Is More Better? Divide between Retailer's and Manufacturers' Preferences for Reviews and Review Monetization

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Abstract

Research on online product reviews has examined a variety of issues ranging from reviewers' motivation to write reviews to impact of reviews on product sales. Implicit in a large number of studies in this research stream is the notion that more reviews is better for sellers and consumers. However, it is unclear whether a retailer, who often controls the review platform, and manufacturers, whose products are being reviewed, prefer more reviews to less. Using a game theoretical model of a context in which a retailer sells competing products from two manufacturers and consumers are uncertain about product qualities, we show that the retailer's and the manufacturers' preferences regarding the number of reviews are not always aligned. If the reviews are costless, then additional reviews benefit the retailer, but manufacturers are hurt by additional reviews when the number of reviews exceeds a threshold. Moreover, if the consumers' own uncertainty about product qualities is not too high, then the manufacturers do not prefer to have reviews at all. On the other hand, if the retailer incurs a sufficiently high cost to generate reviews, he may prefer to have fewer reviews than manufacturers. We show that the divide between the retailer's and the manufacturers' preferences can induce the retailer to monetize reviews by charging a fee to manufacturers in return for a guarantee on the number of reviews that would be made available in the review platform. The retailer's revenue from such monetization follows a U-shape with respect to the cost of generating reviews as well as the extent to which reviews can reveal true product quality, suggesting that the product type and review platform design play a significant role in the attractiveness of review monetization for the retailer. The revenue from "selling" reviews to manufacturers does not exceed the cost of generating reviews, implying the retailer's benefit from reviews, even when they can be monetized, is driven by reviews' positive impacts on the retailer's core business of selling products to consumers. The findings provide economic insights into the nascent efforts of Amazon to monetize reviews through programs such as Amazon Vine.

1 Introduction

Consumer reviews have emerged as an important source of information about product quality in online retail environments. A survey conducted by Pew Research Center finds that 82 percent of consumers read online product reviews before they purchase new products.[1] Recognizing the importance of product reviews in consumers' purchase process, online retailers invest substantial effort in implementing review platforms that facilitate consumers to access and post reviews. The review platforms often allow reviewers to provide both structured ratings and free-form textual reviews. The platforms also consolidate reviewer ratings and provide aggregate information to consumers seeking reviews.

Many studies have shown that online product reviews boost product sales (e.g. Chevalier and Mayzlin, 2006; Clemons et al., 2006), and the reviews' impact is greater when the review volume (i.e., the number of reviews) is larger (Liu, 2006). Consequently, it is generally assumed that an online retailer would encourage more online consumers to contribute reviews (Burtch et al., 2017). On the other hand, only a small percentage of online consumers write reviews and online reviews may be acutely under-provisioned (Avery et al., 1999; Anderson, 1998; Levi et al., 2012). The concern for under-provisioning of reviews is echoed in the same Pew Research survey which shows only around 10 percent of consumers write reviews. The seemingly low rate of review contribution creates a sharp contrast to the substantial majority of consumers' reliance on online reviews. Research on electronic word-of-mouth (eWOM), of which online product reviews is one example, has examined the intrinsic and extrinsic motivations of consumers to contribute contents in an online community (Hennig-Thurau et al., 2004; Gu and Jarvenpaa, 2003; Dellarocas, 2006). A large body of research has also examined the effectiveness of financial and social incentives for online review generation (Stephen et al., 2012; Khern-am nuai et al., 2017; Avery et al., 1999; Cabral and Li, 2015; Fradkin et al., 2018; Wang et al., 2012; Chen et al., 2010; Burtch et al., 2017). This coincides with a trend in online business where retailers have launched several initiatives to encourage consumers to write reviews. For instance, Amazon has established the Vine program and

[1] http://www.pewinternet.org/2016/12/19/online-reviews/

the Early Reviewer program to incentivize customers to write reviews. The Vine program seeks to obtain reviews from a selective group of consumers for a selective set of new and pre-release products. In exchange for reviews, the review writers get the products for free or at a discount. Amazon charges a fee to product manufacturers to be part of the Vine program. On the other hand, the Early Reviewer Program seeks to obtain reviews from those that have purchased the product in return for a small reward, typically a $1-$3 Amazon.com gift card. Alibaba incentivizes users to write reviews about products on Taobao.com and Tmall.com through a score system known as *Taoqizhi* that considers the number of reviews written by users; it offers tangible and intangible rewards for those with high scores.

The extant research and the review programs of Amazon and Alibaba seem to suggest that more reviews is better for the sellers. However, despite the vast body of research about online reviews, the question of whether a multi-product retailer would prefer more or fewer reviews remains open. This question is especially important for the retailer because the review platform, including the number of reviews displayed in it, is typically controlled by the retailer. Equally important is the unexplored question of whether manufacturers, whose products are being reviewed, prefer more or less reviews. The question becomes critical for manufacturers which produce competing products.

In this paper, we examine a retailer's and manufacturers' incentives to generate reviews using a game theoretical model of a context where a retailer sells competing products from two symmetric manufacturers and consumers are uncertain about the products' qualities. We show that the retailer's and the manufacturers' preferences regarding number of reviews are not always aligned. If the reviews are costless, then additional reviews benefit the retailer. On the other hand, the manufacturers benefit from more reviews if the consumers have a high level of own uncertainty about the product qualities and the number of reviews is not too large. If the retailer incurs a cost to generate reviews, then she may prefer to have fewer reviews than the manufacturers when the review cost is sufficiently high. The divide between the retailer's and the manufacturers' preferences regarding the number of reviews provides a strategic opportunity to the retailer to monetize reviews by charging a fee to manufacturers in return for a guarantee on the number of reviews that would be made available in the review platform. While review monetization could provide an additional source of revenue for the retailer, the revenue from "selling" reviews falls short of the cost of generating reviews.

These findings result from several intricate strategic effects of reviews both on the downstream consumers and the upstream manufacturers. The direct effect of reviews is the reduction in consumer uncertainty about product qualities. With more reviews, the signal from reviews becomes more precise (*variance-reducing effect*) and consumer uncertainty becomes less. The improvement in review precision, in turn, causes consumers to rely more on reviews (*review-credibility-enhancing effect*), relative to their own assessment, in assessing product quality. Consequently, with more reviews, consumer assessments tend to be more clustered around the review assessment (*consumer-homogenization effect*). On the manufacturer side, an improvement in review precision causes the reviews to more likely reveal the true quality difference between the products, reducing the likelihood of signaling more differentiation between the two products than the true level (*differentiation-reducing effect*). From the retailer's perspective, all these effects enhance the retailer's incentive to generate reviews - the supply-side impacts lead to an intensification of upstream competition, and the demand-side impacts of consumer homogenization enables the retailer to extract more of the consumer surplus. On the other hand, while the differentiation-reducing effect alone hurts the manufacturers, when combined with the demand-side effects of number of reviews, it can hurt or benefit the manufacturers. A more homogeneous consumer population hurts the manufacturers only if the review precision is high because a more homogeneous consumer population exacerbates the adverse impact of reduced product differentiation on the manufacturers. On the other hand, a more homogeneous consumer population amplifies the positive impact of a large perceived differentiation between the products on the manufacturers. Thus, only when the number of reviews is not too high and the consumers' own uncertainty is sufficiently high, the manufacturers prefer additional reviews.

The significance of our research stems from the implications it offers to academics and practitioners. The finding that the preferences of retailers and manufacturers regarding the number of reviews are not always aligned with each other is new to the academic literature that has examined online reviews. The implicit assumption that sellers would always prefer more reviews to less holds true only for a retailer and also only when reviews are costless. More importantly, the conventional wisdom that competing symmetric manufacturers would not prefer reviews because it would intensify competition does not hold either; a limited number of reviews when consumers are highly uncertain about product quality actually benefits the manufacturers. Additionally, the research identifies and articulates how the intricate impacts of reviews both on the downstream demand (consumer) side and the upstream (product/manufacturer) side lead to the novel findings, which extends the body of knowledge about the diverse impacts of reviews in the retailing context.

The implications of our findings for practice relate to providing a potential explanation for the recent retailer efforts at monetizing reviews and to retailers' choices regarding review platform design. Our research suggests that recent initiatives such as Amazon's Vine program could be the result of a dominant retailer taking advantage of the strategic monetization opportunity created by the misalignment between the retailer's and the manufacturers' preferences regarding the number of reviews. More importantly, the monetization opportunity exists whether the reviews are under-provisioned or over-provisioned. When the products have few reviews, perhaps as a result of the products being new or review generation being costly, the retailer is able to sell more reviews to manufacturers as they could benefit from additional reviews. On the other hand, when the products have excessive reviews, the retailer is in a position to charge the manufacturers a fee in return for curbing reviews by highlighting reviews from a selected group of consumers. Essentially, our findings posit a possible explanation of online retailers recent efforts at transforming their product review platform from one focuses on voluntary contribution of reviews by consumers to one that provides an additional source of revenue from the manufacturers.

The results have practical implications for review platform design as well. Review platform design has traditionally focused on how to effectively transfer the information content of reviews to consumers. Consequently, the emphasis has been on factors such as form and structure (rating scale and review text), rating scales (single dimensional and multi dimensional schemes), review usefulness, and aggregate statistics. Our results suggest that improving the effectiveness of information transfer is appropriate for a retailer if review monetization is not a concern and reviews are costless. On the other hand, when a retailer chooses to monetize reviews, the cost of generating reviews plays a key role in determining whether the retailer should focus on improving the review platform's effectiveness in transferring the information content of reviews to consumers. When the cost is low, the retailer benefits by designing a review platform that has a high degree of effectiveness. On the other hand, when the review cost is high, the retailer is better off not having a high degree of review platform effectiveness. Since the cost of reviews is likely to depend on the product type (e.g., writing reviews for credence goods could be costlier than for experience goods), the results suggest that a uniform strategy for all product types is unlikely to be optimal. Our results also reveal that in addition to review platform design features, the number of reviews to be displayed can be another strategic element that the retailer can use to his advantage when he monetizes reviews. Finally, despite the potential additional revenue a retailer can enjoy through review monetization, the results show that the impact of reviews on the core business - selling products to downstream consumers- is likely to remain the primary reason for the retailer to provide reviews on its platform.

1.1 Related Work

Our paper is most closely related to the vast literature that examines the effects of online product reviews and eWOM. Prior studies have shown that eWOM affects consumer decision making because consumers face product and seller uncertainty in the online market (e.g. Dimoka et al., 2012; Ghose and Huang, 2009; Hong and Pavlou, 2014; Niculescu and Wu, 2014; Sun, 2012), and more importantly, eWOM affects product sales (Chevalier and Mayzlin, 2006; Clemons et al., 2006; Duan et al., 2008; Li et al., 2011). Empirical studies have found that the review volume (Duan et al., 2008, 2009; Liu, 2006), review variance (Clemons et al., 2006), reviewer and product characteristics (Forman et al., 2008; Shen, 2008; Zhu and Zhang, 2010), and text reviews (Archak et al., 2011) impact sales. Studies have also investigated the impacts of the rating system design in review platforms. For instance, studies have examined the effectiveness of multi-dimensional and single-dimensional rating systems (Chen et al., 2017), the rating scale cardinality, the granularity of reviews (Jiang and Guo, 2015), and disclosure of average product rating (Li, 2017). Some studies have focused on the information biases in online reviews: online product reviews could be subject to self-selection biases that impact consumer purchase behavior (Li and Hitt, 2008) and the biases in reviews reflect not only perceived quality but also the perceived value (Li and Hitt, 2010). A few studies have examined the strategic aspects of online product reviews such as the interactions between promotional marketing and product reviews (Chen and Xie, 2008; Lu et al., 2013), interaction between recommendations and product reviews (Jabr and Zheng, 2014), informativeness of consumer reviews (Li et al., 2011), and impact of reviews under competition (Shaffer and Zettelmeyer, 2002; Kwark et al., 2014, 2017).

