Incentives for Selling Two-Sided Markets

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Abstract

Two-sided market platforms, like all businesses, require active selling. Platforms, however, create value differently than traditional products thanks to direct and indirect network effects. How should managers account for network effects in incentive compensation plans offered to agents tasked to mobilize network participants? Should performance based incentives increase or decrease as network effects vary? Which metrics should be used to incentivize agents and what are the implications for profit? We formally investigate these strategic managerial questions by proposing an agency model of platform sales. We find that the agent and the firm respond differently to network effects, because network effects not only enhance the effectiveness of selling efforts, but also increase sales uncertainty. As a result, while network effects always increase the agent’s effort, they can have either a positive impact, a negative impact or no impact on sales commissions depending on the structure of the compensation plan. Moreover, we discover that network effects can erode profit. Why? Because network effects create externalities that cannot be fully internalized by traditional one-dimensional compensation plans. Firms should instead use novel multi-dimensional plans to incentivize agents to fully internalized all network externalities and ultimately restore the beneficial impacts of network effects on profits.
1 Introduction

Platform businesses create value in a fundamentally different way than traditional businesses, thanks to network effects. The value can arise from direct network effects, through interactions between same-side participants (e.g., Facebook users enjoying connecting with their friends); or from indirect network effects, through interactions between participants on opposite sides (e.g., OpenTable diners get value when they book affiliated restaurants, and restaurants derive value from outreach to potential diners). Driven by technologies that support the creation and exchange of value among market participants, platform businesses have disrupted diverse industries such as transportation (e.g., Uber), advertising technology (e.g., Google), media (e.g., Twitter), health care (e.g., American Well), social networking (e.g., Facebook), retail (e.g., Amazon), banking (e.g., CreditKarma) and even education (e.g., Coursera). Firms such as Apple, Google, Facebook and Microsoft for instance surpass traditional companies like General Motors, the Coca-Cola Company and General Electric, not only in terms of brand value, but also in terms of shareholder value.

This paper focuses on managerial considerations related to securing participation on the platform’s network. The participant network constitutes the primary driver of value creation for a platform, and failure to mobilize it can doom success (Evans and Schmalensee, 2010). While network effects fuel growth and sustenance of two-sided markets platforms, the extant literature on platforms and two-sided markets has overlooked that in practice network mobilization requires active selling. OpenTable employs sales people for the non-trivial task of persuading restaurants to adopt the platform. Kyruus, which provides coordination technology to multi-point health systems, hires sales staff to sign up provider organizations, a challenge amplified by barriers to adoption of information technologies in health care. Credit Karma hires sales staff to acquire financial provider firms, rounding out its business objective of serving customers who seek financial products. American Well, an online platform connecting physicians with patients, employs sales agents to reach out to health insurance companies that contract with these physicians. Twitter, like other advertising oriented platforms, employs advertising sales agents to sell advertising space to advertisers.
Selling platforms, however, differs from selling traditional products because network effects distort the agency relationship between the principal and the agent. As a result, insights from the salesforce compensation literature should be revisited. Meanwhile, the literature on platforms does not illuminate the design of incentive plans for agents tasked to mobilize network participants. While this literature covers a rich set of issues such as pricing strategies (Liu and Chintagunta, 2009), product design (Bakos and Katsamakas, 2008), product launch (Lee and O’Connor, 2003), seeding strategies (Dou et al., 2013), compatibility and competition (Farrell and Klemperer, 2007), competition across platforms (Rochet and Tirole, 2003; Chakravorti and Roson, 2006), competition between incumbents and entrants (Katz and Shapiro, 1992; Eisenmann et al., 2011), segmentation (Bhargava and Choudhary, 2004; Jing, 2007), timing of product introduction (Bhargava et al., 2013), and business model design (Parker and Van Alstyne, 2005; Nagi, 2007), the active role of sales people in selling platforms is conspicuously absent.

Given the unobservability of selling efforts by agents, managing a sales force requires to design compensation plans that optimally incentivize efforts. This fundamental issue has been well studied in the salesforce compensation literature (Basu et al., 1985; Coughlan and Sen, 1989; Lal and Srinivasan, 1993; Joseph and Kalwani, 1995; Joseph and Thevaranjan, 1998; Krishnamoorthy et al., 2005; Steenburgh, 2008; Albers and Mantrala, 2008; Mantrala et al., 2010; Misra and Nair, 2011; Jain, 2012; Coughlan and Joseph, 2012; Rubel and Prasad, 2016). This literature, however, is focused on traditional goods without network effects, and does not inform how network effects should enter incentive compensation plans. Therefore, this paper investigates how managers should optimally incentivize sales agents when network effects drive the value of the product. In doing so, we examine not only how the design parameters of the compensation plan must be altered to account for network effects, but whether and how network effects create a need for new compensation designs.

Consequently, we explore a series of novel questions for which initial intuition seems insufficient, i.e., (1) Would the agent inherently work less when network effects are strong because the product is easier to sell, or harder because then each sale has a multiplier effect on net-
work growth? (2) Should network effects cause the firm to decrease commission rates (if the agent works harder at any rate level) or increase commission rates to further leverage network effects? (3) Should the performance metrics that govern incentive compensation plans be the traditional one—sales on the market side that the agent works to cultivate—or, because of cross-market effects, sales on the side the agent has no responsibility for? (4) Would network effects necessarily increase the firm’s profit under optimal designs, or could stronger network effects lead to lower profits? And (5) if the firm needs multiple agents to cover independent territories, should their commissions be dependent on own or each other’s performance?

To shed light on these managerial questions, we propose a principal-agent model of platform sales that takes into account network effects, both direct and cross-sided. The proposed model reveals that the agency relationship arising in the case of platforms differs from the agency relationship that arises in the case of traditional goods. Why? Because contrary to traditional goods, network effects make selling effectiveness and sales uncertainty correlated and generate multiple externalities. An important theoretical contribution of this paper is to classify these externalities into two types—within a given market and between different markets—and to employ this classification as a vital ingredient in designing and understanding optimal compensation plans.

Within-market externalities refer to externalities generated by network effects within the same market, e.g., effects of buyers on buyers in a given market. Between-market externalities capture externalities across two different groups of participants (buyers and sellers for instance). In the current context, within-market externalities explore effects in the market that the agent is actively responsible for. Between-market externalities, on the other hand, capture not only cross-market externalities, but also externalities that exist between distinct sales territories after the firm partitioned a given side into distinct sales territories handed off to two sales agents. An important insight from this classification is that within-market externalities can be internalized with traditional one-sided compensation plans and therefore the firm always earns higher profit from stronger network effects, whereas between-market externalities cannot be internalized with traditional compensation design. Consequently, stronger
network effects can erode firms’ profits when indirect network externalities are managed by standard one-sided compensation plans. More precisely, we show that new, multidimensional, compensation designs are needed to effectively manage risk sharing between the contracting parties, requiring the principal to compensate the agent based not only on the market that she is tasked to mobilize, but also on those that she is not.

