRabbit charges a 20% commission fee on each successful task. The degree to which the intermediary is able to syphon off the gains from trade depends on the agents' outside option (e.g., the probability of matching in the absence of the sharing intermediary; see Weber 2014).

Remark 2. Let $\hat{\gamma} = (1-\rho)\gamma$ be the *intermediated* demand-elasticity parameter, so the effective transaction price becomes $\hat{p}(\vartheta_t, \vartheta_{t-1}) = \hat{\gamma}\,(1-\vartheta_{t-1}) \cdot q(\vartheta_t, \vartheta_{t-1})$. The presence of a sharing intermediary affects the market by changing the price elasticity of demand for potential suppliers.

In the analysis that follows, we assume that all agents discount their payoffs using the same discrete-time discount factor,

$$\delta \triangleq \frac{1}{1+r} \in (0,1),$$

where $r > 0$ denotes the per-period interest (or discount) rate. The (maximum) expected present value V that a type-θ agent, in sharing state x, can obtain over an infinite horizon is the solution of a system of Bellman equations,

$$V(x, \vartheta_t, \vartheta_{t-1}|\theta) = \max_{a \in \mathcal{A}} \{\bar{g}(a, x, \hat{p}(\vartheta_t, \vartheta_{t-1})|\theta) + \delta V(x', \vartheta', \vartheta_t|\theta)\}, \quad \theta \in \Theta, \tag{10}$$

on $\mathcal{X} \times \Theta^2$; the agents' respective sharing states in the economy evolve according to

$$x' = a = \pi(x, \vartheta_t|\theta),$$

where $\pi(\cdot|\theta) : \mathcal{X} \times \Theta \to \mathcal{A}$ is type θ's optimal policy. All agents $\theta \in \Theta$ decide in each time period $t \geq 0$ whether or not to share. Hence, the sharing threshold evolves according to

$$\vartheta' = 1 - \int_0^1 \pi(x, \vartheta_t|\theta)\, d\theta.$$

We refer to the resulting infinite-horizon (super-)game (keeping track of the sharing threshold ϑ_t) as $\mathcal{G}(\vartheta_0)$, where the given initial value of the sharing threshold ($\vartheta_0 \in \Theta$) is common knowledge.

3.1 Adjustment Dynamics

The sharing-state distribution $\xi(\cdot, \vartheta_t)$ in any given period t, introduced in Eq. (5), is fully characterized by the sharing threshold ϑ_t. Thus, to describe the dynamic development of the sharing market, consider the sequence of sharing thresholds $(\vartheta_0, \vartheta_1, \ldots)$, where $\vartheta_0 \in \Theta$ is given. The

[19] Echoing Caillaud and Jullien (2003), Weber (2014, p. 51) notes that intermediated price depends in equilibrium only on a 'commission ratio,' composed of the commissions charged by the intermediary on both sides of the market. Because of this neutrality result one can limit attention to rents extracted on the supply side, setting the buyers' commission to zero.

sequence describes the evolution of the size of the sharing market, in terms of its supply-side liquidity. An agent's transformed optimal policy at time t, denoted by $\hat{\pi}(\theta, \vartheta_t)$, depends on his type θ and on current market participation, i.e., by Eq. (6) on the threshold ϑ_t. Provided the agent uses a threshold policy, it is of the form

$$\hat{\pi}(\theta, \vartheta_t) \triangleq \pi(\xi(\theta, \vartheta_t), \vartheta_t | \theta) = \xi(\theta, \vartheta') = \mathbf{1}_{\{\theta > \vartheta'\}}. \tag{11}$$

With this, next period's sharing threshold ϑ' can be expressed as a function of the current threshold ϑ_t:

$$\vartheta' = \alpha(\vartheta_t) \triangleq \inf\{\theta \in \Theta : \hat{\pi}(\theta, \vartheta_t) = 1\},$$

using the convention that $\inf \emptyset = \sup \Theta = 1$. This defines a (time-invariant) system function $\alpha(\cdot)$ which describes the updates of the current sharing threshold. The optimality of a threshold policy, formally established in Prop. 2, means that an agent of a type higher than θ never prefers an action strictly lower than $\hat{\pi}(\theta, \vartheta_t)$.

3.1.1 Nash Equilibrium

In what follows, we use the concept of subgame-perfect Nash equilibrium by Selten (1965) to derive predictions about the outcome of the dynamic game, where any potential sharer maximizes his expected utility in each period, while taking the other agents' strategies as given. Indeed, for the current sharing threshold ϑ_t, the optimal decision for an agent of type θ (who is currently in the sharing state $\xi(\theta, \vartheta_t)$) is to keep the item if and only if the expected utility of keeping exceeds the expected utility of sharing, given that all other agents follow the equilibrium policy. To determine the subgame-perfect Nash equilibrium of the infinite-horizon dynamic game $\mathcal{G}(\vartheta_0)$ for any given initial value $\vartheta_0 \in \Theta$, we introduce $\hat{V}(\theta, \vartheta_t, \vartheta_{t-1}) \triangleq V(\xi(\theta, \vartheta_t), \vartheta_t, \vartheta_{t-1} | \theta)$ and rewrite Eq. (10) in the form

$$\begin{aligned} \hat{V}(\theta, \vartheta_t, \vartheta_{t-1}) \;\; = \;\; & \bar{g}(\hat{\pi}(\theta, \vartheta_t), \xi(\theta, \vartheta_t), \hat{p}(\vartheta_t, \vartheta_{t-1}) | \theta) \\ & + \delta \underbrace{[\bar{g}(\hat{\pi}(\theta, \vartheta'), \xi(\theta, \vartheta'), \hat{p}(\vartheta', \vartheta_t) | \theta) + \delta \hat{V}(\theta, \alpha(\vartheta'), \vartheta')]}_{\hat{V}(\theta, \vartheta', \vartheta_t)}, \quad \theta \in \Theta. \end{aligned} \tag{12}$$

The time-$(t+1)$ threshold $\vartheta' = \vartheta_{t+1}$ can be obtained as a function of ϑ_t (and, as it turns out, *not* of ϑ_{t-1}) by the one-shot deviation principle (Fudenberg and Tirole 1991). The latter implies a "transversal" equilibrium condition (13) that links the agents' decision problems, which in turn yields ϑ'.[20]

Lemma 1. *For any $t \geq 0$, the next-period sharing threshold $\vartheta' = \alpha(\vartheta_t)$, where*

$$\begin{aligned} \alpha(\vartheta_t) \;\; = \;\; \inf\{\theta \in \Theta : \; & \bar{g}(1, \xi(\theta, \vartheta_t), \hat{p}(\vartheta_t, \vartheta_{t-1}) | \theta) + \delta \bar{g}(\hat{\pi}(\theta, \vartheta'), 1, \hat{p}(\vartheta', \vartheta_t) | \theta) \geq \\ & \bar{g}(0, \xi(\theta, \vartheta_t), \hat{p}(\vartheta_t, \vartheta_{t-1}) | \theta) + \delta \bar{g}(\hat{\pi}(\theta, \vartheta'), 0, \hat{p}(\vartheta', \vartheta_t) | \theta)\}, \end{aligned} \tag{13}$$

and is such that $\hat{\pi}(\vartheta', \vartheta') = 0$.

In addition to providing an expression for the system function, the preceding result describes an invariance property of the threshold policy in Eq. (11), namely that it is always optimal for a marginal type to share, i.e., $\hat{\pi}(\vartheta_t, \vartheta_t) \equiv 0$ for all $t \geq 1$. Agents who find it optimal to share will remain active as suppliers in the sharing economy for all times. This insight is

[20] The details are given in the proof of La. 1 in App. B.

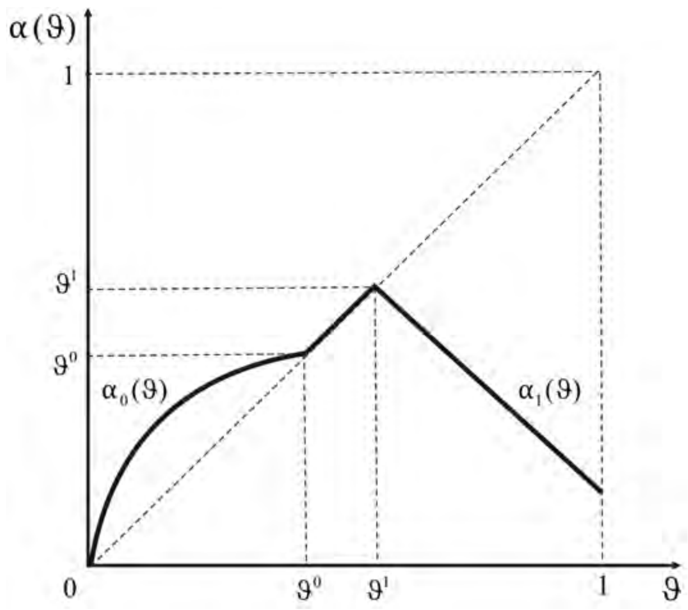

Figure 2: Law of motion for the sharing threshold when $0 < \vartheta^0, \vartheta^1 < 1$.

helpful in pinning down the equilibrium path $(\vartheta_t)_{t=0}^{\infty}$, which depends on the "(lower) invariance threshold,"

$$\vartheta^0 \triangleq \max\left\{0, \frac{\hat{\gamma} - rc_{10}}{\hat{\gamma} + \Delta}\right\}, \tag{14}$$

corresponding to a marginal type, below which all agents are willing to share in the long run. It also depends on the "(upper) invariance threshold,"

$$\vartheta^1 \triangleq \min\left\{1, \frac{\hat{\gamma} + c_{10} + (1+r)c_{01}}{\hat{\gamma} + \Delta}\right\}, \tag{15}$$

corresponding to a marginal type, above which agents would want to exit the sharing market. Agent types in the *invariance region* $\mathcal{R} \triangleq [\vartheta^0, \vartheta^1]$ prefer to remain in their respective *status quo*, so $\vartheta' = \vartheta_t = \bar{\vartheta}$, where $\bar{\vartheta}$ implies a *stationary* sharing-state distribution $\bar{\xi}(\cdot, \bar{\vartheta})$ which stays in place for all times greater than t; see Section 3.3.1 for details.

Remark 3. For all $c_{01}, c_{10} \geq 0$, the lower invariance threshold cannot exceed the upper invariance threshold, i.e., $\vartheta^0 \leq \vartheta^1$. The two thresholds together define an interval for feasible sizes of the sharing economy in the long run. They coincide in a *frictionless economy*, where adjustment costs vanish.

The preceding discussion is now formalized in a full characterization for the equilibrium dynamics of the type threshold in a sharing economy with frictions.

Proposition 1 (Equilibrium Path). *The law of motion for the sharing threshold is $\vartheta' = \alpha(\vartheta)$, for all $t \geq 0$, where the system function is given by*

$$\alpha(\vartheta) \triangleq \max\{\alpha_0(\vartheta), \min\{\vartheta, \alpha_1(\vartheta)\}\} = \begin{cases} \alpha_0(\vartheta), & \textit{if } \vartheta < \vartheta^0, \\ \vartheta, & \textit{if } \vartheta^0 \leq \vartheta \leq \vartheta^1, \\ \alpha_1(\vartheta), & \textit{if } \vartheta > \vartheta^1, \end{cases} \tag{16}$$

with[21]

$$\alpha_0(\vartheta) = \max\left\{0, \frac{(1-\rho)\,p(\vartheta)\,(\vartheta/\alpha_0(\vartheta)) - rc_{10}}{\Delta}\right\}, \tag{17}$$

$$\alpha_1(\vartheta) = \min\left\{1, \frac{(1-\rho)\,p(\vartheta) + c_{10} + (1+r)c_{01}}{\Delta}\right\}, \tag{18}$$

for all $\vartheta \in \Theta$; ϑ^0 and ϑ^1 are specified in Eqs. (14) and (15).

The Nash-equilibrium path $(\vartheta_t)_{t=0}^{\infty}$ in Prop. 1 implies the agents' equilibrium policy $\hat{\pi}$ in Eq. (11) for all types $\theta \in \Theta$, which in turn determines a unique subgame-perfect Nash equilibrium in the supergame $\mathcal{G}(\vartheta_0)$. In this dynamic sharing economy, each agent finds it optimal to implement a threshold-type policy $\hat{\pi}$.

Proposition 2 (Threshold Optimality and Uniqueness of the Equilibrium). *For any given $\vartheta_0 \in \Theta$, the unique subgame-perfect Nash equilibrium of $\mathcal{G}(\vartheta_0)$ is such that $\vartheta_{t+1} = \alpha(\vartheta_t)$ and $\hat{\pi}(\theta, \vartheta_t) = \mathbf{1}_{\{\theta > \alpha(\vartheta_t)\}}$, for all $t \geq 0$ and all $\theta \in \Theta$.*

Fig. 2 illustrates the law of motion of the sharing threshold, starting from any given initial number of suppliers in the economy. The equilibrium path describes the steady-state and non-steady state evolution of the sharing threshold governed by optimizing behavior of the agents in the sharing economy. In the remainder of this section, the salient characteristics of the equilibrium path are discussed in detail.

3.1.2 Equilibrium Dynamics

The different properties of the system function $\alpha(\cdot)$, summarized below, characterize the temporal dynamics of the sharing supply.

Lemma 2. *The system function $\alpha(\cdot)$ has the following properties:*

(i) $\alpha(0) = 0$ *(by continuous completion);*

(ii) For all $\vartheta \in (0, 1]$, the dynamic increment, $\alpha(\vartheta) - \vartheta$, has the (weak) single-crossing property (with respect to $\mathcal{R}$);[22]

(iii) $\alpha''(\vartheta) < 0 < \alpha'(\vartheta)$, *for all $\vartheta \in (0, \vartheta^0)$.*

The system function $\alpha(\cdot)$ corresponds to the law of motion pertaining to the size of the sharing supply, and its characteristics determine the transient dynamics of the sharing economy.

a. No sharing without sharers. By its very nature, the sharing economy needs at least *some* sharers to sustain collaborative consumption. Whenever $\vartheta_t = 0$, then also $\vartheta_{t+1} = 0$, which—by backward induction—means that the economy must have been at zero sharing at all times. In other words, the model does not provide for a driving force other than a positive number of sharers at the beginning. It is agnostic about what might have caused the existence of a positive number of sharers at the initial time.

[21] The lower arc of the system function, $\alpha_0(\vartheta) = \left(-rc_{10} + \sqrt{(rc_{10})^2 + 4\Delta(1-\rho)\vartheta p(\vartheta)}\right)/(2\Delta)$, is obtained in closed form as the unique nonnegative solution of the corresponding fixed-point problem, for all $\vartheta \in [0, \vartheta^0)$.

[22] See, e.g., Athey (2002), p. 190; in particular, the increment vanishes on $\mathcal{R}$.

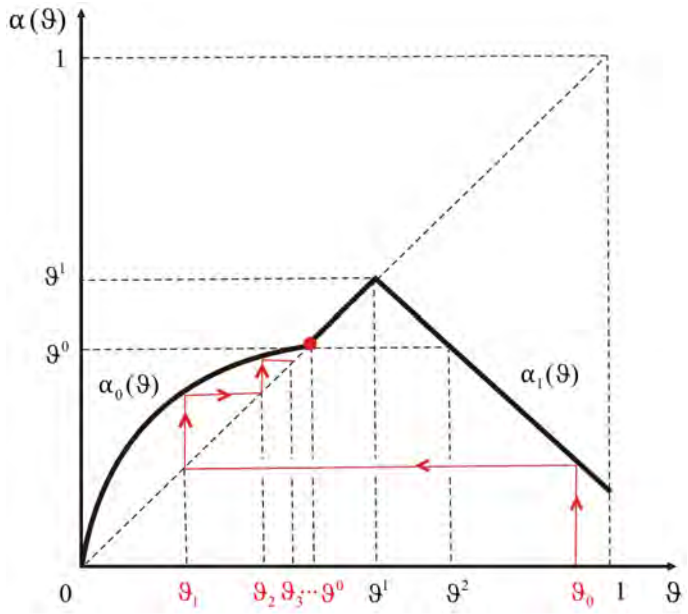

Figure 3: Growth path of the sharing threshold when $0 < \vartheta^0 < \vartheta^2 < \vartheta_0$.

b. Invariance region separates sharing growth from sharing decline and determines the feasible interval for the stationary market coverage. For sharing thresholds ϑ below the invariance region $\mathcal{R}$, the system function describes an upward movement of the sharing supply while for sharing thresholds above $\mathcal{R}$ the system function prescribes a downward movement. This is reflected by the single-crossing property stated in La. 2(ii). This property ensures that, for any nonzero initial condition, the stationary value $\bar{\vartheta}$ of the sharing threshold (and consequently the price and supply of the shared item) can lie only in the invariance region $\mathcal{R}$.

c. Downward adjustments in sharing supply do not persist unless they lead to a steady state. Growth in the sharing economy tends to be more incremental than decline. A one-time decrease in the size of the sharing economy happens above the invariance region $\mathcal{R}$, i.e., for $\vartheta_0 > \vartheta^1$, and *either* leads to a stationary state *or* it is followed by an incremental upward re-adjustment. More specifically, for $\vartheta \in (\vartheta^1, \vartheta^2]$, with the "rest threshold"

$$\vartheta^2 \triangleq \min\{1, \alpha_1^{-1}(\vartheta^0)\} = \min\left\{1, \frac{\hat{\gamma}^2 + c_{10}(\hat{\gamma} + \Delta/\delta) + c_{01}(\hat{\gamma} + \Delta)/\delta}{\hat{\gamma}(\hat{\gamma} + \Delta)}\right\}, \tag{19}$$

the decline of the sharing economy is small enough such that the subsequent sharing threshold $\vartheta' = \alpha(\vartheta)$ rests in the invariance region $\mathcal{R}$ and the economy becomes stationary in finite time. For $\vartheta \in (\vartheta^2, 1]$, however, the sharing economy experiences a steeper downturn, such that the subsequent sharing threshold drops below ϑ^0, i.e., it falls below $\mathcal{R}$. The economy then starts an incremental growth. This possible nonmonotonicity in the evolution of the sharing economy is illustrated in Fig. 3.

d. Growth occurs only below the invariance region, with an S-shaped diffusion pattern. The system function $\alpha(\cdot)$ implies that the sharing economy grows if the supply of the shared item is small enough such that the sharing threshold does not exceed the lower invariance threshold ϑ^0. The sharing economy can go through growth adjustments at varying speeds. The concavity of the system function below the invariance region $\mathcal{R}$ by La. 2(iii) implies that the growth rate of the sharing economy must be unimodal.