Our paper is also related to the research stream that focuses on the motivation of online product reviewers. A large body of empirical evidence shows that intrinsic motivations, including altruism, reciprocation, dissonance reduction, and vengeance, drive consumers to engage in creation of traditional WOM (Sundaram et al., 1998) and eWOM (Hennig-Thurau et al., 2004; Gu and Jarvenpaa, 2003; Dellarocas, 2006). Studies have also identified factors that affect review-posting behavior and review generation in an online context (Dellarocas et al., 2010; Gao et al., 2015; Goes et al., 2014; Lee et al., 2015; Rice, 2012; Zhu and Zhang, 2010). Moreover, the effectiveness of financial incentives (Stephen et al., 2012; Khern-am nuai et al., 2017; Avery et al., 1999; Cabral and Li, 2015; Fradkin et al., 2018; Wang et al., 2012) and social norms (Chen et al., 2010; Burtch et al., 2017) in motivating online product reviewers has been investigated as well.

Another stream of literature has focused on the firms' incentives to provide and/or manipulate online product reviews. Dellarocas (2006) and Chevalier and Mayzlin (2006) suggest that sellers may have an incentive to manipulate reviews of their products to improve their competitive position. Kwark et al. (2014, 2017) examine the effects of reviews on an online retailer and upstream manufacturers in a channel structure, and find that the impacts of reviews on the retailer and manufacturers depend on the upstream pricing scheme, the dominant roles of quality or fit in consumers' assessment of the product, and the precision of reviews.

Our study differs from the extant literature in the following ways. First, we focus on the differential effects of product reviews on competing manufacturers and a dominant retailer in a two-level channel structure. Second, in contrast to the prior studies that assume the review precision to be exogenous, we endogenize the review precision via a mechanism that links number of reviews to effects of reviews on the online retailer and manufacturers. Finally, we are unaware of any other study that has examined the issue of review monetization.

2 Model

We consider an online retailer R that buys products A and B, respectively from manufacturers A and B, and resells the products to consumers. A consumer demands a maximum of one unit of one of the two products. We assume that the marginal production cost is negligible for both manufacturers.

Consumer Type and Utility: Each product is characterized by two attributes that affect consumer utility: quality and fit. Product quality is a vertical attribute in the economic sense that all consumers prefer high quality to low quality. We let $x_i, i \in \{A, B\}$, denote the quality of product i. Product fit is a horizontal attribute in the sense that consumers have heterogeneous preference on this attribute. We use the classic Hotelling model to capture product fit, i.e., consumer preference on the fit dimension is uniformly distributed over a line of unit length, and products A and B are located respectively at locations 0 and 1 on this line. A consumer located at λ , $\lambda \in [0, 1]$, has a misfit cost of λ for product A and a misfit cost of $(1 - \lambda)$ for product B. We define the utility of a consumer located at λ from product i, U_i, as follows.

$$\begin{aligned} U_A &= x_A - \lambda \\ U_B &= x_B - (1 - \lambda) \end{aligned} \tag{1}$$

We distinguish two types of consumers: loyal consumer and shopper. Loyal consumers have complete information about the product to which they are loyal, and buy the product when the net utility, which is defined as utility minus price, is non-negative.[2] Shoppers compare the two products and purchase the product that offers a higher net utility. The difference in the net utility from product A and product B, $V_A - V_B$, for a shopper located at λ is:

$$V_A - V_B = (x_A - x_B) + (1 - 2\lambda) - (p_A - p_B) \tag{2}$$

The term $(x_A - x_B)$ denotes the quality difference. A shopper located at λ will buy product A if $V_A - V_B > 0$ and will buy B otherwise, if she knows the product quality difference. We normalize the number of shoppers to 1. We assume the total number of loyal consumers is h.

Quality Uncertainty and Online Reviews: A loyal consumer knows the true quality of that product to which she is loyal. On the other hand, a shopper is uncertain about the product qualities and uses third-party information such as on-line product reviews, in addition to her own assessment, to estimate the product qualities (Chen and Xie 2008, Kwark et al. 2016, 2017). Since the purchase decision of a shopper depends on the difference between the two qualities, we let x_C and x_R denote a shopper's own assessment and mean review assessment, respectively, of the true quality difference $(x_A - x_B)$. We note that a shopper's own assessment is based on pre-purchase information such as the product specification available on the retailer's website or other sources. On the other hand, mean review assessment is possibly based on reviewers' post-purchase experience with the product.

We assume symmetric products so that $x_A = x_B$.[3] A shopper's own assessment, x_C, comes from a uniform distribution with support $[-\epsilon, \epsilon]$. Thus, $E(X_C) = 0$ and $Var(X_C) = v_C = \frac{\epsilon^2}{3}$.[4] We refer to ϵ as the *degree of shoppers' own uncertainty* about quality difference.

[2]The presence of loyal consumer type in our model serves the sole purpose of ensuring a well-behaved demand function that results in interior prices in equilibrium.

[3]We analyzed the asymmetric case where the true quality difference between the two products is nonzero. While the analysis resulted in a few additional findings, the results regarding the divide in retailer's and manufacturers' preferences regarding the number of reviews and the monetization of reviews remain intact in the asymmetric manufacturer case.

[4]We use upper case letter to refer to the corresponding random variable.

Analogously, quality assessment conveyed in a review i, $x_{r,i}$, comes from a uniform distribution with support $[-\theta\epsilon, \theta\epsilon]$, where $\theta \in (0,1)$. The parameter θ represents the extent to which reviews can reveal the true quality difference. If $\theta = 0$, each review perfectly reveals the true quality difference; on the other hand, if $\theta = 1$, each review is only as good as the consumer's own assessment. Our model about review uncertainty reflects the observation that the reviewers are possibly less uncertain about the quality difference, perhaps due to first-hand experience with the products. The magnitude of θ is likely to depend on various factors including the product type and review platform design. For instance, experience goods will likely have a smaller θ than credence goods. Moreover, a platform that provides rich features for reviewers to provide comprehensive reviews (e.g., multidimensional rating systems) will likely have a smaller θ than the one that provides only limited features (e.g., single-dimensional rating system). We refer to θ as the reviews' *information transfer effectiveness*; a decrease in θ indicates an increase in the effectiveness of reviews to transfer information about the true quality difference. When there are N independent reviews, the mean review assessment x_R is the average of assessments in all reviews, i.e., $x_R = \frac{\sum_{i=1}^{N} x_{r,i}}{N}$. Thus, $E(X_R) = 0$ and $Var(X_R) = v_R = \frac{\theta^2\epsilon^2}{3N}$.[5]

Shoppers incorporate these two sources of information about product quality in their estimation about the quality and utility of the two products. As shown by Bates and Granger (1969), the minimum variance unbiased estimator for the unknown true quality difference based on sources X_C and X_R is a weighted average of the two sources where the weight assigned to each source is its relative precision.[6] Thus, a shopper's expected quality difference, conditional on $X_C = x_C$ and $X_R = x_R$, as follows:

$$E(x_A - x_B | X_C = x_C, X_R = x_R) = \gamma x_C + (1-\gamma) x_R \tag{3}$$

where $\gamma = \frac{v_R}{v_R + v_C} = \frac{\theta^2}{\theta^2 + N}$ is the relative precision of a shopper's own assessment. Clearly, either an increase in N or a decrease in θ increases (decreases) the weight assigned to review

[5] For simplicity and tractability reasons, we treat each review to provide a signal about the quality difference between the two products, rather than a signal about the quality of a single product.

[6] Precision of an estimator is defined as the reciprocal of its variance.

(own) assessment because each has the effect of increasing (decreasing) the relative precision of review (own) assessment.

The retailer controls the review platform including its design and the number of reviews posted in the platform. Even though consumers could provide reviews on their own, review generation may not be costless as evidenced by incentives, financial and otherwise, provided by retail platforms in return for reviews. We let $C(N)$ be the retailer's cost of generating N number of reviews. This cost includes the direct financial incentives the retailer may provide to review contributors and the retailer's managerial effort to provide non-financial rewards to review contributors. We assume, for analytical tractability, the cost is linearly increasing in the number of reviews, i.e., $C(N) = cN$, where c is the marginal cost of generating reviews (hereafter, review cost) and $c > 0$. We assume that the number of possible reviews has an upper bound of $\bar{N}$. The upper bound reflects the observation that there are finite number of reviewers that would be interested in writing product reviews. The marginal review cost is likely to depend on various factors such as the altruism of reviewers, effort required to write reviews, and product complexity (e.g., whether the product is feature-rich or not).[7]

Timing of the Game: Figure 1 shows the game sequence. In stage 1, the retailer chooses the number of reviews, N , that will be made available on the review platform, and X_R is realized which is publicly available to all players. In stage 2, manufacturer i sets the wholesale price w_i. In stage 3, the retailer sets retail prices p_i. In stage 4, consumers make their purchase decisions and payoffs are realized.

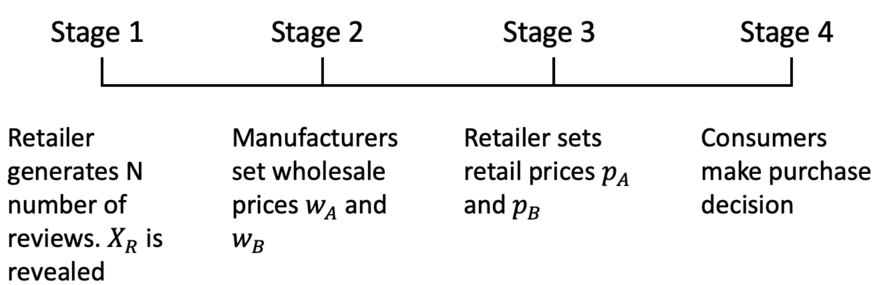


Figure 1: Timeline of events

[7]The cost function can be strictly convex in N. Having a convex cost function does not affect the main results but complicates the analysis significantly.

While the reviews are public and common to all players, a shopper's own assessment of the quality difference is private information. The distribution of shoppers' assessment and review assessment are common knowledge. All players are risk neutral.

