Business models in the Sharing Economy: Manufacturing durable goods in the presence of Peer-to-Peer rental markets

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Abstract

Business models that focus on providing access to assets rather than on transferring ownership of goods have become an important industry trend, representing a challenge for incumbent firms, but also an opportunity to adjust their own business models. This paper analyzes the interaction of a peer-to-peer (P2P) rental market and an original equipment manufacturer (OEM). Our analysis highlights the important role of consumer heterogeneity in usage rates as a driving factor of the mechanisms that explain the different market outcomes. Indeed, both the OEM and consumers can benefit from P2P rental markets for intermediate ranges of consumer heterogeneity in usage rates, but both can be worse-off when the heterogeneity is too low or too high. P2P rental markets have an equalizing effect, as low-usage consumers earn relatively more from P2P rentals than high-usage consumers. We investigate alternative market structures for the OEM, and show that under intermediate consumer heterogeneity in usage rates, it is best for the OEM to operate in the presence of a P2P rental platform. If heterogeneity in usage rates is too low, the OEM prefers to operate as a monopoly, offering sales only, whereas if heterogeneity is too high, the OEM prefers to offer sales and rentals directly to consumers. Further, if a P2P market is unavoidable, the OEM would not necessarily be better-off by introducing its own rentals to compete against P2P. Thus, contrary to what could be expected, the OEM has an incentive to facilitate P2P rentals in a large variety of cases.

Keywords: Business models, sharing economy, peer-to-peer marketplaces, rentals, manufacturing

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1. Introduction

Business models that focus on providing access to assets rather than on transferring ownership of goods have become one of the most fundamental recent industry trends (Bardhi and Eckhardt 2012). The trend is reflected, for example, in what has been called the *sharing economy* (Sundararajan 2016). Recent reports indicate that the five key sectors for the sharing economy—travel, product sharing, finance, staffing, and music and video streaming—have the potential to increase their global revenues from $15 billion in 2015 to approximately $335 billion by 2025 (PricewaterhouseCoopers (PWC) 2015), and that 72% of Americans have engaged in a transaction through the sharing economy or on-demand service companies (Cortese 2016). Similarly, a survey by PWC (2015) establishes that 81% of consumers consider it less expensive to share goods than to own them individually, while 43% agree that "owning today feels like a burden," reflecting the relevance of this consumer trend.

The proliferation of sharing economy companies can represent a challenge for traditional firms. It is sensible to postulate that traditional firms should adapt to compete in this new scenario (Cusumano 2015). However, it is unclear whether the effect of sharing economy companies on incumbent firms is necessarily harmful and uncertain how the latter should react. Peer-to-peer (P2P) rental markets are part of the general sharing economy phenomenon. In these markets, consumers who own assets (such as cars, bikes, and tools) can not only use them whenever needed, but also monetize ownership when they are not using the products, by renting them to other consumers willing to pay for temporary access to these assets. Examples of such marketplaces include Turo, Getaround, or Drivy (P2P car rentals), SpinLister (P2P bike rentals), Boatbound (P2P boat rentals), Airbnb (P2P home rentals), and Zilok (P2P rentals for products and tools). The entry of P2P rental markets could affect consumer ownership preferences and incumbent manufacturers. Although from an OEM perspective it could be hypothesized that the presence of P2P rentals may cannibalize sales, it is also possible that P2P rentals make ownership more attractive, e.g., for consumers with low-usage patterns for which ownership without the option of renting out products in P2P markets may not be even an option. This example also illustrates the potentially important role that heterogeneity in usage rates by consumers can play in terms of market outcomes.

In practice, some OEMs have embraced the idea of P2P rentals. One early example was General Motors partnering with RelayRides (rebranded as Turo in 2015) to facilitate participation of its customers in the P2P rental marketplace in 2013. More recently, some OEMs are even considering launching their own P2P rental platforms, arguing that they could make ownership more attractive. Perhaps the best

illustration of this is Tesla, as stated in Elon Musk's master plan for the upcoming years of the company: "You will also be able to add your car to the Tesla shared fleet just by tapping a button on the Tesla phone app and have it generate income for you while you're at work or on vacation, significantly offsetting and at times potentially exceeding the monthly loan or lease cost. This dramatically lowers the true cost of ownership to the point where almost anyone could own a Tesla" (Musk 2016). Elsewhere, real estate developers are developing buildings designed explicitly for P2P rentals (e.g. the company, Niido). On the other hand, OEMs have also explored a different strategy of offering direct rentals (non-P2P) to consumers. In the automobile industry BMW offers direct rentals through ReachNow; and General Motors offers direct rentals through its recently launched Maven mobility program. Beyond automakers, Home Depot is one example of a retailer offering sales and rentals for a wide range of tools. These cases illustrate that incumbent firms have been actively reacting to the access-based consumption trend in multiple ways.

Motivated by this trend, this paper analyzes the interaction of a P2P rental market with a durable good manufacturer under alternative market structures. The first line of inquiry we pursue is to understand the impact of the entry of a P2P platform on the market, investigating the conditions under which a monopolistic manufacturer and the consumers are better/worse off with the presence of the P2P rental market. In line with industry insights, the analysis reveals the important role played by the heterogeneity in usage rates in the observed market outcomes. Indeed, if consumers are homogeneous in their usage rates, we find that both the firm and consumers are generally worse off with the entry of a P2P rental market. However, when consumers are heterogeneous in their usage rates, both the firm and consumers can be better off with the entry of P2P rentals, if the level of heterogeneity is not too high or too low. One key mechanism that explains this result is that P2P rental markets create an *equalizing effect*—i.e., a reduced disparity in the willingness to pay for the good between low- and high-usage consumers—which is maximized when the level of heterogeneity in usage rates is intermediate, and which explains the more equitable distribution of consumer surplus under P2P. To the best of our knowledge, in the context of P2P rental markets, this equalizing effect has been documented for the first time in this paper.

Second, motivated by the question of whether and how OEMs should adjust their business models, we investigate the equilibrium outcomes under different market structures. To that end, in addition to investigating the sales-only OEM ("Monopoly" operating in the absence of a P2P rental market) and the "P2P-facing" OEM (i.e., the monopolist OEM operating in the presence of the P2P rental market), we investigate two alternative scenarios. First, we analyze the setting where the OEM directly offers sales

and (non-P2P) rentals to consumers (the "Dual" firm). Second, in line with the aforementioned plan of Tesla, we consider an OEM that itself introduces a P2P platform to the market (the "P2P-sponsoring" firm). The results show that the degree of consumer heterogeneity in usage rates continues to play a fundamental role in market outcomes. Indeed, with intermediate levels of consumer heterogeneity in usage rates, the incumbent firm can be better off introducing and embracing the presence of P2P rental markets. In contrast, if heterogeneity in usage rates is too low, the OEM prefers to operate as a sales-only Monopoly, whereas if heterogeneity is too high, the OEM prefers to operate as a Dual firm that offers both sales and rentals directly to consumers. The rationale for the benefit of P2P rental markets in intermediate ranges of heterogeneity is the same as before. With too-high heterogeneity in usage among consumers, the impact of the equalizing effect on sales is diminished, and the firm prefers the Dual strategy, which allows direct control of both sales and rental prices. In contrast, with too-low heterogeneity, the impact of the equalizing effect is again diminished and the cannibalization effect of rentals on sales is high, leading the firm to prefer acting as a sales-only Monopoly. Finally, we consider a third scenario, where the presence of P2P rental markets may be unavoidable, and find that the OEM would not necessarily be better-off by introducing its own rentals to compete against P2P.

We believe that ours is the first paper to contrast P2P rentals with traditional (non-P2P) rental markets in the context of their interaction with product sales. Although in the direct rentals scenario the OEM controls the prices of both sales and rentals, when rentals are P2P instead, the OEM is able to reduce the impact of usage rate heterogeneity by selling only to high value consumers at a higher price. The firm thus faces the trade-off between achieving the benefits associated with direct control of the rental price (Dual strategy) or those arising from the equalizing effect when operating in the presence of P2P rental markets. Our results consider the relative impact of each of these effects, allowing us to determine the conditions under which the OEM would prefer to use different business models. Moreover, we show that if the incumbent firm manages the P2P platform, it would prefer to charge a transaction fee on owners or renters equal to zero, because transaction fees reduce the equalizing effect.

Our results therefore show that, contrary to what could be expected, the OEM has an incentive to facilitate transactions of a P2P rental market in a large variety of cases. We identify the conditions for such scenarios and characterize the mechanisms that drive the results. Although heterogeneity in usage rates is a factor commonly acknowledged in the popular press as one of the most important elements for sharing economy companies (e.g., Badger 2014), the academic literature on P2P rental markets has not emphasized its role in determining market outcomes. The new insights from our paper lead to a better understanding of the conditions and mechanisms by which firms in the traditional economy can benefit

from the entrance of P2P rental marketplaces and have important implications for OEMs' business model design—in particular, for when and why an OEM should adopt different strategies. For example, if an OEM contemplating the introduction of its own P2P platform could choose across multiple markets with varying degrees of heterogeneity, it might introduce the P2P option in markets where the level of consumer heterogeneity is such that the P2P platform would be beneficial. Alternatively, it can choose a sales-plus-rental operation in markets with extremely high degrees of heterogeneity.

We perform several robustness checks to evaluate the generalizability of our findings. First, motivated by the observation that, in practice, some consumers could be unwilling to participate in P2P markets, we examine how the outcomes of the analysis are affected when only a fraction of consumers have access to the P2P market. We find that consumers are better off in such cases because the P2P market increases consumer heterogeneity and reduces the firm's rent extraction capability. Second, in line with the literature on P2P markets, we evaluate how the presence of production costs or transaction costs affects our results. We find that as production costs increase, the P2P-sponsoring strategy becomes best for the firm in an even larger fraction of scenarios, also increasing consumer welfare. Indeed, the P2P market can lead to *allocative efficiency,* through which the firm can serve the needs of the market with fewer goods. Third, regarding transaction costs, we find that as the transaction cost associated with P2P rentals increase, the P2P-sponsoring strategy becomes less attractive (as expected), shrinking the region where the firm would benefit from the presence of P2P rentals. Fourth, we evaluate alternative ways of modeling heterogeneity in consumers' usage rates—e.g., continuous as opposed to discrete distribution—and find largely similar results. Finally, we consider scenarios with additional matching frictions in the P2P market. In all cases, the insights from our main analysis remain qualitatively unchanged.

2. Related Literature

Rental markets have been the focus of considerable attention in Operations Management (OM), Marketing, and Information Systems. Several studies on the topic have focused on aspects such as optimal capacity (Savin et al. 2005), pricing (Gans and Savin 2007), new product introductions (Bassamboo et al. 2009), and inventory management (Slaugh et al. 2016). Part of this literature has contrasted leasing versus selling (Desai and Purohit 1999, Bhaskaran and Gilbert 2005, Agrawal et al. 2012) and the interplay between sales and rentals (Purohit 1997, Gilbert et al. 2014).

Although the roots of this rental markets literature may go back to early developments in the literature on durable goods and secondary markets (e.g., Coase 1972, Bulow 1982), there has been a

recently renewed interest in rental markets for at least two reasons. First, the proliferation of business models that focus on access rather than on transferring ownership of assets has gained attention. The trend toward services in manufacturing, for example, has led to manufacturing-as-a-service business models (servicizing), which have received increasing attention in OM. On the modeling side, papers have analyzed the economic and environmental implications of the servicizing business model (Agrawal and Bellos 2017, Orsdemir et al. 2015). Empirical work has analyzed the value of adding services for OEMs under competition (Guajardo et al. 2016), the implications of servicizing business models for operational performance (Guajardo 2017), and contracting aspects of servicizing in maintenance and operations (Guajardo et al. 2012, Ning et al. 2017). Research in OM has also analyzed this trend in the domain of transportation in particular, e.g., operational aspects of car sharing systems (Bellos et al. 2017).

Second, rentals are at the core of the sharing economy (Einav et al. 2016). Analytical models formulating new theories include Benjaafar et al. (2015) and their characterization of ownership, usage, and matching frictions in P2P sharing platforms, Cachon et al.'s (2016) analysis of the role of surge pricing for matching supply and demand in Uber-type services, and Taylor (2016)'s analysis of operational trade-offs in on-demand service platforms. Similarly, Fraiberger and Sundararajan (2015) develop a dynamic equilibrium model of a sharing market that distinguishes which segments of the market are more prone to switching to rentals. Note also that the temporary P2P aspect of the transactions is distinct from the traditional secondary markets literature, which has explored conditions under which secondary markets can hurt or benefit primary markets (e.g., Hendel and Lizzeri 1999, Chen et al. 2013).

Despite the growing stream of literature attempting to characterize key aspects of the sharing economy, much less is known about how the introduction of P2P markets affects incumbent firms. One exception is Weber (2016), which analyzes consumer purchase decisions with and without the presence of secondary sharing markets, with mixed results depending on the product type. More related to our work, Jiang and Tian (2016) investigate how the entrance of a P2P rental market affects the incumbent's pricing and quality decisions. Their results show that a firm's profits and quantity sold are not monotonically related to the marginal cost of production and the transaction costs in the sharing market. Unlike us, Jiang and Tian (2016) analyze neither the role of uncertainty and heterogeneity in usage rates nor the role of different market structures. Finally, Horton and Zeckhauser (2016) also examine the economic properties of P2P rental markets. Although their model allows for heterogeneity in usage patterns, their results do not refer to customer heterogeneity as a driver of market outcomes nor do they analyze the business model problem for the manufacturer; the same comments apply to Benjaafar

et al. (2015) and Fraiberger and Sundararajan (2015), which similarly do not explicitly consider the manufacturer's production decisions and business model problem.

To the best of our knowledge, in the context of the interaction between an OEM and a P2P rental market, ours is the first paper to highlight the conditions in which the OEM prefers to operate under different business models and the critical role of consumer heterogeneity in usage rates in determining the mechanisms that drive market outcomes. Similarly, ours is also the first paper to contrast the scenario of P2P rentals versus traditional rentals, which adds important new insights. For example, we characterize conditions under which the value and equalizing effects from P2P rentals lead the OEM to be better-off operating in the presence of P2P rentals, rather than offering direct rentals to consumers.

3. Model Setup and Analysis of the P2P Market

We begin with the consumer side. There is a unit mass of consumers in the market. In each instance of consumption of the durable good, consumer i (she) derives a value v_i, where v_i is modeled as a $\mathbb{U}(0,1)$ random variable. Hence, there is heterogeneity across consumers in their valuation for an instance of consumption, although for a given consumer this valuation is time-invariant. In a given period, consumers may experience the need of consumption with probability determined by a Bernoulli(λ_i) random variable. In the general case, we assume that there are two types of consumers in the market, depending on their usage profile: high-usage and low-usage consumers, as characterized by their usage rates $\lambda_i \in \{\lambda_H, \lambda_L\} = \{\lambda, \phi\lambda\}$, where $\phi \in (0,1]$. The low-usage rate is represented as a fraction of the high-usage rate, and the two consumer segments are of equal size. Hence, ϕ is a measure of the market heterogeneity, where $\phi = 1$ denotes a market with homogeneous usage rates.[1]

In the absence of a rental market, a consumer decides whether she wants to purchase the durable good based on her usage rate λ_i, per-period valuation v_i, and the sales price of durable good p_s. In the presence of a rental market, she has an additional option: to rent the durable good when the stochastic need of usage arises at the per-period rental price p_r. When a consumer rents a product, her utility is given by μv_i, where the factor $\mu \in (3/4, 1)$ captures the consumer-side inconvenience for the rental transaction. The parameter μ captures any potential frictions of the P2P sharing market. We do not consider additional transaction cost parameters in the main model.[2]

[1] For analytical simplicity, we adopt equal market size for the two segments. The results do not change qualitatively when we consider two segments of different sizes (see Section 6). In Section 6, we also consider alternative ways of modeling heterogeneity in usage rates.