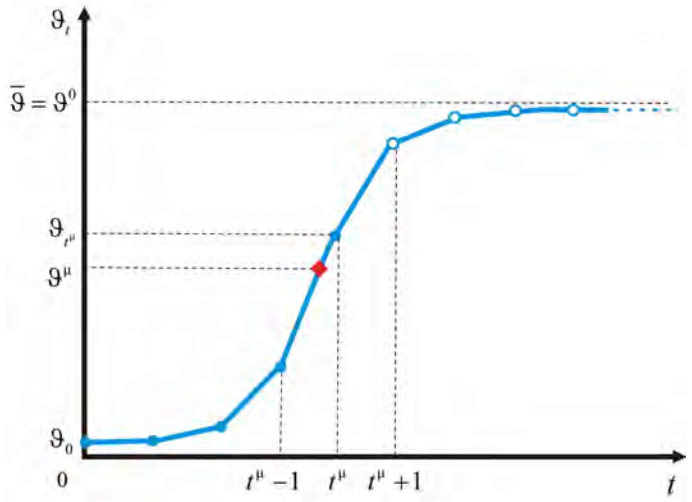

Figure 4: S-shaped diffusion of supply in a sharing economy.

Lemma 3. *Let $rc_{10} < \hat{\gamma}$, so $\vartheta^0 > 0$. Then there is a unique "maximum-diffusion" (sharing) threshold, $\vartheta^\mu \in \arg\max_{\vartheta \in (0,\vartheta^0)}\{\alpha(\vartheta) - \vartheta\}$, at which the growth of the sharing economy is maximal.*

By La. 2(i) and the definition of the lower invariance threshold in Eq. (14), $\alpha(\vartheta)-\vartheta$ vanishes at the boundaries of the growth region $[0, \vartheta^0]$. The concavity of $\alpha(\cdot)$ in the interior of the growth region (by virtue of La. 2(iii)) therefore yields the existence of a unique maximal increment of the sharing supply. This property suggests an S-shaped growth curve of the sharing supply; a pattern of growth in which, in a sufficiently small economy, the population of the sharers increases initially in a positive acceleration phase; but then the growth continues at a decreasing rate, with the sharing economy approaching a steady state. Depending on the initial value ϑ_0 (below the maximum-diffusion threshold), the fastest growth of sharing occurs at a well-defined (finite) maximum-diffusion time

$$t^\mu \in \arg\max_{t \geq 0}\{\alpha(\vartheta_t) - \vartheta_t\}, \tag{20}$$

subject to the law of motion in Prop. 1, which is such that $\alpha_0^{-1}(\vartheta^\mu) \leq \vartheta_{t^\mu} \leq \alpha_0(\vartheta^\mu)$. At this time, the supply-demand mismatch in the economy is maximal. We can immediately infer the following result (therefore stated without proof).

Corollary 1. *If the sharing economy starts with ϑ_0 below the invariance region $\mathcal{R}$, the absolute value of the excess demand z_t is maximized either at $t^\mu > 0$ (for $\vartheta_0 < \vartheta^\mu$) or at $t^\mu = 0$ (for $\vartheta_0 \geq \vartheta^\mu$).*

If the economy starts with a sufficiently small number $n_0 = \vartheta_0$ of sharers, for all $t < t^\mu$ the diffusion of sharing accelerates, whereas for all $t > t^\mu$ it decelerates. The resulting S-shaped diffusion pattern is shown in Fig. 4. This growth pattern of sharing shows the same characteristics as the well-known diffusion model by Bass (1969), which fits the empirically observed adoption paths for a wide range of products (Bass et al. 1994). The finding is remarkable because it is obtained as the equilibrium of a game played by market participants, in the absence of the standard assumptions on the sales of products (usually in the form of a differential equation with quadratic right-hand side); it holds for virtually all nondegenerate parameter values. The robustness of S-shaped diffusion in sharing markets is investigated in App. A.

3.2 Comparative Statics

The invariance region $\mathcal{R} = [\vartheta^0, \vartheta^1]$ determines the participation levels in the sharing economy that can be maintained in the long run. The following result describes how the invariance thresholds ϑ^0 and ϑ^1 delimiting this region depend on the interest rate r, the conversion costs (c_{01}, c_{10}), the utility gain Δ, the intermediary's commission rate ρ, and the demand-elasticity parameter γ (see footnote 14). This determines the asymptotic behavior of the sharing economy in the long run, as a function of the salient model parameters.

Proposition 3 (Monotonicity of the Invariance Thresholds). *Let $\mathcal{R} \subset (0, 1)$. Then the 'interior' invariance thresholds ϑ^0 and ϑ^1 satisfy the following monotonicity properties:*[23]

(i) ϑ^1 *and* ϑ^0 *are decreasing in* Δ*;*

(ii) ϑ^1 *(resp.* ϑ^0*) is increasing (resp. decreasing) in* (c_{01}, c_{10}, r)*;*

(iii) ϑ^1 *and* ϑ^0 *are increasing in* γ *and decreasing in* ρ*.*

Following an increase of the utility difference Δ the invariance region $\mathcal{R}$ shifts downwards, and its size (diameter),

$$|\mathcal{R}| = \vartheta^1 - \vartheta^0 = (1 + r)\, \frac{c_{01} + c_{10}}{\hat{\gamma} + \Delta}, \tag{21}$$

becomes smaller. On the other hand, when the demand-elasticity parameter γ increases or the intermediary's share ρ decreases, the invariance region $\mathcal{R}$ shifts upwards. Corresponding to a smaller price elasticity of demand ε,[24] an increase in $\hat{\gamma}$ tends to expand the number of sharers in an equilibrium steady state. At the same time, a decremental move in the sharing economy is less likely. More individuals become prone to switching sharing states, thus reducing the size of the invariance region where no further adjustments take place.

The monotonicity of the invariance thresholds has practical implications for intermediaries as well as regulators. Indeed, in order to achieve a higher market coverage in the long run and achieve this improved steady state faster, a sharing intermediary can lower the commission rate ρ, especially in markets with elevated conversion-cost levels. Prop. 3 also implies that the size of $\mathcal{R}$ increases both in the magnitude of the conversion cost and in the per-period interest rate. As the conversion costs increase, switching becomes less attractive for a larger fraction of the agents. Another interesting feature is a substitution effect between conversion costs and interest rate, in the sense that for the invariance thresholds to remain unchanged, at a lower interest rate, conversion costs need to be higher. Since switching decisions are taken one period ahead of the actual adjustment of the agents' sharing states, if the potential sharers do not care sufficiently about their future payoffs (because of a high discount rate), they willingly forego the expected future benefits of sharing and remain inactive.

3.3 Steady States

3.3.1 Stationary Sharing-State Distribution

For initial values ϑ_0 in the invariance region $\mathcal{R} = [\vartheta^0, \vartheta^1]$, the sharing economy remains at rest for all times $t \geq 0$, i.e., a steady state $\bar{\vartheta} \in \mathcal{R}$ is attained immediately. In the case of oversharing where the initial value lies above $\mathcal{R}$ but does not exceed the rest threshold, i.e.,

[23] The invariance thresholds are interior, i.e., $\vartheta^0, \vartheta^1 \in (0, 1)$, if and only if $c_{10} < \hat{\gamma}/r$ and $c_{10} + (1+r)c_{01} < \Delta$.

[24] For details, see La. 12 in App. B.

when $\vartheta_0 \in (\vartheta^1, \vartheta^2]$, the sharing economy experiences a downward adjustment to a steady state in $\mathcal{R}$. In all other cases (except for $\vartheta_0 = 0$ according to Sec. 3.1.2.a), the sharing economy never attains a steady state in finite time, i.e., it remains in *dis*equilibrium.

When in a steady state, the sharing market clears and each agent is locked into an 'acceptable' sharing state, given the conversion costs c_{10} and c_{01}, and the collective externality of the other agents' choices. Agents of types $\theta \leq \bar{\vartheta}$ act as suppliers in the sharing economy, for all $t \geq \tau$, where $\tau \in \{0, 1\}$. The stationary transaction volume is $\bar{n} = \bar{\vartheta}$.

Remark 4. For any steady state $\bar{\vartheta} \in \mathcal{R}$, the stationary sharing-state distribution $\bar{\xi} : \Theta \to \mathcal{X}$ is such that $x_t(\theta, \vartheta_t) \equiv \bar{\xi}(\theta, \bar{\vartheta})$, for all $t \geq \tau$. This distribution encapsulates the agents' equilibrium policy:

$$\bar{\xi}(\theta, \bar{\vartheta}) = \pi(\bar{\xi}(\theta, \bar{\vartheta}), \bar{\vartheta}|\theta),$$

for all agent types $\theta \in \Theta$.

Remark 5. If the sharing threshold reaches its steady state $\bar{\vartheta} \in \mathcal{R}$ at time τ, then by Eq. (3) the price reaches its stationary value $\bar{p} = p(\bar{\vartheta})$ at most one period later; the transaction probability for suppliers is always 1.

3.3.2 Stationary Equilibrium Payoffs

In steady state from time τ, a type-θ agent in sharing state $x \in \{0, 1\}$ obtains the "terminal value" $\bar{V}^x(\theta|\bar{\vartheta})$, which is equal to the discounted sum of the per-period rewards in a stationary equilibrium with sharing threshold $\bar{\vartheta} \in \mathcal{R}$. Taking into account the lack of switching in steady-state, the type-θ agent's payoff in Eq. (4), the stationary sharing-state distribution in Remark 4, and the geometric-series formula together imply that in equilibrium $\bar{V}(\theta|\bar{\vartheta}) \triangleq V^{\xi(\theta,\bar{\vartheta})}(\theta|\bar{\vartheta})$, so that

$$\bar{V}^0(\theta|\bar{\vartheta}) = \frac{\bar{g}(0, 0, \bar{p}|\theta)}{1 - \delta} = \frac{\bar{p} + \theta u_0}{1 - \delta}, \quad \theta \in [0, \bar{\vartheta}],$$

and

$$\bar{V}^1(\theta|\bar{\vartheta}) = \frac{\bar{g}(1, 1, \bar{p}|\theta)}{1 - \delta} = \frac{\theta u_1}{1 - \delta}, \quad \theta \in (\bar{\vartheta}, 1],$$

where $\bar{p} = p(\bar{n})$ and $\bar{n} = \bar{\vartheta}$.

Remark 6. In a frictionless economy, a steady state is never attained, unless the sharing economy starts (and then remains) at $\vartheta_0 = \bar{\vartheta}$. Even a small amount of randomness perturbing the initial state of the sharing economy would reduce a steady-state sharing economy to a zero-probability event (at least as long as $\bar{\vartheta} > 0$). For any ϑ_0 different from $\bar{\vartheta}$, the sharing economy remains in perpetual *dis*equilibrium.

Lemma 4. *The steady state $\bar{\vartheta}$ lies in the invariance region $\mathcal{R}$ and satisfies*

$$(\bar{p} - rc_{10})/\Delta \leq \bar{\vartheta} \leq (\bar{p} + (1 + r)c_{01} + c_{10})/\Delta. \tag{22}$$

The preceding result formalizes the observation that an expected per-period utility gain $\bar{\vartheta}\Delta$ for a marginal type must lie between the price $\bar{p}$ in the sharing market plus or minus the relevant per-period rents from the conversion costs, depending on whether the agent is currently sharing (when the price remains above a lower bound, so as to make repossessing the item unattractive) or keeping (when the price remains below an upper bound, so that the required investment c_{10} does not warrant entering the sharing market). As can be verified using Eqs. (1), (14) and (15), inequality (22) is in fact equivalent to $\bar{\vartheta} \in \mathcal{R}$, i.e., it fully characterizes the set of possible steady-state sharing thresholds.

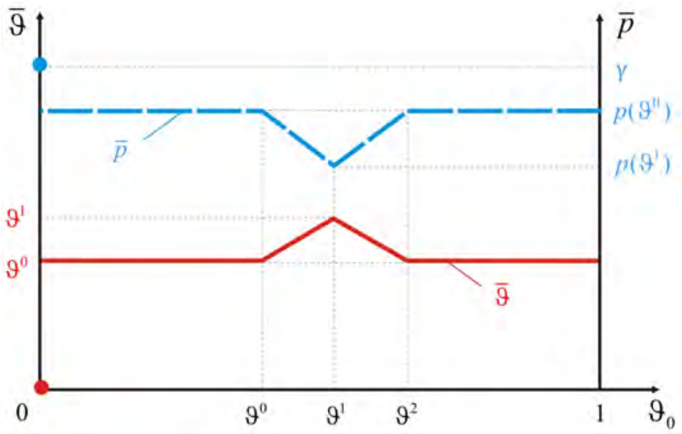

Figure 5: Steady-state values of price and supply as a function of the initial value $\vartheta_0 \in \Theta$.

3.4 Limiting Behavior

For initial values $\vartheta_0 \notin [\vartheta^0, \vartheta^2] \cup \{0\}$, the sharing economy is growing. A steady state is not reached in finite time, but becomes the asymptotic limit of the equilibrium path of the sharing economy for $t \to \infty$. The subgame-perfect equilibrium of $\mathcal{G}(\vartheta_0)$ (in Prop. 2) and the evolution of the sharing threshold ϑ_t on the equilibrium path (in Prop. 1) together describe the rise of the sharing economy for $t > 0$. We now establish a one-to-one mapping from the initial condition to the asymptotic limit of the sharing economy.

Proposition 4 (Sharing Asymptotics). *For any given $\vartheta_0 \in \Theta$ the equilibrium path $(\vartheta_t)_{t=0}^{\infty}$ of sharing thresholds converges to a stationary value $\bar{\vartheta} = \varphi(\vartheta_0) \in \mathcal{R} \cup \{0\}$, such that*

$$\vartheta_0 \mapsto \varphi(\vartheta_0) = \begin{cases} \vartheta_0, & \text{if } \vartheta_0 \in \mathcal{R} \cup \{0\}, \\ \alpha_1(\vartheta_0), & \text{if } \vartheta_0 \in (\vartheta^1, \vartheta^2], \\ \vartheta^0, & \text{otherwise}. \end{cases} \tag{23}$$

Correspondingly, the time when the sharing threshold reaches its steady state is

$$\tau(\vartheta_0) = \begin{cases} 0, & \text{if } \vartheta_0 \in \mathcal{R} \cup \{0\}, \\ 1, & \text{if } \vartheta_0 \in (\vartheta^1, \vartheta^2], \\ \infty, & \text{otherwise}. \end{cases} \tag{24}$$

Note that the price attains its steady state at time $t = 2\tau(\vartheta_0)$.

Remark 7. In a frictionless economy (where $c_{01} = c_{10} = 0$), there is a unique stationary sharing threshold; $\bar{\vartheta} = p(\bar{\vartheta})/\Delta = \gamma/(\gamma + \Delta) \in (0, 1/2]$, and a unique steady-state price $\bar{p} = \gamma\Delta/(\gamma + \Delta)$ in the sharing market.

Fig. 5 depicts the eventual market coverage and the price starting from any initial condition. If the economy is initially *undersharing* with the sharing threshold below $\mathcal{R}$, it converges to the lower invariance threshold ϑ^0 via an infinite sequence of incremental adjustments. In an *oversharing* economy starting from above $\mathcal{R}$, the sharing supply goes through a rapid phase of decline, followed by incremental adjustment, unless it drops to a value in the invariance region $\mathcal{R}$, where the conversion costs prevent all agents from changing their states. In times when the sharing economy gradually expands, the excess supply—as the negative of excess demand in Eq. (7)—is positive. Any positive excess supply is persistent, but diminishes over time.

Corollary 2. *Any excess demand (resp. supply) vanishes asymptotically, i.e.,* $\lim_{t\to\infty} z_t = 0$.

In the absence of macroeconomic shocks, the demand-supply imbalance in the sharing market equilibrates in the long run, reflecting asymptotic market stability. In the presence of shocks, for example, in the form of sudden and unexpected parameter adjustments, the market would readjust and the sharing state would track the updated invariance region.

4 Frictions and Sharing Inertia

4.1 Persistence of Disequilibrium

A disequilibrium in a sharing economy is persistent resulting in perpetual adjustments, as long as the conversion costs are not so large as to render an economic adjustment of their respective sharing states impossible for the agents. This applies *a forteriori* also to a frictionless economy. With high adjustment costs, on the other hand, the sharing economy would attain a steady state quickly, in finite time.