Demand Functions: To facilitate our analysis in the subsequent sections, we first derive the demand functions as perceived by the manufacturers and the retailer. Using equation (2) and equation (3), we can derive a shopper's expected net utility difference between the products as

$$E\left(V_A - V_B | X_C = x_C, X_R = x_r, \lambda\right) = [\gamma x_C + (1-\gamma)x_R] + (1-2\lambda) - (p_A - p_B) \tag{4}$$

Clearly, a shopper buys A if the net utility difference is positive and B otherwise. Consistent with our focus on shoppers' uncertainty about quality, we consider the case in which the quality dominates fit in a shopper's evaluation of the two products in the sense even a shopper that has a perfect fit for a product will not always prefer that product, i.e., a significant quality difference in favor of the other product will cause this shopper to buy the other product. Mathematically, this case implies that for any λ and x_R, the shopper will prefer A if x_C is greater than a threshold that lies in $(-\epsilon, \epsilon)$ and prefer B otherwise. Since X_R is already realized and is known to be x_R, but X_C is a random variable when the manufacturers and retailers set prices, we can formulate the demand for each product from shoppers, from the retailer and manufacturer perspectives, as:

$$\begin{aligned} D_{Ac} &= \int_0^1 \int_{\frac{p_A - p_B - (1-\gamma)x_R - (1-2\lambda)}{\gamma}}^{\epsilon} \tfrac{1}{2\epsilon} dx d\lambda = \tfrac{1}{2} - \tfrac{1}{2\gamma\epsilon}\left[p_A - p_B - (1-\gamma)x_R\right] \\ D_{Bc} &= \int_0^1 \int_{-\epsilon}^{\frac{p_A - p_B - (1-\gamma)x_R - (1-2\lambda)}{\gamma}} \tfrac{1}{2\epsilon} dx d\lambda = \tfrac{1}{2} + \tfrac{1}{2\gamma\epsilon}\left[p_A - p_B - (1-\gamma)x_R\right] \end{aligned} \tag{5}$$

where the integral in product i's demand measures the shoppers who derive a higher net utility from product i than from the other product, $i \in \{A, B\}$. The lower limit of the second integral in D_{Ac} should be greater than $-\epsilon$ and the upper limit of the second integral in D_{Bc} should be less than ϵ in the equilibrium for the quality to dominate fit. We assume the following technical condition to ensures this.

$$\gamma\epsilon > 1, \forall N \implies \epsilon > \frac{\theta^2 + \bar{N}}{\theta^2} \tag{6}$$

We let the demand function from loyal consumers to be linear in price and symmetric across the two products, as the following

$$D_{il} = (\frac{h}{2} - p_i) \tag{7}$$

Thus, the demands for products A and B are computed as:

$$\begin{aligned} D_A &= D_{Ac} + D_{Al} = (\tfrac{1}{2} - \tfrac{1}{2\gamma\epsilon}[p_A - p_B - (1-\gamma)x_R]) + (\tfrac{h}{2} - p_A) \\ D_B &= D_{Bc} + D_{Bl} = (\tfrac{1}{2} + \tfrac{1}{2\gamma\epsilon}[p_A - p_B - (1-\gamma)x_R]) + (\tfrac{h}{2} - p_B) \end{aligned} \tag{8}$$

Finally, to guarantee an interior equilibrium, we impose the following condition on h.

$$h \geq \frac{6\epsilon + 1}{2\epsilon + 1} \tag{9}$$

3 Impact of Reviews on the Retailer and Manufacturers

We use sub game perfect equilibrium as the solution concept for the game. Accordingly, we use backward induction procedure to derive the equilibrium. We first examine the effects of reviews on the retailer and the manufacturers, assuming the reviews have already been generated. Thus, we analyze the sub game starting with stage 2 when the cost of reviews is sunk. Subsequently, in sub section 3.3, we examine the retailer's choice in stage 1 regarding the number of reviews.

3.1 Strategic Effects of Reviews

In stage 3 of the game, the retailer maximizes his profit by setting optimal retail prices, given the wholesale prices. The retailer's profit function is given by:

$$\pi_R = (p_A - w_A)D_A + (p_B - w_B)D_B \tag{10}$$

Maximizing the retailer profit, we find the optimal retail prices as functions of the wholesale prices as follows:

$$\begin{aligned} p_A &= \tfrac{1}{4}\left(2w_a + h + \tfrac{(1-\gamma)x_R}{\gamma\epsilon+1} + 1\right) \\ p_B &= \tfrac{1}{4}\left(2w_b + h - \tfrac{(1-\gamma)x_R}{\gamma\epsilon+1} + 1\right) \end{aligned} \tag{11}$$

In stage 2, anticipating the retailer's optimal retail prices in stage 3, manufacturer i choose the wholesale price, by maximizing his profit given as:

$$\pi_i = w_i D_i \tag{12}$$

Lemma 1. *Given the review assessment x_R, the wholesale prices and retail of products A and B are given by the following:*

$$\begin{aligned} w_A &= \tfrac{\gamma(h+1)\epsilon}{4\gamma\epsilon+1} + \tfrac{(1-\gamma)x_R}{4\gamma\epsilon+3} \\ w_B &= \tfrac{\gamma(h+1)\epsilon}{4\gamma\epsilon+1} - \tfrac{(1-\gamma)x_R}{4\gamma\epsilon+3} \end{aligned} \tag{13}$$

$$\begin{aligned} p_A &= \tfrac{1}{4}\left(\tfrac{(h+1)(6\gamma\epsilon+1)}{4\gamma\epsilon+1} + \tfrac{(1-\gamma)(6\gamma\epsilon+5)x_R}{(\gamma\epsilon+1)(4\gamma\epsilon+3)}\right) \\ p_B &= \tfrac{1}{4}\left(\tfrac{(h+1)(6\gamma\epsilon+1)}{4\gamma\epsilon+1} - \tfrac{(1-\gamma)(6\gamma\epsilon+5)x_R}{(\gamma\epsilon+1)(4\gamma\epsilon+3)}\right) \end{aligned} \tag{14}$$

If x_R is positive, the reviews suggest that product A has a superior quality than product B. Thus, when x_R is positive (negative), the wholesale price and retail price of product A are higher (lower) than the corresponding price of product B.

In stage 2, the expected profits of the retailer and the manufacturers are computed by taking expectation of respective profits over X_R as the following.

$$E(\pi_R) = \frac{1}{8}(2\gamma\epsilon+1)^2\left(\frac{(h+1)^2}{(4\gamma\epsilon+1)^2} + \frac{(1-\gamma)^2 v_R}{\gamma\epsilon(\gamma\epsilon+1)(4\gamma\epsilon+3)^2}\right)$$
$$E(\pi_A) = E(\pi_B) = \frac{(2\gamma\epsilon+1)\left(\gamma^2(h+1)^2\epsilon^2(4\gamma\epsilon+3)^2+(1-\gamma)^2 v_R(4\gamma\epsilon+1)^2\right)}{4\gamma\epsilon(4\gamma\epsilon+1)^2(4\gamma\epsilon+3)^2} \tag{15}$$

The impact of number of reviews on a player's expected profit depends critically on the value of N chosen in stage 1. We define a threshold value N_M, provided in the Appendix as equation (39).

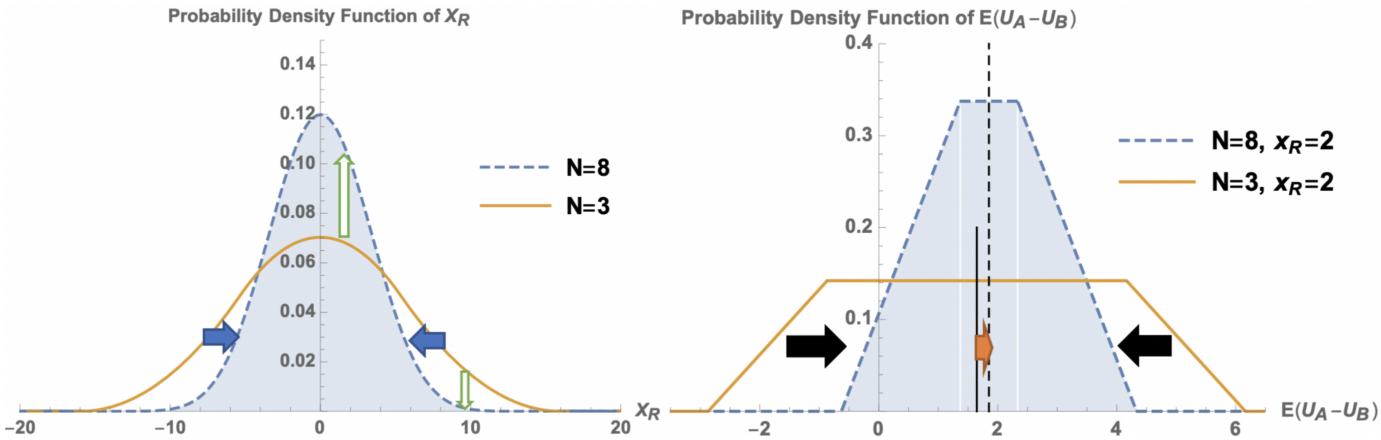
Proposition 1. *a) If $\sqrt{\frac{(2\epsilon+1)(4\epsilon+1)^3}{3(4\epsilon+3)^2}} > h+1$, then (i) both the retailer's profit and manufacturers' profits are increasing in N when $N < N_M$, and (ii) the retailer's profit is increasing in N, but the manufacturers' profits are decreasing in N when $N > N_M$.*
b) If $\sqrt{\frac{(2\epsilon+1)(4\epsilon+1)^3}{3(4\epsilon+3)^2}} \leq h+1$, then the retailer's profit is increasing in N, and the manufacturers' profits are decreasing in N.

Proposition 1 shows that the retailer prefers the maximum possible number of reviews if review generation is costless. On the other hand, the manufacturers desire to limit the number of reviews to N_M. Furthermore, when the degree of shoppers' own uncertainty is less than a threshold (i.e., $\sqrt{\frac{(2\epsilon+1)(4\epsilon+1)^3}{3(4\epsilon+3)^2}} < h+1$), the manufacturers do not prefer to have reviews at all (i.e., $N_M = 0$). Clearly, the manufacturers' and the retailer' preferences regarding number of reviews are not aligned. We explain this finding by recognizing that N affects a player through its direct impact on review variance, v_R. The direct impact, in turn, produces several strategic impacts as illustrated by the following equation (16), which decomposes the marginal impact of N on a player's exptected profit in terms of direct and strategic effects:

$$\frac{dE(\pi(\gamma, v_R))}{dN} = \underbrace{\frac{dv_R}{dN}}_{\text{direct effect}} \underbrace{\left(\frac{\partial E(\pi)}{\partial \gamma}\cdot\frac{\partial \gamma}{\partial v_R} + \frac{\partial E(\pi)}{\partial v_R}\right)}_{\text{strategic effects}} \tag{16}$$

The factor $\frac{\partial v_R}{\partial N}$ represents the direct effect of N, and it is negative. We label this factor as the *review-variance-reducing effect* of N. The left and right arrows shown in Figure 2(a) illustrate the review-variance-reducing effect.[8]

[8]The illustrative figures assume a normal distribution for X_R.



(a) Variance Reducing Effect, Differentiation Reducing Effect

(b) Review Credibility Enhancing Effect, Consumer Homogenization Effect

Figure 2: Effects of reviews ($\epsilon = 20, \theta = 0.8$)

The term $\frac{\partial \gamma}{\partial v_R}$ denotes the strategic effect of N on γ via v_R. We note that a decrease in v_R decreases γ, which implies that an increase in N increases shoppers' reliance on reviews when they compare the two products. We label the effect of N via γ as the *review-credibility-enhancing effect* of N. The review-credibility-enhancing effect is illustrated in Figure 2(b) as a shift in the mean difference in shoppers' valuation of the two products to a value closer to x_R (indicated as the shift from the solid vertical line to a dotted vertical line).