Thus, we make novel contributions that provide a better understanding and guiding principles for the design of salesforce compensation plans for platform businesses. First, we formally establish how network effects should be accounted for in incentive compensation plans, a result absent from the extant platform and salesforce compensation literatures. Second, we articulate how the externalities created by network effects distort the agency relationship between the platform firm and its sales agents, and thereby uncover a variety of different influences on compensation plan design depending on the nature of network effects and the resulting externalities. Third, we find, surprisingly, that network effects can erode firms’ profits when the agent’s plan does not fully internalize all the externalities generated by network effects. Finally, and crucially from a managerial perspective, we construct new compensation plan designs that correct this negative impacts by better internalizing all network externalities.

The remainder of the paper evolves as follows. In Section 2, we derive platform demands, detail the sequence of the game, the agent’s problem and the manager’s optimization program. In Section 3, we present the equilibrium strategies and profit under one-sided compensation plans, which can lead to lower profits due to externalities that are not fully internalized. In Section 4, we detail when to internalize these externalities generated by network effects with multi-sided compensation plans that restore optimal risk sharing between the contracting parties. Finally in Section 5, we conclude.

2 Model

We consider a platform marketplace characterized by direct network effects and cross market (or indirect) network effects. The firm creates the infrastructure and business rules that enable
interactions between the two sides, which we label as $b$ (buyers) and $s$ (sellers). One crucial insight from the economics of platforms is that often the optimal strategy for the platform is to subsidize one side of the market while monetizing the other (Parker and Van Alstyne, 2005; Eisenmann et al., 2006). These are labeled the “subsidy” (or “free”) and the “paying” sides. Commonly, the subsidy side corresponds to buyers, while the paying side corresponds to sellers.

A platform creates value to users via a combination of stand-alone features, direct network effects and cross-network effects. Therefore, sales on a given side of the platform depend on installed bases on both sides, besides pricing and product features on that side. As noted earlier, existing literature on platforms has explored and developed insights for various relationships among these considerations. The novel consideration this paper brings into platform research is the need and role of hired sales agents in the active mobilization of network participants.

In our model, the platform firm relies on an agent to mobilize one side of the platform. In practice, this is usually the paying side of the platform (the sellers), whereas network growth on the non-paying side is primarily achieved organically by word-of-mouth as well as the inherent value (both stand-alone benefits, direct network effects and cross-network benefits) that customers on this side receive from participation. An example is CreditKarma, which provides consumers with a free credit report and earns revenue by directing them to firms that sell financial products, captures consumer-users through word-of-mouth and online advertising, and employs an in-house sales team for signing up financial service providers. Other prototypical examples to illustrate this idea are the aforementioned firms, e.g., OpenTable, Facebook and LinkedIn, which deploy sales agents to recruit members on the paying sides, i.e., restaurants, advertisers, recruiters, respectively. Similarly, advertising platforms task advertising selling agents to sell advertising space to advertisers and not for growing eyeballs.

The model encapsulates several decisions and outcomes, i.e., (a) the design of the agent’s compensation contract negotiated between the agent and the platform, (b) the optimal level

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1 Some platforms, especially ones that have business entities on multiple sides, may require active selling on both sides. This direction is not covered in the present paper, but presents a useful possibility for future research.
of effort exerted by the agent and (c) the network size on both sides of the platform, that is, network participants’ choice of joining the platform. The first step (a) is performed under uncertainty regarding network participation, while steps (b) and (c) are conducted across a period of time so that the equilibrium outcomes will be determined through rational fulfilled expectations (Katz and Shapiro, 1985). Intuitively, the agent’s behavior and optimal effort will be influenced by contract parameters and anticipation of network size; network size will depend on the agent’s effort, and hence on the optimal contract, as well as on various product and market parameters. Consistent with the contract literature, contract’s parameters are chosen to optimize the firm’s profit subject to the agent’s rationality and incentive compatibility constraints.

2.1 Influences on Market Formation

We start with the standard model in the literature for market formation under network effects. Micro-foundations for the model presented below are presented in the appendix. Let $Q_b$ and $Q_s$ represent sales on the buyer and seller sides, respectively. These are affected by stand-alone benefits ($V_b$ and $V_s$ respectively), the direct network benefits consumers anticipate to obtain from the platform (i.e., $\gamma_b Q^a_b$ and $\gamma_s Q^a_s$, where $\gamma_b$ and $\gamma_s$ reflect the intensity of direct network effects, while $Q^a_s$ and $Q^a_b$ represent market participants’ anticipation about mobilization), and finally the cross-network benefits ($\eta_b Q^a_s$ and $\eta_s Q^a_b$, where $\eta_b$ and $\eta_s$ reflect the intensity of cross-network effects). Other influences on $Q_b$ and $Q_s$ are encapsulated in error terms $\epsilon_b$ and $\epsilon_s$, respectively, which are unknown at the time of contracting, and which we assume to be normally distributed (with mean 0 and variance $\sigma^2_b$ and $\sigma^2_s$, respectively). Specifically, the sales agent exerts positive influence on sales commensurate with the level of effort exerted, $w$. Mathematically, these influences are captured by the following model of sales on both markets covered by the platform, i.e.,

\[
Q_b = V_b + \gamma_b Q^a_b + \eta_b Q^a_s + \epsilon_b \quad (1a)
\]
\[
Q_s = V_s + \gamma_s Q^a_s + \eta_s Q^a_b + w + \epsilon_s, \quad (1b)
\]
which are graphically depicted in Fig. 1 below.