Although all agent types $\theta \in [0, \vartheta^0]$ eventually prefer sharing to keeping, higher-type agents in this growth regime prefer to delay their switching, possibly for very long, waiting for the *effective* transaction price $\hat{p}$ in Eq. (9) to increase. That is, because of the demand-supply mismatch in the early periods, the transaction probability q in Eq. (8) is fairly low, so that despite the relatively high market price (corresponding to a scarce supply) the expected revenue is too low to persuade higher-need types to participate in sharing. In other words, agents who are more likely to need the item are more reluctant to incur both the conversion cost and the opportunity cost of sharing, when the risk of not transacting is too high. The following result establishes that the effective transaction price is indeed increasing in the growth portion of the sharing thresholds, below the invariance region $\mathcal{R}$.

Lemma 5. *For all* $\vartheta \in (0, \vartheta^0)$*, the effective transaction price* $\hat{p}(\alpha(\vartheta), \vartheta)$ *increases in the sharing threshold* ϑ.

It turns out that for almost any initial condition, the sharing economy experiences a positive growth diffusion, provided only that conversion costs are low enough, which means that frictions are not too large. The cost c_{01} of repossessing an item plays a role solely in the (somewhat hypothetical) case where the initial number of sharers is very high, exceeding the steady state of sharers in a frictionless economy.

Proposition 5 (Persistence of Sharing Growth). *For any positive initial sharing threshold* $\vartheta_0 \in \Theta \setminus \{\hat{\gamma}/(\hat{\gamma}+\Delta)\}$*, the sharing economy does not converge in finite time if the conversion costs are low enough, such that either*

$$c_{10} < \frac{\delta(\hat{\gamma}+\Delta)}{1-\delta}\left(\frac{\hat{\gamma}}{\hat{\gamma}+\Delta} - \vartheta_0\right),$$

for $\vartheta_0 \in (0, \hat{\gamma}/(\hat{\gamma}+\Delta))$*, or*

$$\begin{bmatrix} c_{01} & c_{10} \end{bmatrix} \begin{bmatrix} 1 & \Delta/\hat{\gamma} \\ \delta & \Delta/\hat{\gamma} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} < \delta(\hat{\gamma}+\Delta)\left(\vartheta_0 - \frac{\hat{\gamma}}{\hat{\gamma}+\Delta}\right),$$

for $\vartheta_0 \in (\gamma/(\gamma+\Delta), 1]$.

Note that $\bar{\vartheta} = \hat{\gamma}/(\hat{\gamma} + \Delta)$ is the long-run sharing threshold in a frictionless economy with intermediary (see Remark 7), and is therefore always in the invariance region $\mathcal{R}$. The impossibility of adjustments when starting at the frictionless threshold explains the dichotomy in Prop. 5. It implies that the economy reaches the stationary regime almost immediately, usually when conversion costs are significant.

4.2 Critical Sharing Thresholds

The three critical sharing thresholds (ϑ^0, ϑ^1, and ϑ^2), discussed in Sec. 3.1.2 create four different regimes, depending on the current number of sharers in the economy $n = \vartheta$.

- $\vartheta \in (0, \vartheta^0)$ ("Growth"): The sharing economy is growing, the excess demand is negative, and the transaction probability is less than 1.
- $\vartheta \in [\vartheta^0, \vartheta^1] \cup \{0\}$ ("Stagnation"): The sharing economy is in the invariance region (or non-existent), with no adjustment at all. The market clears with probability 1, and the excess demand vanishes.
- $\vartheta \in (\vartheta^1, \vartheta^2]$ ("Terminal Adjustment"): The economy declines for one period. The excess demand is positive for one period and vanishes afterwards. The transaction probability equals 1 for the sharers, at all times.
- $\vartheta \in (\vartheta^2, 1]$ ("Growth Adjustment"): The economy declines for one period, followed by an incremental diffusion. The excess demand is positive for the initial period with $q = 1$, and negative afterwards with $q < 1$.

As the conversion costs become large, one or several of these regimes may disappear.

Lemma 6. *It is* $\vartheta^0 \in [0, 1/2]$ *and* $0 < \vartheta^1 \leq \vartheta^2 \leq 1$. *Furthermore,*

(i) $\vartheta^0 > 0$ *if* $c_{10} < \delta\hat{\gamma}/(1-\delta)$*;*

(ii) $\vartheta^1 < 1$ *if* $(c_{01}/\delta) + c_{10} < \Delta$*;*

(iii) $\vartheta^2 < 1$ *if* $c_{10} < \delta\hat{\gamma}/(1-\delta)$ *and* $(1 + \frac{\delta\hat{\gamma}}{\Delta})\, c_{01} + (1 + \frac{\hat{\gamma}}{\Delta})\, c_{10} < \delta\hat{\gamma}$.

The interiority conditions for the critical sharing thresholds ϑ^0, ϑ^1, and ϑ^2 in La. 6 partition the conversion-cost space—with points (c_{01}, c_{10})—into five different regions, denoted by Roman numerals (I through V) in Fig. 6. If the cost c_{10} exceeds $\hat{\gamma}/r$, then $\vartheta^0 = 0$: the sharing economy cannot grow, as no agent is willing to convert his item to sharing, irrespective of the equilibrium price (regions I and II).[25] This can be useful for manufacturers who may find it in their self-interest to either enable or disable sharing markets, as pointed out by Weber (2016). If they can design the item, so that de-personalizing it is sufficiently expensive, then sharing cannot happen at all. Conversely, manufacturers can encourage sharing by subsidizing the conversion cost for prospective sharers. Note also that the viability of a sharing market (i.e., the positivity of ϑ^0) implies an upper bound for a sharing intermediary's commission rate. That is, $\rho \leq 1 - rc_{10}/\gamma$. As a result, an intermediary's take is limited by the sharer's initial conversion cost as well as their level of impatience (measured by the discount rate r). On the other hand,

[25] In regions I and II, the rest threshold $\vartheta^2 = 1$. Therefore, if the economy is "oversharing" with initial condition $\vartheta_0 > \vartheta^1$ in region II, then all agent types between ϑ_0 and $\bar{\vartheta}$ switch from $x = 0$ to $x = 1$, and by Eq. (24) the steady state $\bar{\vartheta}$ is attained at time $\tau = 1$.

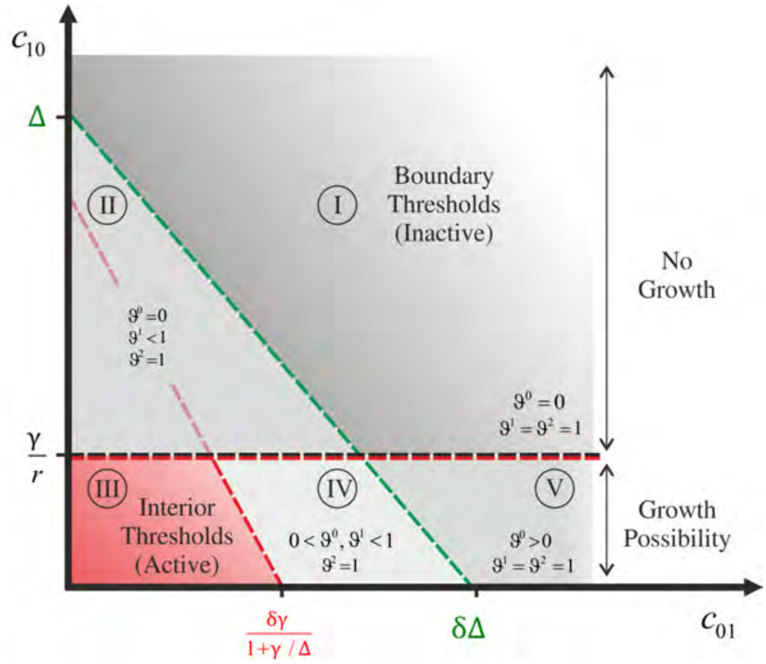

Figure 6: Critical sharing thresholds as a function of conversion costs (c_{01}, c_{10}).

if c_{10} remains sufficiently small with a value less than $\hat{\gamma}/r$ (regions III through V), then the economy can grow with dynamics that also depend on the cost c_{01} of repossessing the item. If c_{01} is small enough, then frictions in the sharing economy are minor (region III), and one obtains the dynamics with asymptotic behavior described by Prop. 4 and the mapping in Eq. (23). As the cost of switching from keeping to sharing increases, the rest threshold ϑ^2 and the upper invariance threshold ϑ^1 increase, and it becomes more unattractive to exit the sharing market. In region IV, the rest threshold hits the upper boundary, and in region V both the rest threshold and the upper invariance threshold saturate, indicating a somewhat utopian situation where all agents are keen sharers.

4.3 Externalities and Aggregate Switching Behavior

In the interesting case where the conversion costs are small enough, so the system can exhibit all types of transitional behavior (see region III in Fig. 6), the critical sharing thresholds are *interior*, i.e., $0 < \vartheta^0, \vartheta^1, \vartheta^2 < 1$. The sharing behavior for each agent type depends on the externality exerted by the agents in the economy who are already sharing. When seen over the entire equilibrium path, there are five possible behavioral patterns,[26] as a function of the initial condition, of which usually two or three coexist for a given initial value ϑ_0. Fig. 7 shows the agents' aggregate switching behavior as a function of the initial condition. The red line highlights the sharing threshold in equilibrium, corresponding to the red line in Fig. 5, above which all agents choose to keep the item for personal use ($\bar{x} = 1$), and below which all agents share the item on the market ($\bar{x} = 0$); see Prop. 4.

If the sharing economy starts below the invariance region $\mathcal{R}$, all agents with $\vartheta_0 \leq \theta \leq \vartheta^0$, switch once from keeping to sharing. However, all switches do not occur at the same time, and at each time t, only agents with $\theta \in [\vartheta_t, \vartheta_{t+1}]$ change their sharing state. As pointed

[26]The possible patterns are: "Share," "Keep $\rightarrow$ Share," "Keep," "Share $\rightarrow$ Keep," and "Share $\rightarrow$ Keep $\rightarrow$ Share."

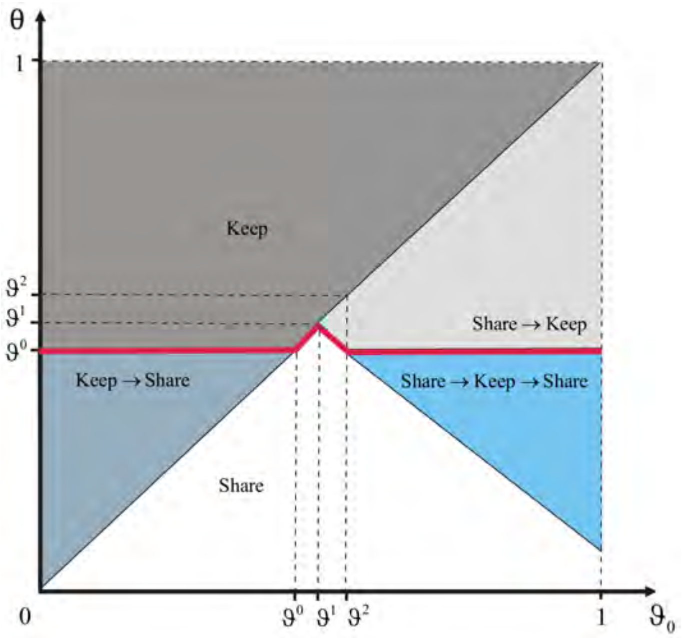

Figure 7: Aggregate switching behavior as a function of the initial condition.

out in Section 3.4, this is directly related to the fact that the effective transaction price in the market is increasing (see La. 5). If the economy starts above the invariance region $\mathcal{R}$ where $\vartheta_0 > \vartheta^1$, then it experiences a temporary decline, and subsequent incremental growth. Agent types $\theta \in [\vartheta_1, \vartheta_0]$ switch from keeping to sharing at $t = 0$. Agent types $\theta \in [\vartheta_1, \bar{\vartheta})$ therefore switch twice, eventually converting back from keeping to sharing; in the long run, all of these agent types participate in the sharing market. Lastly, all other types remain in their *status quo*: agents with $\theta < \vartheta_0 < \vartheta^0$ share at all times, while agents with $\theta > \vartheta_0 > \vartheta^0$ always hold on to their assets for personal use.

4.4 Decision Hysteresis

An agent of type θ prefers to start sharing his item if the effective price $\hat{p}$ in the sharing market is sufficiently high, i.e., exceeds $\hat{p}^0(\theta)$. Since sharing is eventually preferable as long as $\theta < \vartheta^0$, one obtains a (type-specific) *critical sharing price*,

$$\hat{p} > \hat{p}^0(\theta) \triangleq \min\{\hat{\gamma}, \Delta\theta + rc_{10}\}.$$

Similarly, the agent prefers to abandon sharing and switch to using the item if the effective price in the sharing market drops below a critical price $\hat{p}^1(\theta)$. Since keeping proves preferable for $\theta > \vartheta^1$, we obtain a (type-specific) *critical keeping price*,

$$\hat{p} < \hat{p}^1(\theta) \triangleq \max\{0, \Delta\theta - c_{10} - (1+r)c_{01}\}.$$

The two critical prices bracket the agent's expected (incremental) utility of keeping the item, from above and below, respectively:

$$\hat{p}^1(\theta) \le \Delta\theta \le \hat{p}^0(\theta).$$

In a frictionless economy the two thresholds coincide, and agents switch exactly when the expected utility $\Delta\theta$ (in positive or negative direction) is at least offset by the effective transaction

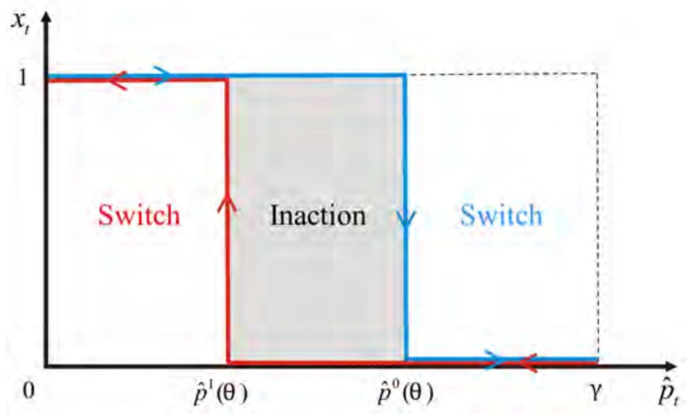

Figure 8: Decision hysteresis for a type-θ agent, with inaction region $\mathcal{P}(\theta) = [\hat{p}^1(\theta), \hat{p}^0(\theta)]$.

price in the sharing market. With conversion costs the gap between the thresholds gives rise to inertia in either direction, also referred to as decision hysteresis (Dixit 1992); see Fig. 8.

For effective transaction prices between the price thresholds, i.e., when

$$\hat{p} \in \mathcal{P}(\theta) \triangleq [\hat{p}^1(\theta), \hat{p}^0(\theta)], \tag{25}$$

a type-θ agent remains inactive. Even though the price in the sharing market might not reflect exactly his optimal sharing state, the conversion costs prevent the agent from taking action. Similar to the behavior of the size of the invariance domain $\mathcal{R}$ in Eq. (21), the width of the agent's inaction region $\mathcal{P}(\theta)$ in the price space,[27]

$$|\mathcal{P}(\theta)| = \hat{p}^0(\theta) - \hat{p}^1(\theta) = \min\{\hat{\gamma}, (1+r)(c_{01} + c_{10})\}, \tag{26}$$

increases in the conversion costs and the discount rate; it is independent of the agent's type (i.e., his probability of need θ). Yet, it is important to note that while the inaction region $\mathcal{P}(\theta)$ does not change its size, it shifts upwards with increasing θ, at the rate Δ.

The sharing economy attains its steady state once $\hat{p}_t \in [\hat{p}^0(\vartheta^1), \hat{p}^1(\vartheta^0)]$. Then the effective transaction price is bracketed by the critical prices relevant for the types that define the invariance thresholds of the sharing economy in Eqs. (14) and (15), respectively. In particular, for all agent types $\theta \in [0, \vartheta^0)$ the effective transaction price is higher than their sharing thresholds, i.e., all the lower-type agents share. Conversely, for all agent types $\theta \in (\vartheta^1, 1]$ the effective price is less than their keeping thresholds, i.e., all higher-type agents keep. Intermediate agent types in the invariance region $\mathcal{R}$ are in their respective inaction regions; see Fig. 9.

5 Examples

To illustrate the results, the equilibrium model of the sharing economy is implemented numerically. The assumed conversion-cost vector $(c_{01}, c_{10}) = (0.01, 0.05)$ reflects the empirical regularity that usually the cost of preparing the item to be shared is greater than the cost of converting it back to one's personal assets. For a unit demand-elasticity parameter ($\gamma = 1$) and in the absence of a sharing intermediary ($\rho = 0$), a per-period interest of $r = 20\%$ (corresponding to $\delta = 0.83$),[28] and utilities $(u_0, u_1) = (-0.6, 0.6)$ (corresponding to $\Delta = 1.2$), Eqs. (14), (15),

[27] Attention is restricted to the interesting case where both thresholds are interior, i.e., in the interval $(0, 1)$.