We label $\frac{\partial E(\pi)}{\partial \gamma}$ as the *consumer-homogenization effect* of N through γ, for the following reason. As mentioned in the previous paragraph, an increase in N increases the weight assigned by shoppers to reviews. Since the mean review assessment is common to all shoppers, i.e., x_R is the same for all shoppers regardless of their own signal, a larger and identical weight assigned to the review assessment by all shoppers causes the expected utility difference for the two products across shoppers to be more similar. Essentially, an increase in N causes a more homogeneous shopper population (with respect to utility difference), which affects the profits of all players. The black horizontal arrows in Figure 2(b) illustrates the consumer-homogenization effect. Effectively, the review-credibility enhancing effect and the consumer homogenization effect cause the shoppers' evaluations to be clustered around the review signal x_R.

Finally, $\frac{\partial E(\pi)}{\partial v_R}$ denotes the impact of review variability on a player's expected profit. We note that when v_R is low, the reviews would more likely reveal the true quality difference of zero. On the other hand, when v_R is high, the reviews would more likely suggest that one product has a higher quality than the other. We label the effect of N through v_R as the (perceived) *differentiation-reducing effect*. This effect is shown as the vertical arrows in Figure 2(a). We note that the probability (density) increases as N increases for realizations close to the true mean difference of zero but the probability decreases as N decreases for realizations that are far from the true mean.

Essentially, the strategic effects of N affect the supply side as well the demand side of the retailer. The differentiation-reducing effect relates to the supply side in that it affects how the two competing products are likely to be presented to and perceived by the consumers - whether they are likely to be similar in quality or different. The review-credibility enhancing effect and the consumer-homogenization effect relate to the demand side in that they affect whether the consumers are likely to be more homogeneous or more heterogeneous in their evaluation of the two products.

The strategic effects have differential impacts on the retailer and the manufacturers. For the retailer, all three strategic effects benefit her. A more homogeneous shopper population enables the retailer to extract more of the consumer surplus. The retailer also benefits when consumers rely more on reviews and the reviews signal a higher quality of one product compared to the other. In this case, the retailer is able to charge a higher price for the product that is perceived to have a superior quality and also enjoy a higher demand for this product, compared to the product that has a perceived inferior quality. Consequently, the retailer enjoys a higher overall expected profit when more consumers rely on the review signal more. Finally, the differentiation-reducing effect creates a more intense price competition between the manufacturers on the average. In this case, the retailer benefits from a reduction in the wholesale prices. Therefore, the retailer prefers to have the maximum possible number of reviews if cost is not a concern.

On the other hand, some of the strategic effects have a different and more subtle impact on manufacturers. A more homogeneous shopper population hurts the manufacturers if the reviews indicate that the products have similar quality. In this case, manufacturers will engage in intense price competition. On the other hand, if the reviews suggest that one product has a superior quality than the other, clustering of the consumers around the review signal softens the price competition between manufacturers. The manufacturer that has a superior perceived quality enjoys a competitive advantage over the other in this case, i.e., the imbalance in competitive positions softens the price competition and benefits the overall expected manufacturer profit. Thus, consumer homogenization and clustering amplifies the manufacturer expected profits when there is a possibility of reviews signaling different qualities for the two products. On the other hand, the differentiation-reducing effect hurts manufacturers because the likelihood of high imbalance in their competitive position is diminished by an increase in N. Essentially, when N is small, the overall impact of the three strategic effect of N is positive for manufacturers, and when N is large, the overall impact turns negative. Thus, only when N is smaller than N_M, manufacturers prefer additional reviews.

The condition under which the manufacturers do not prefer any reviews is intuitive when considered in light of the strategic effects discussed in the previous paragraphs. The discussions reveal that the manufacturers prefer neither too heterogeneous consumers nor too homogeneous consumers. When the the degree of shoppers' own uncertainty is too small so that they are already sufficiently homogeneous, any additional reviews can only hurt the manufacturers by exacerbating the adverse effects of homogenization.

Corollary 1. *The desired number of reviews for the manufacturers, N_M, increases when reviews' information transfer effectiveness decreases, i.e., N_M is increasing in θ.*

The corollary shows that an increase in θ expands the region where the manufacturers' and the retailer's preferences to generate additional reviews are aligned. It shows that when the reviews are less effective in transferring the quality information to consumers, manufacturers would desire for more reviews. This is because when θ increases, the variance of X_R increases. Consequently, while the adverse effects of reviews on the manufacturers are weakened, the positive impacts are strengthened. Consequently, a higher θ leads to a higher marginal benefit from reviews for the manufacturers and thus less of a divide between the manufacturers and the retailer in their incentives to generate reviews. Figure 3 illustrates the Corollary 1, where the line that divides the two regions indicates N_M for various values of θ.

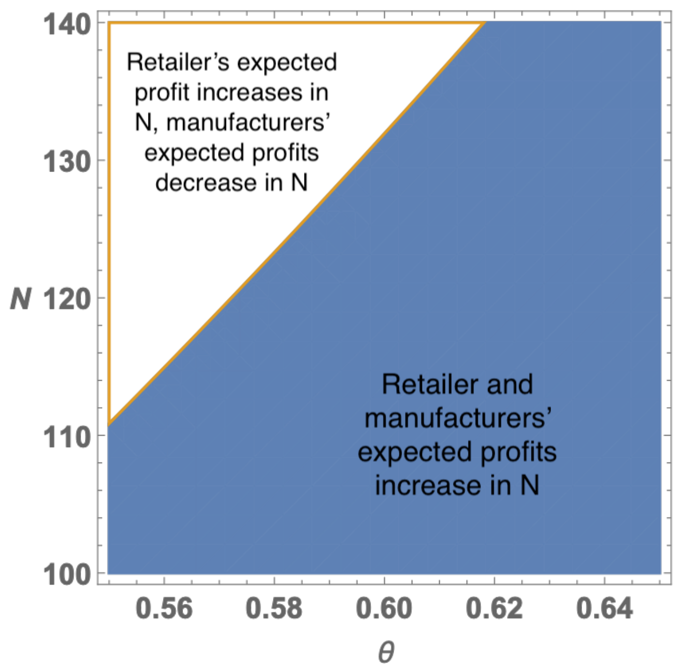


Figure 3: The effect of N on the expected profits of the retailer and the manufacturers. ($\epsilon = 501$, $h = 30$)

The most significant implication of Proposition 1 is that the retailer's and the manufacturers' incentives to have more reviews are not always aligned. While both the retailer and the manufacturers do want to have reviews, the manufacturers prefer to limit the number of reviews whereas the retailer does not in a costless review scenario. Corollary 1 suggests that factors such as the product type and review platform design could play a significant role in determining the extent of the divide in the preferences between the retailer and manufacturers.

3.2 Optimal Number of Reviews

We now examine the retailer's decision regarding the number of reviews in stage 1. We write the retailer's expected profit maximization problem at this stage as follows:

$$\max_{N} \quad E(\pi_R^C) = E(\pi_R) - cN \tag{17}$$

where $E(\pi_R)$ is the expected profit of the retailer if reviews are costless and is given in Equation (15).

Lemma 2. *(i) If* $c < \underline{c} = \frac{1}{72}\theta^2\epsilon\left(\frac{72(h+1)^2\theta^2\epsilon}{\left(\bar{N}+\theta^2(4\epsilon+1)\right)^3} + \frac{8\theta^2\epsilon(4\epsilon+3)}{\left(3\bar{N}+\theta^2(4\epsilon+3)\right)^3} + \frac{3(\epsilon+1)}{\left(\bar{N}+\theta^2(\epsilon+1)\right)^2} - \frac{4(5\epsilon+3)}{\left(3\bar{N}+\theta^2(4\epsilon+3)\right)^2}\right)$, *then the retailer prefers the maximum possible number of reviews* $\bar{N}$.

(ii) If $c > \bar{c} = \frac{1}{24}\epsilon\left(\frac{12(h+1)^2(2\epsilon+1)}{\theta^2(4\epsilon+1)^3} + \frac{1}{\theta^2(\epsilon+1)} - \frac{4(3\epsilon+2)}{\theta^2(4\epsilon+3)^2}\right)$, *then the retailer does not prefer any reviews.*

(iii) Otherwise, the retailer's optimal number of reviews, N_R^*, *is given by the solution to the following equation:*

$$\begin{aligned} c = \frac{1}{24}\epsilon\Big(&\frac{12(h+1)^2(2\epsilon+1)}{(4\epsilon+1)^2(\theta^2+N_R^*+4\theta^2\epsilon)} + N_R^*\Big(-\frac{12(h+1)^2}{(4\epsilon+1)^2(\theta^2+N_R^*+4\theta^2\epsilon)^2} - \frac{1}{(N_R^*+\theta^2(\epsilon+1))^2} \\ &+ \frac{44\epsilon+24}{(4\epsilon+3)(3N_R^*+\theta^2(4\epsilon+3))^2}\Big) + 24\epsilon(N_R^*)^2\Big(-\frac{(h+1)^2}{(4\epsilon+1)^2(\theta^2+N_R^*+4\theta^2\epsilon)^3} - \frac{1}{(4\epsilon+3)(3N_R^*+\theta^2(4\epsilon+3))^3}\Big) \\ &+ \frac{1}{N_R^*+\theta^2(\epsilon+1)} - \frac{4(3\epsilon+2)}{(4\epsilon+3)(3N_R^*+\theta^2(4\epsilon+3))}\Big) \end{aligned} \tag{18}$$

The retailer's expected profit in the absence of review cost is an increasing concave function of the number of reviews. We note that among the four effects of increasing N discussed to explain Proposition 1, only the review-variance-reducing effect is directly affected by N and this effect is concave in N. Given that the cost of generating reviews is linear in N, the marginal benefit from additional reviews dominates the marginal cost if c and N are small and the marginal cost dominates when c and N are large. Since the condition that characterizes the optimal number of reviews when c is neither too small nor too large is complex, we illustrate the optimal number of reviews by plotting the retailer's expected profit as a function of N as shown in Figure 4. For the set of parameter values used to draw Figure 4, the retailer will prefer no reviews when c is greater than 0.112 and prefer maximum possible number of reviews of 180 when c is less than 0.067

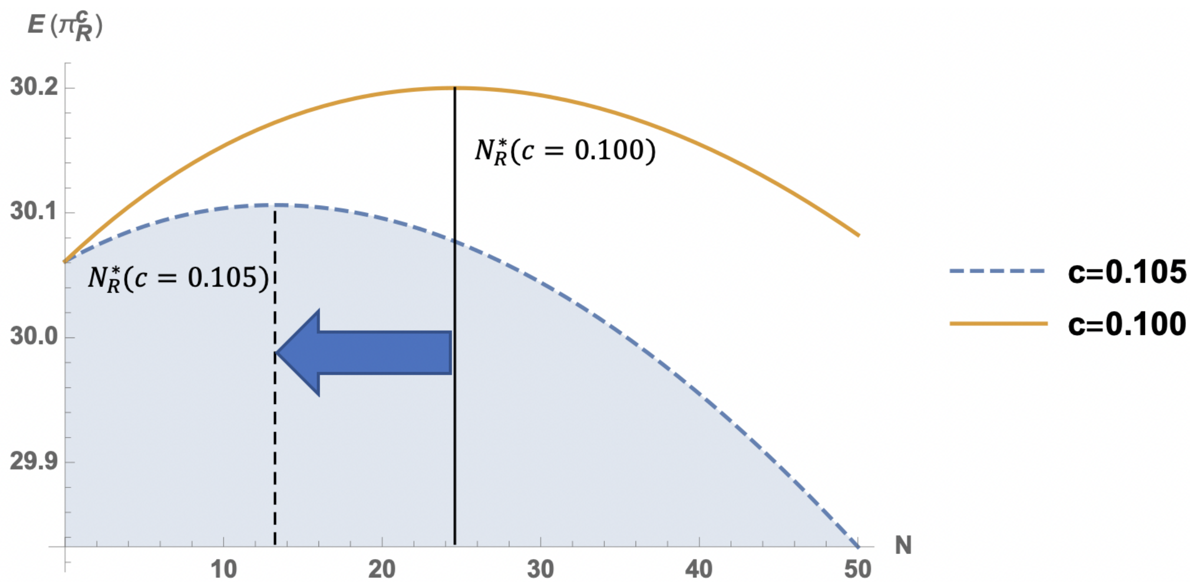


Figure 4: The effect of cost on the retailer's expected profits. ($\epsilon = 501$, $h = 30$, $\theta = 0.6$, $\bar{N} = 180$)

Proposition 2. *The retailer's optimal number of reviews, N_R^*,*

1) increases when the reviews' information transfer effectiveness increases (i.e.,N_R^* is decreasing in θ);

2) decreases when the cost of generating reviews increases (i.e., N_R^* is decreasing in c).