[2] We assume $\mu > 3/4$ to ensure analytical tractability; all our results are valid for $\mu \in (0,1)$. Note that the assumption of an inconvenience costs proportional to valuation is also used in Gilbert et al. (2014) in a similar context, and by Gurvich et al. (2017) in a different context; this assumption permits two-dimensional heterogeneity. In Section 6, we consider an

Without a P2P rental market, a product sits idle when the owner (he) does not need to use it. In the presence of the P2P, the owner has the option of renting out the product to derive a payoff of p_r (p_r is the market clearing price that emerges in equilibrium—see details in section 3.1). The per-period utility of a product owner is given by

$$u_i^B = \begin{cases} \max\{v_i, p_r\} & \text{if owner demands the product} \\ p_r & \text{otherwise.} \end{cases}$$

Similarly, the per-period utility of a non-owner is given by

$$u_i^R = \begin{cases} \mu v_i - p_r & \text{if consumer rents the product} \\ 0 & \text{otherwise.} \end{cases}$$

The P2P market changes the consumer's problem in two distinct ways. First, it increases the utility that owners can derive from potentially renting out a product when they don't need to use it. Secondly, it provides non-owners the option to rent the product. There is full information in the game, and consumers share the same expectation of the rental price. The overall expected utility that consumer i obtains from the different options of owning (U_i^B), renting (U_i^R) and not participating (U_i^0) are given by[3]

$$\begin{aligned} U_i^B &= \frac{\mathbb{E}\left[u_i^B\right]}{1-\delta} - p_s = \frac{\lambda_i \max\{v_i, p_r\} + (1-\lambda_i)p_r}{1-\delta} - p_s, \\ U_i^R &= \frac{\mathbb{E}\left[u_i^R\right]}{1-\delta} = \frac{\lambda_i[\mu v_i - p_r]^+}{1-\delta}, \\ U_i^0 &= 0, \end{aligned} \tag{1}$$

where δ denotes the inter-temporal discount rate common to all consumers. Typically, consumers rent out their products using a platform that charges a fee. For the sake of simplicity, we have ignored the platform fee in the basic model and introduce it in Section 5, where we show that a platform fee does not qualitatively change our insights.

We now turn to the problem facing the manufacturer of the homogeneous durable good. In what follows, let v_H and v_L denote the valuations of the high-usage and low-usage consumers that are indifferent between purchasing and the alternative. This leads to the following characterization of the

extension where additional transaction costs, e.g. reflecting the effect of degradation or moral hazard, are incorporated in the analysis.

[3]The function $[x]^+$ is equal to x if $x > 0$, or 0 otherwise.

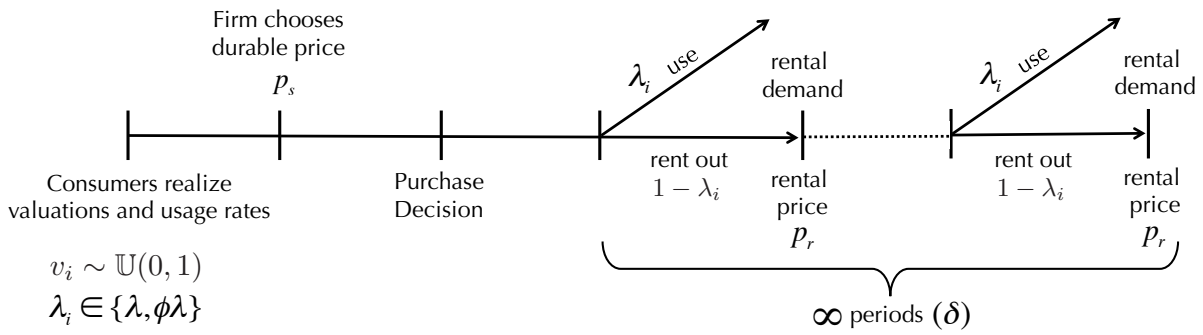

Figure 1: *A timeline of actions in the P2P Market.*

indifferent consumer of type $\theta \in \{L, H\}$ (see the Appendix for an exact expression defining v_H and v_L in equilibrium): $v_\theta = \min\left\{1, \frac{(1-\delta)p_s - p_r}{\lambda_\theta(1-\mu)}\right\}$. All consumers with $v_{i,\theta} > v_\theta$ will purchase. It is easy to show that, for a given p_s, $v_L = \min\left\{1, \frac{v_H}{\phi}\right\}$, hence $v_L < v_H$. Thus, the manufacturer chooses a durable price p_s to maximize its profits, which are given by[4]

$$\pi_s = \frac{((1 - v_H) + (1 - v_L))}{2} p_s. \tag{2}$$

The timeline of the P2P market is reflected in Figure 1. The OEM sets the price of the durable good, p_s. Then, consumers draw their valuation v_i and their type $\lambda_i : \{\lambda, \phi\lambda\}$. Based on this information, they decide whether to purchase or not, and then each period's utility is realized.

3.1 Analysis of the P2P Market

The analysis will use a rational expectations equilibrium (REE) as the solution concept for modeling the consumer choices. Let us assume that consumers predict that the equilibrium rental price is $\hat{p}_r$. Conditional on the predicted rental price $\hat{p}_r$ and the purchase price p_s, consumers evaluate whether they want to buy or rent the product based on Equation (1). At the time of purchase, the valuations of the indifferent consumer thus adjusted to reflect the predicted rental price: $v_\theta = \min\left\{1, \frac{(1-\delta)p_s - \hat{p}_r}{\lambda_\theta(1-\mu)}\right\} \forall \theta \in \{L, H\}$.

In principle, there are two cases: first, where $v_H \geq \hat{p}_r$, and second, $v_H < \hat{p}_r$. In the first case, the per-period utility of the marginal high-type purchaser is greater than the expectation market rental price. Thus, all owners, of any usage rate, anticipate using the good if they demand it. We prove in the

[4]In the main model, we do not consider production costs, to focus the analysis of the main drivers of interest. In Section 6, we extend the analysis to show the impact of positive production costs on the results.

Appendix that the second case, $\hat{p}_r > \{v_H, v_L\}$, is not a stable equilibrium. Under our solution concept, if the expected market rental price is higher than a marginal buyer's per-period usage utility, this attracts additional purchasers, which leads to a lower expected market rental price, until the marginal buyer's per-period usage utility follows the first case, $v_H > \hat{p}_r$. Thus, in equilibrium, product owners will always use the product if they demand it. This simplifies the per-period utility of owners to be v_i if the owner demands the good, rather than $\max\{v_i, p_r\}$.

The demand ($D_r(p_r, v_H, v_L)$) and supply ($S_r(p_r, v_H, v_L)$) functions in the P2P rental market are as follows:

$$\begin{aligned} D_r(p_r, v_H, v_L) &= \frac{1}{2}\left[\lambda[v_H - \frac{p_r}{\mu}]^+ + \phi\lambda[v_L - \frac{p_r}{\mu}]^+\right], \\ S_r(p_r, v_H, v_L) &= \frac{1}{2}\left[(1-\lambda)(1-v_H) + (1-\phi\lambda)(1-v_L)\right]. \end{aligned}$$

In equilibrium, the supply and demand of P2P rentals should coincide, an assumption which we use to derive the equilibrium (similar to other existing papers on the sharing economy, see e.g., Jiang and Tian, 2016). Depending on whether $v_H > p_r/\mu$ or $v_H \leq p_r/\mu$, the high-usage consumers will rent or forego rental in the P2P market. The market clearing rental prices for the P2P platform are thus given by

$$p_r = \begin{cases} \mu\left(\lambda(1+\phi) - (2 - v_H - v_L)\right)/\,\lambda(1+\phi) & \text{when } v_H > p_r/\mu, \\ \mu\left(\lambda(1+\phi) - \lambda v_H - (2 - v_H - v_L)\right)/\,\lambda\phi & \text{otherwise.} \end{cases}$$

For any given p_s, this leads to the equilibrium demand quantities of $(1 - v_H)$ and $(1 - v_L)$, for the high-usage and low-usage consumers, respectively. For the REE to hold, the realized rental price in the P2P market should be equal to the predicted rental price, i.e., $p_r = \hat{p}_r$.

The OEM decides the price of the durable good to maximize its profits (see Equation (2)). By setting the durable good's price, the firm effectively "chooses" the segments of consumer usage it will sell to. Table 1 presents the four feasible outcomes for the overall market. Based on the utilities presented in Equation (1), it is easy to show that, given the incentive compatibility constraints, if anyone purchases it will be the high-usage consumers, if anyone rents it will be the low-usage consumers, and the firm will always want to sell at least to the group of high-usage consumers.

If both types participate in the rental market (outcomes C_1 and C_2), the equilibrium demand for the durable good after solving the REE is: $v_H = \frac{(1-\delta)p_s - p_r}{\lambda(1-\mu)}$, $v_L = \min\left\{1, \frac{v_H}{\phi}\right\}$. If only low-usage

Table 1: *Different Market Outcomes*

	High-usage Buy & Rent	High-usage Buy Only
Low-usage Buy & Rent	C_1	C_3
Low-usage Rent Only	C_2	C_4

consumers participate in the rental market (outcomes C_3 and C_4), we have $v_H = \frac{(1-\delta)p_s - p_r(1-\lambda)}{\lambda}$, $v_L = \min\left\{1, \frac{v_H}{\phi}\right\}$. In Appendix A, we detail the exact expressions for the rental prices and marginal buyers among the four possible market outcomes.

4. Impact of P2P Rentals on a Sales-only OEM

In this section, we present results for an OEM operating in the presence of a P2P rental market, compared to a benchmark scenario where the OEM operates in absence of a P2P rental market. We refer to the OEM in these cases as the P2P-facing firm or the (sales-only) Monopoly firm, respectively. As a brief outline of our insights, we identify four distinct effects of the entry of a P2P rental market on an OEM. Intuitively, there is a potential *cannibalization effect,* because some consumers may switch from purchasing to rentals. On the other hand, the availability of P2P rentals leads to a *participation effect*, because P2P allows participation from consumers who were unable to consume when sales were the only option. These two effects are similar to those present in models of traditional (non-P2P) rental markets, and in the secondary markets literature more generally. Further, P2P markets generate a *value effect*, which refers to the increased value of the ownership as a result of P2P rentals providing a revenue stream for owners. This effect has been documented in the P2P literature, e.g., by Benjaafar et al. (2015) and Jiang and Tian (2016). There is a fourth effect, which we call the *equalizing effect,* which refers to the reduction in the disparity of the willingness to pay for the good by low- and high-usage consumers. Indeed, low-usage consumers benefit more from the entry of a P2P market, because of their higher frequency of non-self-use periods. This effect occurs only when consumers are heterogeneous in their usage rates and leads to key insights in our model, which have not been, to our knowledge, established in the existing P2P literature.

In order to highlight the role of usage rate heterogeneity under P2P rentals, we first discuss the case where consumers are homogeneous in usage rates ($\phi = 1$), to then develop the case of heterogeneous consumer usage rates ($0 < \phi < 1$).

4.1 Homogeneous Usage Rates

We consider the simplified case of an homogeneous usage rate λ common to all consumers. Proposition 1 describes the equilibrium outcomes.

Proposition 1. *The P2P market results in lower firm profits and consumer welfare (relative to the Monopoly case). A fraction $q_s = 1 - \hat{v}$ of consumers purchase the product and a fraction $\hat{v} - \tilde{v}$ rent the product in equilibrium, where*

$$\hat{v} = \frac{1}{2} + \frac{(1-\lambda)\mu}{2\left(\lambda^2(1-\mu)+\mu\right)}, \quad \text{and} \quad \tilde{v} = \frac{1}{2} - \frac{\lambda(1-\lambda)(1-\mu)}{2(\lambda^2(1-\mu)+\mu)}.$$

The quantity purchased is lower than the quantity purchased under Monopoly ($q_s \leq q_m = 1/2$), but a larger fraction of the consumers are able to use a product when they need to.

The entry of a P2P rental market is expected to benefit consumers, as it allows owners to rent out their product when they are not using it (*value effect*) and non-owners to rent when they need to (*participation effect*). However, we find that consumers are actually worse off in the sharing economy. To understand this counterintuitive result, first note that the lifetime utility of ownership can be decomposed into value from own use plus value from receiving rental revenue. Each buyer expects to receive $(1-\lambda)p_r$ in rental revenue each period, with a lifetime expected rental revenue $(1-\lambda)p_r/(1-\delta)$. This rental revenue is identical across all owners. When consumers are homogeneous, the firm is able to raise the durable good price to extract all the value effect. Because of this optimal response by the P2P-facing firm, the buyers with valuations $v_i \in (\hat{v}, 1]$ are worse off with the sharing economy. Although the consumers who switch from purchasing to rentals face a decrease in their per-period consumption utility by a factor of μ, they face a relatively lower price from rentals. As a result, consumers whose valuation $v_i \in [\tilde{v}, \hat{v}]$ are better off with the sharing economy. Because the negative impact on the buyers is larger than the benefit to renters, consumers are overall worse with the sharing economy. This points to a interesting feature of the sharing economy, in which some consumer surplus from the high-valuation consumers is redistributed to the lower-valuation consumers, which is illustrated in Figure 2. Relative to a Monopoly firm, there are additional consumers participating in the market (because $\tilde{v} < 1/2$), reflecting the aforementioned *participation effect*.

It is important to note that the firm uses the P2P market to indirectly segment consumers based on their valuations. The OEM increases prices to sell only to high value consumers ($\hat{v} > 1/2$), while earning rents from the low value consumers indirectly through the inflated durable good price. Even though the P2P-facing firm has a better ability to indirectly segment the market, its profits decrease,

relative to a Monopoly firm. This is because some consumers are now consuming via rentals rather than from purchases (cannibalization effect), which, as noted, provide less per-use utility ($\mu < 1$), all else equal. Figure 2 provides a visualization of the comparison.[5]

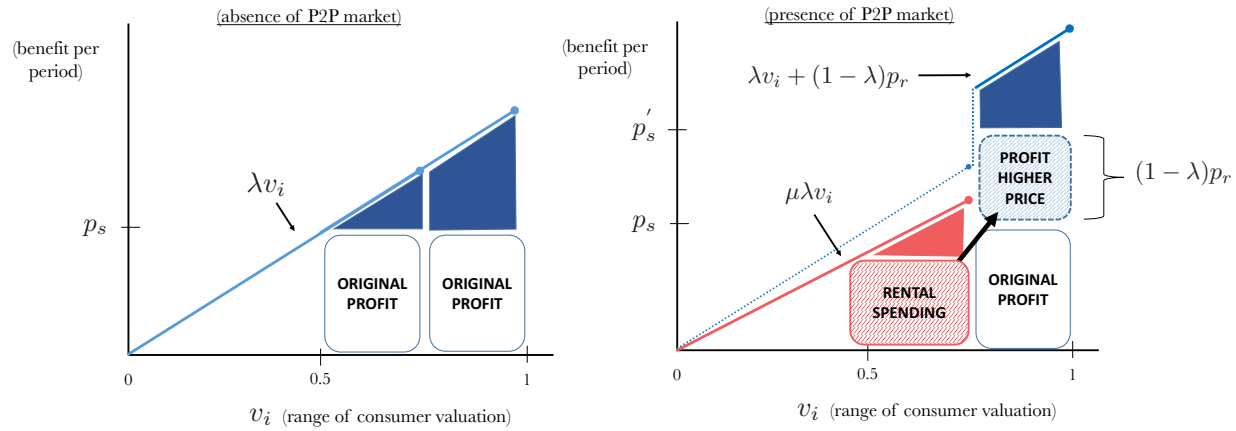

Figure 2: *In the presence of a P2P rental market, a smaller fraction of consumers buy, compared to a monopolistic setting in the absence of P2P, but at a higher price because the firm raises the price to reflect the constant value effect across owners.*

4.2 Heterogeneous Usage Rates

When consumers are heterogeneous in their usage rates, P2P rentals reduce the difference in high and low-usage consumers' willingness to pay (equalizing effect). Indeed, without a P2P rental market, low-usage consumers have a lower willingness to pay, because they experience more frequent non-use periods ($1-\phi\lambda > 1-\lambda$). The introduction of a P2P rental market benefits low-usage consumers proportionally more by creating value during their non-use periods. We first begin by examining how the P2P market affects the consumption choices of the low-usage and high-usage consumers, which is presented in the proposition below.

Proposition 2. *In a market with heterogeneous usage rates, in equilibrium:*

(i) There are always high-usage consumers who purchase the durable good. Low-usage consumers purchase the good only when their usage rate is sufficiently high, i.e., $\phi \geq \hat{\phi}$.

(ii) There are always low-usage consumers who rent the good. High-usage consumers rent for low λ or when usage heterogeneity is very high/low.

[5]Note that one could posit a Pareto optimal equilibrium when no owners rent out goods in the P2P market and the market degenerates to the Monopoly case. However, this is not a credible equilibrium, as owners who do not need to use the product will always rent out their product in the P2P market.

Figure 3 illustrates the regions in which the high and low-usage consumers buy and/or rent. A fraction of high-usage consumers always purchase the durable good, as the OEM will always serve these consumers in equilibrium. As the usage rate of the low-usage consumers ($\phi\lambda$) decreases, they gradually move from buying to renting. Beyond a certain threshold $\hat{\phi}$, none of the low-usage consumers ($\phi < \hat{\phi}$) buy the product, because their usage rate is too low. This threshold $\hat{\phi}$ is decreasing in λ because a higher value of λ not only reduces the available rental supply (pushing up the rental price and the relative value from buying), but also leads to increased usage of low-usage consumers, incentivizing purchases. Next, let us consider the rental behavior of high-usage consumers. For sufficiently low values of λ, rental prices are sufficiently low because the larger rental supply encourages high-usage consumers to rent. Similarly, high-usage consumers rent under low heterogeneity (high value of ϕ)—because low-usage consumers are supplying rentals—and high heterogeneity (low value of ϕ)—because there is infrequent low-usage rental demand. Otherwise, as seen in Figure 3, high-usage consumers will be priced out and consume only through purchases.