![Figure 1: Platform Sales](image)

Next, following the extant platform research, we assume that market participants form rational expectations, which means that participants’ anticipations about network size are correct. As is standard in the literature (Katz and Shapiro, 1985), the term expectation in the expression “rational expectations” means that $Q_b = Q_b^a = q_b$ and $Q_s = Q_s^a = q_s$, which is different from the expectation operator $\mathbb{E}$ in statistics. As a result, we obtain that at the time of contracting the equilibrium levels $q_s$ and $q_b$ on the two sides are defined as

$$q_b = \frac{V_b(1 - \gamma_s) + V_s \eta_b + w \eta_b}{(1 - \gamma_b)(1 - \gamma_s) - \eta_b \eta_s} + \frac{\epsilon_b((1 - \gamma_b)\gamma_s + \eta_b \eta_s) + \epsilon_s \eta_b}{(1 - \gamma_b)(1 - \gamma_s) - \eta_b \eta_s}$$

$$q_s = \frac{V_b \eta_s + (1 - \gamma_b)(V_s + w)}{(1 - \gamma_b)(1 - \gamma_s) - \eta_b \eta_s} + \frac{\epsilon_b \eta_s + \epsilon_s((1 - \gamma_b)\gamma_s + \eta_b \eta_s)}{(1 - \gamma_b)(1 - \gamma_s) - \eta_b \eta_s},$$

both of which incorporate uncertainty and depend on product parameters, as well as the agent’s effort level. Note that (2) comport with the extant economics of platforms and two-sided markets literature. Moreover, when network effects are set to zero, i.e., $\gamma_s = \gamma_b =$
\( \eta_s = \eta_b = 0 \), demands (2) provide the classical sales response functions used in numerous studies that investigate sales force compensation and moral hazard (see, e.g., Holmstrom and Milgrom 1987; Hauser et al. 1994; Kalra et al. 2003; Mishra and Prasad 2005; Krishnamoorthy et al. 2005; Chung et al. 2014; Syam et al. 2016). The manager’s problem is then to find the compensation plan, i.e., \( \omega(q_s) \), that incentivizes the agent to work optimally.

### 2.2 Contract Design between the Platform and the Agent

Following the extant sales force literature, we consider the linear one-dimensional compensation contract, whereby the agent is incentivized on sales

\[
\omega(q_s) = \alpha_0 + \alpha_1 q_s
\]

where \( \alpha_0 \) is the agent’s fixed salary and \( \alpha_1 \) is the commission on sales. Linear contracts received much attention because of their robustness (see, e.g., Holmstrom and Milgrom (1987)) and simplicity in practice. Similarly, we follow the extant literature and model the agent’s utility as

\[
U(\omega(q_s), w) = -e^{-\rho(\omega(q_s) - C(w))},
\]

where \( \rho \) is the agent’s risk aversion coefficient and \( C(w) \) is the cost of effort (with \( C'(w) > 0 \) and \( C''(w) > 0 \)). Based on our discussions with sales leaders at various platform companies, the profile of sales agents in such companies does not differ from the profile of sales agents in more traditional industries. Thus, we follow the extant analytical (see, e.g., Syam et al. 2016), empirical (see, e.g., Misra and Nair 2011; Chung et al. 2014) and experimental (see, e.g., Chen and Lim 2017) literatures on salesforce incentives and consider the specific convex function \( C(w) = \frac{w^2}{2} \).

The firm’s decision of what compensation contract to offer the agent requires specification of the agent’s effort and participation strategy. Given the contract \( \omega(q_s) = \alpha_0 + \alpha_1 q_s \), the agent’s optimal level of effort is determined by maximizing the certainty equivalent of her
utility function, i.e.,

$$w^* = \arg \max_w \underbrace{E[\omega(q_s)] - \frac{1}{2} Var[\omega(q_s)] - C(w)}_{U_{CE}}, \quad (5)$$

The optimal effort $w^*$ forms the incentive compatibility constraint in the firm’s compensation design problem. Furthermore, the agent’s participation or individual rationality constraint is that the agent receives non-negative net utility in expectation (as we normalized the value of the agent’s outside option to zero without loss of generality). Subject to the agent’s decisions (i.e., agent’s effort strategy and participation), the principal determines the contract parameters $\alpha_0$ and $\alpha_1$ that maximize the expected value of the firm’s profit, i.e., $\Pi = mE[q_s] - E[\omega(q_s)]$, where $m$ is the per-unit margin on the paying side, normalized to 1 without loss of generality.

The market outcomes are determined as a function of the agent’s effort, the contract parameters, product characteristics, and the realized values of the random variables, which become available as time unfolds. Hence, consistent with the salesforce literature (see, e.g., Caldieraro and Coughlan (2009)), the sequence of the game is specified below.

- Stage 1, the principal offers the agent a linear contract, composed of a fixed salary and a commission rate, i.e., $\alpha_0$ and $\alpha_1$, respectively.
- Stage 2, the agent accepts or rejects the offer.
- Stage 3, the agent exerts effort, i.e., $w$.
- Stage 4, market participants mobilize and payments are made.

As noted earlier, there is market uncertainty when the principal and the agent agree on the compensation contract, and this uncertainty resolves over the time period during which the agent mobilizes the network and participants make decisions. Naturally, therefore, the compensation parameters are chosen with respect to expectations about outcomes in the later stages. Moreover, the game sequence implies that the contract parameters influence mobilization through the agent’s decisions to accept the contract (or not) and to work hard (or not).
3 Standard Compensation Plan

This section explores how network effects should influence the optimal configuration of the classic linear compensation contract $\omega(q_s) = \alpha_0 + \alpha_1 q_s$ with a fixed salary $\alpha_0$ and a commission rate $\alpha_1$ based on network mobilization on the market’s side that the agent is hired to mobilize. We develop results and insights regarding which kinds of network effects will cause changes in configuration of the compensation. Besides specifying payments, the optimal compensation contract characterizes how risks and rewards from network mobilization are shared between the two contracting parties.

3.1 Optimal Strategies

Given the plan structure, we examine the following questions, i.e., (i) would the agent work less or more due to network effects (given the same commission rate incentive)? (ii) what influence does each type of network effect exert, if any, on the level of incentive the firm offers to the agent? and (iii) given the agent’s actions and the optimal contract specified by the firm, is the platform always able to positively leverage stronger network effects?