[28] This is consistent with a 6-month interest rate in a high-risk environment.

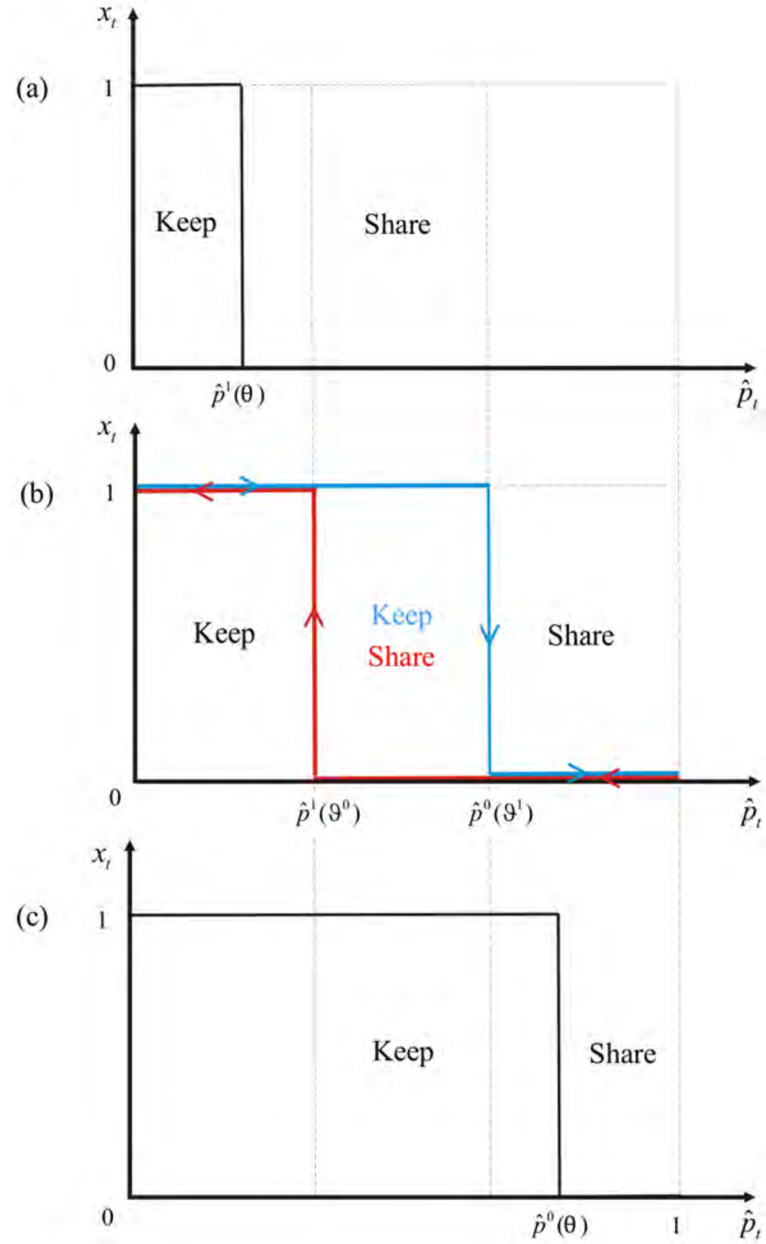

Figure 9: Agents' threshold decisions: (a) for $\theta < \vartheta^0$; (b) for $\vartheta^0 \le \theta \le \vartheta^1$; (c) for $\theta > \vartheta^1$.

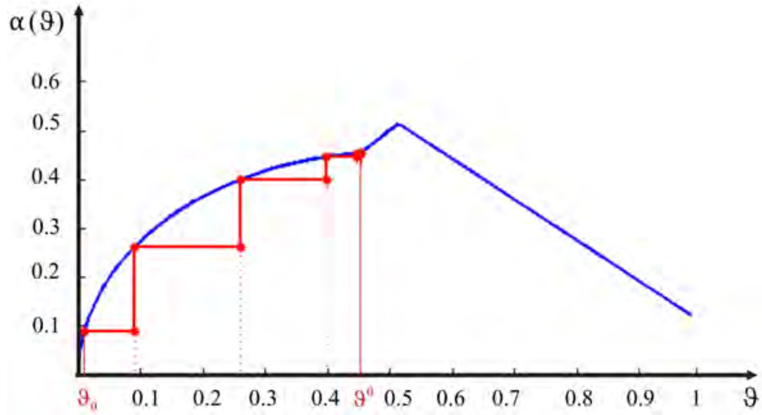

Figure 10: Price dynamics starting from the initial condition $\vartheta_0 = 0.01$.

and (19) yield the critical sharing thresholds,

$$(\vartheta^0, \vartheta^1, \vartheta^2) = (0.450, 0.482, 0.522),$$

as discussed in Sec. 3.1.2. Depending on the initial fraction of sharers ($\vartheta_0 \in (0, 1]$), market participation will eventually attain a fraction of sharers in the invariance region $\mathcal{R} = [45.0\%, 48.2\%]$. Fig. 10 depicts the law of motion (based on Prop. 1) and a particular trajectory $(\vartheta_t)_{t=0}^{\infty}$ of the sharing threshold for the initial value $\vartheta_0 = 0.01$, i.e., when the economy starts with $\vartheta_0 = 1\%$ of sharers.

We now examine the model behavior for two archetypical scenarios, "undersharing," when the economy starts below the invariance region (i.e., when $\vartheta_0 < \vartheta^0$), and "oversharing," when the economy begins with a fraction of sharers above the invariance region (i.e., when $\vartheta_0 > \vartheta^1$).

5.1 Undersharing

Suppose that the sharing economy initially starts with only 1% of the population actively sharing. Fig. 11 shows the resulting incremental diffusion. As the sharing supply increases, the market price drops to about half of its initial level, whereas the effective transaction price monotonically increases to about five times its initial level. This great discrepancy is due to the strong variation of the transaction probability, from around 10% initially to close to 100% near the steady state. Despite the very small number of initial sharers, the lag in the price-formation process (see Eq. (3)) causes the price to be too high for all the suppliers to be matched, so that only about 1/10 of the initial population of sharers actually transact, while 9/10 of them incur the disutility of having converted the asset to a sharing state which makes private use difficult. In the long run, supply is matched with demand, and by Eq. (23) participation in the sharing market approaches the steady-state level of $\bar{\vartheta} = \vartheta^0 \approx 45.0\%$. The transient excess supply (minus the excess demand in Eq. (7)) is positive but nonmonotonic. Overall, the diffusion of sharing exhibits an S-shaped growth pattern, driven mainly by the (quasi-)concavity of the law of motion (by La. 3 and Corollary 1). The latter features a maximum-diffusion sharing threshold, $\vartheta^\mu \approx 13.0\%$.[29] Correspondingly, the maximum-diffusion time (when the sharing economy grows fastest) is by Eq. (20) at $t^\mu = 2$, i.e., after about 1 year (equal to 2 six-month periods).

[29] An S-shaped growth generally obtains for initial values ϑ_0 below the maximum-diffusion threshold ϑ^μ.

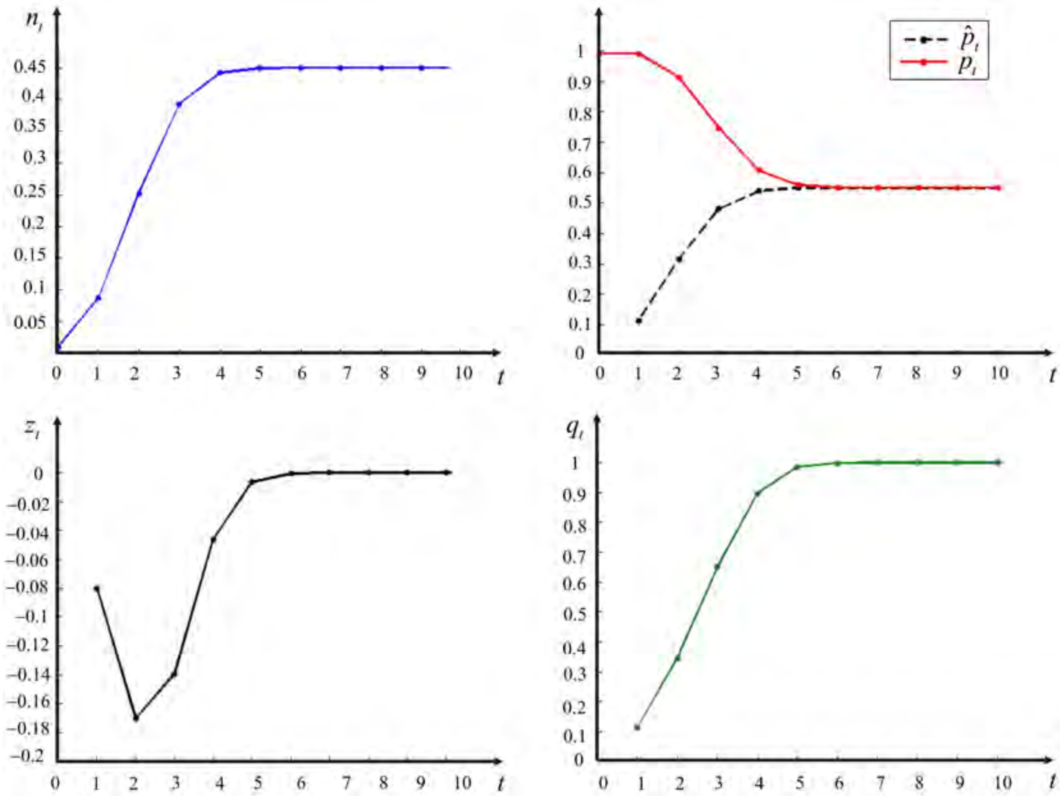

Figure 11: Adjustment dynamics starting from the initial condition $\vartheta_0 = 0.01$.

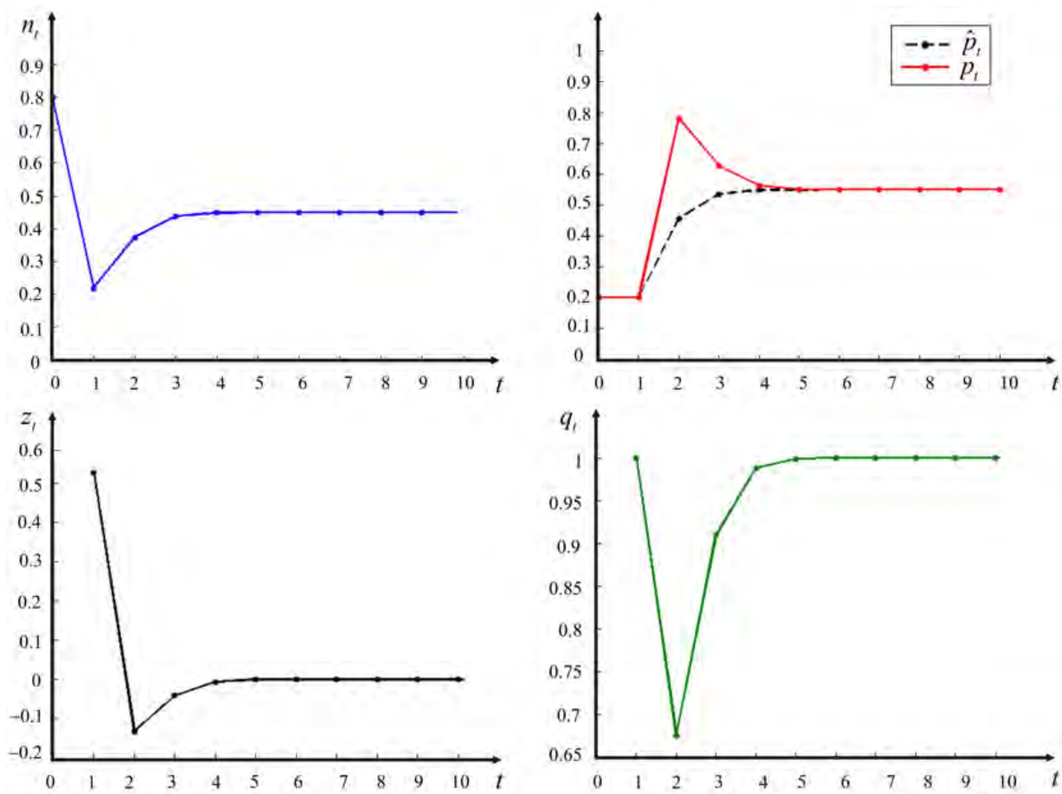

Figure 12: Adjustment dynamics starting from the initial condition $\vartheta_0 = 0.8$.

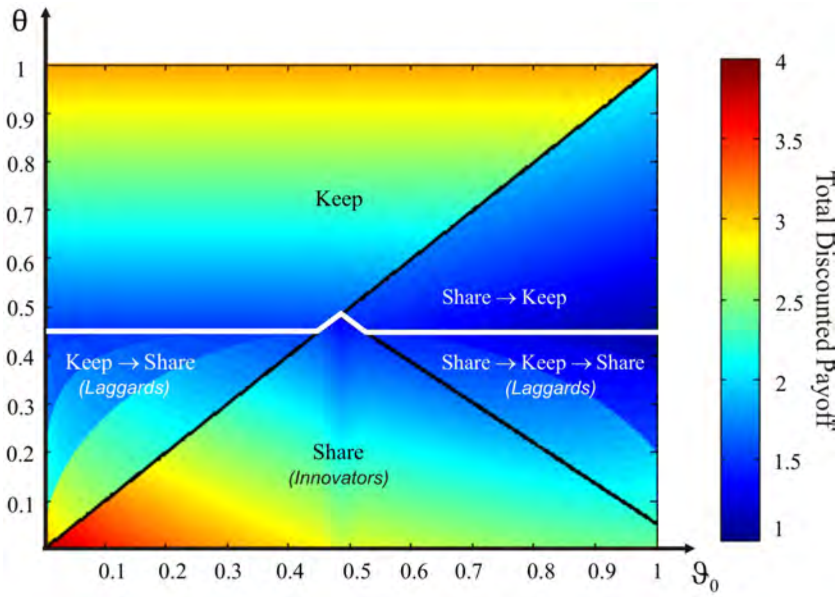

Figure 13: Equilibrium value $\hat{V}(\theta|\vartheta_0)$ for all $(\theta, \vartheta_0) \in \Theta \times \Theta$.

5.2 Oversharing

Fig. 12 depicts the dynamics of an oversharing economy, where $\vartheta_0 = 80\% \in (\vartheta^2, 1]$. The sharing supply declines immediately, where in the first period approximately 75% of the initial sharers leave the market. It is then followed by an incremental diffusion; by Eq. (23) participation in the sharing market approaches the steady-state level of $\bar{\vartheta} = \vartheta^0 = 45.0\%$. Following the massive decline of the sharing economy in the first period, the transaction price increases to about 4 times its initial value. It then decreases monotonically towards its steady-state level, which amounts to more than twice the initial price. The effective transaction price starts at the same initial value as the actual market price, but monotonically increases towards its steady-state level. The transients depend on the substantial variations of the transaction probability. In the first period, where the transaction price is low, the transaction probability for suppliers equals 1. Following the sudden price increase in the sharing market, the transaction probability falls below 70%, where approximately 1/3 of the sharers do not get the chance to actually transact. The transaction probability increases monotonically afterwards and approaches 100% again in the long run. Due to the initial oversharing, there is a positive excess demand in the first period. From $t = 2$ onwards, the transient excess supply (minus the excess demand in Eq. (7)) becomes positive and decreases monotonically towards 0. The monotonicity follows from the fact that the minimum amount of supply (at $t = 1$) corresponds to about 20% market participation, which is still greater than the maximum-diffusion sharing threshold, $\vartheta^\mu \approx 13.0\%$. Correspondingly, by Eq. (20) the maximum-diffusion time is $t^\mu = 2$, i.e., the fastest growth of the sharing economy takes place immediately after its sharp decline, resulting in marked volatility.

5.3 Agents' Payoffs

The discounted payoff (value) for a type-θ agent can be computed, based on his externality- and initial-condition-induced switching behavior, depending on the initial sharing state x_0. The latter is fully determined by θ and the initial value ϑ_0, resulting in five possible lifetime switching patterns (Fig. 13).