The retailer's optimal number of reviews depends on the trade-off between the marginal benefit and the marginal cost of generating reviews. The variance of product quality difference reflected in reviews is high when the reviews do not transfer information effectively, e.g. θ is high. Higher variance means the consumers are less homogeneous, rely less on reviews, and perceive high differentiation in product quality on average. The positive benefits of all the strategic impacts of reviews on the retailer are weakened by a decrease in the effectiveness of reviews' information transfer. Therefore, the retailer's optimal number of reviews increases when θ decreases.

On the other hand, increasing the marginal cost of generating reviews drives the retailer to generate lower number of reviews, which is intuitive. Clearly, the review generation cost reduces the retailer's incentive to generate reviews. In the costless review generation scenario, only the manufacturers would prefer to limit the number of reviews. However, in the scenario where the retailer incurs a cost to generate reviews, the retailer would also prefer to limit the number of reviews. In fact, it is possible that the retailer would prefer fewer reviews than manufacturers if the cost to generate review is sufficiently high. Thus, the nature of the divide in the retailer's and manufacturers' incentives to have more reviews depends critically on the cost to generate reviews.

More importantly, a comparison of Corollary 1 with Proposition 2 reveals another sharp contrast or divide in the retailer's and manufacturers' preferences. While the optimal number of reviews preferred by the manufacturers decreases with the reviews' information transfer effectiveness, the optimal number of reviews preferred by the retailer increases with the reviews' information transfer effectiveness. This finding also implies that when the cost of reviews is not too high, the retailer would prefer to have a retail platform that is highly effective in transferring the information content of reviews, but the manufacturers prefer a platform that is not highly effective.

4 Monetization of Reviews

The misalignment of preferences of the manufacturers and the retailer regarding the number of reviews creates a strategic opportunity for the retailer to monetize reviews because the retailer often controls the review platform in online marketplaces. When the retailer is dominant in the marketplace and has bargaining power over manufacturers, he can offer a take-it-or-leave-it contract to the manufacturers in which he commits to a certain number of reviews that would be made available in the platform in return for a fee to be paid by manufacturers. Early indications of the efforts by a retailer to monetize reviews are found in Amazon's Vine and Early Review programs. In these programs, Amazon controls the contract parameters such as the fees and number of reviews. Furthermore, in some cases, Amazon also selects the sellers whose products will be reviewed and the reviewers that can contribute reviews. In effect, through these programs, Amazon is seeking to transform its product review platform from one that traditionally operated under voluntary contribution of reviews by consumers to a money-generating business. Our results in the previous section suggest that the divergent incentives of players regarding reviews could potentially play a role in Amazon's initiatives to monetize reviews. We examine the retailer's review monetization problem in this section.

4.1 Efficient Number of Reviews

We first examine the number of reviews that maximizes the expected industry profit, which is the sum of the expected retailer's profit and the expected manufacturers' profits. We refer to it as the *efficient number of reviews.* Clearly, all sellers, including the retailer and the two manufacturers, will be better off under a contract that yields the efficient number of reviews and each gets at least as much payoff as he will get in the absence of a contract.

The efficient number of reviews, N^*, is obtained by solving the following problem:

$$\max_{N} \quad E(\pi_{Industry}) = E(\pi_R) + E(\pi_A) + E(\pi_B) - cN \tag{19}$$

Lemma 3. *The efficient number of reviews, N^*, is given by the solution to the following equation:*

$$c = \frac{1}{24}\epsilon\Big(\frac{24(h+1)^2\epsilon}{(4\epsilon+1)^2(N^*+\theta^2(4\epsilon+1))} + N^*\Big(-\frac{48(h+1)^2\epsilon}{(4\epsilon+1)^2(N^*+\theta^2(4\epsilon+1))^2} - \frac{1}{(N^*+\theta^2(\epsilon+1))^2}$$
$$+\frac{4(\epsilon+3)}{(4\epsilon+3)(3N^*+\theta^2(4\epsilon+3))^2}\Big) + 24\epsilon(N^*)^2\Big(\frac{(h+1)^2}{(4\epsilon+1)^2(N^*+\theta^2(4\epsilon+1))^3} + \frac{1}{(4\epsilon+3)(3N^*+\theta^2(4\epsilon+3))^3}\Big)$$
$$+\frac{1}{N^*+\theta^2(\epsilon+1)} - \frac{4(\epsilon+1)}{(4\epsilon+3)(3N^*+\theta^2(4\epsilon+3))}\Big) \tag{20}$$

Analogous to the retailer's benefit, the industry's benefit (sum of the expected retailer's profit and the expected manufacturers' profits without accounting for the cost of reviews) from reviews is an increasing concave function of the number of reviews. Consequently, when the cost of generating reviews is linear in N, the marginal benefit dominates the marginal cost when $N < N^*$, and the marginal cost dominates the marginal benefit when $N > N^*$.

Proposition 3. *The efficient number of reviews, N^*,*

1) increases when the degree of information transferrability in reviews increases (θ decreases);

2) decreases when the cost of generating reviews increases (c increases).

The intuition for Proposition 3 is same as that for Proposition 2 because the expected industry profit and the expected retailer profit have the same qualitative behavior with respect to change in the number of reviews. Even though an increase in θ benefits the manufacturers but hurts the retailer, the net marginal gain for the industry from increasing the number of reviews is smaller under a larger θ. Therefore, the efficient number of reviews decreases when θ increases. On the other hand, since the expected manufacturers' profit does not change when the review cost increases, the impact of review cost on efficient number of reviews is qualitatively identical to that on N_R^*, i.e., the efficient number of reviews decreases as c increases.

Proposition 4. *Define $\hat{c}$ as given in equation (58) in the Appendix. The retailer's optimal number of reviews is smaller than the efficient number of reviews if and only if the marginal cost of generating review is higher than $c > \hat{c}$, i.e., $N_R^* < N^*$ if and only if $c > \hat{c}$.*

Proposition 4 reveals that, relative to the efficient number of reviews, a self-interested retailer over-provisions reviews when the review cost is less than a threshold value and under-provisions reviews when the review cost is greater than the threshold value. It is intuitive the retailer would have a large incentive to provide reviews when review cost is small. When the review cost is sufficiently low, the retailer prefers to have more reviews than the manufacturers. Recall from Proposition 1 that the retailer prefers more reviews than the manufacturers when reviews are costless; therefore, when the review cost is small, the marginal revenue dominates the marginal cost for the retailer, resulting in the retailer preferring more reviews than the manufacturers. However, at the point where the number of reviews maximizes the self-interested retailer's expected profit, the manufacturers' expected profit is declining in number of reviews. Essentially, when the number of reviews is greater than that preferred by manufacturers, the marginal impact of number of reviews on expected industry profit is less than that on expected the retailer's profit. Consequently, the number of reviews that maximizes the expected industry profit is less than N_R^* when the optimal number of reviews is greater for the retailer than the manufacturers, i.e., when the review cost is sufficiently small. On the other hand, when the review cost is large, a self-interested retailer prefers to have fewer reviews than the manufacturers. In this case, at the point where the number of reviews maximizes the expected retailer's profit, the expected manufacturers' profit is increasing in the number of reviews. Thus, when the number of reviews is less than that preferred by the manufacturers, the marginal impact of number of reviews on the expected industry profit is greater than that on the expected retailer's profit. Consequently, the number of reviews that maximizes the expected industry profit is greater than N_R^* when the optimal number of reviews is less for the retailer than the manufacturers, i.e., when the review cost is sufficiently high. Figure 5 illustrates Proposition 4.

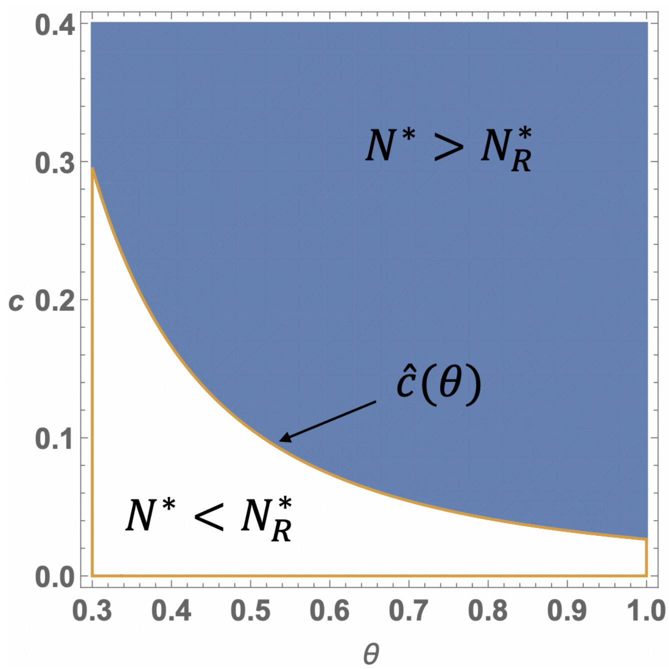


Figure 5: Comparison of the retailer's optimal numbers of reviews N_R^* and the efficient number of reviews N^* ($\epsilon = 501, h = 30$)

Corollary 2. *The threshold for the marginal cost of generating reviews, $\hat{c}$, is decreasing in θ.*

Furthermore, as shown in Corollary 2, the region where the efficient number of reviews exceeds the retailer's number of review shrinks as the reviews' information transfer effectiveness increases (i.e., θ decreases). It suggests that in a scenario with a low review cost and highly effective review platform (in transferring the information content), a retailer could significantly over-provision reviews.

Proposition 4 and Corollary 2 have significant implications for the retailer. In particular, it implies that a dominant retailer can exploit the misalignment in her own incentives to generate reviews and that of manufacturers (and the industry) by "selling" reviews to the manufacturers for a fee. Furthermore, the incentive to sell reviews exists regardless of whether he over-provisions or under-provisions reviews. Suppose the marginal cost of reviews is large and the information transfer effectiveness is low such that the retailer under-provisions reviews on her own. In this case, the manufacturers desire more reviews than what the retailer would provide. Thus, the manufacturers would be willing to pay a fee to the retailer in return for more reviews. This fee increases the retailer's incentive to provide more reviews and raise the number of reviews closer to the efficient level. Conversely, when the review cost is small and the information transfer effectiveness is high, the retailer over-provisions reviews and provides more reviews than that desired by the manufacturers. In this case, the manufacturers would be willing to pay a fee to the retailer if the retailer guarantees, in return, to limit the number of reviews. A dominant retailer would thus exploit the manufacturer's willingness to pay for (either increasing or limiting the number of) reviews by designing a contract that would just extract the manufacturer surplus. We examine such a contract next.