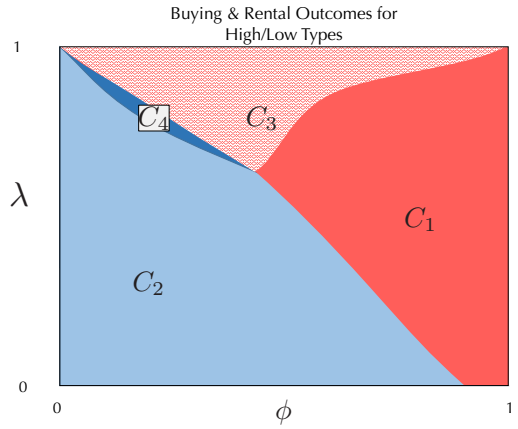

Figure 3: *Illustration of consumer buying and renting behavior under different market conditions (assuming $\mu = 0.8$). Regions C_1 and C_3 represent parameter regions where low-usage consumers purchase durable goods, and regions C_2 and C_4 represent no low-usage consumers purchase.*

Now that we have examined the consumption behavior by different types of consumers, we present the proposition that describes profits for the P2P-facing firm.

Proposition 3. *The OEM's profits are higher in the presence of a P2P rental market than in the Monopoly case when ϕ assumes intermediate values, i.e., $\phi_1^\pi < \phi < \phi_2^\pi$. If the difference in usage rates is very big ($\phi < \phi_1^\pi$) or very small ($\phi_2^\pi < \phi$), the OEM's profits are lower when there is a P2P rental market.*

Figure 4 provides a useful illustration. The intuition behind this proposition is as follows. In a

market with heterogeneous consumers ($\phi < 1$), the Monopoly firm can set only one price for a durable good, as it cannot distinguish between consumers (the consumer type information is private). Hence, the firm sets a price such that only high-usage consumers buy when $\phi < 1/3$, or a price such that both high- and low-usage consumers buy when $\phi \geq 1/3$. Due to the disparity between the high and low-types, the firm is unable to extract rents from consumers who have high expected values from using the product (either high valuations or high usage rate).

When the P2P rental market is introduced, the firm is more able to segment the consumers based on valuation into buyers (high v_i) and renters (low v_i), as shown in Proposition 1. In addition, the equalizing effect brings the willingness to pay for the high and low-usage consumers closer together. The P2P-facing firm can then sell to *both* high and low-usage owners (with high valuations) at a high price. Previously, when facing consumer heterogeneity, the substantially lower willingness to pay of low-usage consumers required the Monopoly firm to set a relatively much lower price to ensure sales to the heterogeneous range of consumers. This double whammy can help the P2P-facing firm generate more revenues than a Monopoly, when the heterogeneity in the market is sufficiently high. Even when the usage rates of low-usage consumers are too low to lead them to purchases (hence, no impact from the equalizing effect), the P2P-facing firm can benefit from a participation effect too, where additional consumers participate in the market (rentals by low-usage consumers) relative to the Monopoly firm. When heterogeneity is very high (ϕ is very low), low-usage consumers do not purchase, nor do they rent frequently enough for the participation effect to be meaningful. In this case, the market degenerates to a homogeneous market comprising of just the high-usage types and half the size.

We believe that this result is quite unique in the literature, as it demonstrates that an OEM can be better off in the presence of a P2P rental market, even without the efficiency gains obtained from positive production costs. In Appendix A, we discuss the mathematical expression of the equalizing effect and a demonstrative example of its impact on consumer willingness to pay.

Finally, Proposition 4 describes the effect of the P2P rental market on consumer welfare (an illustration is provided in Figure 4).

Proposition 4. *When $\phi_0 < \phi < \frac{1}{3}$, consumer welfare is larger under P2P than under Monopoly. Otherwise, consumer welfare decreases with P2P. ϕ_0 is decreasing in λ.*

It is easy to show that, when heterogeneity is high ($\phi < 1/3$), it is optimal for the Monopoly firm to price such that only high-usage consumers purchase, with low-usage consumers priced out of the market, leading to dead weight loss for those consumers. If there is a P2P rental market though, these low-usage consumers benefit from the participation effect, as they can consume via rentals. This results

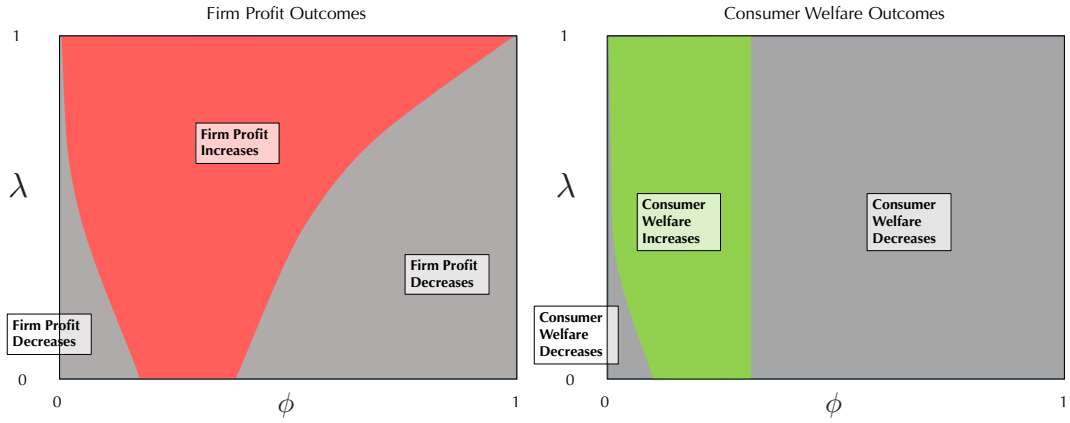

Figure 4: *On the left, the firm's profit in the presence of a P2P rental market (relative to Monopoly). On the right, consumer welfare in the presence of P2P (relative to under a Monopoly firm).*

in a net increase in the total consumer welfare, as long as the participation effect is sufficiently high ($\phi_0 < \phi < 1/3$)[6]. If $\phi < \phi_0$, the demand of low-usage consumers is extremely infrequent, resulting in a minimal participation effect. As a result, participating consumers are essentially homogeneous in usage rate. In this case, consumer welfare is reduced (consistent with the results of Proposition 1). However, when $\phi \geq 1/3$, overall consumer welfare is reduced with a P2P market compared to with a Monopoly firm. When $\phi \geq 1/3$, the Monopoly firm would sell to both consumer types at a relatively low, single price, but with the equalizing effect from a P2P market, the firm extracts more rents from the now two market prices: a high durable price for high expected value consumers and the P2P rental price.

In either case ($\{\phi > \frac{1}{3}, \phi \leq \frac{1}{3}\}$), the entry of a P2P rental market shifts welfare from high-usage consumers to low-usage consumers. High-usage consumers no longer benefit from the relatively low, single price set by a Monopoly firm, whereas low-usage consumers benefit from both the participation and equalizing effects.

5. The OEM's Business Model Problem

The analysis so far has focused on comparing outcomes for a monopolistic OEM operating in isolation or in the presence of a P2P rental market. In this section, we introduce two alternative OEM strategies. First, a "P2P-sponsoring" firm, where the OEM directly operates the P2P platform. The P2P-sponsoring firm derives revenue both from the sales of the durable good and from any transaction fees it charges in the P2P platform; note that, in this case, p_r continues to be the market clearing price and is not

[6]When λ is very high, low-usage consumers also purchase in the presence of a P2P rental market if $\phi < 1/3$. This implies that there can also be an equalizing effect in these cases.

set by the firm. This is in the spirit of, for example, the Tesla network[7]. The second strategy is a "Dual" firm offering both sales and direct rentals (non-P2P), setting the price for both. Recent notable implementations of this business model have been BMW's and General Motor's introduction of urban car rental programs, ReachNow and Maven, respectively, in several US cities.[8]

In this section, we investigate the conditions under which these alternative strategies (P2P-sponsoring, Dual) are most profitable for the OEM when compared to the P2P-facing and Monopoly scenarios analyzed in the previous section.

5.1 The P2P-sponsoring Firm

An OEM can proactively introduce its own P2P rental platform. This can increase profits not only for the reasons highlighted so far, but also by directly capturing platform fees for each P2P rental transaction. The P2P-sponsoring firm obtain profits from both sales and P2P rental operations, charging a percentage fee α per P2P rental ($\alpha \in [0, 1)$). The firm's profit function is thus:

$$\pi^N = p_s^N q_s^N + \frac{\alpha p_r^N q_r^N}{1-\delta},$$

where $\{p_s^N, q_s^N\}$ are the durable good price and quantity sold, and $\{p_r^N, q_r^N\}$ are the equilibrium rental price and quantity of rentals in each period, respectively. We assume that the α percent fee is incurred by the owners of the good. Thus, the utility of ownership is now:

$$U_{i,N}^B = \frac{\lambda_i \max\{v_i, p_r\} + (1-\lambda_i) p_r^N (1-\alpha)}{1-\delta} - p_s^N$$

The utility of renting remains the same for consumers. When there is no platform fee ($\alpha = 0$), this model matches exactly the scenario described in previous sections for the P2P-facing firm. Surprisingly, we find that it is optimal for the firm to give away the P2P platform for free.

Lemma 1. *If the OEM supports the availability of P2P rentals by managing the P2P rental platform, it is optimal for the firm not to charge a platform fee (i.e., $\alpha = 0$).*

To understand why the firm should not charge a fee for P2P rental transactions, note that the firm is already able to capture a significant part of the rental revenues indirectly, by raising the durable good

[7] https://techcrunch.com/2016/10/26/musk-on-tesla-network-its-not-tesla-vs-uber-its-the-people-vs-uber/

[8] https://www.press.bmwgroup.com/global/article/detail/T0266044EN/bmw-group-expands-its-us-premium-car-sharing-service-reachnow-to-brooklyn-new-york-and-launches-four-new-mobility-services-as-a-pilot?language=en
http://media.gm.com/media/us/en/gm/home.detail.html/content/Pages/news/us/en/2016/Jan/0121-maven.html

price. When the firm charges a fee for P2P rentals, the low-usage owners are hurt disproportionately, as there is a larger reduction in their value effect $\big($size of reduction $= (1-\phi\lambda)p_r^N(1-\alpha)/(1-\delta)\big)$. This reduces the equalizing effect, making the P2P market less efficient. Hence, it is optimal for the firm to provide the platform for free. The increase in the platform fee hurts also the consumers. An increase in the α reduces sales to low-usage consumers, leading to a reduction in the rental supply. This in turn hurts consumers who participate in the rental market, reducing overall consumer welfare.

5.2 The Dual Firm

The Dual OEM offers sales and rentals directly to consumers. Under this business model, the firm sets the price p_s for the durable good and p_R for the per-use rental. The consumer problem is similar to the one described for the P2P-facing firm. However, in this case, owners derive value only from their own use and do not gain additional utility from rental revenue. The Dual firm's direct rentals also have per-use rental frictions, represented as ν, where consumers receive νv_i utility per rental, with no additional transaction costs associated with the supply of rentals. In the comparison of Section 5.3, we assume that $\nu = \mu$ to keep the comparison parsimonious between the Dual firm and the P2P-sponsoring strategy, but relax this assumption in Section 5.4. The consumer utility for the different choices is thus given by

$$\begin{aligned} U_i^B &= \frac{\mathbb{E}\left[u_i^B\right]}{1-\delta} - p_s = \frac{\lambda_i v_i}{1-\delta} - p_s, \\ U_i^R &= \frac{\mathbb{E}\left[u_i^R\right]}{1-\delta} = \frac{\lambda_i[\nu v_i - p_R]^+}{1-\delta}, \quad U_i^0 = 0. \end{aligned} \tag{3}$$

Let q_R^D denote the rental units in the market for the Dual firm. The Dual firm's profit is given by $\pi^D = \frac{1}{2}p_s\left[(1-v_L^D)+(1-v_H^D)\right] + \frac{p_R q_R^D}{1-\delta}$.

Here, $\{v_L^D, v_H^D\}$ are the valuations of the marginal low-usage and high-usage purchasers, such that all low-type (high-type) consumers with valuation $v_i \geq v_L^D$ $\left(v_H^D\right)$ purchase the good. The firm simultaneously chooses $\{p_s, p_R\}$. We note that under some market conditions, a Dual firm prefers to forego rentals and provides only sales. Indeed, as indicated in Lemma 2 below, it is possible to show that the firm offers both sales and rentals only when usage rates are sufficiently heterogeneous.

Lemma 2. *The Dual firm chooses to operate both sales and rentals when the difference in usage rates across consumers is sufficiently large, i.e., $\phi \leq \tilde{\phi}^D = \frac{3-\nu-\sqrt{(9-\nu)(1-\nu)}}{2\mu}$. Otherwise, the Dual firm chooses a sales-only operation.*

The introduction of rentals allows the firm to segment the market by targeting sales of the durable good to the high-usage consumers and per-use rentals to consumers who have low-usage rates but a high valuation from consumption (v_i). However, rentals can cannibalize the sales of durable goods. When the difference between the high-usage consumers and low-usage consumers is small ($\phi \geq \tilde{\phi}^D$), the firm is unable to set the optimal rental price for low-usage consumers without cannibalizing some sales to high-usage consumers. Hence, the firm decides to offer only sales and acts as a Monopoly firm. On the other hand, with significant heterogeneity in usage rates ($\phi < \tilde{\phi}^D$), the firm is able to realize the benefits of segmentation, offering both sales (to high-usage consumers) and rentals (to low-usage consumers).

5.3 Comparison Between the Different Strategies

Having outlined alternative strategies that the firm can adopt, we now compare the conditions under which the firm prefers the different business models. We first note that, given Lemma 1, the analysis of the P2P-sponsoring firm is equivalent to the analysis of the P2P-facing firm in Section 4; hence, in the subsequent discussion, we will refer to the P2P-sponsoring firm. We first highlight the comparative advantages of each firm strategy relative to a Monopoly. Adopting the Dual strategy allows the firm to segment high and low-usage consumers into separate markets, by directly controlling both the sales and rental prices. The P2P-sponsoring firm uniquely benefits from the equalizing effect of a P2P rental market, resulting in a more homogeneous set of consumers (in terms of their willingness to pay) and improved rent extraction from high value consumers. The choice between these three firm strategies depends on which of these comparative advantages is most relatively impactful, which in turn depends on the heterogeneity in usage rates in the market (as determined by ϕ and λ). The comparison between the different strategies is formalized in the following proposition.

Proposition 5. *The OEM prefers (i) the Dual strategy when the heterogeneity in the market is sufficiently high, (ii) the Monopoly/sales-only strategy when the heterogeneity in usage rates is sufficiently low, and (iii) the P2P-sponsoring strategy if the heterogeneity in usage rates is intermediate and the usage rate for the high-type consumers is sufficiently high $\lambda \geq \lambda_{min}$.*

Figure 5 shows the optimal strategy for the firm under different market conditions. When λ is sufficiently high ($\lambda \geq \lambda_{min}$), we observe that the Dual strategy works better for low ϕ, the P2P-sponsoring model is best for intermediate ϕ, and Monopoly is preferred for very large ϕ. Let's first examine the region with intermediate ϕ. Because of a sufficiently high λ, the high-usage consumers do not supply the P2P market often, which keeps P2P rental prices sufficiently high and encourages

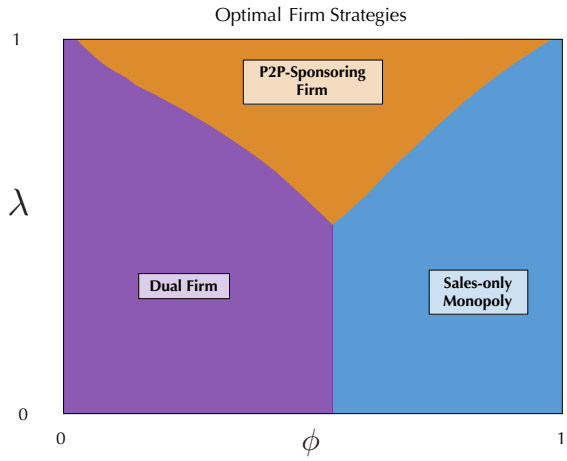

Figure 5: *Varying consumer conditions* $\{\lambda, \phi\}$ *and* $\mu = \nu = 0.8$. *The P2P-sponsoring firm strategy is best under higher* λ *and intermediate* ϕ.

low-usage consumers to purchase, resulting in impact from the equalizing effect. The profitability of the P2P-sponsoring firm here is due to two factors: one, the equalizing effect enables improved rent-extraction, by allowing the P2P-sponsoring firm to face a relatively more homogeneous range of willingness to pay, and two, because low-usage consumers are purchasing, they receive their full usage utility rather than a discounted utility via renting.