(i) *Initial keepers:* $x_0 = 1$*, i.e.,* $\theta > \vartheta_0$. For types greater than the lower invariance threshold ϑ^0, keeping is always best; otherwise there will be one switch to sharing at the type-dependent sharing-switch time, after which an agent stays a sharer for the future,

$$\bar{t}(\theta) = \sup\{t \in \mathbb{N} : \theta > \vartheta_t\}, \tag{27}$$

where we set, as is customary, $\sup \emptyset \triangleq \infty$. Thus,

$$V(\theta|\vartheta_0) = \begin{cases} \sum_{t=0}^{\infty} \delta^t \theta u_1, & \text{if } \max\{\vartheta_0, \vartheta^0\} < \theta \leq 1, \\ \sum_{t=0}^{\bar{t}(\theta)} \delta^t \theta u_1 - \delta^{\bar{t}(\theta)} c_{10} + \sum_{t=\bar{t}(\theta)+1}^{\infty} \delta^t (\hat{p}_t + \theta u_0), & \text{if } \vartheta_0 < \theta \leq \vartheta^0, \end{cases}$$

where $\hat{p}_0 \triangleq (1-\rho)\, p(n_0) = \hat{\gamma}\,(1-\vartheta_0)$ (with $n_0 = \vartheta_0$).
(ii) *Initial sharers:* $x_0 = 0$*, i.e.,* $\theta \leq \vartheta_0$. The situation is more intricate when agents start out as sharers. Types above the lower invariance threshold ϑ^0 switch to own use immediately and never switch back, whereas types below $\alpha_1(\vartheta_0)$ will always participate in the sharing market. Lastly, an intermediate type θ first switches to keeping and then, at the sharing-switch time $\bar{t}(\theta)$ in Eq. (27), back to sharing, so

$$\hat{V}(\theta|\vartheta_0) = \begin{cases} \sum_{t=0}^{\infty} \delta^t (\hat{p}_t + \theta u_0), & \text{if } 0 \leq \theta \leq \alpha_1(\vartheta_0), \\ \hat{p}_0 + \theta u_0 - c_{01} + \sum_{t=1}^{\bar{t}(\theta)} \delta^t \theta u_1 - \delta^{\bar{t}(\theta)} c_{10} + \sum_{t=\bar{t}(\theta)+1}^{\infty} \delta^t (\hat{p}_t + \theta u_0), & \text{if } \alpha_1(\vartheta_0) < \theta \leq \vartheta^0, \\ \hat{p}_0 + \theta u_0 - c_{01} + \sum_{t=1}^{\infty} \delta^t \theta u_1, & \text{if } \vartheta^0 < \theta \leq 1. \end{cases}$$

It is remarkable that the value function on the equilibrium path,

$$\hat{V}(\theta|\vartheta_0) = \bar{g}(\hat{\pi}(\theta, \vartheta_0), \xi(\theta, \vartheta_0), p(\vartheta_0)|\theta) + \delta \hat{V}(\theta, \alpha(\vartheta_0), \vartheta_0),$$

as a solution to the system of Bellman equations (12) and the transversal equilibrium condition (13) for all θ, ϑ_0 in Θ, exhibits generic discontinuities at the boundaries of the switching regions. In standard optimization problems, by the Berge maximum theorem (Berge 1963, p. 116), the continuity of the objective functions in parameters implies the continuity of the value function. The discontinuities here are driven by the fact that $\hat{V}$ depends not only on the agent's own type θ but also on the aggregate switching behavior in the market. The latter externality is encapsulated by the sharing threshold ϑ_t, i.e., the number of sharers in the market at any given time t.

Fig. 13 depicts the value function as a function of (θ, ϑ_0). The black lines partition the space according to the aggregate switching behavior of the agents, as in Fig. 7. For agents above the white line, who always prefer keeping to sharing, the total discounted payoffs are increasing in their respective types, whereas for the eventual sharers below the white line, the payoffs are decreasing in their respective types. For a given initial value, the *innovators* of the sharing economy are the agents of types $\theta \in (0, \min\{\vartheta_0, \vartheta_1\})$; they are present on the sharing market from the very beginning, never switch to keeping, and tend to gain the highest discounted payoffs, which may amount to a value up to 4 times greater than the discounted payoffs for the *laggards*, i.e., agents who prefer to share but would rather delay participating in the market as much as possible. Each agent's decision affects other agents' payoffs, which in general creates payoff-discontinuities in the (θ, ϑ_0)-space, driven by the fulfilled-expectations character of the equilibrium.

6 Conclusion

The infinite-horizon equilibrium model for the diffusion of sharing in an economy with heterogeneous agents and intermediary introduced in this paper is the first to analyze transient sharing-market dynamics in the presence of frictions. It makes three main contributions. First, the closed-form expressions of the dynamic market growth path (as a unique subgame-perfect Nash equilibrium in Prop. 1 and 2) provide structural insights into the behavior of agents and the dependence of this behavior on key parameters. For example, as the price elasticity of demand increases,[30] the overall participation in the sharing market decreases, whereas an increase in the discount rate tends to have the opposite effect (see Prop. 3). The participation in the sharing market also decreases in the intermediary's commission rate. The complex dynamics captured in the equilibrium path feature generally nonmonotonic growth behavior. Second, the model allows for a persistent disequilibrium between supply and demand, which manifests itself usually in the form of excess supply, driven by a realistic lag of price-adjustments in the sharing market. The supply-demand mismatch produces a phenomenon also observed in finite-horizon models of sharing markets (Weber 2015). Agents' incentives to incur costs when switching the status of their possessions (from keeping to sharing or vice-versa) depend on the *effective* transaction price, as the product of transaction probability and intermediated market price. A positive diffusion of sharing is driven by an increase in the effective transaction price (see La. 5). Third, the model generates S-shaped diffusion patterns endogenously, using natural primitives.[31] The characteristic change in growth rates is obtained via the (quasi-)concavity of the system function, which is a robust model property that does not depend on particular parameter values. It continues to hold for nonlinear demand specifications and is qualitatively insensitive with respect to changes in the speed of price adjustments (as shown in App. A). Thus, whenever the economy starts with an initial number of sharers below a "maximum-diffusion threshold" (see La. 3), the expansion path of the sharing economy is S-shaped.

Beyond these insights, the model presented here lays the foundations for the systematic estimation of sharing-market dynamics, via identification of the model parameters. For example, an estimation of the frictional diffusion model allows for the independent estimation of the conversion-cost parameters, similar in spirit to the estimation of frictional costs by Hann and Terwiesch (2003). A good estimation of the market dynamics in highly risky sharing markets, as implied by significant growth rates (see footnote 3) together with the well-known boom-bust behavior of concomitant capacity expansions, can help in estimating the market potential and ultimately the value of business models that aim at capitalizing on the sharing of durable goods.

From a managerial (and possibly regulatory) viewpoint, the model clearly indicates that while an intermediary may enable sharing transactions through its provision of a platform, including trust, transaction, and matching technologies, it also slows down the diffusion of sharing in two ways: firstly, the intermediary's commission decreases the speed of adjustment (as determined by the law of motion in Prop. 1); secondly, it also decreases the maximum level of sharing in the economy (as determined by the relevant invariance threshold and by Prop. 3). This echoes earlier findings for search intermediaries, who pass on efficiency improvements to market participants—only in much reduced form—by creating an endogenous obfuscation, knowingly displaying suboptimal search results to increase the number of clicks and thus their own revenues (Weber and Zheng 2007). Similarly, by increasing the commission rate, sharing

[30]This corresponds to a decrease in the demand-elasticity parameter $\hat{\gamma}$; see footnote 24.

[31]This is in contrast to the extremely successful diffusion model by Bass (1969), where the right-hand side of a differential equation of the change of an installed base is essentially pre-assumed to be quadratic, resulting in an S-shaped logistic growth curve; see also footnote 7.

becomes more difficult and evolves more slowly. This is also reflected in the model by Einav et al. (2015) where, in a P2P market with free entry, the intermediary is the only one to come away with a positive surplus. Finally, our model implies a simple viability condition for sharing markets: the per-period value created by a sharing transaction minus the intermediary's take must exceed the "total servicing cost," which includes all expected transaction and maintenance expenses in addition to the current interest on the capital required to convert the item to a sharing state.

Regarding limitations, the model rests on a number of simplifying assumptions. Firstly, all agents are assumed to be risk-neutral and all items on the sharing market are homogeneous. A model of a dynamic sharing economy with differentiated goods and/or risk-averse agents is left for future research. Secondly, the diffusion dynamics represent a supply-side response to a stationary demand function, and it reflects the partial-equilibrium notion that the demand function is not in itself affected by supply. Relaxing the stationarity assumption would pose challenges to being able to derive a closed-form solution, since the law of motion would need to carry an explicit time dependence. Thus, the behavior of the sharing economy over time would reflect the imposed time dynamics. This raises the question of how time-dependencies are transmitted through the economy, for instance in terms of the impulse response of a linearized version of the model to unit demand shocks. This type of analysis is common for dynamic general equilibrium models, which are notoriously difficult to solve, even computationally. Lastly, the full universe of available buyer and seller decisions was assumed to be extremely simple. In each period sellers could decide to enter or exit the sharing market, and buyers could decide whether to rent or not. Realistically, some buyers may decide to become owners, thus increasing the potential supply, and decreasing demand. This in turn would trigger a dynamic and strategic optimization by retailers of the prices at which goods are sold, which can be expected to give rise to complex dynamics, the exact nature of which will depend on the particular assumptions made. We leave it for future research to tackle the purchase/sale decision as well as the dynamic price optimization and possible short-term capacity planning by intermediaries to match supply with demand; these deserve their own theoretical investigations.

Appendix A: Robustness

The empirical relevance of the model depends on the sensitivity of its properties on the key assumptions. In particular, the Nash equilibrium and the characteristic S-curve diffusion pattern of sharing depend on the validity of La. 1 and La. 3 because the latter guarantee the transversal equilibrium condition and the unimodality of the growth increments in the sharing supply, respectively. We now show that these results are robust with respect to relaxation of the assumptions about the price formation.[32] Thus, the main results exhibit structural stability, as they continue to hold away from the nominal model assumptions which were used to obtain closed-form expressions of the transitory sharing equilibrium. Specifically, the qualitative behavior of the model remains unchanged when demand is nonlinear but sufficiently elastic and/or prices can adjust at a different time scale. In Sec. A.3, we discuss how to incorporate an expected per-period servicing cost for remaining in the sharing market into the model without materially affecting the results.

A.1 Nonlinear Demand

The first model relaxation to consider is that instead of the linear (inverse) demand $p(\vartheta)$ in Eq. (1) we allow for any downward-sloping demand curve $D(p)$ (i.e., any downward-sloping inverse demand $p(\vartheta)$) which is elastic at the lower invariance threshold ϑ^0; for such a nonlinear demand,[33] the lower and upper invariance thresholds are (uniquely) defined in terms of fixed points, as shown below.

Lemma 7. *Let $y \in \mathbb{R}$. There exists a unique $\varphi = \varphi(y)$ such that $\varphi = (p(\varphi) + y)/\Delta$.*

As for the results in the main text, all proofs are provided in App. B. By La. 7, the fixed-point mapping $\varphi(\cdot)$ is well-defined as a single-valued function. For nonlinear demand, the lower and upper invariance thresholds in Eqs. (14) and (15) become

$$\vartheta^0 \triangleq \varphi(-rc_{10}) \tag{14'}$$

and

$$\vartheta^1 \triangleq \varphi(c_{10} + (1+r)c_{01}), \tag{15'}$$

respectively. With this, we can formalize the aforementioned demand-elasticity condition,

$$\varepsilon(\vartheta) = -\frac{p(\vartheta)}{\vartheta} \cdot \frac{\partial D(p)}{\partial p} = -\frac{p(\vartheta)}{\vartheta^0 p'(\vartheta)} \geq 1, \quad \vartheta \in (0, \vartheta^0). \tag{A}$$

Requirement (A) is key to extending the results establishing the subgame-perfect Nash equilibrium in Sec. 3.1.1 to nonlinear demand functions.

Lemma 8. *Provided the demand-elasticity condition (A) holds, the conclusion of La. 1 remains valid, i.e., for any $t \geq 0$, the next-period sharing threshold $\vartheta' = \alpha(\vartheta_t)$ is such that $\hat{\pi}(\vartheta', \vartheta') = 0$.*

[32] La. 1 implies that Eqs. (17) and (18) continue to hold, ensuring that the law of motion in Eq. (16) provided by Prop. 1 (without the explicit expressions for α_0, α_1) and the subgame-perfect Nash equilibrium in Prop. 2 of $\mathcal{G}(\vartheta_0)$ remain valid.

[33] If the minimum possible payoff of joining the market exceeds the owner's utility difference, i.e., $p(1) \geq \Delta$, then $\vartheta^0 = 1$ and all agents are willing to share in steady state (regardless of the need state) such that full sharing penetration is attained in finite time. As stipulated by Eq. (2), we concentrate on the interesting case where $p(1) \leq \Delta$.

Persistent growth in the sharing economy is a robust result given any (downward-sloping) demand which is elastic in an undersharing economy.

Remark 8. Important classes of demand specifications satisfy the elasticity condition (A), as illustrated by the following three representatives:[34]

- *constant-elasticity demand:* $p(\vartheta) = \gamma\vartheta^{-1/\eta}$, for $\eta \geq 1$ and $\gamma > 0$;
- *semi-logarithmic demand:* $p(\vartheta) = \gamma_0 - \gamma_1 \ln(\vartheta)$, for $\gamma_0 \geq \gamma_1 > 0$;
- *quasi-affine demand:* $p(\vartheta) = \gamma_0 - \gamma_1\vartheta^{1/\eta}$, for $\eta \geq (\gamma_1/\gamma_0)/(1-(\gamma_1/\gamma_0))$ and $\gamma_0 > \gamma_1 > 0$.

It is easy to check that all of the preceding demand curves are globally elastic, i.e., such that $\varepsilon(\vartheta) \geq 1$, and the relevant inequalities in Eq. (2) can readily be satisfied. Note also that these classes of demand specifications have been widely used in practice; see, e.g., Bulow and Pfleiderer (1983).

Remark 9. The elasticity requirement in Eq. (A) means that a sharing equilibrium is attained in an elastic part of the demand curve. This is reminiscent of the monopoly pricing rule (Tirole 1988, p. 66) that a monopolist's relative markup must be equal to the inverse elasticity (or Lerner index), which implies that the demand elasticity must exceed 1 at the optimal monopoly price. This also means that important for the results is not the linearity of demand but the fact that demand is elastic, i.e., responsive to price movements, when only small quantities are available on the sharing market.

We now show that for all these three general classes of demand curves, the diffusion pattern in the sharing economy is unimodal (i.e., quasi-concave).

Lemma 9. *For constant-elasticity, semi-logarithmic, and quasi-affine demand, as specified in Remark 8, the conclusion of La. 3 remains valid, i.e., there is a unique "maximum-diffusion" (sharing) threshold ϑ^μ in the interval $(0, \vartheta^0)$, at which the growth of the sharing economy is maximal.*

The finding also obtains for any inverse demand which is concave.

A.2 Change of the Time Scale for Price Adjustments

The second model relaxation concerns the intertemporal price-adjustment process in Eq. (3), where we allow for any positive price-adjustment time scale,

$$dt \equiv t_{k+1} - t_k > 0, \tag{B}$$

for $k \in \{0, 1, 2, \ldots\}$, instead of the normalized interval length with $t_k \equiv k$ and $dt = 1$ in the main text. Without loss of generality, we can restrict attention to a *decrease* of the price-adjustment time scale because the unit of the original time scale is arbitrary. By reducing dt, the per-period rate interest also drops. Intuitively, as dt decreases, adjustments take place more frequently, but the variation of size of these increments is not so easy to guess. To understand the change in the intertemporal model behavior in response to faster price updates, one needs to assess the effect of the time-scale compression on the system function *and* on the invariance threshold.

[34]Without loss of generality, set $\rho = 0$; for $\rho \in (0, 1)$, it is sufficient to replace $(\gamma, \gamma_0, \gamma_1)$ by $(1-\rho)(\gamma, \gamma_0, \gamma_1)$.

Lemma 10. *By compressing the price-adjustment time scale in (B) to $dt = \lambda \in (0, 1)$, the growth arc of the system function becomes $\hat{\alpha}_0(\vartheta)$ and the lower invariance threshold becomes $\hat{\vartheta}^0$ which are such that*

$$0 < \hat{\alpha}_0(\vartheta) - \alpha_0(\vartheta) < \left(r - \frac{(1+r)^\lambda - 1}{\lambda} \right) \frac{c_{10}}{\Delta} < (r - \ln(1+r)) \frac{c_{10}}{\Delta},$$

for all $\vartheta \in (0, \vartheta^0)$, and

$$0 < \hat{\vartheta}^0 - \vartheta^0 = \left(r - \frac{(1+r)^\lambda - 1}{\lambda} \right) \frac{c_{10}}{\hat{\gamma} + \Delta} < (r - \ln(1+r)) \frac{c_{10}}{\hat{\gamma} + \Delta}.$$

An important implication of the preceding result is that a faster time scale for price adjustment does *not* imply that adjustments in the economy become smaller or that they disappear altogether. While the faster adjustments tend to become larger, the asymptotic limits in the sharing market also become larger. In other words, it is true that the adjustment steps are larger (but always finite, bounded uniformly) and take place more frequently, but also the distance from a given initial value ϑ_0 (below $\mathcal{R}$) to the time-compressed invariance threshold $\hat{\vartheta}^0$ is larger than the original adjustment distance from ϑ_0 to ϑ^0. The latter effect somewhat balances out the former effect. This remains true in the limit, for $\lambda \to 0^+$. Thus, even with close-to-infinite adjustment speed, the economy remains in perpetual disequilibrium, so that the effects discussed in this paper for $dt = 1$ are robust with respect to the choice of the length of the price-adjustment period. The latter may be subject to the effects of technological change which may, possibly with the aid of an intermediary, improve the price discovery on the sharing market.

Remark 10. The upper arc $\alpha_1(\cdot)$ of the system function becomes equal to 1 when the time-scale compression factor λ is sufficiently small. This also applies to the invariance threshold ϑ^1. Thus, in a regime with rapid price adjustments a sharing economy tends to either grow or stagnate; technology improvements which lead to faster price adjustments therefore bypass negative supply shocks and the concomitant (deterministic) price volatility.