For this analysis, we add another stage to the time line as shown in Figure 6. In stage 1, the retailer announces a take-it-or-leave-it contract to generate N_p reviews for a fee of pN_p from each manufacturer. The parameter p represents the fee per review paid by each manufacturer. If both manufacturers accept the contract, then the retailer generates N_p number of reviews and gets $2pN_p$ from the manufacturers in stage 2. On the other hand, if either manufacturer does not accept the contract, then the retailer generates N_R^* number of reviews as in the no monetization scenario. Depending on the number of reviews generated by the retailer, X_R is revealed to the public. Conditional on the realized value of X_R, the manufacturers decide the wholesale prices in stage 3, and then the retailer decides the retail prices in stage 4. Lastly, consumers make purchase decisions in stage 5.

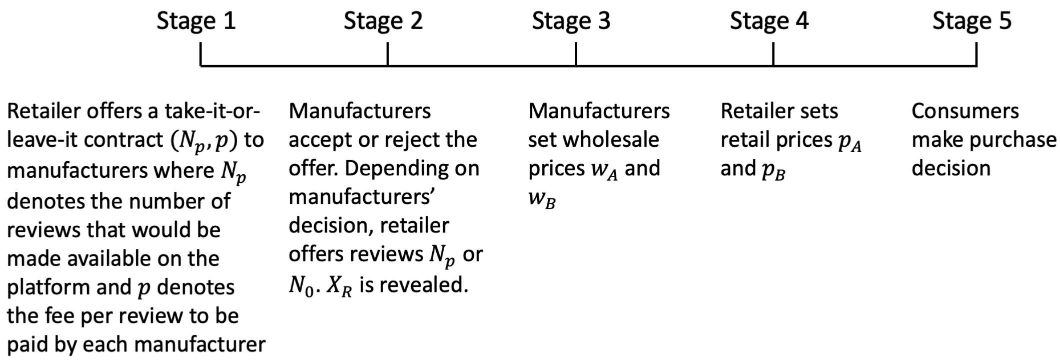


Figure 6: Time line of events in the Review Monetization Case

We define the retailer's expected profit in the case of review monetization as follows:

$$E(\pi_R^M) = E(\pi_R(N_p)) + 2pN_p - cN_p \tag{21}$$

Clearly, as the first-mover, the retailer will design a contract such that the individual rationality (IR) constraint for the manufacturers is satisfied and they accept the contract, i.e., the manufacturers are not worse off by accepting the contract, because such a contract maximizes the retailer's payoff. The retailer would design a contract that would provide the manufacturers an expected payoff that is slightly larger than the expected payoff they could obtain if they do not accept the contract. Therefore, the contract that the retailer would offer in the equilibrium would maximize the expected industry profit with the manufacturers receiving the expected profit they would have received in the absence of such a contract and the retailer keeping the remaining part of the expected industry profit. That is, in equilibrium, $N_p^* = N^*$ and $p^*N_p^* = E(\pi_i|N = N^*) - E(\pi_i|N = N_R^*)$. We can also check that the retailer is not worse off from such a contract, compared to the no monetization case.

Proposition 5. *The fee paid by the manufacturers, $p^*N_p^*$,*

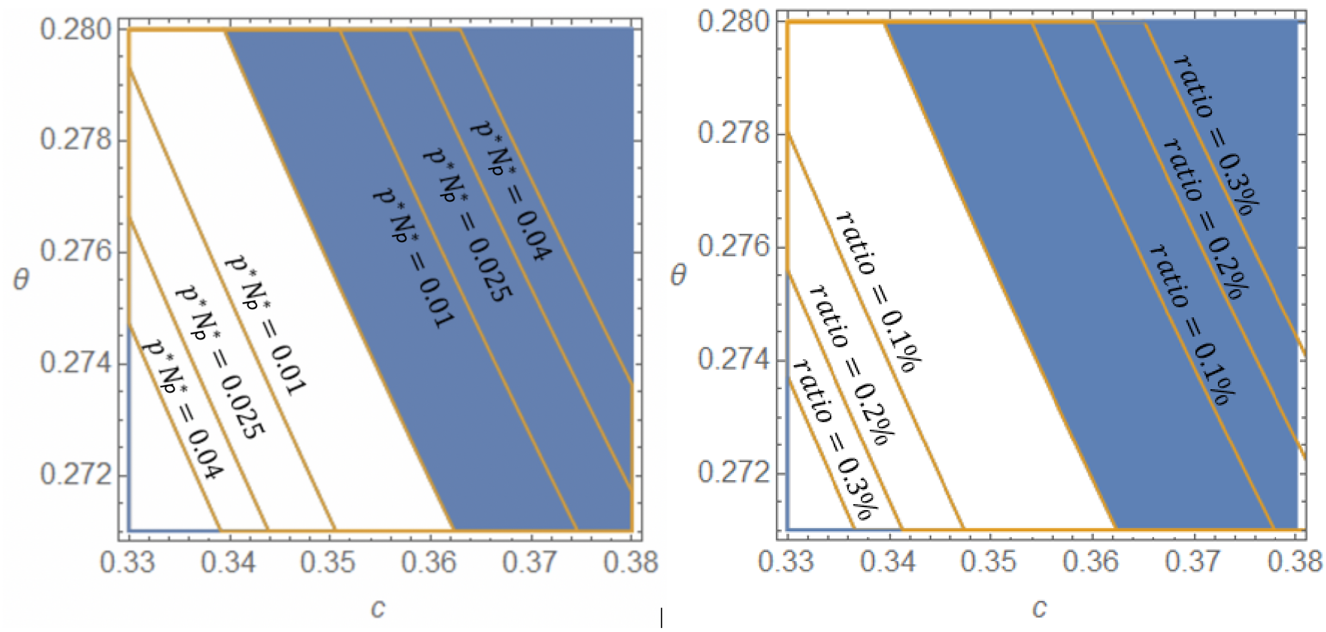
a) decreases in c if $c < \hat{c}$ and increases in c, otherwise.

b) decreases in θ if $c < \hat{c}$.

Recall that when c is less than $\hat{c}$, the retailer has an excess incentive to provide reviews and therefore the manufacturers are willing to pay to limit the number of reviews. As c increases and approaches $\hat{c}$, the retailer's incentive to offer excess reviews diminishes and the retailer would, on her own, provide reviews that are closer to the manufacturers' optimal number. Therefore, the manufacturers' willingness to pay, and hence the total fee charged by the retailer, also decreases. On the other hand, When c is greater than $\hat{c}$, the retailer has an incentive to under-provision reviews and therefore the manufacturers are willing to pay to increase the number of reviews. As c increases and moves away from $\hat{c}$, the retailer's incentive to under-provision reviews also increases, and therefore, the manufacturers' willingness to pay, and hence the total fee charged by the retailer, also increases. A similar reasoning applies for the impact of θ on the fee paid by manufacturers. Recall that an increase in θ reduces the benefit of reviews for the retailer, and thus diminishes the incentive to provide reviews. Therefore, when the retailer has an incentive to over-provision reviews (i.e., $c < \hat{c}$), an increase in θ reduces that incentive and hence the fee paid by manufacturers; on the other hand, though we are unable to prove analytically, the numerical experimentation suggests that when the retailer has an incentive to under-provision reviews (i.e., $c > \hat{c}$), an increase in θ exacerbates under provisioning, causing manufacturers to be willing to pay more for reviews. Figure 7(a) illustrates Proposition 5.

Proposition 5 reveals that the retailer's revenue from review monetization is a a U-shaped function of c as well as θ. It suggests that review monetization is especially attractive to the retailer in a scenario where the review cost is low and the information transfer effectiveness is high and in a scenario where the review cost is high and the information transfer effectiveness is low.

Proposition 5 and Figure 7(a) show how the review cost and review' information transfer effectiveness affect the absolute revenue the retailer can enjoy by monetizing reviews. However, the revenue from monetization is just one source of benefit from reviews. As we discussed in Section 3, reviews have an impact on the expected retailer's profit even in the absence of monetization. Consequently, we examine how the revenue from monetization as a fraction of the total expected retailer profit under monetization. Since we are unable to



(a) Fee, $p^* N_p^*$, from each manufacturer

(b) Ratio of the revenue from review monetization and the retailer's expected profit, $\frac{2p^* N_p^*}{E(\pi_R^M)}$

Figure 7: Effects of c and θ on fee and ratio ($\epsilon = 501, h = 30$)

analytically characterize this fraction, we use Figure 7(b) to numerically illustrate it. We find that the impacts of c and θ on the revenue from monetization as a percentage of the total expected retailer profit are qualitatively identical to those on the absolute revenue. This finding provides further support regarding the scenarios suggested by Proposition 5 where the retailer finds review monetization to be attractive.

The preceding analysis of review monetization shows that a dominant retailer can indeed use her review platform as a source of additional review. A natural question that arises is whether the review monetization alone can be a profitable standalone business for the retailer if he incurs a cost to generate reviews; in other words, does the revenue from review monetization cover the cost of generating reviews? We answer this question in Proposition 6.

Proposition 6. *The fee collected by the retailer from the manufacturers is less than the cost of generating reviews, $2p^* < c$, under review monetization.*

Proposition 6 shows that even though the retailer gets an additional source of revenue from review monetization, the retailer's gain from "selling" reviews cannot compensate the cost of generating reviews. This suggests that the retailer's profitability from reviews is driven by their positive impacts on the retailer's core business of selling the products to downstream consumers, and review monetization is not profitable to the retailer if it is evaluated based solely on the direct revenue it brings and the direct cost incurred to generate the reviews.

5 Discussion and Conclusion

In this paper, we seek to a fill a gap in the vast literature that has examined online reviews. Specifically, we show that the retailer's preference and the manufacturers' preferences regarding the number of reviews are not always aligned. If the reviews are costless, while the retailer prefers to have the maximum possible number of reviews, the manufacturers do not want the number of reviews to exceed a threshold, and in some cases, do not prefer to have any review. If the retailer incurs a cost to generate reviews, he may prefer to have fewer reviews than the manufacturers when the review cost is sufficiently high. The divide between the retailer's and the manufacturers' preferences regarding the number of reviews can enable the retailer to monetize reviews by charging a fee to the manufacturers in return for a guarantee on the number of reviews that would be made available in the review platform. While review monetization could provide an additional source of revenue for the retailer, the revenue from "selling" reviews does not exceed the cost of generating reviews, implying the retailer's profitability from reviews even when they can be monetized is driven by the reviews' positive impacts on the expected retailer's profit from product sales to downstream consumers. We identify several subtle effects of reviews both on downstream consumers and upstream products that give rise to the results.

Our work has implications for academics and practitioners. The extant academic literature on online product reviews makes the implicit assumption that more reviews lead to higher seller profitability. By examining the underlying mechanism that links the number of reviews to their impacts on profitability, we show that the assumption may not hold. In particular, we offer insights about the differences among the retailer and the manufacturers' incentives to generate reviews. These insights point to an alternative view that over-provisioning of reviews is a distinct possibility in an online retail environment. The research also offers a hitherto unexplored perspective related to online reviews, specifically how a retailer can monetize the review platform.