For the first question, consider the motivation of the agent. The agent chooses the level of effort $w$ based on the risk and return of the compensation contract, i.e., $\text{Var}[\omega(q_s(w))] = \alpha_1^2 \text{Var}[q_s]$ and $\mathbb{E}[\omega(q_s(w))] = \alpha_0 + \alpha_1 \mathbb{E}(q_s)$ in (5), respectively. Intuitively, both direct and cross-market network effects enhance the agent’s selling effectiveness, i.e., number of sellers mobilized for some level of effort. This not only provides a direct financial reward to the agent, but also an indirect one due to the feedback loop of sellers on buyers and back to sellers, thus enhancing the value of a marginal seller for the agent. More financial rewards implies that, as the intensity of network effect increases, there is an increase in the effort level at which the marginal benefit (commission) equals the agent’s marginal costs. Therefore, at any level of commission $\alpha_1$, the agent should work more as any network effect parameter increases, i.e., $\frac{\partial \omega^*}{\partial \gamma_b} > 0$, $\frac{\partial \omega^*}{\partial \gamma_s} > 0$, $\frac{\partial \omega^*}{\partial \eta_b} > 0$ and $\frac{\partial \omega^*}{\partial \eta_s} > 0$. This intuitive result can be verified by computing the
agent’s optimal effort level,

$$w^* = \frac{(1 - \gamma_b) \alpha_1}{(1 - \gamma_b)(1 - \gamma_s) - \eta_b \eta_s}.$$  \hspace{1cm} (6)

Based on the agent’s effort strategy, how should the firm set the agent’s commission rate as network effects vary? The firm sets the contract parameters to maximize its expected profit, balancing the compensation cost against the reward from the agent’s effort. Formally, it picks $\alpha_1^* = \arg \max_{\alpha_1} E[\Pi]$, subject to the agent’s IC and IR conditions, i.e., $w = w^*$ and $U_{CE}(w^*) \geq 0$, where $E[\Pi] = (1 - \alpha_1)E[q_s] - \alpha_0$. From the firm’s perspective, the agent’s selling effort has a spillover effect on sales growth, beyond the gain that would accrue for a traditional good, because of the feedback loop of sellers on buyers and back to sellers, thus enhancing the value of a marginal seller for the firm. As a result, every unit of commission offered to the agent not only generates extra margin from sellers but has a multiplier effect on overall profit. This intuition would suggest that an increase in the intensity of network effects should motivate the firm to increase the agent’s commission rate $\alpha_1$. Our formal analysis, however, contradicts this intuition.

**Proposition 1** The optimal commission rate

$$\alpha_1^* = \frac{(1 - \gamma_b)^2}{(1 - \gamma_b)^2 (1 + \rho \sigma_s^2) + \rho \eta_s^2 \sigma_b^2},$$  \hspace{1cm} (7)

is independent of $\eta_b$ and $\gamma_s$ and inversely related to $\gamma_b$ and $\eta_s$.

Proposition 1 demonstrates that the agent and the principal respond differently to network effects and uncovers in particular that the firm does not respond to certain types of network effects since $\frac{\partial \alpha_1^*}{\partial \gamma_s} = \frac{\partial \alpha_1^*}{\partial \eta_b} = 0$. More surprisingly, the firm decreases the agent’s commission rate when the intensity of certain network effects increases, since $\frac{\partial \alpha_1^*}{\partial \gamma_b} < 0$, $\frac{\partial \alpha_1^*}{\partial \eta_s} < 0$, despite the fact that all types of network effects increase the agent’s productivity and effort. Note that in the case of standard goods, i.e., goods without network effects, the extant sales force literature recommends to increase the agent’s commission rate as her effectiveness increases. We find the opposite because in the case of platforms, network effects not only increase the agent’s
selling effectiveness, but also increase sales uncertainty as

\[ \text{Var}[q_s] = \frac{(\eta_s \sigma_b)^2 + (1 - \gamma_b)^2 \sigma_s^2}{((1 - \gamma_b)(1 - \gamma_s) + \eta_b \eta_s)^2}, \tag{8} \]

which implies that \( \frac{\partial \text{Var}[q_s]}{\partial \eta_s} > 0, \frac{\partial \text{Var}[q_s]}{\partial \eta_b} > 0, \frac{\partial \text{Var}[q_s]}{\partial \gamma_s} > 0 \) and \( \frac{\partial \text{Var}[q_s]}{\partial \gamma_b} > 0 \). Thus, contrary to traditional goods where selling effectiveness and sales uncertainty are independent from each other, network effects make the agent’s effectiveness and sales uncertainty “correlated”.

This interdependence between selling effectiveness and sales uncertainty creates a tension since the manager cannot fully capitalize on the fact that the agent’s selling efforts are more effective due to network effects. Increasing the commission rate in response to stronger network effects would increase the agent’s compensation risk in two ways instead of only one way in the case of standard goods. Specifically, the compensation risk imposes a cost on the agent, i.e.,

\[ \frac{\rho}{2} \text{Var}[\omega(q_s)] = \frac{\rho}{2} \alpha_1^2 \text{Var}[q_s] \tag{9} \]

in the certainty equivalent of her utility function. Thus, when network effects increase, so does \( \text{Var}[q_s] \). Then, if the manager were to respond to the agent’s higher selling effectiveness by increasing \( \alpha_1 \) (as would be recommended in the case of a traditional good), this would unnecessarily amplify \( \frac{\rho}{2} \alpha_1^2 \text{Var}[q_s] \), i.e., by increasing both \( \alpha_1 \) and \( \text{Var}[q_s] \), instead of just increasing \( \alpha_1 \) in the case of a standard good. In equilibrium, the manager’s optimal response is then to not make \( \alpha_1^* \) increase in network effects, and actually to reduce \( \alpha_1^* \) as \( \gamma_b \) and \( \eta_s \) increase.

To further understand why only \( \gamma_b \) and \( \eta_s \) enter the optimal commission rate, we present \( \alpha_1^* \) for two polar cases to disentangle the externalities generated by direct network effects from the externalities generated by indirect network effects. The optimal commission rates under the two extreme cases are

\[ \alpha_1^*|_{\eta_b=\eta_s=0} = \frac{1}{1 + \rho \sigma_s^2} \quad \text{and} \quad \alpha_1^*|_{\gamma_b=\gamma_s=0} = \frac{1}{1 + \rho \left( \sigma_s^2 + \eta_s^2 \sigma_b^2 \right)}. \]

\[ \text{direct network effects only} \quad \text{and} \quad \text{indirect network effects only} \]
To clarify why network effects do not enter the optimal commission rate in the first case, but do in the second case, consider the situation of a standard product without network effect, with sales equation \( q = V + \beta \times w + \epsilon \), where \( \beta \) is the agent’s selling effectiveness and \( \epsilon \) are Normally distributed demand shocks with zero mean and variance \( \sigma^2 \). In such a case, the agent’s optimal effort strategy is \( w^* = \beta \alpha_1 \) and the firm’s optimal commission rate is \( \alpha_1^* = \frac{\beta^2}{\beta^2 + \rho \sigma^2} \).