In contrast, let us examine the region with low ϕ. As ϕ decreases, there is increasing separation between the willingness-to-pay of high and low-usage consumers, diminishing the impact the equalizing effect has on firm profits—to zero if the equalizing effect is insufficient to encourage low-usage consumers to purchase. Additionally, as ϕ decreases, the frequency of use for low types decreases, and there is a smaller total benefit of owning to avoid the μ rental frictions. Lastly, the lower ϕ leads to a decrease in rental demand from low-usage consumers, decreasing the P2P rental price and the value effect for buyers. In sum, as ϕ decreases, the P2P-sponsoring firm is no longer able to sufficiently profit from the equalizing effect. The Dual strategy becomes the preferred mode of operation, because the Dual firm can control both prices effectively, optimally pricing for both groups of consumers separately.

In the region with high ϕ's, the usage rate heterogeneity is smaller. In this setting, any type of rentals (firm or P2P) attractive to a portion of low-usage consumers would also attract a sizable portion of high-usage consumers, severely cannibalizing sales and introducing more rental frictions. Furthermore, profit gain from the equalizing effect is reduced when the difference in usage is small, because the gap in willingness to pay is already small. Hence, for high values of ϕ, the prevention of cannibalization is most relatively impactful and the Monopoly strategy is most profitable.

Alternatively, when $\lambda < \lambda_{min}$, the high-usage owners rent out to the P2P market more frequently,

increasing rental supply and reducing rental prices. With low rental prices, it becomes more attractive for low-usage consumers to rent, reducing the impact of the equalizing effect. As seen in Proposition 1 and Figure 3, at low values of λ there is a larger range of ϕ where low-usage consumers do not purchase, meaning there is a greater range with zero profit impact of the equalizing effect; thus, the Dual firm's ability to optimize both prices is more attractive. With low values of λ, low-usage consumers purchase only when consumers are sufficiently homogeneous, so a sales-only Monopoly strategy is still more optimal to avoid the cannibalization of sales. In Table 2 of Appendix A, we provide a numerical illustration of the various discussed effects for markets with low ($\phi = 0.9$), intermediate ($\phi = 0.5$), and high ($\phi = 0.1$) levels of heterogeneity in usage rates.

With respect to consumer welfare, note first that in the results of Section 4 (Proposition 4), we show that consumer welfare is generally worse off with a P2P-sponsoring firm rather than a Monopoly firm. In contrast, when comparing to a Dual firm, total consumer welfare can be higher for the P2P-sponsoring firm if and only if usage rates $\{\lambda, \phi\lambda\}$ are sufficiently high such that some low-usage consumers purchase (outcomes C_1 and C_3). On the other hand, if usage rates are low or heterogeneity is high such that low-usage consumers do not purchase, then total consumer welfare would be relatively lower under a P2P-sponsoring firm. The cause of this result is two-fold. One, if low-usage consumers are able to purchase, they do not face the rental frictions from μ per use. Two, if low-usage consumers are sufficiently valuable to the P2P-sponsoring firm such that it sells to both low and high-usage consumers, then the durable price it chooses is lower than the one the firm would set if it were selling to just high-usage consumers (i.e., the Dual firm's durable price). This lower price benefits high-usage owners. Regarding the welfare of low-usage consumers in the P2P-sponsoring firm case, if the P2P market rental price is higher than it would be under a Dual firm ($\frac{\mu}{2}$), then low-usage consumers will be worse off than under a Dual firm, because of reduced participation. Vice versa, low-usage consumers will be better off, whether they purchase or not.[9]

5.4 Dual Firm In The Presence of P2P

In the analysis so far, we have considered three business models: sales-only in the absence of P2P (Monopoly), sales+rentals in the absence of P2P (Dual), and sales-only with P2P (P2P-sponsoring). In this section, we consider a fourth model. The presence of P2P rental markets may be unavoidable. In such a scenario, an OEM can also consider offering its own rentals *to compete against* P2P rentals.

[9] A similar observation occurs for high types if low types are not purchasing. If high types purchase more than they would under a Dual firm, then overall high-type welfare is higher. Vice versa, if high types purchase less than they would under a Dual firm, then overall high-type welfare is lower.

We refer to the new, latter strategy as the "Dual+P2P" firm strategy, since the firm is offering both sales+rentals in the presence of a P2P rental market. In other words, when P2P rental markets are unavoidable, the OEM can consider whether to introduce their own direct rentals (Dual+P2P strategy) or not (P2P-sponsoring strategy).

In this case, we relax the assumption that $\nu = \mu$ and instead assume that the Dual firm's rental frictions are lower than the P2P rental frictions, i.e., $\mu < \nu < 1$. This is an important distinction to make in this model because it is a main avenue for the Dual firm to compete with P2P rentals. We do not make this distinction in the earlier subsection 5.3, in order to parsimoniously analyze the trade-off between the Dual and P2P-sponsoring strategies. In this subsection's analysis however, in order to solve the model, we make this additional assumption that implies that direct rentals from the Dual firm are more attractive than P2P rentals.

A consumer with valuation v_i and usage λ_i thus has the following consumer problem:

$$\begin{aligned} U_i^B &= \frac{\mathbb{E}\left[u_i^B\right]}{1-\delta} - p_s = \frac{\lambda_i v_i + (1-\lambda_i)p_r}{1-\delta} - p_s, \\ U_i^{R,firm} &= \frac{\mathbb{E}\left[u_i^{R,firm}\right]}{1-\delta} = \frac{\lambda_i[\nu v_i - p_R]^+}{1-\delta}, \qquad (4) \\ U_i^{R,P2P} &= \frac{\mathbb{E}\left[u_i^{R,P2P}\right]}{1-\delta} = \frac{\lambda_i[\mu v_i - p_r]^+}{1-\delta}, \quad U_i^0 = 0. \qquad (5) \end{aligned}$$

The firm chooses the prices $\{p_s, p_R\}$ to maximize its combined profit from sales and direct rentals, similarly to the Dual firm in section 5.2, taking the market P2P price p_r into consideration. We numerically examine the firm's profits, prices, and consumer decisions for this strategy, and discuss the results and insights of this exercise. We note that, if market usage rate and consumer heterogeneity are sufficiently high, the Dual+P2P strategy leads to higher profits; otherwise, the firm prefers the P2P-sponsoring strategy. The presence and size of the region where the Dual+P2P strategy is preferred depends on the difference in rental frictions $(\nu - \mu)$ being sufficiently large, due to a larger difference enabling the profit-increasing role of an alternative rentals option amongst sufficiently heterogeneous consumers.

Consider the choice between adopting a P2P-sponsoring strategy versus a Dual+P2P strategy. At lower levels of heterogeneity (higher values of ϕ) and sufficiently high overall usage λ, the P2P-sponsoring strategy is preferable because low-usage consumers are purchasing a notable amount, due to the equalizing effect. However, as the level of heterogeneity increases, the number of purchases from low-usage consumers decreases and the number of low-usage consumers relying on rentals increases. Once this

ratio is sufficiently low, the OEM prefers to forgo some or all of its sales to low-usage consumers in exchange for introducing its own direct rentals, which provide a third price point and improves efficiency for renters by reducing rental frictions (consumers receive νv_i per firm rental rather than μv_i). The Dual+P2P firm sets a higher rental price (p_R) than it would using a Dual strategy, but it sells fewer goods than it would under a P2P-sponsoring strategy. When the Dual+P2P strategy is preferred, the direct firm rentals are usually demanded by only low-usage consumers.

Furthermore, we find that, even if the firm has control over the presence of P2P markets and thus a choice of any of considered business models, in some conditions, the Dual+P2P strategy may be preferred to the Dual strategy. In such cases, if ν is sufficiently larger than μ, the Dual+P2P strategy represents a "bridge" strategy between the Dual and the P2P-sponsoring strategies, in situations where the amount of low-usage consumers is low, but not low enough such that the Dual strategy is optimal. The reduced sales under the Dual+P2P strategy counters the downward pressure on the P2P market price created by a drop in P2P demand, and subsequently counters the downward pressure on the value effect. An example of this is shown in Figure 6.

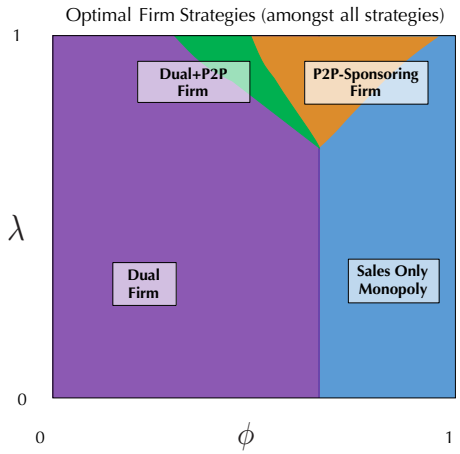

Figure 6: *Varying consumer conditions* $\{\lambda, \phi\}$*, and assuming* $\mu = 0.8$ *and* $\nu = 0.9$*. Each color represents the optimal strategy in the corresponding parameter space, when all business models are available.*

6. Robustness and Extensions

In this section we analyze the robustness of the main results to modeling variations including (i) market size of the P2P rental market, (ii) production costs, (iii) transaction costs in the P2P market, (iv) alternative ways of modeling consumer heterogeneity, and (v) matching frictions in the P2P market.

6.1 Market Size

In the base model, all consumers (potentially) participate in the P2P economy. However, it is possible that in reality only a fraction of consumers might be willing to participate in the P2P market, because of factors like risk-aversion, lack of physical proximity to sharing markets, high transaction costs, and psychological factors. To examine this effect, we extend our base model to allow only a fraction γ of the consumers to participate in the P2P rental market, i.e., a fraction (1-γ) of consumers are not affected by the entrance of a P2P market (consumers in the latter group would not participate in the P2P market either as supply or demand, regardless of the economics of that market). The case of $\gamma = 1$ analyzed so far is thus a special case when the whole population is potentially part of the sharing economy. The firm's profit is given by

$$\pi_s = p_s \left\{ (1-\gamma) \left[(1 - v_H^m) + (1 - v_L^m) \right] + \gamma \left[(1 - v_H^s) + (1 - v_L^s) \right] / 2 \right\}, \tag{6}$$

where $\{v_H^s, v_L^s\}$ denote the valuations of the high-usage and low-usage consumers who are indifferent between purchasing and renting/not consuming. Similarly, $\{v_H^m, v_L^m\}$ represent the indifferent consumers among those who are not willing to participate in P2P transactions. As a special case, one can consider the homogeneous usage rate case (homologously to section 4.1). The next proposition shows how in this case the sharing economy can benefit consumers even if there is no usage-rate heterogeneity in the market.

Proposition 6. *When consumer have a homogeneous usage rate ($\phi = 1$) and the fraction of consumers not participating in the P2P market is sufficiently large, $\gamma < \tilde{\gamma}$, consumer welfare increases under the sharing economy as compared to the Monopoly case, and $\tilde{\gamma} = \frac{4\lambda^2\mu^2 - 5\lambda^2\mu + \lambda^2 - 4\mu^2 + \mu}{4\lambda^2\mu^2 - 4\lambda^2\mu - 4\mu^2 + \mu}$.*

Intuitively, this occurs because a larger non-sharing market mediates the increase in the durable good price caused by a P2P rental market. Comparing the $\gamma = 1$ and $\gamma < 1$ cases, it is easy to see that consumers in the sharing segment are better off when $\gamma < 1$, because the firm is not able to increase the durable price as much as if γ =1. In this case, consumers in the sharing market benefit in two ways. Consumers purchasing the product get a higher utility because of their rental revenues, which can no longer be extracted completely by the OEM, because $\gamma < 1$. Furthermore, there is an increase in the number of consumers who rent, which also increases consumer surplus. Although consumers in the non-sharing segment are slightly hurt by a price increase, we show that the consumer surplus gained from the sharing segment will outweigh the lost surplus in the non-sharing segment when $\gamma < \tilde{\gamma}$.

Additionally, we show numerically that if $\gamma < 1$, the parameter region of $\{\lambda, \phi\}$ where consumer

welfare is largest under a P2P-sponsoring firm is expanded (compared to a Dual or sales-only firm). The same intuition follows: if a fraction of consumers do not participate in the P2P rental market and only choose whether to buy or not, this mediates the P2P-sponsoring firm's durable price increase, also mediating the rent extraction from consumers. Correspondingly, as γ is reduced, the parameter region where the P2P-sponsoring firm is the most profitable business model shrinks. However, we also show that when $\gamma < 1$, it can now be optimal for the P2P-sponsoring firm to charge $\alpha > 0$ platform fees on its P2P platform. This occurs because the firm can capture rents from the value effect via platform transaction fees instead of raising the durable price as much, which because $\gamma < 1$, hurts sales to the γ fraction of non-participating consumers. This suggests that a secondary option for the P2P-sponsoring firm is to use its control over the P2P platform if its ability to raise the durable price is restricted.

6.2 Production Costs

Until now, in the analysis of a P2P-facing firm we have assumed zero production costs for analytical tractability. We now turn to the scenario where the firm incurs a positive production cost, which is indeed a factor that received some attention in related literature (e.g., Benjaafar et al. 2015, Jiang and Tian 2016). In this case, the firm's profit function is given by $\pi_s = (p_s - c)\left\{\left[(1 - v_H) + (1 - v_L)\right]/2\right\}$, where c represents the marginal cost of production. The P2P market helps in the redistribution of the good when it is not being used by the owner. We term this gain in efficiency resulting from the temporary redistribution of the asset as *allocative efficiency.* The next proposition represents the efficiency gains brought by the sharing economy when the marginal costs are non-zero. To develop our intuition, we focus on the homogeneous case, i.e., $\phi = 1$, but the finding is also valid $\forall\, \phi \in (0, 1)$.

Proposition 7. *When the marginal cost of production is sufficiently high, i.e.,* $c \geq \frac{\lambda}{(1-\delta)(1+\lambda)} - \sqrt{\frac{\lambda^4+\lambda^2\mu-\lambda^4\mu}{(1-\delta)^2(1+\lambda)^2}}$*, both firm profit and consumer welfare increase because of the P2P economy, relative to the Monopoly scenario.*

The firm benefits from cost efficiencies when it produces fewer durable goods. Thus, for a sufficiently large c, the efficiencies from reduced production costs allow the firm to profit from the presence of a P2P rental market. The P2P market reduces the quantity sold, because of both a more competitive alternative and the increased durable good price. This allocative efficiency is also reflected in increased consumer welfare when production costs are sufficiently high. This shows that, consistent with previous literature (e.g., Jiang and Tian 2016), the sharing economy can lead to a *win-win* scenario when production costs are sufficiently high even without heterogeneity in usage rates. Further, if consumers have

heterogeneous usage rates, we can show that as ϕ decreases, the P2P-facing strategy is preferred in a larger parameter region. In other words, when the firm faces positive production costs, Propositions 2-5 continue to hold and the regions where profits and consumer welfare increase in the presence of a P2P market become larger. As shown earlier, when durable goods are more costly to create, the firm benefits from allocative efficiency. In addition, production costs reduce the overall demand for the durable good, increasing the number of consumers who demand rentals. This leads to an increase in the value effect and the equalizing effect, which makes the P2P-sponsoring strategy more profitable.

6.3 Transaction Costs

Transaction costs in the sharing economy can arise in many different forms. For owners, they can represent a moral hazard cost (as in Jiang and Tian 2016), a depreciation cost caused by the increased usage (as in Fraiberger and Sundararajan 2015), or a fee charged by the P2P platform. For renters, transaction costs can arise, because of inconvenience or the presence of commission fees paid to the P2P platform. These costs reduce the incentive for owners and consumers to participate in the P2P market. In our prior analysis we have already considered the inconvenience cost on the renter side (μ). In this section, we include a transaction cost, tc, on the owner side when he rents out the product. The owner's per-period utility in this case is given by

$$u_i^B = \begin{cases} \max\{v_i, p_r - tc\} & \text{if owner demands the product} \\ p_r - tc & \text{otherwise} \end{cases}$$

The expected utility from purchasing is given by $U_i^B = \frac{\lambda_i \max\{v_i, p_r - tc\} + (1-\lambda_i)(p_r - tc)}{1-\delta} - p_s$.

Our results are qualitatively robust to transaction costs. Higher transaction costs shrink the parameter region where the P2P-sponsoring firm is the most profitable strategy; however, the shapes of the regions remain the same, where a P2P-sponsoring strategy is best for intermediate values of ϕ and sufficiently high values of λ. The intuition behind this is that a transaction cost will reduce the purchase incentives, but this effect is mitigated by the simultaneous upward pressure on the rental price. Hence, if suppliers of rentals incur transaction costs, the region where P2P-sponsoring is a preferred strategy shrinks, but neither disappears nor experiences any fundamental changes in substantive terms.

6.4 Alternative Ways of Modeling Heterogeneity

In our main model, we assume that there are low-usage consumers (rate = $\phi\lambda$) and high-usage consumers (rate = λ). In this section, we consider alternative ways of modeling heterogeneity in usage rates.