From a practitioner perspective, the results provide insights into the nascent efforts of retailers like Amazon to monetize reviews through initiatives such as Vine program. Specifically, the results identify the conditions under which review monetization is likely to be attractive for a dominant retailer. The findings also reveal that the retailer may need to think more strategically about the design of review platform if he monetizes the review platform. For example, the debate over the value of multi-dimensional rating systems centers around their informational advantage over the single-dimensional rating system as multi-dimensional ratings transfer more granular product information to consumers. However, when the retailer considers the revenue from review monetization, implementing a multi-dimensional rating system is not always desirable. Moreover, since the product type is likely to influence the review platform's effectiveness and possibly, our findings show that the attractiveness of review monetization can vary across product categories.

As a first study in review monetization, we used a stylized model that only accounted for the most salient aspects of the problem. The paper can be extended in several directions. For instance, we considered symmetric manufacturers. Clearly, the preferences of a manufacturer that has a superior quality product can differ from those of a manufacturer that has an inferior quality product. Modeling asymmetry between product qualities could also offer insights into why the retailer might prefer to sell reviews only to a sub set of manufacturers. We assumed a wholesale pricing model for our analysis. Examining the agency pricing model which is common in online retailing environments could offer additional insights. Finally, we considered a scenario in which product quality dominates fit in consumers' evaluation of products. Analysis of the scenario where fit dominates quality is another potential direction for future research.

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6 Appendix

Proof of Lemma 1

Proof. Substituting the demand functions D_A and D_B in Equation (8), we can rewrite the retailer profit function π_R in Equation (10) as:

$$\pi_R = \frac{1}{(2\gamma\epsilon)}[(p_A - w_A)((1+h)\gamma\epsilon + (1-\gamma)X_R - (1+2\gamma\epsilon)p_A + p_B) + (p_B - w_B)((1+h)\gamma\epsilon - (1-\gamma)x_R + p_A - (1+2\gamma\epsilon)p_B)] \quad (22)$$

Taking the first derivative of retailer's profit π_R with respect to retail prices p_A and p_B, we charaterize the retailer's optimalization problem in stage 3 as follows:

$$\begin{aligned} \frac{\partial \pi_R}{\partial p_A} &= \frac{2p_B - w_B - 2(2\gamma\epsilon+1)p_A + w_A(2\gamma\epsilon+1) + \gamma(h+1)\epsilon + (1-\gamma)X_R}{2\gamma\epsilon} = 0 \\ \frac{\partial \pi_R}{\partial p_B} &= \frac{2p_A - w_A - 2(2\gamma\epsilon+1)p_B + w_B(2\gamma\epsilon+1) + \gamma(h+1)\epsilon - (1-\gamma)X_R}{2\gamma\epsilon} = 0 \end{aligned} \quad (23)$$

from which we can derive the retailer's optimal retail prices given w_A and w_B:

$$\begin{aligned} p_A &= \frac{1}{4}\left(2w_A + h + \frac{(1-\gamma)x_R}{\gamma\epsilon+1} + 1\right) \\ p_B &= \frac{1}{4}\left(2w_B + h - \frac{(1-\gamma)x_R}{\gamma\epsilon+1} + 1\right) \end{aligned} \quad (24)$$

Substituting the retail prices, we rewrite the manufacturer's profit functions in Equation (12):

$$\begin{aligned} \pi_A &= \frac{w_A(w_B - w_A(2\gamma\epsilon+1) + \gamma(h+1)\epsilon + (1-\gamma)x_R)}{4\gamma\epsilon} \\ \pi_B &= \frac{w_B(w_A - w_B(2\gamma\epsilon+1) + \gamma(h+1)\epsilon - (1-\gamma)x_R)}{4\gamma\epsilon} \end{aligned} \quad (25)$$

Taking the first derivative of the manufacturer profit functions with respect to w_A and w_B, we characterize the manufacturers' optimalization problems in stage 2:

$$\begin{aligned} \frac{\partial \pi_A}{\partial w_A} &= \frac{w_B - 2w_A(2\gamma\epsilon+1) + \gamma(h+1)\epsilon + (1-\gamma)x_R}{4\gamma\epsilon} = 0 \\ \frac{\partial \pi_B}{\partial w_B} &= \frac{w_A - 2w_B(2\gamma\epsilon+1) + \gamma(h+1)\epsilon - (1-\gamma)x_R}{4\gamma\epsilon} = 0 \end{aligned} \quad (26)$$

from which we can drive the manufacturers' optimal wholesale prices:

$$\begin{aligned} w_A &= \tfrac{\gamma(h+1)\epsilon}{4\gamma\epsilon+1} + \tfrac{(1-\gamma)x_R}{4\gamma\epsilon+3} \\ w_B &= \tfrac{\gamma(h+1)\epsilon}{4\gamma\epsilon+1} - \tfrac{(1-\gamma)x_R}{4\gamma\epsilon+3} \end{aligned} \tag{27}$$

Substituting the optimal wholesale prices into the retailer's optimal retail prices:

$$\begin{aligned} p_A &= \tfrac{1}{4}\left(\tfrac{(h+1)(6\gamma\epsilon+1)}{4\gamma\epsilon+1} + \tfrac{(1-\gamma)(6\gamma\epsilon+5)x_R}{(\gamma\epsilon+1)(4\gamma\epsilon+3)}\right) \\ p_B &= \tfrac{1}{4}\left(\tfrac{(h+1)(6\gamma\epsilon+1)}{4\gamma\epsilon+1} - \tfrac{(1-\gamma)(6\gamma\epsilon+5)x_R}{(\gamma\epsilon+1)(4\gamma\epsilon+3)}\right) \end{aligned} \tag{28}$$

Hence, we show Lemma 1. ■

Proof of Proposition 1

Proof. By substituting the optimal wholesale prices and optimal retail prices from Lemma 1 into the profit functions of the retailer and the manufacturers:

$$\pi_R = \frac{1}{8}(2\gamma\epsilon+1)^2\left(\frac{(h+1)^2}{(4\gamma\epsilon+1)^2} + \frac{(1-\gamma)^2 x_R^2}{\gamma\epsilon(\gamma\epsilon+1)(4\gamma\epsilon+3)^2}\right) \tag{29}$$

$$\begin{aligned} \pi_A &= \tfrac{(2\gamma\epsilon+1)(\gamma(h+1)\epsilon(4\gamma\epsilon+3)+(1-\gamma)(4\gamma\epsilon+1)x_R)^2}{4\gamma\epsilon(4\gamma\epsilon+1)^2(4\gamma\epsilon+3)^2} \\ \pi_B &= \tfrac{(2\gamma\epsilon+1)(\gamma(h+1)\epsilon(4\gamma\epsilon+3)-(1-\gamma)(4\gamma\epsilon+1)x_R)^2}{4\gamma\epsilon(4\gamma\epsilon+1)^2(4\gamma\epsilon+3)^2} \end{aligned} \tag{30}$$

The expected profits of the retailer and the manufacturers in stage 1 can be derived by taking the expectation of x_R, where $E(X_R) = 0$ and $E(X_R^2) = Var(X_R) = v_R$:

$$\begin{aligned} E(\pi_R) &= \tfrac{1}{8}(2\gamma\epsilon+1)^2\left(\tfrac{(h+1)^2}{(4\gamma\epsilon+1)^2} + \tfrac{(1-\gamma)^2 v_R}{\gamma\epsilon(\gamma\epsilon+1)(4\gamma\epsilon+3)^2}\right) \\ E(\pi_A) = E(\pi_B) &= \tfrac{(2\gamma\epsilon+1)\left(\gamma^2(h+1)^2\epsilon^2(4\gamma\epsilon+3)^2+(1-\gamma)^2 v_R(4\gamma\epsilon+1)^2\right)}{4\gamma\epsilon(4\gamma\epsilon+1)^2(4\gamma\epsilon+3)^2} \end{aligned} \tag{31}$$

Substituting $\gamma = \frac{v_R}{v_R+v_C}$ and $v_C = \frac{\epsilon^2}{3}$, we rewrite the expected profits of the retailer and the manufacturers in stage 1 in term of v_R:

$$
\begin{aligned}
E(\pi_R) &= \frac{1}{8}\left(\frac{6v_R\epsilon}{3v_R+\epsilon^2}+1\right)^2\left(\frac{(h+1)^2}{\left(\frac{12v_R\epsilon}{3v_R+\epsilon^2}+1\right)^2}+\frac{\epsilon^3\left(3v_R+\epsilon^2\right)^2}{27(3v_R(\epsilon+1)+\epsilon^2)(v_R(4\epsilon+3)+\epsilon^2)^2}\right)\\
E(\pi_A) = E(\pi_B) &= \frac{\epsilon\left(v_R(6\epsilon+3)+\epsilon^2\right)\left(81(h+1)^2v_R^3(4\epsilon+3)^2+9v_R^2\epsilon^2\left(72h(h+2)\epsilon+54h(h+2)+16\epsilon^2+80\epsilon+55\right)+3v_R\epsilon^4(27h(h+2)+8\epsilon+29)+\epsilon^6\right)}{108(3v_R(4\epsilon+1)+\epsilon^2)^2(v_R(4\epsilon+3)+\epsilon^2)^2}
\end{aligned} \tag{32}
$$

Substituting $v_R = \frac{\theta^2\epsilon^2}{3N}$, we rewrite the expected profits of the retailer and the manufacturers in stage 1 in term of N:

$$
\begin{aligned}
E(\pi_R) &= \frac{1}{8}\left(\frac{2\theta^2\epsilon}{\theta^2+N}+1\right)^2\left(\frac{(h+1)^2}{\left(\frac{4\theta^2\epsilon}{\theta^2+N}+1\right)^2}+\frac{N\epsilon\left(\theta^2+N\right)^2}{3(N+\theta^2(\epsilon+1))(3N+\theta^2(4\epsilon+3))^2}\right)\\
E(\pi_A) = E(\pi_B) &= \frac{\epsilon\left(N+\theta^2(2\epsilon+1)\right)\left(\theta^2N^2(27h(h+2)+8\epsilon+29)+\theta^4N\left(72h(h+2)\epsilon+54h(h+2)+16\epsilon^2+80\epsilon+55\right)+3(h+1)^2\theta^6(4\epsilon+3)^2+N^3\right)}{12(N+\theta^2(4\epsilon+1))^2(3N+\theta^2(4\epsilon+3))^2}
\end{aligned} \tag{33}
$$

Taking the first order derivative of the retailer's expected profit in Equation (33) with respect to N, we characterize the retailer's optimization problem in stage 1:

$$
\begin{aligned}
\frac{dE(\pi_R)}{dN} = \frac{1}{24}\epsilon\Big(&\frac{12(h+1)^2(2\epsilon+1)}{(4\epsilon+1)^2(\theta^2+N+4\theta^2\epsilon)}+N\Big(-\frac{12(h+1)^2}{(4\epsilon+1)^2(\theta^2+N+4\theta^2\epsilon)^2}-\frac{1}{(N+\theta^2(\epsilon+1))^2}\\
&+\frac{44\epsilon+24}{(4\epsilon+3)(3N+\theta^2(4\epsilon+3))^2}\Big)+24\epsilon(N)^2\Big(-\frac{(h+1)^2}{(4\epsilon+1)^2(\theta^2+N+4\theta^2\epsilon)^3}-\frac{1}{(4\epsilon+3)(3N+\theta^2(4\epsilon+3))^3}\Big)\\
&+\frac{1}{N+\theta^2(\epsilon+1)}-\frac{4(3\epsilon+2)}{(4\epsilon+3)(3N+\theta^2(4\epsilon+3))}\Big)
\end{aligned} \tag{34}
$$

We notice that $\frac{dE(\pi_R)}{dN} > 0$ since $\epsilon > \frac{\theta^2+\bar{N}}{\theta^2}$.