Using the canonical formula \( \alpha_1^* = \frac{\beta^2}{\beta^2 + \rho \sigma^2} \), we first replace \( \beta \) and \( \sigma^2 \) by the agent’s enhanced selling effectiveness and sales uncertainty in the case of platforms (as obtained from the sales equations) when \( \eta_s = \eta_b = 0 \), i.e., \( 1/(1-\gamma_s) \) and \( \sigma^2_s/(1-\gamma_s)^2 \) respectively, and obtain that

\[
\alpha_1^* = \frac{\beta^2}{\beta^2 + \rho \sigma^2_s} \Rightarrow \alpha_1^* = \frac{\left( \frac{1}{1-\gamma_s} \right)^2}{\left( \frac{1}{1-\gamma_s} \right)^2 + \rho \frac{\sigma^2_s}{(1-\gamma_s)^2}} \Rightarrow \alpha_1^* = \frac{1}{1 + \rho \sigma^2_s}.
\] (10)

The optimal commission rate in this case is identical to \( \alpha_1^* = \frac{\beta^2}{\beta^2 + \rho \sigma^2_s} \) with \( \beta = 1 \) as normalized earlier. The independence of \( \alpha_1^* \) with respect to network effects in this case comes from the fact that the externalities (i.e., enhanced selling effectiveness and enhanced sales uncertainty) generated by network effects on the agency relationship occur within the same market, i.e., sellers on sellers in this case.

Conversely, when only \( \gamma_s = \gamma_b = 0 \), the agent’s enhanced selling effectiveness and sales uncertainty are \( 1/(1-\eta_s \eta_b) \) and \( (\sigma^2_s + \eta^2_s + \sigma^2_b) / (1-\eta_b \eta_s)^2 \), respectively, and yield that

\[
\alpha_1^* = \frac{\beta^2}{\beta^2 + \rho \sigma^2} \Rightarrow \alpha_1^* = \frac{\left( \frac{1}{1-\eta_s} \right)^2}{\left( \frac{1}{1-\eta_s} \right)^2 + \rho \sigma^2_s + \eta^2_s \sigma^2_b / (1-\eta_s \eta_b)^2} \Rightarrow \alpha_1^* = \frac{1}{1 + \rho (\sigma^2_s + \eta^2_s \sigma^2_b)}.
\] (11)

In this case, the optimal commission rate differs from the canonical formula because of between-market externalities, i.e., \( \eta_s \) imports additional risk coming from the buyers’ side in the sellers’ side, i.e., \( \eta^2_s \times \sigma^2_b \), which is not balanced by the enhanced selling effectiveness and thus necessitates to adjust the commission rate.

Finally, when all network effects are present, we obtain that the agent’s enhanced selling
effectiveness is \( \frac{1 - \gamma_b}{(1 - \gamma_b)(1 - \gamma_s) - \eta_b \eta_s} \), with sales uncertainty presented in (8) which yield the optimal commission rate presented in Proposition 1, i.e.,

\[
\alpha_i^* = \frac{(1 - \gamma_b)^2}{(1 - \gamma_b)^2(1 + \rho \sigma_s^2) + \rho \eta_s^2 \sigma_b^2},
\]  

(12)

In this case, when both direct and indirect network effects are present and \( \alpha_i^* \) now depends on both \( \eta_s \) and \( \gamma_b \). Similar to (11), the dependence on \( \eta_s \) comes from the fact that this parameter imports additional risk in the sellers’ side, which is not balanced by the agent’s enhanced selling effectiveness. At the same time, the dependence on \( \gamma_b \) comes from the fact that this network effect parameter amplifies the risk from the buyers’ side.

To summarize, network effects enter differently in the optimal commission rate based on the externalities they generate and based on how the agent’s enhanced selling effectiveness and sales’ amplified risk balance each other. These externalities alter the risk and reward balance that the commission rate is supposed to achieve. When \( \eta_s = \eta_b = 0 \), these externalities occur within the same market and balance each other as formally shown in (10). However, they do not when the externalities occur between markets (or sides in this case), which as a result necessitates to make the optimal commission decrease with network effects as formally shown in (11) and (12).

3.2 Equilibrium Profit and the Value of Selling in Platforms

Now we investigate how the strengths of network effects impact equilibrium profits. Intuitively, and based on the extant platform literature, stronger network effects make the platform more attractive to consumers as they enhance the value customers derive from it. Moreover, as noted, stronger network effects increase the agent’s productivity and effort level. As a result, the firm’s profit should increase as network effects increase. Surprisingly, however, this is not always true.

**Proposition 2** The platform’s profit may decrease when network effects are stronger.
To understand why network effects can have a negative impact on profit, we first recall
that in the case of a traditional good, the equilibrium profit is

$$
\Pi^* = \underbrace{V}_{\text{Value of the product}} + \frac{\beta^4}{2(\beta^2 + \rho\sigma^2)} \underbrace{1}_{\text{Value of Selling}},
$$

which informs that the firm’s profit is derived from two sources. The first term $V$ captures
the profit created due to the product features and the second term captures the agent’s
contribution to the equilibrium profit, i.e., the value of selling. For instance, when $\beta = 0$, the
value of selling is zero and the only source of profit comes for the product features.

With network effects, consider again the two polar cases. When only direct network effects
exist, i.e., $\eta_s = \eta_b = 0$, the equilibrium profit is

$$
\Pi^*|_{\eta_s = \eta_b = 0} = \underbrace{\frac{V_s}{(1 - \gamma_s)}}_{\text{Value of the Product}} + \frac{1}{2(1 + \rho\sigma^2_s)(1 - \gamma_s)^2} \underbrace{1}_{\text{Value of Selling}}.
$$

The first part demonstrates, as one might expect, that network effects make the product more
valuable and increase the profit contribution of the product. The second part identifies the
agent’s contribution to the firm’s profit, and shows that this contribution always increases
with direct network effects. The key insight is that in this case the compensation plan fully
internalizes the externalities generated by network effects as $\frac{\partial \Pi^*|_{\eta_s = \eta_b = 0}}{\partial \gamma_s} > 0$

However, this anticipated outcome does not hold when indirect network effects exist, which
becomes evident by setting the direct effects $\gamma_s, \gamma_b$ to 0. Then the equilibrium profit is

$$
\Pi^*|_{\gamma_s = \gamma_b = 0} = \underbrace{\frac{V_s + V_b\eta_s}{1 - \eta_s\eta_b}}_{\text{Value of the Product}} + \frac{1}{2(1 - \eta_s\eta_b)^2} \underbrace{1 + \rho(\eta^2_s\sigma^2_b + \sigma^2_s)}_{\text{Value of Selling}},
$$

which reveals that while network effects always increase the profit contribution due to the
product, they can decrease the value of selling. In equilibrium, at sufficiently high $\eta_s$, the net
profit can even decrease as $\eta_s$ increases. The possible deleterious effect of $\eta_s$ on profit indicates that the compensation plan does not fully internalize the externalities generated by network effects and in particular between-market externalities.