First, we model consumer usage as being uniformly distributed between these two rates. In this case, consumer i demand rate is λ_i, which is a draw from a $\mathbb{U}[\phi\lambda, \lambda]$ variable. The utility of buying and renting remain the same, as a function of $\{v_i, \lambda_i\}$. Both are increasing functions of $\{v_i, \lambda_i\}$, with the utility of buying increasing at a faster rate in both variables. The consumers indifferent between buying and the alternative are a function of a consumer's expected value per period, $EB_i = \lambda_i v_i$. The consumer indifferent between buying and renting ($\bar{EB}$) is:

$$\begin{aligned} U_i^R(\bar{EB}) &= U_i^B(\bar{EB}) \\ \frac{\mu\bar{EB} - \lambda_i p_r}{1-\delta} &= \frac{\bar{EB} + (1-\lambda_i)p_r}{1-\delta} - p_s \\ \bar{EB} &= \frac{p_s(1-\delta) - p_r}{(1-\mu)} \end{aligned}$$

Thus, if $\lambda_i v_i = EB_i > \bar{EB}$, a consumer will purchase the durable good. For the P2P-sponsoring firm, this can be simplified into functions for the marginal indifferent consumers, given either usage rate or valuation: $\hat{v}(\lambda) = \frac{p_s(1-\delta)-p_r}{(1-\mu)\lambda}$ and $\hat{\lambda}(v) = \frac{p_s(1-\delta)-p_r}{(1-\mu)v}$. Similarly, for the Monopoly firm, the purchasing consumers are those whose $\lambda_i v_i = EB_i > p_s(1-\delta)$. Using such equations, we identify the P2P-sponsoring purchase quantities, rental supply, and rental demand:

$$\begin{aligned} Q_{purchased} &= \int_{\phi\lambda}^{\lambda} \min\left\{[1-\hat{v}(\lambda)]^+, 1\right\}\ d\lambda \\ S_r = \int_{\phi\lambda}^{\lambda} Q_{purchased} \cdot (1-\lambda)\ d\lambda \quad &; \quad D_r = \int_{\phi\lambda}^{\lambda} \min\left\{\left[\hat{v}(\lambda) - \frac{p_r}{\mu}\right]^+, 1\right\} \cdot \lambda\ d\lambda \end{aligned}$$

We numerically compare the optimal profits for the P2P-sponsoring and Monopoly firm strategies, and the results are qualitatively similar to Proposition 2, except instead of an intermediate region of ϕ where P2P-sponsoring is optimal, there is now a threshold below which the P2P-sponsoring strategy is optimal. This occurs because the P2P-sponsoring firm continues to benefit from the equalizing and participation effects, but unlike in Proposition 2, the sales-only firm can no longer segment consumers when $\phi < \frac{1}{3}$, selling only to discrete high-usage consumers. Hence, if consumer usage rates are uniform-continuously distributed and sufficiently heterogeneous, the P2P-sponsoring firm is optimal over the Monopoly firm.

Similarly, we replicate the comparison in Section 5 and compare Dual, Monopoly, and P2P-sponsoring strategies, with continuous consumer usage rates, and our main insights remain the same. The Dual strategy is limited by consumers having continuous usage rates, because its optimality in Section 5 relies on segmenting discrete high-usage consumers into purchases and low-usage consumers into rentals. Thus, under continuous consumer usage rates, we find that the Dual strategy is optimal for low values of λ and above a threshold of heterogeneity (small values of ϕ), while the P2P-sponsoring strategy is optimal above a threshold of heterogeneity and with sufficiently high values of λ.

Alternatively, we considered different models of consumer heterogeneity. For example, a model where high-usage consumers have a usage rate $\lambda + \phi$ and low-usage consumers have a usage rate of $\lambda - \phi$, or a model where high-usage consumers have a usage rate of λ/ϕ and low-usage consumers have a usage rate of $\phi\lambda$. Our main results are robust to these alternative models of consumer heterogeneity. Moreover, our main model captures a superset of the possible market conditions represented by these alternative formulations.

6.5 Further Robustness and Extensions

We consider several smaller modifications to the model or extensions to the firm strategies. For example, we considered a case with different proportions of high and low-usage consumers (i.e., markets with a split different than 50% of each type), and our results hold qualitatively. In addition, we considered a model without multiplicative rental frictions by setting $\mu \overset{lim}{\to} 1$ for the P2P-sponsoring and Dual firms, but including positive flat transaction costs instead. In this simplified case, we find that the qualitative insights of Section 4 and 5 continue to hold; the parameter region where P2P-sponsoring is optimal is larger but also more sensitive to transaction costs.

In another interesting case, we considered the role of matching frictions in the P2P rental market. This is motivated by the observation that in reality, instances of need for the good are potentially correlated (e.g., it could be more difficult to rent products if instances of need of renters and owners are correlated), or alternatively, the P2P-sponsoring firm could choose to restrict the rental supply in some way. We model this by including a parameter denoting the fraction of time where a product owner is unable to match with a renter. Overall, for different rates of mismatch, the results showed no substantial qualitative change to the parameter regions in Section 4 and 5. However, if a P2P-sponsoring firm strategically chooses the matching rate, then it could notably expand the regions of profitability of the P2P-sponsoring strategy. The expansion occurs by shrinking the region of profitability of the Monopoly. By introducing mismatch, the P2P-sponsoring firm is able to again "bridge" strategies

between the base P2P-sponsoring and Monopoly strategies. Intuitively, if the P2P-sponsoring firm is able to control the matching percentage from 0% to 100%, it is essentially enabling a spectrum for the firm between a sales-only strategy (0% match rate) and a "full" P2P-sponsoring strategy (100% match rate). Although it seems that introducing a rate of mismatch would hurt the P2P-sponsoring firm, it can benefit the firm by restricting the rental supply and raising the rental price, enabling the firm to sell more products at a slightly lower price. In fact, we find that for a majority of the parameter region, there is an optimal amount of mismatch rate (i.e., P2P platform matching inefficiency) between 0% and 100%. This suggests that the P2P-sponsoring firm may be able to increase its profits by introducing some risk of non-matching for owners.

7. Conclusion

The sharing economy represents a substantial change in the structure of a market, because consumers also become the providers of goods and services. Our analysis showed that the manufacturer's profit maximizing response depends a great deal on the heterogeneity in consumer usage rates. When consumers are homogeneous in usage rates, surprisingly, both the firm and consumers can be worse off with a P2P rental market. However, when there is heterogeneity in the usage rates, the P2P market makes the willingness to pay of high and low-usage consumers more similar (the equalizing effect). This effect is maximized when the level of heterogeneity in usage rates is intermediate, and explains why the firm and consumers benefit most from P2P rentals precisely in that case. Similarly, we considered the business model problem for the manufacturer and characterized the conditions under which the OEM would prefer to operate offering sales-only, sales plus direct rentals, and sales-only plus P2P rentals. To the best of our knowledge, this is the first paper to analyze the business model problem for an OEM in the presence of P2P rentals, and to highlight the heterogeneity in usage rates as an important driver of market outcomes in P2P rental markets.

The analysis leads to new insights with important implications for OEMs—for example, to decide when and why to adopt different business models. To illustrate, if an OEM is evaluating the introduction of its own P2P platform and can choose across multiple markets with varying degrees of heterogeneity, it may introduce the P2P option (or equivalently, facilitate the operation of an external P2P platform) in markets where the level of consumer heterogeneity is such that the P2P platform would be beneficial. Alternatively, it can choose to operate a sales-plus-rental operation in markets with very large degrees of heterogeneity. The analysis also suggests several avenues for future research; for example, the impact

of competition between OEMs and the analysis of the interaction between incumbent firms and sharing economy companies in settings different than P2P rentals (e.g., on-demand services) are two natural extensions to consider. Similarly, empirical work can be useful to test some of the implications from our research. For example, it could be examined empirically whether the entrance of a P2P rental market affects prices, quantities, and consumer segments, in a way consistent with our findings, e.g., whether sales prices indeed increase when a manufacturer introduces a sponsored P2P platform.

Overall, our findings indicate that OEMs can benefit from P2P rentals in a variety of situations, consistent with many industry examples. For instance, some OEMs are promoting car-sharing as a means of increasing sales. In addition to the example of Tesla mentioned earlier, in 2016 BMW Mini announced a feature to make it easier for buyers to share their vehicles when they are not in use. To quote a board member at BMW, "when your product is not needed because you're away, you can hand over your product to our car-sharing concept so your product can work for you and make money for you" (Schwarzenbauer 2016). Beyond cars, VanMoof (a bicycle manufacturer) partnered with SpinLister (a P2P rental market for bicycles) to sell smart bikes that are easily entered and accessible via SpinLister, and marketed this as an added incentive for purchases. Similarly, real-estate developer Niido partnered with Airbnb for branding to promote their new apartments that explicitly facilitate P2P-sharing for infrequent usage owners. Brunswick (a boat manufacturer) partnered with the P2P boat rental platform Boatbound, based on the common belief that "the P2P boat rental model encourages boating participation and trial, while allowing boat owners to offset some of their ownership costs," which, among other things, offers Brunswick "access to an emerging segment of boating consumers" (Brunswick 2014).

The entry of sharing economy companies is challenging traditional incumbent firms in several industries. We hope that this paper contributes to a better understanding of the potentially beneficial interactions that traditional firms can experience in this new scenario where consumers are not only always right, but also competing to supply.

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Appendix A

Equalizing Effect To more clearly illustrate the equalizing effect, we present how the entry of P2P rental markets affects the willingness-to-pay (WTP) of heterogeneous consumers. The willingness-to-pay for a consumer in the Monopoly firm scenario is: $WTP_i^M = Value_{buying,i}^M - Value_{outside,i}^M = \frac{\lambda_i v_i}{1-\delta} - 0$, where $\lambda_i \in \{\lambda, \phi\lambda\}$ and $Value$ represents the utility of a consumption choice ignoring cost. For that same consumer i, the WTP after the entry of a P2P rental market is: $WTP_i^{P2P} = Value_{buying,i}^{P2P} - \max\{Value_{renting,i}^{P2P}, 0\}$. This simplifies to $WTP_i^{P2P} = WTP_i^M + \frac{1}{1-\delta}(p_r - \mu v_i \lambda_i)$.

Thus, the change in a consumer's willingness-to-pay attributable to the availability of a P2P rental market is $\Delta WTP_i = WTP_i^{P2P} - WTP_i^M = \frac{1}{1-\delta}(p_r - \mu v_i \lambda_i)$. The equalizing effect refers to ΔWTP_i being larger for consumers with lower usage rates and lower per-use valuations v_i, because $\Delta WTP_i|_{\lambda_i=\phi\lambda} > \Delta WTP_i|_{\lambda_i=\lambda}$. This intuition also holds for more general models of consumer usage rates. As a result, the WTP for the durable good is now more "equalized" between high and low-type consumers with P2P markets, enabling increased rent extraction by the firm compared to a sales-only Monopoly strategy.

To visualize this, we present one particular numerical example of the equalizing effect of P2P rentals, with market parameters: $\{\lambda = 0.8, \phi = 0.5, \mu = 0.8\}$. In Figure 7, on the x-axis, the consumers are represented by their per-use valuation $v_i \in [0, 1]$, and on the y-axis, their corresponding WTP for purchasing, $\{WTP_i^M, WTP_i^{P2P}\}$, is plotted. It's clear that the WTP of consumers is much closer together in the presence of P2P rental markets than in the absence of P2P markets. The horizontal lines recommend the respective optimal prices by the P2P-facing and Monopoly firm strategies.

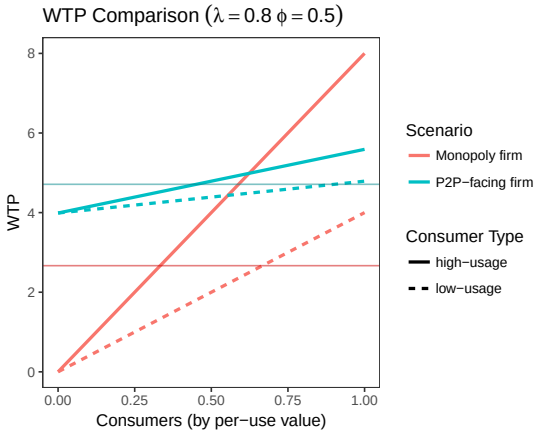

Figure 7: *A comparison of WTP amongst the range of consumer valuation between a firm in the presence or absence of P2P rental markets. The horizontal lines recommend the respective optimal prices by the Monopoly and P2P-facing firms.*

Numerical Illustration of Proposition 5 In Table 2 of this Appendix, we provide a numerical illustration of the various discussed effects of Proposition 5, for markets with low ($\phi = 0.9$), intermediate ($\phi = 0.5$), and high ($\phi = 0.1$) levels of consumer heterogeneity in usage rates. We observe that the firm's profit is highest under the P2P-sponsoring strategy for intermediate ϕ, but Dual provides the highest profit under low ϕ, whereas Monopoly provides the highest profit under high ϕ.

Strategy	$1-\hat{v}_L$ (sales)	$1-\hat{v}_H$ (sales)	Total Sales	Total Rentals	Durable Price	Rental Price	Firm Profit	Consumer Surplus
$\boldsymbol{\phi = 0.1}$								
Monopoly	0.00	0.50	0.25	–	3.75	–	0.938	0.47
Dual	0.00	0.50	0.25	0.02	3.75	0.400	1.013	0.51
P2P-s	0.00	0.42	0.21	0.05	4.75	0.388	1.008	0.50
$\boldsymbol{\phi = 0.5}$								
Monopoly	0.33	0.67	0.50	–	2.50	–	1.250	0.95
Dual	0.00	0.50	0.25	0.09	3.75	0.400	1.312	0.66
P2P-s	0.06	0.51	0.28	0.08	4.69	0.398	1.334	0.67
$\boldsymbol{\phi = 0.9}$								
Monopoly	0.47	0.53	0.50	–	3.56	–	1.78	0.90
Dual	0.28	0.54	0.41	0.09	3.48	0.371	1.75	0.95
P2P-s	0.34	0.41	0.37	0.11	4.71	0.382	1.74	0.88

Table 2: *Market outcomes for Monopoly, Dual, and P2P-sponsoring firm strategies* ($\lambda = 0.75, \mu = 0.8$).

Equilibrium Rental Prices and Marginal Purchasers In Section 3, we discuss the four possible market outcomes $\{C_1, C_2, C_3, C_4\}$. In this appendix subsection, we detail the exact, analytical equilibrium expressions and marginal purchasers (for low and high-usage consumers), when the firm chooses p_s optimally, in terms of the market parameters $\{\delta, \mu, \phi, \lambda\}$.

The marginal high-type purchasers are[10]:

$$v_H = \begin{cases} \frac{\phi(\lambda\phi(2\lambda(\mu-1)+\mu)+(\lambda-4)\mu)}{2(\phi+1)(\lambda^2(\mu-1)\phi-\mu)} & if\ C_1 \\ \frac{\lambda(\phi+1)(\lambda(\mu-1)+\mu)-2\mu}{2(\lambda^2(\mu-1)(\phi+1)-\mu)} & if\ C_2 \\ \frac{(\lambda-1)\lambda(\mu-1)\mu\phi(\lambda\phi+\lambda-2)-\text{A1}}{\lambda^3(\mu-1)\phi^2+(\lambda-1)^2\lambda(\mu-1)\mu\phi-\lambda\mu} & if\ C_3 \\ \frac{\lambda^2\phi+(\lambda-1)\mu(\lambda(\phi+2)-2)}{2\lambda^2\phi+2(\lambda-1)^2\mu} & if\ C_4. \end{cases}$$

The marginal low-type purchasers are $v_L = \min\{v_H/\phi, 1\}$, except for outcomes $\{C_2, C_4\}$ where $v_L = 1$

[10] where $A1 = \frac{\lambda\left(\lambda(\mu-1)\phi^2-\mu\right)\left((\mu-1)\phi^2(\lambda(\mu-2)-\mu)+\mu\phi(\lambda(-\mu)+\lambda+\mu)+\mu\right)}{2(\phi+1)(\mu(\phi-1)-\phi)}$

by definition since low-usage consumers are not buying in those outcomes.