A.3 Servicing Cost

Suppose that in addition to the conversion cost c_{10}, there is also a per-period servicing cost $\kappa \geq 0$ associated with a continued presence in the sharing market as a supplier, which includes expected outlays for cleaning, maintenance, repair, and transportation. Transportation costs may include sending the item through mail services, or bringing it personally to an agreed place of exchange. In this context, Caillaud and Jullien (2003) showed that in intermediated markets, such costs are usually borne by the supplier, similar to the intermediary's commission rate. Note that an agent incurs the per-period servicing cost only if a transaction takes place. Thus, κ changes the effective transaction price seen from the perspective of any supplier. Incorporating the fixed cost in the model, the effective transaction price, including the effect of intermediation, becomes

$$\hat{p}(\vartheta_t, \vartheta_{t-1}) = (\hat{\gamma} - \kappa - \hat{\gamma}\vartheta_t)\, q(\vartheta_t, \vartheta_{t-1}).$$

Thus, the results in the main part of the paper are unaffected, in particular the shape of the market growth curve.

Lemma 11. *For all $\kappa \geq 0$, the conclusion of La. 3 remains valid, i.e., the sharing economy exhibits an S-shaped diffusion pattern.*

The main effect of the servicing cost is to reduce the intermediated demand-elasticity parameter $\hat{\gamma}$, which also determines the maximal payoff from a sharing transaction for a potential supplier.

Remark 11. As in the base case without servicing cost, the lower invariance threshold,

$$\vartheta_0 = \frac{\hat{\gamma} - \kappa - rc_{10}}{\hat{\gamma} + \Delta},$$

is located in the elastic part of the demand curve.

For a sharing market to be viable, the lower invariance threshold ϑ^0 must be positive, so $\gamma > (\kappa + rc_{10})/(1-\rho)$. This limits the sharing intermediary's commission to be strictly less than $\bar{\rho} \triangleq 1 - (\kappa + rc_{10})/\gamma$. For this commission to be positive, necessarily $\gamma > \kappa + rc_{10}$, i.e., the per-period value (as measured by the quality parameter γ) must exceed the per-period "total servicing cost," which includes all transaction and maintenance costs *and* the cost of servicing the capital needed for converting the item to its sharing state.

Appendix B: Proofs

Proof of Lemma 1. By the one-shot deviation principle, to establish a subgame-perfect equilibrium, it is sufficient to check that no agent has an incentive to deviate from the equilibrium path in any single period (Fudenberg and Tirole 1991).[35] Accordingly, consider an agent of type θ at time $t > 0$. At that time, the agent prefers to keep the item in the next period, i.e., $\hat{\pi}(\theta, \vartheta_t) = 1$, if and only if

$$\begin{aligned} &\bar{g}(1, \xi(\theta, \vartheta_t), \hat{p}(\vartheta_t, \vartheta_{t-1})|\theta) + \delta \bar{g}(\hat{\pi}(\theta, \vartheta'), 1, \hat{p}(\vartheta', \vartheta_t)|\theta) + \delta^2 \hat{V}(\theta, \alpha(\vartheta'), \vartheta') \geq \\ &\quad \bar{g}(0, \xi(\theta, \vartheta_t), \hat{p}(\vartheta_t, \vartheta_{t-1})|\theta) + \delta \bar{g}(\hat{\pi}(\theta, \vartheta'), 0, \hat{p}(\vartheta', \vartheta_t)|\theta) + \delta^2 \hat{V}(\theta, \alpha(\vartheta'), \vartheta'). \end{aligned} \tag{28}$$

All other agent types follow the equilibrium threshold policy $\hat{\pi}$. Hence, the time-$(t+1)$ type threshold becomes

$$\begin{aligned} \vartheta' \;=\; \inf\{\theta \in \Theta : &\; \bar{g}(1, \xi(\theta, \vartheta_t), \hat{p}(\vartheta_t, \vartheta_{t-1})|\theta) + \delta \bar{g}(\hat{\pi}(\theta, \vartheta'), 1, \hat{p}(\vartheta', \vartheta_t)|\theta) \geq \\ &\; \bar{g}(0, \xi(\theta, \vartheta_t), \hat{p}(\vartheta_t, \vartheta_{t-1})|\theta) + \delta \bar{g}(\hat{\pi}(\theta, \vartheta'), 0, \hat{p}(\vartheta', \vartheta_t)|\theta)\}, \end{aligned} \tag{29}$$

independent of agent θ's choice. The last condition defines the next-period type as an externality for all agents, and it provides a transversal equilibrium condition linking the individuals' payoff-maximization problems in Eq. (12). Thus, $\vartheta' = \alpha(\vartheta_t)$ is such that

$$\begin{aligned} &\bar{g}(1, \xi(\vartheta', \vartheta_t), \hat{p}(\vartheta_t, \vartheta_{t-1})|\vartheta') + \delta \bar{g}(\hat{\pi}(\vartheta', \vartheta'), 1, \hat{p}(\vartheta', \vartheta_t)|\vartheta') = \\ &\quad \bar{g}(0, \xi(\vartheta', \vartheta_t), \hat{p}(\vartheta_t, \vartheta_{t-1})|\vartheta') + \delta \bar{g}(\hat{\pi}(\vartheta', \vartheta'), 0, \hat{p}(\vartheta', \vartheta_t))|\vartheta'). \end{aligned} \tag{30}$$

To show that $\hat{\pi}(\vartheta', \vartheta') = 0$, suppose that, on the contrary, $\hat{\pi}(\vartheta', \vartheta') = 1$. Depending on whether the sharing market is about to stagnate, to expand, or to contract, we distinguish three cases.

1. If $\vartheta_t = \vartheta'$, the result is immediate. The threshold policy in Eq. (11) yields that $\hat{\pi}(\vartheta', \vartheta') = \hat{\pi}(\vartheta_t, \vartheta_t) = \mathbf{1}_{\{\vartheta_t > \vartheta'\}} = 0$.

[35] The continuity-at-infinity condition (Fudenberg and Tirole 1991, Def. 4.1) for the application of the one-shot deviation principle in an infinite-horizon dynamic game is satisfied because the stage-game payoffs are uniformly bounded and future payoffs are discounted geometrically.

2. $\vartheta_t < \vartheta'$. By Eq. (5), it is $\xi(\vartheta', \vartheta_t) = 1$, and by hypothesis $\hat{\pi}(\vartheta', \vartheta') = 1$. By Eq. (30) ϑ' is such that

$$\bar{g}(1, 1, \hat{p}(\vartheta_t, \vartheta_{t-1})|\vartheta') + \delta\bar{g}(1, 1, \hat{p}(\vartheta_t, \vartheta')|\vartheta') = \bar{g}(0, 1, \hat{p}(\vartheta_t, \vartheta_{t-1})|\vartheta') + \delta\bar{g}(1, 0, \hat{p}(\vartheta_t, \vartheta')|\vartheta'),$$

or equivalently,

$$\vartheta' u_1 + \delta\vartheta' u_1 = \vartheta' u_1 - c_{10} + \delta((1-\rho)\, p(\vartheta_t)\, q(\vartheta', \vartheta_t) + \vartheta' u_0 - c_{01}).$$

We therefore obtain that

$$\vartheta' = \frac{(1-\rho)\, p(\vartheta_t)\, q(\vartheta', \vartheta_t) - c_{01} - (1+r)c_{10}}{\Delta}. \tag{31}$$

The time-$(t+1)$ type threshold ϑ' can exceed ϑ_t, as long as $\vartheta_t \in [0, \omega)$, where ω solves a stationary version of Eq. (38), corresponding to a fixed-point problem with $q(\omega, \omega) = 1$, so

$$\omega = \min\left\{1, \frac{\hat{\gamma} - c_{01} - (1+r)c_{10}}{\hat{\gamma} + \Delta}\right\} \quad \left(= \frac{(1-\rho)\, p(\omega) - c_{01} - (1+r)c_{10}}{\Delta}\right).$$

Note that $\omega \leq \hat{\gamma}/(\hat{\gamma} + \Delta) \leq 1/2$, since $\hat{\gamma} \leq \Delta$ by Eq. (2). On the other hand, differentiation of ϑ' in Eq. (38) with respect to ϑ_t yields

$$\frac{\partial \vartheta'}{\partial \vartheta_t} = \frac{\hat{\gamma}(1 - 2\vartheta_t)}{2\Delta\vartheta' + c_{01} + (1+r)c_{10}}.$$

Thus, $\vartheta_t < \omega$ $(\leq 1/2)$, which in turn implies that $\partial\vartheta'/\partial\vartheta_t > 0$. As a result, for all $\vartheta_t \in [0, \omega)$, the next-period sharing threshold ϑ' is increasing in ϑ_t, and

$$\vartheta_t < \vartheta' < \vartheta'' < \omega \leq 1/2.$$

By Eq. (11) therefore $\hat{\pi}(\vartheta', \vartheta') = \mathbf{1}_{\{\vartheta' > \vartheta''\}} = 0$, in contradiction to our hypothesis.

3. $\vartheta_t > \vartheta'$. In this case, $q_t(\vartheta', \vartheta_t) = \min\{1, \vartheta_t/\vartheta'\} = 1$, and $\xi(\vartheta', \vartheta_t) = 1$. Thus, as before, by Eq. (30) it is

$$\vartheta' = \frac{(1-\rho)\, p(\vartheta_t) + rc_{10}}{\Delta}. \tag{32}$$

Moreover, since by hypothesis $\hat{\pi}(\vartheta', \vartheta') = 1$, Eq. (11) implies that $\vartheta' > \vartheta''$, where $\vartheta' = \vartheta_{t+1}$ and $\vartheta'' = \vartheta_{t+2}$. Thus, as before, by Eq. (37) one obtains

$$\vartheta'' = \frac{(1-\rho)\, p(\vartheta') + rc_{10}}{\Delta}. \tag{33}$$

Since $\vartheta' > \vartheta''$, Eqs. (32)–(33) imply that $p(\vartheta_t) > p(\vartheta')$. As $p(\cdot)$ is decreasing, it is $\vartheta_t < \vartheta'$, which contradicts the initial assumption. Thus, necessarily $\hat{\pi}(\vartheta', \vartheta') = 0$.

The three cases taken together imply that $\hat{\pi}(\vartheta', \vartheta') = 0$, completing our proof. □

Proof of Proposition 1. Depending on the direction of movement from ϑ_t to $\vartheta' = \vartheta_{t+1}$, three cases are examined separately.

Case 1: $\vartheta_t < \vartheta'$. By Eq. (5), the time-$(t+1)$ marginal type $\theta = \vartheta'$ is still out of the sharing market at time t, that is $\xi(\vartheta', \vartheta_t) = 1$. However, by La. 1, this agent type will join the market in the next time period: $\hat{\pi}(\vartheta', \vartheta') = 0$. As a result, Eq. (13) becomes

$$\bar{g}(1, 1, \hat{p}(\vartheta_t, \vartheta_{t-1})|\vartheta') + \delta\bar{g}(0, 1, \hat{p}(\vartheta_t, \vartheta')|\vartheta') = \bar{g}(0, 1, \hat{p}(\vartheta_t, \vartheta_{t-1})|\vartheta') + \delta\bar{g}(0, 0, \hat{p}(\vartheta_t, \vartheta')|\vartheta'),$$

or equivalently: $\vartheta' u_1 + \delta(\vartheta' u_1 - c_{10}) = \vartheta' u_1 - c_{10} + \delta((1-\rho)p(\vartheta_t)q(\vartheta', \vartheta_t) + \vartheta' u_0)$, which yields $\vartheta' = \alpha_0(\vartheta_t)$ as in Eq. (17). The marginal type ϑ' creates an action threshold, below which all agents are willing to share. As shown in the proof of La. 1, the action threshold is valid for $\vartheta_t \in [0, \vartheta^0)$, where the lower invariance threshold ϑ^0 is given in Eq. (14).

Case 2: $\vartheta_t > \vartheta'$. The sharing-state distribution in Eq. (5) implies that $\xi(\vartheta', \vartheta_t) = 0$ and by La. 1 it is $\hat{\pi}(\vartheta', \vartheta') = 0$. That is, the time-$(t+1)$ marginal type finds it optimal to exit the sharing market for one period, and rejoin afterwards. Furthermore, by Eqs. (8)–(9), the transaction probability equals 1, and $\hat{p}(\vartheta', \vartheta_t) = (1-\rho)p(\vartheta_t)$. Thus, Eq. (13) becomes

$$\bar{g}(1, 0, \hat{p}(\vartheta_t, \vartheta_{t-1})|\vartheta') + \delta\bar{g}(0, 1, \hat{p}(\vartheta', \vartheta_t)|\vartheta') = \bar{g}(0, 0, \hat{p}(\vartheta_t, \vartheta_{t-1})|\vartheta') + \delta\bar{g}(0, 0, \hat{p}(\vartheta', \vartheta_t)|\vartheta'),$$

or equivalently,

$$\hat{p}(\vartheta_t, \vartheta_{t-1}) + \vartheta' u_0 - c_{01} + \delta(\vartheta' u_1 - c_{10}) = \hat{p}(\vartheta_t, \vartheta_{t-1}) + \vartheta' u_0 + \delta((1-\rho)p(\vartheta_t) + \vartheta' u_0).$$

We therefore obtain that $\vartheta' = \alpha_1(\vartheta_t)$ as in Eq. (18). Analogous to Case 1, the action threshold is valid for $\vartheta_t \in (\vartheta^1, 1]$, where the upper invariance threshold ϑ^1 is specified in Eq. (15).

Case 3: $\vartheta_t \in [\vartheta^0, \vartheta^1]$. For type thresholds between the invariance thresholds, it is optimal for all agents to remain in their current sharing state, so $\vartheta' = \vartheta_t = \bar{\vartheta}$, where $\bar{\vartheta}$ implies a stationary sharing-state distribution $\bar{\xi}(\cdot, \bar{\vartheta})$ which stays in place for all times greater than t; see Section 3.3.1 for details.

Cases 1–3 together establish the law of motion in Eq. (16). $\square$

Proof of Proposition 2. Let $\vartheta_0 \in \Theta$. The fact that the agents' strategies $\hat{\pi}$ describe a subgame-perfect equilibrium of $\mathcal{G}(\vartheta_0)$ follows from the discussion in the main text, specifically because the agents' strategy profile implements value functions which together satisfy the system of Bellman equations (10). By application of the one-shot deviation principle in (28) we know that at time t, a type-θ agent's binary decision is to keep the item if and only if the following inequality holds:

$$\begin{aligned}\bar{g}(1, \xi(\theta, \vartheta_t), \hat{p}(\vartheta_t, \vartheta_{t-1})|\theta) + \delta\bar{g}(\hat{\pi}(\theta, \vartheta'), 1, \hat{p}(\vartheta', \vartheta_t)|\theta) \geq \\ \bar{g}(0, \xi(\theta, \vartheta_t), \hat{p}(\vartheta_t, \vartheta_{t-1})|\theta) + \delta\bar{g}(\hat{\pi}(\theta, \vartheta'), 0, \hat{p}(\vartheta', \vartheta_t)|\theta).\end{aligned}$$

The entries in Table 6 show the difference between the left-hand side and the right-hand side of the last inequality, which corresponds to the differential benefit of keeping ($a_t = 1$) over sharing ($a_t = 0$) for any given type $\theta \in \Theta$, conditional on the current sharing state and next period's optimal action.

	$\xi(\theta, \vartheta_t) = 0$	$\xi(\theta, \vartheta_t) = 1$
$\hat{\pi}(\theta, \vartheta') = 0$	$-c_{01} + \delta(\theta\Delta - \hat{p}_{t+1} - c_{10})$	$c_{10} + \delta(\theta\Delta - \hat{p}_{t+1} - c_{10})$
$\hat{\pi}(\theta, \vartheta') = 1$	$-c_{01} + \delta(\theta\Delta - \hat{p}_{t+1} + c_{01})$	$c_{10} + \delta(\theta\Delta - \hat{p}_{t+1} + c_{01})$

Table 1: Payoff difference of keeping over sharing for an agent of type $\theta \in \Theta$, contingent on the current-period sharing state $\xi(\theta, \vartheta_t)$ and next period's equilibrium action $\hat{\pi}(\theta, \vartheta')$.

For all contingencies, the payoff difference is increasing in θ, so that a threshold strategy must necessarily be optimal, at any time t.[36] On the other hand, our equilibrium construction, beginning with the threshold policy in Eq. (11), leads to a unique strategy profile as subgame-perfect Nash equilibrium, so that the equilibrium is indeed unique. $\square$

Proof of Lemma 2. (i) We need to show that the limit of $\alpha(\cdot)$ vanishes when ϑ tends towards the lower end of the type space Θ, i.e., that $\lim_{\vartheta\to 0^+}\alpha_0(\vartheta) = 0$. For $\vartheta^0 = 0$ the result is immediate. Consider now the interesting case where $\vartheta^0 \in (0,1)$. For $0 < \vartheta < \vartheta^0$, by Prop. 1 it is $\alpha(\vartheta) = \alpha_0(\vartheta) > 0$, and using the definition of the transaction probability in Eq. (8),

$$\alpha_0(\vartheta) = \frac{(1-\rho)\,p(\vartheta)(\vartheta/\alpha_0(\vartheta)) - rc_{10}}{\Delta}.$$

Multiplying both sides by $\alpha_0(\vartheta) > 0$, and then taking the right-sided limit, for $\vartheta \to 0^+$, yields

$$0 \le \lim_{\vartheta\to 0^+} \alpha_0^2(\vartheta)\Delta \;=\; \lim_{\vartheta\to 0^+}((1-\rho)\,p(\vartheta)\vartheta - rc_{10}\alpha_0(\vartheta)) = -\lim_{\vartheta\to 0^+} rc_{10}\alpha_0(\vartheta) \le 0,$$

which immediately implies that $\alpha_0(0) = 0$, by continuous completion.