4 Multi-Dimensional Compensation Plan

The previous section highlighted that the misalignment of the risks and rewards of selling when network effects exist causes the firm to potentially lose from stronger network effects, despite their otherwise beneficial nature. The question, then, is what could the firm do to reverse this negative impact of stronger network effects? We develop a new, multi-dimensional, compensation design in this section, and demonstrate that it restores the desirable positive impact of stronger network effects on profit. We do so, first when only cross-sided network effects exist, and next when only direct network effects exist but when the sellers’ side is split into two different territories that are managed by two different selling agents.

4.1 Multi-Sided Plan under Indirect Network Effects Only

Consider a platform characterized by cross-sided network effects only, with the following demands, i.e.,

\[ q_b = \frac{V_b + V_s \eta_b + \omega \eta_b}{1 - \eta_b \eta_s} + \frac{\epsilon_b \eta_b \eta_s + \epsilon_s \eta_b}{1 - \eta_b \eta_s} \]  
(16a)

\[ q_s = \frac{V_b \eta_s + V_s + \omega}{1 - \eta_b \eta_s} + \frac{\epsilon_b \eta_s + \epsilon_s \eta_b \eta_s}{1 - \eta_b \eta_s}, \]  
(16b)

which are special cases of (2) after setting $\gamma_s = \gamma_b = 0$. Expanding beyond the classic plan structure, we now consider a two-sided compensation plan where the agent’s incentive compensation plan is informed not only by how many sellers joined the platform, but also by network mobilization on the buyers’ side. The agent’s compensation formula is

\[ \omega(q_s, q_b) = \alpha_0 + \alpha_1 q_s + \alpha_2 q_b, \]  
(17)
which consists, as before, of a fixed salary $\alpha_0$ and a commission rate $\alpha_1$ on sales from the sellers’ side, and a new parameter $\alpha_2$, which adjusts the agent’s compensation based on the number of buyers who join the platform. The agent chooses the optimal effort level such that

$$w^* = \arg \max_w \mathbb{E}[\omega(q_b, q_s)] - \frac{\rho}{2} \text{Var}[\omega(q_b, q_s)] - C(w),$$ (18) 

where $\omega(q_s, q_b) = \alpha_0 + \alpha_1 \mathbb{E}[q_s] + \alpha_2 \mathbb{E}[q_b]$ with

$$(\mathbb{E}[q_s]; \mathbb{E}[q_b]) = \left( \frac{V_b \eta_s + V_s + \frac{w}{1 - \eta_b \eta_s}}{1 - \eta_b \eta_s} ; \frac{V_b + V_s \eta_b + w \eta_b}{1 - \eta_b \eta_s} \right); \quad (\text{Var}[q_s]; \text{Var}[q_b]) = \left( \frac{\eta_s^2 \sigma^2_b + \sigma^2_s}{(1 - \eta_s \eta_b)^2}; \frac{\sigma^2_b + \eta_s^2 \sigma^2_s}{(1 - \eta_s \eta_b)^2} \right).$$

The firm’s expected profit then becomes

$$\mathbb{E}[\Pi] = (1 - \alpha_1) \mathbb{E}[q_s] - \alpha_2 \mathbb{E}[q_b] - \alpha_0.$$ (19) 

As a result, we find that under the two-sided compensation plan, the agent’s optimal effort strategy is to work more as network effects increase, specifically,

$$w^* = \frac{\alpha_1 + \alpha_2 \eta_b}{1 - \eta_s \eta_b},$$ (20) 

and the principal then chooses $\alpha_1$ and $\alpha_2$ to maximize $\mathbb{E}[\Pi]$ subject to the agent’s IC and IR conditions. The following result is obtained.

**Proposition 3** The optimal two-sided compensation plan is such that

$$(\alpha_1^*; \alpha_2^*) = \left( \frac{1}{(1 - \eta_b \eta_s)} \times \frac{1}{1 + \rho \sigma^2_s}; -\eta_s \alpha_1^* \right).$$ (21) 

With this plan, stronger network effects always increase equilibrium profit,

$$\Pi^* = \frac{V_s + V_b \eta_s}{1 - \eta_s \eta_b} + \frac{1}{2(1 - \eta_s \eta_b)^2} \times \frac{1}{1 + \rho \sigma^2_s}.$$ (22)
With this new plan structure, the analysis produces three insights that contrast with the results obtained under the one-sided compensation plan. Previously, Proposition 1 revealed that under the one-sided compensation plan, the optimal commission rate $\alpha^*_1$ on the sellers’ side varies only in $\eta_s$ and more specifically decreases. In contrast, with the two-dimensional plan, $\alpha^*_1$ always increases in both $\eta_s$ and $\eta_b$. Hence, with a two-dimensional compensation plan, the principal is now empowered to fully capitalize on network effects. The intuition for this desirable property comes from the second insight revealed by Proposition 3.

This second insight is that network mobilization on the buyers’ side enters negatively in the agent’s compensation plan. While surprising, this result can be understood by examining the total compensation risk to which the agent is exposed to under the two-sided compensation plan, i.e.,

$$Var(\omega(q_s; q_b)) = \alpha_1^2 \frac{\eta_s^2 \sigma_b^2 + \sigma_s^2}{(1 - \eta_b \eta_s)^2} + 2\alpha_1 \alpha_2 \frac{\eta_s \sigma_b^2 + \eta_b \sigma_s^2}{(1 - \eta_b \eta_s)^2} + \alpha_2^2 \frac{\sigma_b^2 + \eta_b^2 \sigma_s^2}{(1 - \eta_b \eta_s)^2}.$$  

With all other terms being positive, we see that setting $\alpha_2^* < 0$ allows the principal to reduce and diversify the compensation risk faced by the agent. As a result, the principal has “more room” to provide stronger incentives to the agent for mobilizing sellers as shown by

$$\alpha_1^*|_{\alpha_2=0} < \alpha_1^*|_{\alpha_2<0}.$$  

In equilibrium, and despite $\alpha_2^* < 0$, we obtain that the agent works more under the two-sided compensation plan than under the one-sided compensation plan since

$$\mathbf{w}^*|_{\alpha_2=0} < \mathbf{w}^*|_{\alpha_2<0}.$$  

Managerially, $\alpha_2^* < 0$ can be interpreted as a royalty that the principal extracts from the agent because of the platform’s contribution—buyer-side growth—to seller-side growth, sales that create incentive compensation for the agent. This negative compensation could be viewed as lower fixed salary but more powered incentives, for working for a popular platform. In
fact, though, it has an additional benefit, the risk associated with this negative compensation cancels out some of the risk associated with the positive compensation linked to seller-side sales. Thus, the two-sided compensation plan allows the firm to not only capitalize on network effects, but also incentivize the agent to work harder.