Similarly, the equilibrium rental price is slightly different depending on the market outcomes, due to variations in the supply and demand[11]:

$$p_r = \begin{cases} \frac{\mu(\mu(\phi(2\lambda(\lambda\phi+\lambda-1)-1)-1)-2\lambda\phi(\lambda\phi+\lambda-1))}{2(\phi+1)(\lambda^2(\mu-1)\phi-\mu)} & if\ C_1 \\ \frac{\mu(\lambda(\mu-1)(2\lambda(\phi+1)-1)-\mu)}{2(\lambda^2(\mu-1)(\phi+1)-\mu)} & if\ C_2 \\ \frac{\mu\left((\mu-1)\phi^3(A2)-(\mu-1)\phi^2(A3)+2\lambda^2(1-\mu)^2\phi^4+\mu\phi(1-2(\lambda-2)\lambda(\mu-1))+\mu\right)}{2(\phi+1)(\mu(\phi-1)-\phi)(\lambda^2(\mu-1)\phi^2+(\lambda-1)^2(\mu-1)\mu\phi-\mu)} & if\ C_3 \\ \frac{2\lambda^2\mu\phi+(\lambda-1)\mu((\lambda-1)\mu+\lambda)}{2\lambda^2\phi+2(\lambda-1)^2\mu} & if\ C_4. \end{cases}$$

With these rental prices, it is easy to identify the marginal renter in each outcome; the marginal renter is $\tilde{v} = \frac{p_r}{\mu}$.

Lastly, incorporating these consumer and rental market outcomes, the P2P-sponsoring firm's profits are thus:

$$\pi_s \cdot (1-\delta) = \begin{cases} \frac{2\lambda\phi+\mu\phi+\mu-2\lambda\mu\phi}{2\phi+2} & if\ C_1 \\ \frac{1}{2}(\lambda+\mu-\lambda\mu) & if\ C_2 \\ \frac{(\mu-1)\phi^2(\lambda(\mu-2)-\mu)+\mu\phi(\lambda+\mu-\lambda\mu)+\mu}{2(\phi+1)(\mu(1-\phi)+\phi)} & if\ C_3 \\ \frac{1}{2}(\lambda+\mu-\lambda\mu) & if\ C_4. \end{cases}$$

Or, in terms of the firm's durable price choice:

$$\pi_s(p_s) = \begin{cases} \frac{\lambda p_s(\phi(\mu-2\lambda(\mu-1))+\mu+(\delta-1)p_s(\phi+1))}{2\lambda^2(1-\mu)\phi+2\mu} & if\ C_1 \\ \frac{\lambda p_s(\phi+1)(-\lambda\mu+\lambda+\mu+(\delta-1)p_s)}{2(\lambda^2(1-\mu)(\phi+1)+\mu)} & if\ C_2 \\ \frac{\lambda p_s\left((\mu-1)\phi^2(-\lambda(\mu-2)+\mu+(\delta-1)p_s)-\phi(\mu(\lambda(-\mu)+\lambda+\mu)+(\delta-1)p_s)+\mu(-\delta p_s+p_s-1)\right)}{2\lambda^2(\mu-1)\phi^2+2(\lambda-1)^2(\mu-1)\mu\phi-2\mu} & if\ C_3 \\ \frac{\lambda p_s\phi(-\lambda\mu+\lambda+\mu+(\delta-1)p_s)}{2\lambda^2\phi+2(\lambda-1)^2\mu} & if\ C_4. \end{cases}$$

Appendix B

Proof of Section 3 Owner Supply Behavior: In the main text of Section 3, we state that, in equilibrium, $\{v_H, v_L\} \geq p_r$ such that all product owners will utilize the good in periods when they demand it. In this subsection, we prove this must always hold in equilibrium. v_H represents the per-use valuation of the marginal high-type purchaser, and it's clear that $v_L > v_H$. Consider the edge case where $v_H = \hat{p}_r^0$, where $\hat{p}_r^0$ is the consumer expected market rental price. High-usage consumers do

[11] where $A2 = \lambda^2(\mu-2)(\mu+1) - 2\lambda\left(\mu^2-1\right) + (\mu-1)\mu$ and $A3 = \lambda^2\left(\mu^2+\mu+2\right) - 2\lambda\left(\mu^2+1\right) + \mu^2 + \mu$.

not demand rentals in this case, so the rental supply and demand are: $D_r = \phi\lambda\left[v_L - \frac{\hat{p}_r^0}{\mu}\right]^+$, $S_r = (1-\lambda)(1-v_H) + (1-\phi\lambda)(1-v_L)$. As discussed in the main text, the resulting equilibrium market rental price is some p_r^0, and following the rational expectations equilibrium, $p_r^0 = \hat{p}_r^0$. Now, consider a perturbation to the expected rental price $\hat{p}_r^{'} = \hat{p}_r^0 + \epsilon$ while v_H is unchanged, such that $v_H < p_r^{'}$ and $\epsilon = p_r^{'} - v_H$ owners rent out the good in all periods. The rental demand remains unchanged, but the rental supply is now: $S_r^{'} = (1-\lambda)(1-v_H) + (1-\phi\lambda)(1-v_L) + \lambda\epsilon$. The resulting market rental price does not satisfy the rational expectations equilibrium; the resulting market price is $p_r^{'} = p_r^0 - \frac{2\epsilon\mu}{\phi}$. In fact, $p_r^{'} < v_H < \hat{p}_r^{'}$. Thus, there cannot be an equilibrium market outcome where some owners expect to rent out the good in all periods; if that were to occur, with rational expectations, this would attract additional consumers to buy into the market and subsequently lower the rental price such that there were no longer owners who bought purely to rent out. All rational expectations equilibrium will have $v_H \geq p_r$. ■

Proof of Proposition 1: In Proposition 1, consumers have homogeneous stochastic usage rates, λ. Let $\hat{v}$ represent the marginal buying consumer, indifferent between buying and renting/outside option, such that consumers with valuation $v_i > \hat{v}$ will purchase. The demand and supply for rentals are then: $D_r = \lambda\left(\hat{v} - \frac{p_r}{\mu}\right); S_r = (1-\lambda)(1-\hat{v})$. The resulting equilibrium P2P rental price is: $p_r = \frac{\mu(\lambda-(1-\hat{v}))}{\lambda}$. The valuation of the marginal purchasing consumer is given by (where $p_s^{'} = p_s(1-\delta)$): $\hat{v} = \frac{p_s^{'}-p_r}{\lambda(1-\mu)} = \frac{\lambda p_s^{'}+\mu(1-\lambda)}{\lambda^2(1-\mu)+\mu}$. The P2P-facing firm's profit problem is: $\pi^P = \left(1 + \frac{\mu-\lambda p_s^{'}+\mu(1-\lambda)}{\lambda^2(1-\mu)+\mu}\right)p_s$ and thus the profit-maximizing durable price: $p_s = \frac{1}{2}\left(\lambda(1-\mu)+\mu\right)$. The resulting number of sales under the P2P-faicng firm is $1-\hat{v} = \frac{1}{2} - \frac{\mu(1-\lambda)}{2(\lambda^2(1-\mu)+\mu)}$, while under the Monopoly firm, number of sales is $\frac{1}{2}$, so it's clear that fewer consumers purchase under a P2P-facing firm. The marginal renter is $\tilde{v} = \frac{p_r}{\mu} = \frac{1}{2} - \frac{\lambda(1-\lambda)(1-\mu)}{2(\lambda^2(1-\mu)+\mu)}$, so it's also clear that there is more increased participation under a P2P-facing firm than in the absence of a P2P market, i.e., $\tilde{v} < \frac{1}{2} = \hat{v}_{Mon}$.

We also prove that, with homogeneous consumer usage rate, the P2P-facing firm's profit and resulting consumer welfare are lower than under the Monopoly firm. The Monopoly firm's profit function is: $\pi^M = (1 - \frac{p^M(1-\delta)}{\lambda})p_s^M$, so the optimal price is $p_s^M = \frac{\lambda}{2}(1-\delta)$ and the Monopoly firm's optimal profit is $\frac{\lambda}{4(1-\delta)}$. On the other hand, the optimal profit for a P2P-facing firm is: $\frac{\lambda}{4(1-\delta)}\left(\frac{(\lambda+\mu-\lambda\mu)^2}{(\lambda^2(1-\mu)+\mu)}\right)$. Thus, $\pi^P - \pi^M$ is always negative, because $\frac{(\lambda+\mu(1-\lambda))^2}{(\lambda^2+\mu(1-\lambda^2))} \leq 1$. Under a Monopoly firm, consumer welfare comes only from purchases, while under a P2P-facing firm, welfare comes from both purchases and rentals. Monopoly consumer welfare is: $CS^M = \frac{\lambda}{8(1-\delta)}$. P2P-facing consumer welfare is: $CS^P = \frac{\lambda(\lambda+\mu-\lambda\mu)^2}{8(1-\delta)(\lambda^2(1-\mu)+\mu)}$. Following this, $CS^P < CS^M \rightarrow 0 < (1-\lambda)^2\left(\mu + \lambda^2(1-\mu)\right)(1-\mu)\mu$, which always

holds true if $0 < \mu < 1$. ■

Proof of Proposition 2: In this Proposition, we identify the buying and renting behaviors of consumers under different market conditions $\{\lambda, \phi, \mu\}$, as visualized in the main paper. These are described as four possible market outcomes $\{C_1, C_2, C_3, C_4\}$. These outcomes occur because of the piecewise buying and renting nature of low and high-usage consumers, e.g. since $v_L = \min\{1, \frac{v_H}{\phi}\}$ or cases when $v_H < \frac{p_r}{\mu}$. The firm recognizes this and optimally prices to maximize profits. In this proof, we prove that there exists thresholds of the parameters $\{\lambda, \phi\}$ where low-type consumers do not purchase and high-type consumers do not rent. First, note that as ϕ approaches 1 (i.e., $\phi \to 1$), the market outcome degenerates to C_1, where both types buy and both types rent because low and high-type consumers become homogeneous. Similarly, as $\phi \to 0$, the market degenerates to condition C_2 because the low-type consumers become nonexistent and thus have no willingness to pay to purchase the good. This ensures the existence of at least one threshold of ϕ where low-usage consumers do not purchase.

Next, we show that there must exist only one such threshold, given other market parameters. C_1 and C_3 are market outcomes where low-usage consumers purchase; in those outcomes, low-usage purchases monotonically decrease as ϕ decreases:[12]

$$\frac{\partial \hat{v}_{L,1}}{\partial \phi} = -\frac{2\lambda^4(\mu-1)^2\phi^2 + \lambda^3(\mu-1)\mu(\phi+1)^2 - 2\lambda^2(\mu-1)\mu(4\phi+1) + 4\mu^2}{2(\phi+1)^2\left(\mu - \lambda^2(\mu-1)\phi\right)^2} < 0.$$

$$\frac{\partial \hat{v}_{L,3}}{\partial \phi} = \frac{1}{2}\left(B1 - \frac{\lambda\mu}{\lambda^2(\mu-1)\phi^2 + (\lambda-1)^2(\mu-1)\mu\phi - \mu} - \frac{2(\mu-1)}{(2\mu-1)(\phi+1)^2} + \frac{\mu(\mu-1)}{(2\mu-1)(\mu(-\phi)+\mu+\phi)^2}\right) < 0$$

For given $\{\lambda, \mu\}$, there exists some $\phi = \hat{\phi}$, below which ϕ is sufficiently low such that low-usage consumers either stop purchasing or the firm opts to not sell to low-usage consumers, so that C_2 or C_4 is the resulting market outcome. We show this is unique, given market parameters, by showing that the rental price under C_2 decreases as ϕ decreases: $\frac{\partial p_{r,2}}{\partial \phi} = \frac{\lambda^2(1-\mu)\mu(\lambda(1-\mu)+\mu)}{2(\mu-\lambda^2(\mu-1)(\phi+1))^2} > 0$ and $\frac{\partial p_{r,4}}{\partial \phi} = \frac{(\lambda-1)\lambda^2\mu(\lambda(\mu-1)-\mu)}{2(\lambda^2\phi+(\lambda-1)^2\mu)^2} > 0$. Because the rental price decreases as ϕ decreases for all $\phi < \hat{\phi}_2$, the market rental price is lower than $p_r|_{\phi=\hat{\phi}}$, further reducing incentive for low-usage consumers to purchase. Thus, low-usage consumers forego purchases $\forall \phi \leq \hat{\phi}$.

We also prove that at sufficiently high values of λ, high-usage consumers do not rent in the market (outcomes C_3, C_4). Consider ϕ decreasing from 1, under outcome C_1,, then there is a unique value of $\phi = \tilde{\phi}_1$ where, in equilibrium, $\hat{v}_{H,1} = p_{r,1}/\mu$:

$$\tilde{\phi}_1 = \frac{\lambda(2-3\mu) + 3\mu - 2\lambda^2(1-\mu) - \sqrt{(\lambda(2\lambda(\mu-1) - 3\mu + 2) + 3\mu)^2 - 4\lambda\mu^2}}{2\lambda\mu}.$$

[12] where $B1 = \frac{\mu(\lambda(\mu-1)\phi((\lambda-2)\lambda(\mu-2)+\mu)+\mu(\lambda(-(\lambda-4)\lambda(\mu-1)-5\mu+3)+2(\mu-1)))}{\left(\lambda^2(\mu-1)\phi^2+(\lambda-1)^2(\mu-1)\mu\phi-\mu\right)^2}$

It is easy to show numerically that if λ is sufficiently large, $\tilde{\phi}_1 > \hat{\phi}$, and the market outcome first transitions from C_1 to C_3 as ϕ decreases. If so, as ϕ further decreases, for $\phi < \tilde{\phi}_1$, the market must first transition to outcome C_4 (since C_1 is suboptimal for $\phi < \tilde{\phi}_1$), and subsequently, as ϕ decreases even further, $\phi \to 0$, to outcome C_2. This proves the existence of $\tilde{\phi}_2$, such that high-usage consumers do not rent for $\tilde{\phi}_2 < \phi < \tilde{\phi}_1$ and sufficiently high λ. ■

Proof of Proposition 3: In the proof of this Proposition, we use the intermediate value theorem to prove that the P2P-facing firm's profit is larger than the Monopoly firm's profit for intermediate regions of ϕ. To prove this, we first establish the Monopoly firm's optimal pricing strategy and profits, for market parameters $\{\lambda, \phi\}$, facing heterogeneity in consumer usage. For the Monopoly firm, the optimal profit from selling to both types of consumers is $\frac{\lambda\phi}{2(1-\delta)(1+\phi)}$ and the optimal profit from selling to just high-type consumers is $\frac{\lambda}{8(1-\delta)}$; it is easy to see that when $\phi < \frac{1}{3}$, the Monopoly firm prices so that they sell just to high type consumers. Further, the Monopoly firm's profit is monotonically increasing in ϕ for $\phi \in [\frac{1}{3}, 1]$ and is constant for $\phi \in [0, \frac{1}{3})$: $\frac{\partial \pi^M}{\partial \phi}|_{\phi \in [\frac{1}{3},1]} = \frac{-\lambda}{2(\delta-1)(\phi+1)^2}$; $\quad \frac{\partial \pi^M}{\partial \phi}|_{\phi \in [0,\frac{1}{3})} = 0$.

Region of $\phi \in [\frac{1}{3}, 1]$. The P2P-facing firm has lower profits than the Monopoly firm at $\phi = 1$ (as shown in Proposition 1), but at $\phi = \frac{1}{3}$, the P2P-facing firm has higher profits than the Monopoly firm, no matter which market outcome $C_1 - C_4$ occurs. Using this and showing that both the P2P-facing firm and Monopoly firm's profits are monotonically increasing in ϕ, the intermediate value theorem proves there must exist a single crossover point of the profits of these two firms. This crossover point represents the rightmost value of the intermediate value of ϕ where the P2P-facing firm is more profitable. At $\phi = \frac{1}{3}$, the differences in the firm strategy profits are:

$$\pi^{P,1} - \pi^M|_{\phi=1/3} = \frac{\lambda\mu((\lambda-4)\lambda(\mu-1)+4\mu-3)}{8(\delta-1)\left(\lambda^2(\mu-1)-3\mu\right)},$$
$$\pi^{P,2} - \pi^M|_{\phi=1/3} = \frac{\lambda\mu(4(\lambda-2)\lambda(\mu-1)+4\mu-3)}{8(\delta-1)\left(4\lambda^2(\mu-1)-3\mu\right)},$$
$$\pi^{P,3} - \pi^M|_{\phi=1/3} = \frac{\lambda\mu\left(\lambda^2(\mu-1)((\mu+7)\mu+4)-2\lambda(\mu-1)((\mu+12)\mu+8)+((\mu+16)\mu+4)\mu-12\right)}{8(\delta-1)(2\mu+1)\left(\lambda^2(\mu-1)(3\mu+1)-6\lambda(\mu-1)\mu+3(\mu-4)\mu\right)},$$
$$\pi^{P,4} - \pi^M|_{\phi=1/3} = \frac{(\lambda-1)\lambda\mu(\lambda(\mu-5)-\mu+3)}{8(\delta-1)\left(\lambda^2+3(\lambda-1)^2\mu\right)},$$

Each of the above differences are analytically positive for any $\{0 < \lambda < 1, \frac{3}{4} \leq \mu < 1\}$. Thus, the upper envelope of the possible profits at $\phi = \frac{1}{3}$, regardless of the market outcome, is greater than the Monopoly firm's profit. Lastly, we show that each of the P2P-facing firm's profit functions are increasing in ϕ.