(ii) If $\hat\gamma < rc_{10}$, then by Eq. (14) the lower invariance threshold vanishes, so for all $\vartheta \in (0, \vartheta^1]$ the sharing threshold lies in $\mathcal{R}$, and therefore $\alpha(\vartheta) \equiv \vartheta$. For all $\vartheta \in (\vartheta^1, 1]$, Prop. 1 yields $\alpha(\vartheta) = \alpha_1(\vartheta)$. Since $\alpha_1(\vartheta^1) = \vartheta^1$ and $\alpha_1'(\vartheta) < 0$, it follows that $\alpha_1(\vartheta) < \vartheta$ for all ϑ above $\mathcal{R}$. This implies the single-crossing property, as claimed. For $\hat\gamma > rc_{10}$, the lower invariance threshold $\vartheta^0 > 0$. By part (i), it is $\alpha_0(0) = 0$, and by La. 2 we know that $\alpha_0(\vartheta)$ has a positive slope for all $\vartheta \in (0, \vartheta^0)$. Thus, $\alpha_0(\vartheta) > \vartheta$ for $0 < \vartheta < \vartheta^0$. As for $\hat\gamma < rc_{10}$, $\alpha_1(\vartheta) < \vartheta$ for $\vartheta > \vartheta^1$. Hence, $\alpha(\vartheta) > \vartheta$ for $0 < \vartheta < \vartheta^0$ and $\alpha(\vartheta) \le \vartheta$ for $\vartheta \ge \vartheta^0$, so the forward-difference of the sharing thresholds, $\alpha(\vartheta) - \vartheta$, has the (weak) single-crossing property for all $\vartheta \in (0,1]$, as claimed.

(iii) By Prop. 1, $\alpha(\vartheta) = \alpha_0(\vartheta)$ for all ϑ below the lower invariance threshold. For $\hat\gamma \le rc_{10}$, by Eq. (14) the lower invariance threshold ϑ^0 vanishes, so there is nothing to prove. For $\hat\gamma > rc_{10}$, we first consider the monotonicity of the system function. Note that $\vartheta^0 \le \hat\gamma/(\hat\gamma + \Delta) \le 1/2$. Differentiation of α_0 in Eq. (17) with respect to ϑ yields

$$\alpha_0'(\vartheta) = \frac{\hat\gamma(1-2\vartheta)}{2\Delta\alpha_0(\vartheta) + rc_{10}} > 0, \tag{34}$$

for all $\vartheta \in (0, \vartheta^0)$. Next we consider the concavity of the system function. Taking into account that $0 < \vartheta < \vartheta^0 \le 1/2$, Eq. (34) yields

$$\alpha''(\vartheta) = -\frac{2\hat\gamma(2\Delta\alpha(\vartheta) + rc_{10}) + 2\Delta\hat\gamma(1-2\vartheta)\alpha'(\vartheta)}{(2\Delta\alpha(\vartheta) + rc_{10})^2} < 0,$$

for all $\vartheta \in (0, \vartheta^0)$, completing the proof. $\square$

Proof of Lemma 3. Let $\phi(\vartheta) \triangleq \alpha(\vartheta) - \vartheta$ for all $\vartheta \in [0, \vartheta^0]$ be the threshold increment, where we use the fact that by La. 2(i) the system function $\alpha(\cdot)$ is well-defined (by continuous completion) on the entire domain of ϕ. Then, $\phi(0) = \phi(\vartheta^0) = 0$, and by Rolle's theorem there exists a $\vartheta^\mu \in (0, \vartheta^0)$ such that $\phi'(\vartheta^\mu) = 0$. Since by La. 2 it is $\phi'' < 0$ on $(0, \vartheta^0)$, the maximum-diffusion threshold ϑ^μ is the unique maximizer of the threshold increment ϕ on $[0, \vartheta^0]$. $\square$

[36] If type θ prefers keeping over sharing, then all types $\hat\theta > \theta$ also prefer keeping over sharing.

Proof of Proposition 3. (i),(ii): The claims follow directly from the definition of the invariance thresholds in Eqs. (14)–(15). (iii): Differentiating ϑ^0 in Eq. (14), with respect to γ and σ, respectively, yields

$$\frac{\partial\vartheta^0}{\partial\hat{\gamma}}\frac{\partial\hat{\gamma}}{\partial\gamma} = (1-\rho)\frac{\Delta + rc_{10}}{(\hat{\gamma}+\Delta)^2} > 0 \quad \text{and} \quad \frac{\partial\vartheta^0}{\partial\hat{\gamma}}\frac{\partial\hat{\gamma}}{\partial\rho} = -\gamma\frac{\Delta + rc_{10}}{(\hat{\gamma}+\Delta)^2} < 0,$$

as claimed. Similarly, differentiating ϑ^1 in Eq. (15), with respect to $\hat{\gamma}$ yields

$$\frac{\partial\vartheta^1}{\partial\hat{\gamma}}\frac{\partial\hat{\gamma}}{\partial\gamma} = (1-\rho)\frac{\Delta - c_{10} - (1+r)c_{01}}{(\hat{\gamma}+\Delta)^2} > 0 \quad \text{and} \quad \frac{\partial\vartheta^0}{\partial\hat{\gamma}}\frac{\partial\hat{\gamma}}{\partial\rho} = -\gamma\frac{\Delta - c_{10} - (1+r)c_{01}}{(\hat{\gamma}+\Delta)^2} < 0,$$

which is true for all $\vartheta^1 < 1$. □

Proof of Lemma 4. As in the one-shot deviation principle applied to the general case in Eq. (28), the steady state is maintained in a (subgame-perfect) Nash equilibrium if no agent has an incentive to deviate (in any proper subgame of $\mathcal{G}(\vartheta_0)$). Given the optimality of a threshold-type strategy by Prop. 2, as well as optimality of sharing for the next-period sharing threshold by La. 1, $\bar{\vartheta}$ is stationary if

$$\delta\bar{V}^0(\theta|\bar{\vartheta}) \geq -c_{01} + \delta(\theta u_1 - c_{10}) + \delta^2\bar{V}^0(\theta|\bar{\vartheta}), \quad \theta \in [0, \bar{\vartheta}],$$

and

$$\delta\bar{V}^0(\theta|\bar{\vartheta}) - c_{10} \leq \delta\bar{V}^1(\theta|\bar{\vartheta}), \quad \theta \in (\bar{\vartheta}, 1].$$

Combining the last two inequalities with the previous identities yields

$$\Big(\theta \in [0, \bar{\vartheta}] \;\Rightarrow\; (1+r)c_{01} + c_{10} \geq \theta\Delta - \bar{p}\Big) \quad \text{and} \quad \Big(\theta \in (\bar{\vartheta}, 1] \;\Rightarrow\; -rc_{10} \leq \theta\Delta - \bar{p}\Big).$$

Hence, for $\theta = \bar{\vartheta}$ one obtains

$$(\bar{p} - rc_{10})/\Delta \leq \bar{\vartheta} \leq (\bar{p} + (1+r)c_{01} + c_{10})/\Delta.$$

This completes the proof. □

Proof of Proposition 4. (i) If $\vartheta_0 = 0$, then by La. 2(i) the sequence of sharing thresholds is stationary, and $\vartheta_t \equiv 0$. (ii) If $0 < \vartheta_0 \leq \vartheta^0$, then by La. 2 and Corollary 1, $(\alpha(\vartheta_t) - \vartheta_t)_{t=t^\mu}^{\infty}$ is a monotonically decreasing sequence, bounded from below by 0. Thus, by the monotone convergence theorem (see, e.g., Rudin 1976, p. 55)

$$\lim_{t\to\infty}(\alpha(\vartheta_t) - \vartheta_t) = \big(\alpha(\bar{\vartheta}) - \bar{\vartheta}\big)\big|_{\bar{\vartheta}=\vartheta^0} = 0,$$

so that by continuity of $\alpha(\cdot)$, it is $\lim_{t\to\infty}\alpha(\vartheta_t) = \vartheta^0$. (iii) If $\vartheta^0 \leq \vartheta_0 \leq \vartheta^1$, then by Prop. 1, ϑ_0 lies in the invariance region $\mathcal{R}$, which implies the result. (iv) If $\vartheta^1 \leq \vartheta_0 \leq \vartheta^2$, then by Prop. 1, $\vartheta^0 \leq \alpha(\vartheta_0) = \alpha_1(\vartheta_0) \leq \vartheta^1$, so that $\alpha(\vartheta_0) \in \mathcal{R}$, which yields the claim. (v) If $\vartheta^2 \leq \vartheta_0 \leq 1$, then by Prop. 1, $\vartheta^0 \geq \alpha(\vartheta_0) = \vartheta_1$, and for $t \geq 1$ onwards the threshold ϑ_t lies in the interval $[\alpha_1(\vartheta_0), \vartheta^0]$. As in part (ii), we obtain that $\lim_{t\to\infty}\vartheta_t = \vartheta^0$. □

Proof of Lemma 5. Let $\vartheta \in (0, \vartheta^0)$. By Prop. 1 and Eq. (8), we can rewrite the effective transaction price in Eq. (9) in the form

$$\hat{p}(\vartheta, \alpha(\vartheta)) = \hat{\gamma}(1-\vartheta)\vartheta/\alpha_0(\vartheta).$$

Differentiating the effective price with respect to the sharing threshold yields

$$\hat{p}'(\vartheta) = \frac{\hat{\gamma}(1-2\vartheta)}{\alpha_0^2(\vartheta)} \left(\alpha_0(\vartheta) - \frac{\hat{\gamma}\vartheta(1-\vartheta)}{2\Delta\alpha_0(\vartheta) + rc_{10}} \right). \tag{35}$$

Taking into account that $\vartheta^0 \leq 1/2$, to show that the effective price is increasing in ϑ, i.e., $\hat{p}'(\vartheta) > 0$, it is sufficient that the bracketed expression in Eq. (35) is positive, i.e.,

$$\alpha_0(\vartheta)(2\Delta\alpha_0(\vartheta) + rc_{10}) > \hat{\gamma}\vartheta(1-\vartheta). \tag{36}$$

By definition of $\alpha_0(\vartheta)$ in Prop. 1, for all $\vartheta \in (0, \vartheta^0)$ we obtain

$$\alpha_0(\vartheta)\left(\Delta\alpha_0(\vartheta) + rc_{10}\right) = \hat{\gamma}(1-\vartheta)\vartheta.$$

By substituting the last equation, inequality (36) is equivalent to

$$\Delta\alpha_0^2(\vartheta) > 0;$$

the latter holds for all $\vartheta \in (0, \vartheta^0)$, which completes our proof. □

Proof of Proposition 5. By Prop. 4, for any nonzero initial condition the economy attains its steady state in finite time if and only if $\vartheta_0 \in [\vartheta^0, \vartheta^2]$. By Eq. (14), $\vartheta_0 < \vartheta^0$ if $rc_{10} < \hat{\gamma} - \vartheta_0(\hat{\gamma} + \Delta)$. Since $r = \delta/(1-\delta)$, rearranging the terms yields

$$c_{10} < \frac{\delta(\hat{\gamma}+\Delta)}{1-\delta} \left(\frac{\hat{\gamma}}{\hat{\gamma}+\Delta} - \vartheta_0 \right),$$

as claimed. Further, by Eq. (19), $\vartheta_0 > \vartheta^2$ if and only if

$$c_{10}(\hat{\gamma} + \Delta/\delta) + c_{01}(\hat{\gamma}+\Delta) < \delta(\hat{\gamma}+\Delta) \left(\vartheta_0 - \frac{\hat{\gamma}}{\hat{\gamma}+\Delta} \right),$$

which, once written in matrix form, yields the result immediately. □

Proof of Lemma 6. By Eq. (14), $0 \leq \vartheta^0 \leq \hat{\gamma}/(\hat{\gamma}+\Delta) \leq 1/2$, and by Remark 3 it is $\vartheta^0 \leq \vartheta^1 \leq 1$. Since $\alpha_1(\vartheta)$ is decreasing in ϑ, it follows that $\vartheta^1 = \alpha_1^{-1}(\vartheta^1) \leq \alpha_1^{-1}(\vartheta^0) = \vartheta^2$, as claimed. We now prove the three remaining claims. (i) By Eq. (14), $\vartheta^0 > 0$ if $\hat{\gamma} > rc_{10}$. Since $r = \delta/(1-\delta)$, the result follows immediately. (ii) Using the definition of ϑ^1 in Eq. (15) and rearranging the terms yields the desired result. (iii) If $\hat{\gamma} > rc_{10}$, since $\alpha_1(\vartheta)$ is a decreasing function, for $\vartheta^2 < 1$ we require $\alpha_1(1) < \vartheta^0$, i.e.,

$$\frac{c_{10} + (1+r)c_{01}}{\Delta} \leq \frac{\hat{\gamma} - rc_{10}}{\hat{\gamma}+\Delta}.$$

Rearranging the terms yields $c_{10}(1 + \frac{\hat{\gamma}}{\Delta}) + c_{01}(1 + \delta\frac{\hat{\gamma}}{\Delta}) < \delta\hat{\gamma}$. If $\hat{\gamma} > rc_{10}$, the lower invariance threshold vanishes and $(c_{10} + (1+r)c_{01})/\Delta \geq 0$ implies that $\vartheta^2 = 1$. □

Proof of Lemma 7. Let $y \in \mathbb{R}$ be fixed, and let $\Phi \subset \mathbb{R}$ be a nonempty, convex, compact set (i.e., a closed interval). Consider the mapping $H : \Phi \to \mathbb{R}$ with $H(\varphi) \triangleq (p(\varphi) + y)/\Delta$ for all $\varphi \in \Phi$. Then $H(\cdot)$ is continuous as a composition of continuous functions. By a similar argument it is also differentiable, and

$$H'(\varphi) = \frac{p'(\varphi)}{\Delta} \leq 0,$$

for all $\varphi \in \mathbb{R}$, since the inverse demand function $p(\cdot)$ is downward-sloping by assumption. Let $\Phi = [0, 1 + y/\Delta]$, and let $h(x) \triangleq H(x) - x$. Since $h(1 + y/\Delta) < 0 < h(0)$, by the intermediate value theorem there exists a solution $\varphi = \varphi(y, \Delta)$ (in the set Φ) such that $h(\varphi) = 0$, or equivalently

$$\varphi = H(\varphi) = \frac{p(\varphi) + y}{\Delta}.$$

The uniqueness of $\vartheta(y, \Delta)$ follows from the fact that the left-hand side of the last equation is strictly increasing whereas its right-hand side is (weakly) decreasing. □

Proof of Lemma 8. Similar to the procedure undertaken in the proof of La. 1, we use the one-shot deviation principle to establish a subgame-perfect equilibrium, when the economy is about to (i) stagnate; (ii) expand; or (iii) contract. Accordingly we distinguish the three cases (i)–(iii). The proofs of cases (i) and (iii) are analogous to the ones in La. 1, since the arguments only depend on the basic non-Giffen-good property, i.e., that the demand curve is downward-sloping. For case (ii), we now check that in an expanding sharing economy, no agent has an incentive to deviate from the equilibrium path in any single period, and $\vartheta' = \alpha(\vartheta_t)$ is such that

$$\begin{aligned}\bar{g}(1, \xi(\vartheta', \vartheta_t), \hat{p}(\vartheta_t, \vartheta_{t-1})|\vartheta') + \delta\bar{g}(\hat{\pi}(\vartheta', \vartheta'), 1, \hat{p}(\vartheta', \vartheta_t)|\vartheta') = \\ \bar{g}(0, \xi(\vartheta', \vartheta_t), \hat{p}(\vartheta_t, \vartheta_{t-1})|\vartheta') + \delta\bar{g}(\hat{\pi}(\vartheta', \vartheta'), 0, \hat{p}(\vartheta', \vartheta_t))|\vartheta').\end{aligned} \tag{37}$$

We establish this by contradiction. Suppose that $\hat{\pi}(\vartheta', \vartheta') = 1$. By Eq. (5), it is $\xi(\vartheta', \vartheta_t) = 1$, so that by Eq. (30):

$$\vartheta' = \frac{p(\vartheta_t)q(\vartheta', \vartheta_t) - c_{01} - (1 + r)c_{10}}{\Delta}. \tag{38}$$

The time-$(t+1)$ type threshold ϑ' can exceed ϑ_t, as long as $\vartheta_t \in [0, \omega)$, where ω solves a stationary version of Eq. (38), corresponding to a fixed-point problem with $q(\omega, \omega) = 1$, so

$$\omega = \max\left\{0, \frac{p(\omega) - c_{01} - (1 + r)c_{10}}{\Delta}\right\} \leq \vartheta^0. \tag{39}$$