As a result, we obtain our last insight that contrasts with Proposition 2 in that the equilibrium profit always increases as cross-sided network effects increase. Why? Because the two-sided compensation plan allows the principal to internalize all the externalities generated by network effects. More specifically, the two instruments ($\alpha_1$ and $\alpha_2$) allow the principal to manage the two sources of uncertainty that impact the agency relationship, i.e., the uncertainty from the buyers’ side and the uncertainty from the sellers’ side, which was not possible under the one-sided compensation plan. As a result, the equilibrium profit under the two-sided compensation plan exceeds the equilibrium profit under the one-sided compensation plan as easily verified by comparing (15) and (22).

### 4.2 Multiple Selling Territories

Next, we analyze the design of a two-sided compensation plan when only direct network effects are present and when the sellers’ side is split into two different territories that are managed by two different selling agents. Our goal here is twofold. First, it is to show that even when only direct network effects exist, between-market externalities can still arise and erode profit. Second, it is to show that in such cases, even when only direct network effects are present, two-sided compensation plans dominate one-sided compensation plans. The setting we consider is that the seller-side market is partitioned into two distinct selling territories $i = 1, 2$ managed by two distinct agents.
4.2.1 Benchmark Case: Compensating Agent $i$ on her Own Sales Only

In the benchmark case, the agent’s compensation is only informed by sales in her market, such that agent $i$’s compensation is

$$
\omega_i(q_{si}) = \alpha_0 + \alpha_1 q_{si}.
$$

Focusing on direct network effects only, let $Q_{si}$ represent sales in the selling territory $i = 1, 2$, respectively, which are affected by stand-alone benefits ($V_{si} = V$) and the direct network benefits consumers anticipate to obtain from the platform, i.e., $\gamma (Q_{si}^a + Q_{sj}^a)$, with $j = 3 - i$. As previously, $\gamma$ represents the intensity of direct network effects, while $Q_{si}^a$ represents market participants’ anticipation about mobilization on territory $i$. Finally, other influences on $Q_{si}$ are encapsulated in error terms $\epsilon_{si}$, which are unknown at the time of contracting, and which we assume to be Normally distributed (with mean 0 and variance $\sigma_{si}^2$). The sales agents then exert effort $w_i$ to mobilize market participants in their respective territories. Mathematically,

$$
Q_{si} = V + \gamma (Q_{si}^a + Q_{sj}^a) + w_i + \epsilon_{si},
$$

Examining again the equilibrium where market participants form rational expectations about network size, i.e., $Q_{si} = Q_{si}^a = q_{si}$ for both territories, we find that at the time of contracting and determination of the effort strategies, equilibrium demands are characterized by

$$
q_{si} = \frac{V + w_i (1 - \gamma) + w_j \gamma + \epsilon_i (1 - \gamma) + \epsilon_j \gamma}{1 - 2 \gamma},
$$

with $i = 1, 2$ and $j = 3 - i$. Furthermore, agent $i$’s optimal effort strategy is characterized by

$$
\mathbf{w}_i^* = \arg \max_{w_i} \mathbf{E}[\omega(q_{si})] - \rho \frac{1}{2} \mathbf{Var}[\omega(q_{si})] - C(w_i),
$$
which yields that agent’s $i$ optimal effort strategy is

$$w_i^* = \alpha_{1i} \frac{1 - \gamma}{1 - 2\gamma},$$

which similar to our previous results means that the agent works more as network effects increase and as her commission rate increases. The principal then determines the contracts’ parameters to maximize the firm’s expected profit, i.e.,

$$\mathbb{E}[\Pi] = (1 - \alpha_{11})q_{s1} + (1 - \alpha_{12})q_{s2} - \alpha_{01} - \alpha_{02},$$

subject to the agents’ IC and IR conditions. Thus, the market outcomes are determined as a function of the agents efforts, the contract parameters, product characteristics, and the realized values of the random variables, which become available as time unfolds. As a result, we obtain the following proposition.

**Proposition 4** The firm’s equilibrium profit and optimal commission rate for the agent serving territory $i$ are

$$\Pi^* = \frac{2V}{1 - 2\gamma} + \sum_{i=1}^{2} \frac{\alpha_{1i}^*}{(1 - 2\gamma)^2} \left( (1 - \gamma) - \alpha_{1i}^* \frac{(1 - \gamma)^2 (1 + \rho \sigma_i^2) + \rho \gamma^2 \sigma_{3-i}^2}{2} \right)$$

and

$$\alpha_{1i}^* = \frac{1 - \gamma}{(1 - \gamma)^2 (1 + \rho \sigma_i^2) + \gamma \rho \sigma_{3-i}^2},$$

respectively.

The most important insight that Proposition 4 offers is that the equilibrium profit can decrease with direct network effects when two agents serve two distinct selling territories that are influenced by each other through $\gamma$. This insight contrasts from what we found when there is only one territory and one agent (i.e., see (14)), in which case network effects increased both the contribution of $V$ to the firm’s profit and the value created by the agent. From Proposition 4, we learn that network effects still increase the contribution of $V$ to the firm’s
profit, but can decrease the value created by the agent. Why? Because, owing to the multi-
territory structure, between-market externalities emerge in this case, which the one-sided
compensation plan cannot fully internalize. We circumvent this challenge by proposing a new
compensation plan structure.

4.2.2 Optimal Plan with Two Independent Sales Territories

Specifically, we now extend the compensation of agent \(i\) by incorporating agent \(j\)’s performance
into agent’s \(i\) compensation plan such that,

\[
\omega_i(q_{si}) = \alpha_{0i} + \alpha_{1i}q_{si} + \alpha_{2i}q_{sj}
\]  

(34)

Note that this compensation plan can be rewritten as being based on individual performance
as well as group’s performance, i.e., \(\omega_i(q_{si}) = \theta_{0i} + \theta_{1i}q_{si} + \theta_{2i}(q_{si} + q_{sj})\), where \(\theta_{0i} = \alpha_{0i}\),
\(\theta_{1i} = \alpha_{1i} + \alpha_{2i}\) and \(\theta_{2i} = \alpha_{2i}\). Following the same steps as before, we obtain that the optimal
effort strategy of agent \(i\) is

\[
w^*_i = \frac{\alpha_{1i}(1 - \gamma) + \gamma\alpha_{2i}}{1 - 2\gamma},
\]  

(35)

and next report in the following proposition the equilibrium profit and commission rates of
the firm

**Proposition 5** The firm’s equilibrium profit and commission rates are

\[
\Pi^* = \frac{2V}{1 - 2\gamma} + \frac{2 + \rho(\sigma_1^2 + \sigma_2^2)}{2(1 - 2\gamma)^2(1 + \rho\sigma_1^2)(1 + \rho\sigma_2^2)}
\]  

(36)

and

\[
(\alpha_{1i}; \alpha_{2i}^*) = \left(\frac{1 - \gamma}{(1 - 2\gamma)(1 + \rho\sigma_1^2)}; \frac{\gamma}{1 - \gamma}\alpha_{1i}^*\right)
\]  

(37)

respectively.