The derivative of the profits with respect to ϕ is given by

$$\frac{\partial \pi^{P,1}}{\partial \phi} = \frac{\lambda\left(2\lambda^3(1-\mu)^2\phi + 2\lambda\mu(1-\mu)(\phi+2) + \mu(\phi+1)(\mu-\lambda^2(1-\mu))\right)(\mu((2\lambda-1)\phi-1)-2\lambda\phi)}{8(1-\delta)(\phi+1)^2\left(\lambda^2\phi-\mu^2\lambda^2\phi\right)^2},$$

$$\frac{\partial \pi^{P,2}}{\partial \phi} = \frac{\lambda\mu(\lambda+\mu-\lambda\mu)^2}{8(1-\delta)\left(\mu-\lambda^2(\mu-1)(\phi+1)\right)^2},$$

$$\frac{\partial \pi^{P,3}}{\partial \phi} = \frac{\lambda\left((1-\lambda)\mu^2(\phi-1)\phi+\mu\left(\lambda\left(\phi-3\phi^2\right)+\phi^2+1\right)+2\lambda\phi^2\right)(L_1)}{8(1-\delta)(\phi+1)^2(\mu+\phi-\mu\phi)^2\left(\mu\left(\lambda^2\phi^2+(\lambda-1)^2(-\phi)-1\right)-\lambda^2\phi^2+(\lambda-1)^2\mu^2\phi\right)^2},$$

where $L_1 = (\mu-1)^2\phi^4\left(2\lambda^3+(\lambda-1)^3\mu^3-(\lambda-1)((\lambda-4)\lambda+1)\mu^2-(\lambda(\lambda+3)-2)\lambda\mu\right)$

$+\mu^2\phi\left(\left((5-2\lambda)\lambda^2-3\right)\mu^2+(\lambda-1)^3\mu^3+\lambda(\lambda(\lambda+2)-16)\mu+\lambda(13-4\lambda)+9\mu-4\right)$

$+(\mu-1)\mu\phi^3\left(\lambda^3(\mu-1)((\mu-5)\mu+7)-\lambda^2(3\mu-4)((\mu-3)\mu+3)+\lambda(\mu-1)(3(\mu-3)\mu+8)-(\mu-4)(\mu-1)\mu\right)$

$--3(\mu-1)\mu\phi^2(\lambda(\mu-1)-\mu+2)\left((\lambda-1)^2\mu^2-2(\lambda-1)\lambda\mu+\lambda\right)+\mu^2(\mu((\lambda-4)\lambda(\mu-1)+3\mu-1)-1),$

$$\frac{\partial \pi^{P,4}}{\partial \phi} = \frac{(\lambda-1)^2\lambda\mu(\lambda(-\mu)+\lambda+\mu)^2}{8(1-\delta)\left(\lambda^2\phi+(\lambda-1)^2\mu\right)^2}.$$

It is easy to show that these derivatives are positive, which is intuitive because ϕ increases total demand. Thus, the actual firm profit, determined by the upper envelope of these four profit functions when they are valid, $\hat{\pi}^P = \max\{\pi^{P,1}, \pi^{P,2}, \pi^{P,3}, \pi^{P,4}\}$, will also be monotonically increasing in ϕ. Because $\hat{\pi}^P$ is lower than the Monopoly profit at $\phi = 1$, above the Monopoly profit at $\phi = \frac{1}{3}$, and both are monotonically increasing, by the intermediate value theorem, given some value of $\{\lambda, \mu\}$, there must exist one unique crossover point between these two profits, which we denote by ϕ_2^{π}.

Region of $\phi \in [0, \frac{1}{3})$. We already have proved above that the P2P-facing firm's profit will always be larger than the Monopoly firm's profit at $\phi = \frac{1}{3}$. At $\phi = 0$, as we show in Proposition 2, the P2P-facing firm's market outcome must be C_2, where low types do not purchase. It's easy to show that the difference between the P2P-facing firm and the Monopoly firm is negative: $\pi^{P,2} - \pi^M|_{\phi=0} = \frac{-(\lambda-1)^2\lambda(1-\mu)\mu}{8(1-\delta)(\lambda^2(1-\mu)+\mu)}$. As shown above, the P2P-facing firm's profits are monotonically increasing in ϕ for any λ and $\frac{3}{4} \leq \mu < 1$. Using the intermediate value theorem again, for any given $\{\lambda, \mu\}$, there must exist one unique crossover point between the P2P-facing firm's profit $\hat{\pi}^P$ and the Monopoly firm's profit, which we denote by ϕ_1^{π}.

Combining the proofs about the two regions $\{\phi \in [0, \frac{1}{3}), \phi \in [\frac{1}{3}, 1]\}$, we thus prove that, for any given λ, there must exist two and only two crossover values of ϕ, between the Monopoly firm profit and the P2P-facing firm profit, one in the region of $\phi \in [0, \frac{1}{3})$ and one in the region of $\phi \in [\frac{1}{3}, 1]$. In the intermediate region between these two points, the P2P-facing firm is more profitable than the Monopoly firm. ■

Proof of Proposition 4: Similarly to the proof of Proposition 3, we show that for the region of $\phi \in [\frac{1}{3}, 1]$, consumer welfare is lower under the P2P-facing firm than under the Monopoly firm, in all of

the four possible market conditions. Vice versa, we also show that for the region of $\phi \in [0, \frac{1}{3})$, there is a threshold of ϕ, above which, consumer welfare is higher under the P2P-facing firm.

Region of $\phi \in [\frac{1}{3}, 1]$. For any given λ, in this region of ϕ, under the Monopoly firm, the consumer welfare is the surplus obtained by purchasers: $CS^M = \frac{\lambda(1-(1-\phi)\phi)}{2(1-\delta)(1+\phi)}$. Consumer welfare under the P2P-facing firm is the surplus obtained by both purchasers and renters:

$$
\begin{aligned}
CS^{P,1} &= \lambda\left(4\lambda^2\phi((\phi-1)\phi+1)+\mu^2\left(\phi\left(4\lambda\left(\lambda\phi^2-(\lambda+1)\phi+\lambda-1\right)-3\phi+10\right)-3\right)\right. \\
&\quad \left.+4\mu(1+\phi(1-\lambda(2\lambda((1-\phi)\phi)+\phi+1)+\phi-2))\right)/ \\
&\quad 16(\delta-1)(\phi+1)\left(\lambda^2(\mu-1)\phi-\mu\right), \\
CS^{P,2} &= \frac{\lambda(\phi+1)(\lambda+\mu-\lambda\mu)^2}{16(1-\delta)\left(\lambda^2(1-\mu)(\phi+1)+\mu\right)}, \\
CS^{P,3} &= \frac{\lambda\left(2\mu^3(\phi-1)\phi\left(\lambda^2\phi\left(2\phi^2-5\phi+5\right)+\lambda\left(8\phi^2-11\phi-1\right)-5\phi^2+4\phi+3\right)-L_2\right)}{16(\delta-1)(\phi+1)(\mu(\phi-1)-\phi)\left(\mu\left(\lambda^2\phi^2+(\lambda-1)^2(-\phi)-1\right)-\lambda^2\phi^2+(\lambda-1)^2\mu^2\phi\right)}, \\
&\quad \text{where } L_2 = \mu^2\left(12\lambda^2\phi^5-\left(23\lambda^2-14\lambda+11\right)\phi^4+2\left(13\lambda^2-21\lambda+8\right)\phi^3-\left(11\lambda^2-18\lambda-6\right)\phi^2+2(\lambda-4)\phi+1\right) \\
&\quad +4\mu\phi\left(3\lambda^2\phi^4-\left(4\lambda^2-\lambda+1\right)\phi^3-\left(\lambda^2-\lambda-1\right)\phi+(1-2\lambda)^2\phi^2-1\right) \\
&\quad -4\lambda^2\phi^3\left(\phi^2-\phi+1\right)+3(\lambda-1)^2\mu^4(\phi-1)^2\phi^2, \\
CS^{P,4} &= \frac{\lambda\phi(\lambda+\mu-\lambda\mu)^2}{16(1-\delta)\left(\lambda^2\phi+(1-\lambda)^2\mu\right)}.
\end{aligned}
$$

It is easy to show that each of these values are smaller than CS^M for any value of $\{\lambda, \mu\}$ in the region of $\phi \in [\frac{1}{3}, 1]$, hence whichever market outcome occurs under a P2P-facing firm, the consumer surplus will be smaller.

Region of $\phi \in [0, \frac{1}{3})$. For any given λ, in this region of ϕ, the Monopoly firm sells only to high-type consumers, with a resulting consumer welfare of: $CS^M = \frac{\lambda}{16(1-\delta)}$. Using the same consumer surplus functions from the previous paragraph, for any $0 < \lambda < 1$ and $\frac{3}{4} \leq \mu < 1$, if $\{\lambda, \phi\}$ are sufficiently high such that market outcome C_1 or C_3 occurs, then consumer welfare is always higher under the P2P-facing firm. Otherwise, if market outcome C_2 occurs—which always occurs as $\phi \to 0$—consumer welfare is higher under the P2P-facing firm when $\frac{(1-\mu)(1-\lambda)^2}{\lambda^2\mu-\lambda^2-2\lambda\mu+2\lambda+\mu} < \phi < \frac{1}{3}$. If market outcome C_4 occurs, consumer welfare is higher under the P2P-facing firm when $\frac{1-\lambda}{\mu(1-\lambda)+2\lambda} < \phi < \frac{1}{3}$. ■

Proof of Lemma 1: In Lemma 1, we prove that the P2P-sponsoring firm's profit is either not affected by α or has an optimal value of $\alpha = 0$. With market outcomes C_1 and C_3, the optimal firm profits are not affected by α (because earnings from α end up being canceled out by an equal decrease in the durable good price); hence the profit functions are the same as in Appendix A.

Otherwise, the firm profit, under C_1, with revenue from α fees, is:

$$\pi^{P,1} = -\frac{\lambda(\mu-1)\left(\mu^2\left(L_3\right)+4\lambda\mu\phi(\phi+1)\left(L_4\right)-4\lambda^2\phi^2(\phi+1)^2\right)}{8(\delta-1)(\phi+1)\left(\mu^2\left(\alpha^2(\phi-1)^2+(\phi+1)^2\left(\lambda^2\phi-1\right)\right)-\mu(\phi+1)^2\left(2\lambda^2\phi-1\right)+\lambda^2\phi(\phi+1)^2\right)},$$

where $L_3 = \alpha^2(\phi-1)^2\left(4\lambda^2\phi^3+\left(8\lambda^2-8\lambda-1\right)\phi^2+\left(4\lambda^2-8\lambda+2\right)\phi-1\right)$

$-2\alpha(\phi+1)(\phi-1)^2\left(2\lambda^2\phi^2+\left(2\lambda^2-2\lambda-1\right)\phi-1\right)-(\phi+1)^2(-2\lambda\phi+\phi+1)^2,$

and $L_4 = \alpha(\phi-1)^2(\lambda\phi+\lambda-1)+(\phi+1)((2\lambda-1)\phi-1).$

Using Mathematica, we show that $\frac{d\pi^{P,1}}{d\alpha} \leq 0$ for any $0 < \lambda < 1$ and $1/2 \leq \mu \leq 1$; hence the optimal value of α is 0. Similarly, the firm profit under C_3 is:

$$\pi^{P,3} = \frac{\lambda\left(\mu^2\left(L_5\right)+L_6\right)}{8(\delta-1)\left(L_7\right)}$$

where $L_5 = -\alpha^2(\phi-1)^2\left(4\lambda^2\phi^3+\left(3\lambda^2-6\lambda-1\right)\phi^2-2(\lambda-1)\phi-1\right)$

$+2\alpha(\phi-1)\left(4\lambda^2\phi^4-\left(5\lambda^2+2\lambda+1\right)\phi^3+\left(-3\lambda^2+4\lambda+1\right)\phi^2+(2\lambda-1)\phi+1\right)$

$+\lambda^2\phi^2\left(13\phi^2-10\phi+1\right)+2\lambda\phi\left(-5\phi^3+3\phi^2-3\phi+1\right)+\left(\phi^2+1\right)^2,$

$L_6 = 2(\alpha-1)\mu^3(\phi-1)\phi\left(\alpha(\phi-1)\left(\lambda^2\phi(2\phi+1)-\lambda(2\phi+1)-\phi+1\right)+(\lambda-1)\left(\lambda(3\phi-1)\phi-\phi^2-1\right)\right)$

$+(\alpha-1)^2(\lambda-1)^2\mu^4(\phi-1)^2\phi^2$

$+4\lambda\mu\phi^2\left(\alpha\left((\phi-1)^2-\lambda\left(\phi^3-2\phi^2+1\right)\right)+\lambda\left(\phi-3\phi^2\right)+\phi^2+1\right)+4\lambda^2\phi^4,$

$L_7 = -\mu^2\left(L_8\right)+\mu\phi\left(\phi^2\left(-2\alpha\lambda^2+2\lambda-1\right)+2\phi\left((\alpha-1)\lambda^2+\lambda-1\right)+2\lambda^2\phi^3-1\right)$

$+(\alpha-1)(\lambda-1)^2\mu^3(\phi-1)\phi(\alpha(\phi-1)+\phi+1)-\lambda^2\phi^3(\phi+1),$

and $L_8 = \alpha^2(\phi-1)^2\left((\lambda-1)^2\phi+1\right)-2\alpha(\phi-1)\left(\lambda^2\phi^2+(\lambda-1)^2(-\phi)-1\right)$

$+(\phi+1)\left(\lambda^2\phi\left(\phi^2-3\phi+1\right)+2\lambda\phi(2\phi-1)-2\phi^2+1\right).$

Again, using Mathematica, we show that $\frac{d\pi^{P,3}}{d\alpha} \leq 0$ for any $0 < \lambda < 1$ and $1/2 \leq \mu \leq 1$, hence the optimal value of α is 0. ■

Proof of Lemma 2: To prove Lemma 2, we first show that for $\phi < \tilde{\phi}^D$, the Dual firm is able to optimally segment consumers, independently using monopolistic pricing for both high and low-usage consumers, such that half of each group participates, without any high-usage consumers being attracted to rent. Second, we prove that this is the most profitable option for the Dual firm for $\phi < \tilde{\phi}^D$ and otherwise, sales-only Monopoly firm profits are most profitable for $\phi \geq \tilde{\phi}^D$.

The Dual firm has three possible optimal profits under different market outcomes: $\pi^{D,1} = \frac{\lambda(\nu-1)\phi(\phi+1)}{2(\delta-1)((2-4\nu)\phi+\phi^2+1)}$ when both high and low-usage consumers both buy and rent, $\pi^{D,2} = \frac{\lambda\nu\phi+\lambda}{8-8\delta}$ when the Dual firm optimally segments consumers, selling optimally to high types and renting optimally to low types, and $\pi^{D,3} = \frac{2\lambda\phi(\nu-\phi-1)}{(\delta-1)((\nu^2-9\nu+8)\phi+4\phi^2+4)}$ when both types buy, but only low types rent. These profit equations are derived using multivariate analytical maximization on the Dual firm's choice of both

the durable good price and rental price.

Part 1. In this part, we identify values of ϕ where $\pi^{D,2}$ is a valid equilibrium profit and where it is not. Intuitively, if ϕ is sufficiently large, it is invalid because the firm cannot separately optimally price rentals and sales for low/high-usage consumers respectively, without high-usage consumers being attracted away to rentals. Those optimal monopolistic prices are a durable price of $\frac{\lambda}{2(1-\delta)}$ and a per-rental price of $\frac{\nu}{2}$. If consumers can be segmented by the Dual firm, half of high-usage consumers would buy and half of the low-usage consumers would rely on rentals, with the other two halves with the outside option. We identify the value of ϕ such that the Dual firm can segment consumers. The net utility of renting and buying for high-usage consumers is is $U^{B,H} = \frac{2\lambda v - \lambda}{2(1-\delta)}$ (buying) and $U^{R,H} = \frac{2\nu\lambda v - \nu\lambda}{2(1-\delta)}$ (renting). The net utility of low-usage consumers is $U^{B,L} = \frac{2\phi\lambda v - \lambda}{2(1-\delta)}$ and $U^{R,L} = \frac{2\mu\phi\lambda v - \nu\phi\lambda}{2(1-\delta)}$. For low-usage consumers, we identify that $U^{B,L} < U^{R,L}$ when $\phi < \frac{1}{\nu + 2(1-\nu)v}$. The low-usage consumer with the highest valuation ($v = 1$) will not purchase at the optimal prices if $\phi < \frac{1}{2-\nu}$. For high-usage consumers, at these monopolistic optimal prices, we know that $U^{B,H} - U^{R,H} \geq 0 \ \forall v > \frac{1}{2}$ and $U^{R,H} < 0 \ \forall v < \frac{1}{2}$, such that consumers with valuation $v_i \geq \frac{1}{2}$ will buy and those with $v_i < \frac{1}{2}$ will prefer the outside option, no matter the value of ϕ. Therefore, if $\phi < \frac{1}{2-\nu}$, consumers are sufficiently heterogeneous such that the Dual firm can sustain an equilibrium that separately optimally prices for both segments of consumers.