By La. 7, there exists a unique $\omega \in \Theta$ that satisfies Eq. (39). Differentiating ϑ' in Eq. (38) with respect to ϑ_t and evaluating at $\vartheta_t = \omega$ yields, together with the demand-elasticity condition (A),

$$\left.\frac{\partial\vartheta'}{\partial\vartheta_t}\right|_{\vartheta_t=\omega} = \left.\frac{p'(\vartheta_t)\vartheta_t + p(\vartheta_t)}{2\Delta\vartheta' + c_{01} + (1 + r)c_{10}}\right|_{\vartheta_t=\omega} = \frac{\omega(\Delta + p'(\omega)) + c_{01} + (1 + r)c_{10}}{2\Delta\vartheta' + c_{01} + (1 + r)c_{10}} \geq 0,$$

since $\varepsilon(\omega) \geq 1$ means that $p'(\omega)\omega + p(\omega) \geq 0$. Hence, for all $\vartheta_t \in [0, \omega)$, the next-period sharing threshold ϑ' is increasing in ϑ_t, and

$$\vartheta_t < \vartheta' < \vartheta'' < \omega.$$

The strict inequalities are a consequence of the uniqueness of the fixed point ω. Thus, by virtue of Eq. (11) it is $\pi(\vartheta', \vartheta') = \mathbf{1}_{\{\vartheta' > \vartheta''\}} = 0$, in direct contradiction to our hypothesis, so that the claim holds for case (ii). □

Proof of Lemma 9. The result is obtained in a manner analogous to the proof of La. 3. We restrict attention to establishing the convexity of $\alpha_0(\vartheta)$ as claimed by La. 2. The demand-elasticity condition (A) implies that

$$\alpha_0'(\vartheta) = \frac{p'(\vartheta)\vartheta + p(\vartheta)}{2\Delta\alpha_0(\vartheta) + rc_{10}} \geq 0, \quad \vartheta \in (0, \vartheta^0). \tag{40}$$

Eq. (40) yields the second derivative,

$$\alpha_0''(\vartheta) = \frac{(p''(\vartheta)\vartheta + 2p'(\vartheta))(2\Delta\alpha_0(\vartheta) + rc_{10}) - 2\Delta\alpha_0'(\vartheta)(p'(\vartheta)\vartheta + p(\vartheta))}{(2\Delta\alpha_0(\vartheta) + rc_{10})^2}, \quad \vartheta \in (0, \vartheta^0).$$

To obtain the convexity of the system function for all $\vartheta \in (0, \vartheta^0)$, it is enough to show that $\hat{p}''(\vartheta)\vartheta \leq -2\hat{p}'(\vartheta))$, which can be accomplished by considering each demand class separately:

- *constant-elasticity demand:* $p''(\vartheta)\vartheta = \left(1 + \frac{1}{\eta}\right)\frac{\gamma}{\eta}\vartheta^{-1/\eta} \leq 2\frac{\gamma}{\eta}\vartheta^{-1/\eta} = -2p'(\vartheta)$, for $\eta \geq 1$ and $\gamma > 0$;
- *semi-logarithmic demand:* $p''(\vartheta)\vartheta = \frac{\gamma_1}{\vartheta} \leq 2\frac{\gamma_1}{\vartheta} = -2p'(\vartheta)$, for $\gamma_0 \geq \gamma_1 > 0$;
- *quasi-affine demand:* $p''(\vartheta)\vartheta = \left(1 - \frac{1}{\eta}\right)\frac{\gamma_1}{\eta}\vartheta^{1/\eta} \leq 2\frac{\gamma_1}{\eta}\vartheta^{1/\eta} = -2p'(\vartheta)$, for $\eta \geq (\gamma_1/\gamma_0)/(1-(\gamma_1/\gamma_0))$ and $\gamma_0 > \gamma_1 > 0$.

Thus, the mapping $\alpha_0(\cdot)$ is strictly concave in each case, which establishes our claim. □

Proof of Lemma 10. Suppose that the utilities u_0, u_1 and the quality γ are measured over the normalized adjustment-interval length $dt = 1$, as in the main text. Assume also that the demand-elasticity parameter $\gamma > 0$ is given. If prices can adjust after the reduced time $dt' = \lambda dt < dt$, where $\lambda \in (0,1)$, the per-period utilities adjust accordingly to λu_1 and λu_0, while the consumers perceive the quality $\gamma' = \lambda\gamma$. Furthermore, the per-period interest rate for the compressed time scale becomes

$$r' = (1+r)^\lambda - 1. \tag{41}$$

As in Prop. 1, the system function for the compressed time scale can be written in the implicit form

$$\hat{\alpha}_0(\vartheta) = \frac{(1-\rho)\lambda p(\vartheta)\vartheta/\hat{\alpha}_0(\vartheta) - r'c_{10}}{\lambda\Delta} = \frac{(1-\rho)p(\vartheta)\vartheta/\hat{\alpha}_0(\vartheta) - rc_{10}}{\Delta} + \left(r - \frac{(1+r)^\lambda - 1}{\lambda}\right)\frac{c_{10}}{\Delta}. \tag{42}$$

The first term on the right-hand side of Eq. (42) is identical to the right-hand side in the implicit definition of the original (not time-compressed) system function $\alpha_0(\cdot)$ in Prop. 1. Using elementary methods, one can show that the second term is nonnegative, decreasing in λ, and increasing in r. Moreover, using implicit differentiation it becomes clear that the value of the time-compressed system function $\hat{\alpha}_0(\vartheta)$ is increasing in the value of this second term, so that

$$\alpha_0(\vartheta) < \hat{\alpha}_0(\vartheta) \quad \text{and} \quad \max\left\{\frac{\partial\hat{\alpha}_0(\vartheta)}{\partial\lambda}, \frac{\partial\hat{\alpha}_0(\vartheta)}{\partial r}\right\} < 0,$$

for all $\vartheta \in (0, \vartheta^0)$. Even in the limit, for $\lambda \to 0^+$, the difference between the compressed and uncompressed system function stays positive (for $r > 0$):

$$\lim_{\lambda\to 0^+}\left[\hat{\alpha}_0(\vartheta) - \alpha_0(\vartheta)\right] = \frac{(1-\rho)p(\vartheta)\vartheta}{\Delta}\left(\frac{1}{\hat{\alpha}_0(\vartheta)} - \frac{1}{\alpha_0(\vartheta)}\right) + (r - \ln(1+r))\frac{c_{10}}{\Delta} > 0.$$

Because the first term on the right-hand side is negative, the second term is an upper bound for the deviation between the compressed and uncompressed system functions, i.e.,

$$0 < \hat{\alpha}_0(\vartheta) - \alpha_0(\vartheta) < (r - \ln(1+r))\frac{c_{10}}{\Delta},$$

for any time-compression factor $\lambda \in (0, 1)$. Similarly, the time-compressed lower invariance threshold $\hat{\vartheta}^0$ can be computed using Eq. (14),

$$\hat{\vartheta}^0 = \frac{\lambda\hat{\gamma} - r'c_{10}}{\lambda\hat{\gamma} + \lambda\Delta} = \vartheta^0 + \left(r - \frac{(1+r)^\lambda - 1}{\lambda}\right)\frac{c_{10}}{\hat{\gamma} + \Delta} > \vartheta^0,$$

for all $\lambda \in (0, 1)$. Using the same limit-argument as before, we can therefore conclude that

$$0 < \hat{\vartheta}^0 - \vartheta^0 < (r - \ln(1+r))\,\frac{c_{10}}{\hat{\gamma} + \Delta},$$

for all $\lambda \in (0, 1)$. This completes the proof. □

Proof of Lemma 11. To establish the claim, it is sufficient to show that Las. 1 and 2 hold for a generic affine demand function of the form $p(\vartheta) = \gamma_0 - \gamma_1\vartheta$, where $\gamma_0 = (1-\rho)\gamma - \kappa$ and $\gamma_1 = (1-\rho)\gamma$. By replacing ω with

$$\hat{\omega} = \min\left\{1, \frac{\gamma_0 - c_{01} - (1+r)c_{10}}{\gamma_1 + \Delta}\right\},$$

La. 1 obtains, since $\hat{\omega} \le \gamma_0/(\gamma_1 + \Delta) \le 1/2$, and

$$\frac{\partial\vartheta'}{\partial\vartheta_t} = \frac{\gamma_0 - 2\gamma_1\vartheta_t}{2\Delta\vartheta' + c_{01} + (1+r)c_{10}} > 0,$$

for all $\vartheta_t \in [0, \hat{\omega})$. The rest of the proof of La. 1 remains essentially unchanged. Furthermore, parts (i) and (ii) of La. 2 can be proved as before, with the condition $\hat{\gamma} < rc_{10}$ replaced by the condition $\gamma_0 < rc_{10}$ in part (ii) of the proof. We can therefore restrict attention to showing that $\alpha''(\vartheta) < 0 < \alpha'(\vartheta)$, as claimed in part (iii) of La. 2. Since $\vartheta^0 \le \gamma_0/(\gamma_1 + \Delta)$,

$$\alpha_0'(\vartheta) = \frac{\gamma_0 - 2\gamma_1\vartheta}{2\Delta\alpha_0(\vartheta) + rc_{10}} > 0, \tag{43}$$

for all $\vartheta \in [0, \vartheta^0)$. We now establish the convexity of the system function. Differentiating Eq. (43) yields

$$\alpha_0''(\vartheta) = -\frac{2\gamma_1(2\Delta\alpha_0(\vartheta) + rc_{10}) + 2\Delta(\gamma_0 - 2\gamma_1\vartheta)\alpha_0'(\vartheta)}{(2\Delta\alpha_0(\vartheta) + rc_{10})^2} < 0,$$

for all $\vartheta \in [0, \vartheta^0)$, as claimed. The generalized versions of La. 1 and La. 2 together imply La. 11. □

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Lemma 12. *The time-t price elasticity of demand ε_t is (weakly) decreasing in $\hat{\gamma}$ on the equilibrium path.*

Proof of Lemma 12. Consider the interesting case where $\vartheta_0 > 0$. By footnote 14 and Eq. (6), the price elasticity of demand at time t is $\varepsilon_t = (1 - n_t)/n_t = (1 - \vartheta_t)/\vartheta_t$, provided $\vartheta_t > 0$. Differentiating ε_t with respect to the demand-elasticity parameter $\hat{\gamma}$ yields

$$\frac{\partial\varepsilon_t}{\partial\hat{\gamma}} = -\frac{1}{\vartheta_t^2}\left(\frac{\partial\vartheta_t}{\partial\vartheta_{t-1}} \cdot \frac{\partial\vartheta_{t-1}}{\partial\vartheta_{t-2}} \cdot \ldots \cdot \frac{\partial\vartheta_1}{\partial\hat{\gamma}}\right). \tag{44}$$

If $\vartheta_0 \in \mathcal{R}$, by Prop. 4 the sharing economy becomes stationary, independent of the demand-elasticity parameter $\hat{\gamma}$, i.e., $\partial\varepsilon_t/\partial\hat{\gamma} = 0$. For all $\vartheta_0 \in (0, \vartheta^0)$, on the equilibrium path given by Prop. 1, it is

$$\frac{\partial\vartheta_1}{\partial\hat{\gamma}} = \frac{\partial}{\partial\hat{\gamma}}\left(\frac{p(\vartheta_0)\cdot\vartheta_1/\vartheta_0 - rc_{10}}{\Delta}\right) = \frac{(1-\vartheta_0)\cdot\vartheta_1/\vartheta_0}{\Delta + \hat{\gamma}(1-\vartheta_0)} > 0, \tag{45}$$

which, together with La. 2, implies that $\partial\varepsilon_t/\partial\hat{\gamma} < 0$. Similarly, for all $\vartheta_0 \in (\vartheta^1, 1)$ we obtain

$$\frac{\partial\vartheta_1}{\partial\hat{\gamma}} = \frac{1-\vartheta_0}{\Delta} > 0. \tag{46}$$

If $\vartheta_0 \in (\vartheta^1, \vartheta^2]$, then $\partial\vartheta_t/\partial\vartheta_{t-1} = 1$ for all $t \geq 1$. Hence, $\partial\varepsilon_t/\partial\hat{\gamma} = \partial\vartheta_1/\partial\hat{\gamma} < 0$. On the other hand, for $\vartheta_0 \in (\vartheta^2, 1]$, La. 2 implies that $\partial\vartheta_t/\partial\vartheta_{t-1} > 0$, and the right-hand side of Eq. (44) becomes negative, thus completing the proof. $\square$

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Appendix C: Notation

Symbol	*Description*	*Domain/Definition*
a	Agent's adjustment decision	$\mathcal{A} = \{0, 1\}$
c_{01}	Conversion cost from "sharing" to "keeping"	$\mathbb{R}_+$
c_{10}	Conversion cost from "keeping" to "sharing"	$\mathbb{R}_+$
$F(\cdot)$	Cumulative distribution function for types $\theta \in \Theta$	$F : \Theta \to [0, 1]$
$\bar{g}(\cdot)$	Agent's current-period expected net payoff	$\bar{g} : \mathcal{A} \times \mathcal{X} \times \mathbb{R}_+ \to \mathbb{R}$
n	Available supply of the shared item	$[0, 1]$
$\bar{n}$	Steady-state supply of the shared item	$[0, 1]$
$p(\cdot)$	Price of the shared item as a function of the available supply	$p : [0, 1] \to [0, \hat{\gamma}]$
$\hat{p}^0$	Critical (effective) sharing price	$[0, \hat{\gamma}]$
$\hat{p}^1$	Critical (effective) keeping price	$[0, \hat{\gamma}]$
$\hat{p}$	Effective transaction price	$\hat{p} = p \cdot q$
$\bar{p}$	Steady-state price	$[0, \hat{\gamma}]$
$\mathcal{P}(\theta)$	Agent θ's inaction region in the price space	$[\hat{p}^1(\theta), \hat{p}^0(\theta)]$
q	Transaction probability	$[0, 1]$
r	Per-period interest (or discount) rate	$\mathbb{R}_+$
$\mathcal{R}$	Invariance region	$[\vartheta^0, \vartheta^1]$
s	Agent's need state	$\mathcal{S} = \{L, H\}$
t	Time	$\mathbb{N}$
$\bar{t}$	Type-dependent sharing-switch time	$\mathbb{N} \cup \{\infty\}$
t^μ	Maximum-diffusion time	$\mathbb{N}$
$u(\cdot)$	Utility as a function of the agent's need state and his sharing state	$u : \mathcal{S} \times \mathcal{X} \to \mathbb{R}$
u_0	Agent's (*dis*)utility from not having the item at disposal when needed	$\mathbb{R}_-$
u_1	Agent's utility from having the item at disposal when needed	$\mathbb{R}_+$
$V(\cdot\|\theta)$	Type-θ agent's value function	$V : \mathcal{X} \times \Theta^2 \to \mathbb{R}$
$\hat{V}(\cdot)$	Agent's transformed value function	$\hat{V} : \Theta^3 \to \mathbb{R}$
$\bar{V}^0(\theta\|\bar{\vartheta})$	Terminal payoff of a type-θ agent when sharing, at the steady-state $\bar{\vartheta}$	$\mathbb{R}$
$\bar{V}^1(\theta\|\bar{\vartheta})$	Terminal payoff of a type-θ agent when keeping, at the steady-state $\bar{\vartheta}$	$\mathbb{R}$
x	Sharing state	$\mathcal{X} = \{0, 1\}$
z	Excess demand	$[0, 1]$
$\alpha(\cdot)$	Law of motion of the sharing threshold (system function)	$\alpha : \Theta \to \Theta$
$\alpha_0(\cdot)$	Law of motion of the sharing threshold below the invariance region	$\alpha_0 : [0, \vartheta^0] \to [0, \vartheta^0]$
$\alpha_1(\cdot)$	Law of motion of the sharing threshold above the invariance region	$\alpha_1 : [\vartheta^1, 1] \to [0, \vartheta^1]$
γ	Perceived quality by renters / demand-elasticity parameter	$\mathbb{R}_{++}$
$\hat{\gamma}$	Maximal sharing payoff / intermediated demand-elasticity parameter	$\mathbb{R}_{++}$
δ	Discount factor	$(0, 1)$
Δ	Utility gain for use of shared item when needed, $u_1 - u_0$	$[\hat{\gamma}, \infty)$
ε	Price elasticity of demand for the shared item	$\mathbb{R}_+$
ϑ_t	Sharing threshold at time t	Θ
ϑ^0 / ϑ^1	Lower / upper invariance threshold	Θ
ϑ^2 / ϑ^μ	Rest / maximum-diffusion threshold	Θ
$\bar{\vartheta}$	Steady-state sharing threshold	Θ
θ	Agent's type (probability of being in high-need state)	Θ
Θ	Type space for θ	$[0, 1]$
κ	Per-period servicing cost	$\mathbb{R}_+$
λ	Compression factor for price-adjustment time scale	$(0, 1]$
$\pi(\cdot\|\theta)$	Agent θ's optimal policy as a function of own sharing state and sharing threshold	$\pi : \mathcal{X} \times \Theta \to \mathcal{A}$
$\hat{\pi}(\cdot)$	Agent's transformed optimal policy as a function of type and sharing threshold	$\hat{\pi} : \Theta^2 \to \mathcal{A}$
ρ	Sharing intermediary's commission rate	$[0, 1]$
$\xi(\cdot)$	Agent's sharing state as a function of type and sharing threshold	$\xi : \Theta^2 \to \mathcal{X}$
$\bar{\xi}(\cdot)$	Stationary sharing-state distribution	$\bar{\xi} : \Theta \to \mathcal{X}$
$\tau(\cdot)$	Time of convergence as a function of the initial condition	$\tau : \Theta \to \mathbb{N}$
$\varphi(\cdot)$	Mapping from the initial state of the economy to its limit	$\varphi : \Theta \to \Theta$

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