The three main insights offered by Proposition 5 are as follows. First, agent \(i\)’s commissions
now depend only on \(\sigma_1^2\), and not \(\sigma_2^2\) anymore, as what we found in Proposition 4 under the
one-sided compensation plan. Second, we find that the commission rate received by agent $i$ from the selling territory $j$ is negative, not unlike what we found in Proposition 3. Similar to the intuition provided in the case of Proposition 3’s result, the negative commission $\alpha_{2i}^*$ is not only proportional to $\alpha_{1i}^*$, but also used to diversify agents’ compensation risks. As the result, the agent’s equilibrium effort strategy is $w_i^* = \alpha_{1i}^*/(1 - \gamma)$. Finally, and most importantly, Proposition 5 reveals that under the two-sided compensation plan in this case, profit always increase as network effects increase, which was not the case in Proposition 4. This result indicates again that the two-sided compensation plan fully internalizes the externalities imposed on the agent relationship by network effects.

5 Conclusion

Platforms are an exciting aspect of business today. The positive feedback created by network effects, the immense popularity of many new platforms and excellent financial indicators, have created enormous interest for this business model. However, setting up platforms and securing participation of key players is difficult and requires concerted selling effort. To our knowledge, the present paper is the first to examine selling strategy and salesforce incentives for platforms and two-sided markets. Our analysis demonstrates that the existence of network effects indisputably alters the management of sales force compensation plans and most importantly, that ignoring them when designing performance based incentives can hurt profits. We offer five propositions that are pertinent for platform business and which answer our initial research questions.

Specifically, we first asked whether the agent will work less or more as network effects increase. Our analysis shows that for any commission rate, the agent will always work more as network effects increase because they enhance her selling effectiveness. We then asked whether stronger network effects should then cause the firm to decrease or increase commission rates as network effects vary? The answer to this question depends on the type of compensation plan used by the firm. On the one hand, Proposition 1 and Proposition 4 show that under
one-sided compensation plans, optimal commission rates should decrease with network effects. On the other hand, Proposition 3 and Proposition 5 establish that under two-sided compensation plans, commission rates should increase as network effects increase. We explained that these results are due to the fact that one-sided compensation plans do not fully internalize all the externalities generated by network effects, whereas two-sided compensation plans do. As a result, we are able to not only inform which metrics should be used to inform agents’ compensation plans, but also to discover that profits can go down under one-sided compensation plans as network effects increase, whereas profits always go up with network effects under the two-sided compensation plans. Why? Because two-sided compensation better internalize between-market externalities that arise due to network effects.

With these results in place, our work creates possibilities for future research. For instance, it would be useful to endogenize the platform’s standalone quality and intensity of network effects, to explore the optimal design of platforms that subsequently need to be sold by sales agents under moral hazard. Second, considering price as well as personal selling would be crucial to see how moral hazard can change known pricing strategies for platforms. Finally, managers often use other marketing instruments such as advertising to grow platforms (Sridhar et al., 2011), often using different instruments on different sides. Hence, considering more than one marketing instruments would be valuable to design marketing budgeting and allocation strategies.
Appendix

Micro Foundations of Demands

We present how demands (2) might be derived from micro foundations. Given the direct and cross-platform network effects that are present, the utility obtained by a participant on side $i = \{b, s\}$ of the market is influenced by product characteristics (which affect standalone benefit and network benefits), network and market parameters (which influence the scale of network benefits), and agent effort (if a sales agent is deployed on that side). The standalone benefit is denoted by $V_i$ (a proxy for intrinsic product quality), less a “cost” $tx_i$ faced by participants. This cost may represent the cost of product adoption and use or simply a heterogeneous misfit cost based on personal preferences. Network benefits depend on network sizes ($q_i$ and $q_j$ respectively) and intensity of direct and cross-network effects ($\gamma_i$ and $\eta_i$ respectively). The network size, $q_i$, on side $i$ is the sum of participants who join in this manner, i.e., are affected by the agent and have positive net utility and a second group of size $M_i$ who are risk-taking innovators or early adopters of the product and are not influenced by the agent’s effort. We refer to the first group as “followers.” To be specific, the participation decisions of followers are influenced by $M_i$ (and other parameters in the model), while those of the second group are not. The firm and the agent negotiate a compensation contract before the agent starts mobilizing network participants, making $M_i$ a random variable at the time of contracting, which we assumed to be Normally distributed with mean $\mu_i$ and variance $\sigma_i^2$, but which is certain when network participants mobilize. Formally, utility functions for followers on the $b$ and $s$ sides of the market are as defined below,

$$U_b = V_b - t \times x_b + \gamma_b(x_b + M_b) + \eta_b \times (x_s + M_s) \quad (38)$$

$$U_s = V_s + w - t \times x_s + \gamma_s(x_s + M_s) + \eta_s \times (x_b + M_b). \quad (39)$$

Normalizing the misfit cost to one, i.e., $t = 1$, and equating the two utility functions to zero, we solve for $x_b, x_s$, which represent the equilibrium marginal followers on both sides and the
sizes of the follower groups.

\[
x_b = \frac{(1 - \gamma_s)(V_b + \gamma_b M_b) + \eta_b (M_s + V_s + \Bar{w}) + \eta_b \eta_s M_b}{(1 - \gamma_b)(1 - \gamma_s) - \eta_b \eta_s}
\]

\[
x_s = \frac{(1 - \gamma_b)(V_s + \gamma_s M_s + \Bar{w}) + \eta_s (M_b + V_b) + \eta_s \eta_b M_b}{(1 - \gamma_b)(1 - \gamma_s) - \eta_b \eta_s}
\]

Thus, at the time of contracting, expected demands are \( \mathbb{E}[q_b] = \mathbb{E}[x_b + M_b] \) and \( \mathbb{E}[q_s] = \mathbb{E}[x_s + M_s] \), which comports with (2).
References