Building on this segmenting result, we identify the threshold $\tilde{\phi}^D$ where $\pi^{D,2} < \pi^M$ by comparing the two respective profit functions. This threshold is when $\nu < \frac{3\phi - 1}{\phi^2 + \phi}$, or $\tilde{\phi}^D = \frac{1}{2\nu}\left\{3 - \left(\sqrt{(9-\nu)(1-\nu)} + \nu\right)\right\}$. We show that $\tilde{\phi}^D$ is the binding threshold; i.e. that $\tilde{\phi}^D < \frac{1}{2-\nu}$ and for $\phi < \tilde{\phi}^D$, the Dual firm is able to sustain the $\pi^{D,2}$ equilibrium with separately optimal prices. This is clear because $\tilde{\phi}^D < \frac{1}{2-\nu} \Rightarrow -\sqrt{\frac{(9-\nu)(1-\nu)}{\nu^2}} - \frac{2}{2-\nu} - 1 + \frac{3}{\nu} < 0$ and the term on the left will always be below zero for any $0 < \nu < 1$.

Part 2. Using the above results of Part 1 of this proof, we now prove that $\pi^{D,2}$ is most profitable for $\phi < \tilde{\phi}^D$ and π^M is most profitable for $\phi \geq \tilde{\phi}^D$. Consider $\pi^{D,2} > \pi^{D,1}|_{\phi \in [0, \tilde{\phi}^D)}$ and $\pi^{D,2} > \pi^{D,3}|_{\phi \in [0, \tilde{\phi}^D)}$. Using Mathematica, it is clear that both of these inequalities will always hold true in the region of $\phi \in [0, \tilde{\phi}^D)$. Alternatively, for $\phi \in [\tilde{\phi}^D, 1]$, we instead compare π^M to $\{\pi^{D,1}, \pi^{D,3}\}$. Using Mathematica on the equations presented initially, it is clear that the following equations will always be true in the region of $\phi \in [\tilde{\phi}^D, 1]$: $\left\{\pi^M > \pi^{D,1}|_{\phi \in [\tilde{\phi}^D, 1]} \pi^M > \pi^{D,3}|_{\phi \in [\tilde{\phi}^D, 1]}\right\}$. ■

Proof of Proposition 5: To prove Proposition 5, we show that the P2P-sponsoring firm's profits are lower than a segmenting Dual firm's profits ($\pi^{D,2}$, defined in the proof of Lemma 2) when the market outcome is C_2 or C_4, but when the market outcome is C_1 or C_3 and if the market $\lambda > \tilde{\lambda}_1$ (or $\lambda > \tilde{\lambda}_3$, respectively)—where the sufficient threshold depends on ϕ—then the P2P-sponsoring firm's

profits will be higher than a Dual firm's. Combined with the upper threshold value of ϕ_2^{π} above which the P2P-sponsoring firm's profits are less than a Monopoly firm's, this will prove the result. (Note that μ represents the rental frictions for both the Dual and P2P-sponsoring firm in this proof.)

Outcomes C_2 and C_4. It is easy to show that when the P2P-facing firm has a market outcome where low types do not buy (C_2 or C_4), then the profits for a P2P-facing firm are weakly smaller than those of a segmenting Dual firm:

$$\pi^{D,2} - \pi^{P,2} = \frac{\lambda(1-\mu)\mu(\lambda\phi+\lambda-1)^2}{8(1-\delta)\left(\lambda^2(1-\mu)(1+\phi)+\mu\right)} \geq 0,$$
$$\pi^{D,2} - \pi^{P,4} = \frac{\lambda\mu(\lambda\phi+\lambda-1)^2}{8(1-\delta)\left(\lambda^2\phi+(\lambda-1)^2\mu\right)} \geq 0.$$

Outcome C_3. The difference in profits between the P2P-sponsoring and segmenting Dual strategy is

$$\pi^{P,3} - \pi^{D,2} = \frac{1}{8(1-\delta)}\left\{\frac{\lambda\left((\lambda-1)\mu^2(\phi-1)\phi+\mu\left(\lambda\left(\phi-3\phi^2\right)+\phi^2+1\right)+2\lambda\phi^2\right)^2}{(\phi+1)(\mu(\phi-1)-\phi)\left(\mu\left(\lambda^2\phi^2-\phi(1-\lambda)^2-1\right)-\lambda^2\phi^2+(1-\lambda)^2\mu^2\phi\right)}+\lambda\mu\phi+\lambda\right\}.$$

It is easy to show that this difference in profit is 0 when $\lambda = 0$. The only other λ root for this difference, i.e. the value of λ where $\pi^{P,3} - \pi^{D,2} = 0$, is:

$$\begin{aligned}\widetilde{\lambda}_3 = &\left(2\mu^4(\phi-1)\phi+\mu^3\left(2\phi^3-5\phi^2+3\phi-2\right)-\right.\\
&\sqrt{-(\mu-1)^2\mu(\phi+1)^2(\mu(-\phi)+\mu+\phi)^2\left(2\mu^2(\phi-1)\phi+\mu\left(-3\phi^2+3\phi-2\right)-3\phi+1\right)}\\
&\left.+\mu^2\left(-4\phi^3+2\phi^2-4\phi+2\right)+\mu\phi\left(2\phi^2+\phi+3\right)\right)/\\
&\left(2\mu^4(\phi-1)\phi+\mu^3\left(\phi^4+4\phi^3-9\phi^2+3\phi-1\right)+\right.\\
&\left.+\mu^2\left(-2\phi^4-12\phi^3+10\phi^2-3\phi+1\right)+\mu\phi\left(\phi^3+11\phi^2-4\phi+2\right)-3\phi^3+\phi^2\right)\end{aligned}$$

If $\lambda < \tilde{\lambda}_3$, under market outcome C_3, the P2P-sponsoring firm's profits is less than those of the Dual firm, otherwise, the P2P-sponsoring firm is more profitable. When $\phi = 0$, then $\tilde{\lambda}_3 > 1$ and $\tilde{\lambda}_3$, for any $\mu \geq 1/2$, is analytically monotonically decreasing in ϕ. Thus as ϕ increases, this sufficient value of λ is decreasing. This indicates that the firm will adopt a P2P-sponsoring strategy when market parameters result in outcome C_3 and for $\lambda \geq \tilde{\lambda}_3$.

Outcome C_1. The difference in profits under the two strategies is given by

$$\pi^{P,1} - \pi^{D,2} = \frac{1}{8(1-\delta)}\left\{\frac{\lambda(\mu((2\lambda-1)\phi-1)-2\lambda\phi)^2}{(1+\phi)\left(\mu\left(1-\lambda^2\phi\right)+\lambda^2\phi\right)}-\lambda\mu\phi-\lambda\right\}.$$

Solving for the value of λ where $\pi^{P,1} - \pi^{D,2} = 0$ has one valid value $\in [0,1]$:

$$\tilde{\lambda}_1 = \frac{2\mu^2\phi(\phi+1) + \sqrt{(\mu-1)^2\mu\phi\left(3\phi^2+2\phi-1\right)(\mu\phi+1)} - 2\mu\phi(\phi+1)}{(\mu-1)\phi\left(\mu\phi^2+(5\mu-3)\phi+1\right)}.$$

If the market parameters $\{\lambda, \phi, \mu\}$ are such that $\lambda > \tilde{\lambda}_1$, then a P2P-sponsoring firm (with outcome C_1) is more profitable than a Dual firm and is preferred. We also show that $\frac{\partial\tilde{\lambda}_1}{\partial\phi} > 0$ for $\mu \geq 1/2$, so this sufficiently high value of λ is increasing as ϕ increases, unlike $\tilde{\lambda}_3$.

Additionally, we identify the value λ_{min}, above which there exists an intermediate region of ϕ that P2P-sponsoring is preferred. We know that $\pi^{P,1} - \pi^{D,2}$ and $\pi^{P,3} - \pi^{D,2}$ are increasing in ϕ, so the largest difference in profits occurs at $\phi = \tilde{\phi}^D$ (the largest ϕ where the Dual firm is segmenting)—for $\phi > \tilde{\phi}^D$, the Monopoly firm profits are the appropriate comparison and that difference is decreasing in ϕ as shown in Proposition 3. Substituting $\phi = \tilde{\phi}^D$ into $\pi^{P,1} - \pi^{D,2}$ and solving for the minimum λ at which a P2P-sponsoring strategy is preferred results in:

$$\lambda_{min} = \frac{9 - 5\mu - 3A - \sqrt{6\left(8A-39\right)\mu - 8\mu^3 - 8\left(A-11\right)\mu^2 - 54\left(A-3\right)}}{2\left(\mu^2+\left(A-8\right)\mu - 3A + 9\right)},$$

where $A = \sqrt{(1-\mu)(9-\mu)}$. It can be shown that at $\left\{\lambda = \lambda_{min}, \phi = \tilde{\phi}^D\right\}$ and any $\mu \geq \frac{1}{2}$, the P2P-sponsoring firm's market outcome is C_1, so we do not need to identify λ_{min} for outcome C_3. Hence, for any value of $\lambda < \lambda_{min}$, the firm never adopts a P2P-sponsoring strategy, regardless of ϕ.

Summary. The firm chooses the Monopoly strategy for $\left\{\lambda, \phi : \lambda < \lambda_{min}, \phi \geq \tilde{\phi}^D\right\}$ and the Dual strategy for $\left\{\lambda, \phi : \lambda < \lambda_{min}, \phi < \tilde{\phi}^D\right\}$. Otherwise, if $\lambda \geq \lambda_{min}$ and $\phi \geq \tilde{\phi}^D$, the firm chooses the Monopoly strategy when consumers are sufficiently homogeneous ($\phi > \phi_2^\pi$, where ϕ_2^π comes from the proof of Proposition 3) and the P2P-sponsoring strategy otherwise. When $\lambda \geq \lambda_{min}$ and $\phi < \tilde{\phi}^D$, the firm chooses the Dual strategy if $\lambda < \tilde{\lambda}_1(\tilde{\lambda}_3)$ when P2P sharing results in C_1 (C_3); otherwise, when $\lambda \geq \tilde{\lambda}_1(\tilde{\lambda}_3)$, it chooses the P2P-sponsoring strategy. ■

Proof of Proposition 6: In the proof of Proposition 6, we introduce a new parameter γ that adjusts the fraction of consumers who are exposed to the P2P rental market. For homogeneous consumer usage rate, the valuations of marginal purchasers are: $\hat{v}^{P2P} = \frac{\frac{(\lambda-1)\lambda^2\mu(-\gamma+\lambda+1)-\lambda^4}{(\gamma-1)(\lambda^2-1)\mu+\lambda^2}+2(\lambda-1)\mu}{2(\lambda^2(\mu-1)-\mu)}$ and $\hat{v}^{NoP2P} = \frac{(\lambda-1)\mu(\gamma-\lambda-1)+\lambda^2}{2((\gamma-1)(\lambda^2-1)\mu+\lambda^2)}$. The valuation of the marginal renter is $\tilde{v}^{P2P} = \frac{\lambda\left(\frac{\lambda}{(\gamma-1)(\lambda^2-1)\mu+\lambda^2}+\frac{1}{\lambda^2(\mu-1)-\mu}+2\right)}{2(\lambda+1)}$. Subsequently, the firm's optimal profit is $\frac{\lambda\left((\lambda-1)\mu(\gamma-\lambda-1)+\lambda^2\right)^2}{4(\delta-1)(\lambda^2(\mu-1)-\mu)((\gamma-1)(\lambda^2-1)\mu+\lambda^2)}$. Using Mathematica we can show that this profit is smaller than the Monopoly profit for all $\gamma : 0 < \gamma \leq 1$, consistent with Proposition 1.

For consumer welfare, the welfare of the γ fraction of consumers is $\frac{\lambda^2(2(\lambda-1)\mu-\lambda)\left(-2(\gamma-1)(\lambda-1)^2(\lambda+1)\mu^2+(\lambda-1)\lambda\mu(2\gamma\lambda+\gamma-3\lambda-1)+\lambda^3\right)^2}{8(\delta-1)(\mu-\lambda^2(\mu-1))^2((\gamma-1)(\lambda^2-1)\mu+\lambda^2)^2}$ (from buying) plus $\frac{(1-\lambda)^2\lambda\mu\left(-2(\gamma-1)(\lambda-1)^2(\lambda+1)\mu^2+(\lambda-1)\lambda\mu(2\gamma\lambda+\gamma-3\lambda-1)+\lambda^3\right)^2}{8(1-\delta)(\mu-\lambda^2(\mu-1))^2((1-\gamma)(\lambda^2-1)\mu+\lambda^2)^2}$ (from renting). The welfare of the $1-\gamma$ fraction of non-exposed consumers is $\frac{\left((1-\lambda)\lambda\mu(2\gamma\lambda+\gamma-\lambda-1)+\lambda^3\right)^2}{8(1-\delta)\lambda((1-\gamma)(1-\lambda^2)\mu+\lambda^2)^2}$. We can show that there is a threshold of γ sufficiently below 1 such that consumer welfare is higher under a P2P-facing firm than a Monopoly firm. This sufficiently low value is:$\tilde{\gamma} = \frac{4\lambda^2\mu^2-5\lambda^2\mu+\lambda^2-4\mu^2+\mu}{4\lambda^2\mu^2-4\lambda^2\mu-4\mu^2+\mu}$. ■

Proof of Proposition 7: In the presence of production costs and homogeneous consumer usage rate, the valuation of the marginal indifferent purchaser is: $\hat{v}_{|c>0} = \frac{1}{2} + \frac{\mu(1-\lambda)}{2(\lambda^2(1-\mu)+\mu)} + \frac{\lambda c(1-\delta)}{2(\lambda^2(1-\mu)+\mu)}$ (similar to the term in the proof of Proposition 1, with an additional third term). As expected, increased production costs reduces sales. The valuation of the marginal renter is: $\tilde{v}_{|c>0} = \frac{1}{2} - \frac{\lambda(1-\lambda)(1-\mu)}{2(\lambda^2(1-\mu)+\mu)} + \frac{c(1-\delta)}{2(\lambda^2(1-\mu)+\mu)}$.

Subsequently, the Monopoly and P2P-facing firm profits with positive production costs are $\pi^M|_{c>0} = \frac{(\lambda-(1-\delta)c)^2}{4(1-\delta)\lambda}$; $\pi^P|_{c>0} = \frac{\lambda(\lambda+\mu-c(1-\delta)-\lambda\mu)^2}{4(1-\delta)(\lambda^2(1-\mu)+\mu)}$. Thus, $\pi^P > \pi^M$ as long as:

$$\frac{\lambda}{(1-\delta)(1+\lambda)} - \sqrt{\frac{\lambda^4+\lambda^2\mu-\lambda^4\mu}{(1-\delta)^2(1+\lambda)^2}} < c < \frac{\lambda}{(1-\delta)(1+\lambda)} + \sqrt{\frac{\lambda^4+\lambda^2\mu-\lambda^4\mu}{(1-\delta)^2(1+\lambda)^2}}$$

The upper bound of this inequality is not binding though, because the Monopoly firm will not produce any goods if $c > c^M_{max} = \frac{\lambda}{1-\delta}$, and $c^M_{max} = \frac{\lambda}{1-\delta} < \frac{\lambda}{(1-\delta)(1+\lambda)} + \sqrt{\frac{\lambda^4+\lambda^2\mu-\lambda^4\mu}{(1-\delta)^2(1+\lambda)^2}}$. Thus, the upper boundary of c in the inequality comparing $\pi^P > \pi^M$ is not binding, because that upper boundary is above where the Monopoly firm would stop operating. Thus, for any $\{\lambda, \mu\}$, the firm will be more profitable under P2P-facing firm than Monopoly when: $c > \frac{\lambda}{(1-\delta)(1+\lambda)} - \sqrt{\frac{\lambda^4+\lambda^2\mu-\lambda^4\mu}{(1-\delta)^2(1+\lambda)^2}}$.

In the presence of production costs, the consumer welfare under a P2P-facing firm and a Monopoly firm (absent P2P) are $CS^M = \frac{(c(\delta-1)+\lambda)^2}{8(1-\delta)\lambda}$; $CS^P = \frac{\lambda(c(\delta-1)-\lambda\mu+\lambda+\mu)^2}{8(\delta-1)(\lambda^2(\mu-1)-\mu)}$. Comparing these two values, we identify the same threshold value of c as the threshold for where firm profits increase under P2P-facing. It is easy to show that consumers are better off under a P2P-facing firm than under a Monopoly firm if, for any $\{\lambda, \mu\}$: $c > \frac{\lambda}{(1-\delta)(1+\lambda)} - \sqrt{\frac{\lambda^4+\lambda^2\mu-\lambda^4\mu}{(1-\delta)^2(1+\lambda)^2}}$. ■