Taking the second derivative of the retailer's expected profit with respect to N:

$$
\begin{aligned}
\frac{d^2E(\pi_R)}{dN^2} = \frac{1}{12}\epsilon\Big(&36N^2\epsilon\Big(\frac{(h+1)^2}{(4\epsilon+1)^2(N+\theta^2(4\epsilon+1))^4}+\frac{3}{(4\epsilon+3)(3N+\theta^2(4\epsilon+3))^4}\Big)\\
&-\frac{12(h+1)^2(\epsilon+1)}{(4\epsilon+1)^2(N+\theta^2(4\epsilon+1))^2}+N\Big(-\frac{12(h+1)^2(2\epsilon-1)}{(4\epsilon+1)^2(N+\theta^2(4\epsilon+1))^3}+\frac{1}{(N+\theta^2(\epsilon+1))^3}\\
&-\frac{12(13\epsilon+6)}{(4\epsilon+3)(3N+\theta^2(4\epsilon+3))^3}\Big)-\frac{1}{(N+\theta^2(\epsilon+1))^2}+\frac{8(5\epsilon+3)}{(4\epsilon+3)(3N+\theta^2(4\epsilon+3))^2}\Big)
\end{aligned} \tag{35}
$$

We notice that $\frac{d^2E(\pi_R)}{dN^2} < 0$ since $\epsilon > \frac{\theta^2+\bar{N}}{\theta^2}$.

Taking the derivative of the manufacturers' expected profit in Equation (32) with respect to v_R, we characterize the effect of variance of reviews on the manufacturers profit:

$$\frac{dE(\pi_A)}{dv_R} = \frac{dE(\pi_B)}{dv_R} = \frac{\begin{array}{r}\epsilon^3(81v_R^4(4\epsilon+3)(3h^2(4\epsilon+3)^2+6h(4\epsilon+3)^2-8(4\epsilon(4\epsilon+5)+3)\epsilon^2\\ +58\epsilon+26)+108v_R^3\epsilon^2(3h^2(\epsilon+3)(4\epsilon+3)^2+6h(\epsilon+3)(4\epsilon+3)^2\\ -8\epsilon(\epsilon(8\epsilon(2\epsilon+3)-9)-26)+78)+54v_R^2\epsilon^4(8(9h(h+2)+2)\epsilon^2\\ +6(27h(h+2)+23)\epsilon+81h(h+2)-32\epsilon^3+78)\\ +12v_R\epsilon^6(81h(h+2)\epsilon+81h(h+2)-8\epsilon^2+68\epsilon+78)\\ +\epsilon^8(81h(h+2)-2\epsilon+78))\end{array}}{108(3v_R(4\epsilon+1)+\epsilon^2)^3(v_R(4\epsilon+3)+\epsilon^2)^3} \tag{36}$$

Equating $\frac{dE(\pi_A)}{dv_R}=0$, and solving the equality, we get:

$$h = \bar{h} = \frac{1}{9}\sqrt{\frac{\begin{array}{r}41472v_R^4\epsilon^5+82944v_R^4\epsilon^4+62208v_R^4\epsilon^3+22032v_R^4\epsilon^2+3726v_R^4\epsilon+243v_R^4\\ +13824v_R^3\epsilon^6+25920v_R^3\epsilon^5+15552v_R^3\epsilon^4+3780v_R^3\epsilon^3+324v_R^3\epsilon^2\\ +1728v_R^2\epsilon^7+3024v_R^2\epsilon^6+1296v_R^2\epsilon^5+162v_R^2\epsilon^4+96v_R\epsilon^8\\ +156v_R\epsilon^7+36v_R\epsilon^6+2\epsilon^9+3\epsilon^8\end{array}}{(3v_R+\epsilon^2)\left(4v_R\epsilon+3v_R+\epsilon^2\right)^3}}-1 \tag{37}$$

We notice $\frac{dE(\pi_A)}{dv_R}>0$ and $\frac{dE(\pi_B)}{dv_R}>0$ if $h>\bar{h}$, $\frac{dE(\pi_A)}{dv_R}<0$ and $\frac{dE(\pi_B)}{dv_R}<0$ if $h<\bar{h}$.

Taking the first order derivative of $\bar{h}$ with respect to v_R:

$$\frac{d\bar{h}}{dv_R} = \frac{4\epsilon^3(3v_R(4\epsilon+1)+\epsilon^2)^2(3v_R^2(4\epsilon+3)(\epsilon(4\epsilon+11)+4)+2v_R(\epsilon(20\epsilon+29)+12)\epsilon^2+(3\epsilon+4)\epsilon^4)}{3(3v_R+\epsilon^2)^2(v_R(4\epsilon+3)+\epsilon^2)^4\sqrt{\frac{(3v_R(4\epsilon+1)+\epsilon^2)^3(3v_R(2\epsilon+1)(4\epsilon+3)+(2\epsilon+3)\epsilon^2)}{(3v_R+\epsilon^2)(v_R(4\epsilon+3)+\epsilon^2)^3}}}>0 \tag{38}$$

Substituting $v_R=\frac{\theta^2\epsilon^2}{3N}$ in $\bar{h}$:

$$\bar{h} = \frac{\sqrt{3}}{3}\sqrt{\frac{\left(N+\theta^2(4\epsilon+1)\right)^3\left(N(2\epsilon+3)+\theta^2(2\epsilon+1)(4\epsilon+3)\right)}{(\theta^2+N)\left(3N+\theta^2(4\epsilon+3)\right)^3}}-1 \tag{39}$$

Since $\frac{dv_R}{dN}<0$, $\frac{d\bar{h}}{dN}=\frac{\partial\bar{h}}{\partial v_R}\cdot\frac{dv_R}{dN}<0$. There exists a N corresponding to $\bar{h}$. We denote N_M as the number of review that makes $h=\bar{h}$, and N_M satisfies:

$$81(\bar{h}\quad+\quad 1)^2 \quad=\quad \frac{(27((N_M+(1+4\epsilon)\theta^2))^3(N_M(3+2\epsilon)+(1+2\epsilon)(3+4\epsilon)\theta^2))}{((N_M+\theta^2)((3N_M+(3+4\epsilon)\theta^2))^3)}. \tag{40}$$

from which, we know when $N < N_M$, $h < \bar{h}$, thus $\frac{dE(\pi_A)}{dv_R} < 0$, and $\frac{dE(\pi_A)}{dN} = \frac{\partial E(\pi_A)}{\partial v_R} \cdot \frac{dv_R}{dN} > 0$; when $N > N_M$, $h > \bar{h}$, thus $\frac{dE(\pi_A)}{dv_R} > 0$, and $\frac{dE(\pi_A)}{dN} = \frac{\partial E(\pi_A)}{\partial v_R} \cdot \frac{dv_R}{dN} < 0$.

Taking the second derivative of manufacturers' profit functions in Equation (32) with respect to N, we have

$$\frac{d^2E(\pi_A)}{dN^2} = \frac{d^2E(\pi_B)}{dN^2} = \frac{1}{6}\epsilon(18N^2\epsilon(-\frac{(h+1)^2}{(4\epsilon+1)^2(N+\theta^2(4\epsilon+1))^4} - \frac{3}{(4\epsilon+3)(3N+\theta^2(4\epsilon+3))^4}) - \frac{3(h+1)^2(2\epsilon-1)}{(4\epsilon+1)^2(N+\theta^2(4\epsilon+1))^2} + 3N(\frac{(h+1)^2(8\epsilon-1)}{(4\epsilon+1)^2(N+\theta^2(4\epsilon+1))^3} + \frac{14\epsilon+3}{(4\epsilon+3)(3N+\theta^2(4\epsilon+3))^3}) - \frac{8\epsilon+3}{(4\epsilon+3)(3N+\theta^2(4\epsilon+3))^2}) \quad (41)$$

We notice that $\frac{d^2E(\pi_A)}{dN^2} < 0$, since $\epsilon > \frac{\theta^2+\bar{N}}{\theta^2}$.

Since $\frac{d\bar{h}}{dN} < 0$, $\bar{h}$ has its maximum value when $N = 0$ and has its minimum value at $N = \bar{N}$. Substituting $N = \bar{N}$ and $N = 0$, we get the range for h:

$$h \in (\frac{\sqrt{3}}{3}\sqrt{\frac{\left(\bar{N}+\theta^2(4\epsilon+1)\right)^3\left(\bar{N}(2\epsilon+3)+\theta^2(2\epsilon+1)(4\epsilon+3)\right)}{\left(\theta^2+\bar{N}\right)\left(3\bar{N}+\theta^2(4\epsilon+3)\right)^3}} - 1, \sqrt{\frac{(2\epsilon+1)(4\epsilon+1)^3}{3(4\epsilon+3)^2}} - 1) \quad (42)$$

which implies that $\frac{dE(\pi_A)}{dN} = 0$ and $\frac{dE(\pi_A)}{dN} = 0$ when $N = N_M$ if $h < \sqrt{\frac{(2\epsilon+1)(4\epsilon+1)^3}{3(4\epsilon+3)^2}} - 1$; and $\frac{dE(\pi_A)}{dN} < 0$ and $\frac{dE(\pi_A)}{dN} < 0$ $\forall N, 0 < N < \bar{N}$ if $h \geq \sqrt{\frac{(2\epsilon+1)(4\epsilon+1)^3}{3(4\epsilon+3)^2}} - 1$.

Hence, we show Proposition 1.

■

Proof of Corollary 1

Proof. Moving the LHS of Equation (40) to the RHS, we define R_3 as follows:

$$R_3 = \frac{(27((N_M+(1+4\epsilon)\theta^2))^3(N_M(3+2\epsilon)+(1+2\epsilon)(3+4\epsilon)\theta^2))}{((N_M+\theta^2)((3N_M+(3+4\epsilon)\theta^2))^3)} - 81(\bar{h}+1)^2. \quad (43)$$

Taking the partial derivatives of R_3 with respect to θ and N_M respectively, we get:

$$\begin{aligned}\frac{\partial R_3}{\partial N_M} &= -\frac{216\theta^2\epsilon\left(N_M+\theta^2(4\epsilon+1)\right)^2\left(2\theta^2 N_M(\epsilon(20\epsilon+29)+12)+3N_M^2(3\epsilon+4)+\theta^4(4\epsilon+3)(\epsilon(4\epsilon+11)+4)\right)}{(\theta^2+N_M)^2(3N_M+\theta^2(4\epsilon+3))^4} < 0 \\ \frac{\partial R_3}{\partial \theta} &= \frac{432\theta N_M\epsilon\left(N_M+\theta^2(4\epsilon+1)\right)^2\left(2\theta^2 N_M(\epsilon(20\epsilon+29)+12)+3N_M^2(3\epsilon+4)+\theta^4(4\epsilon+3)(\epsilon(4\epsilon+11)+4)\right)}{(\theta^2+N_M)^2(3N_M+\theta^2(4\epsilon+3))^4} > 0\end{aligned} \quad (44)$$

By implicit function theorem, $\frac{dN_M}{d\theta} = -\frac{\frac{\partial R_3}{\partial \theta}}{\frac{\partial R_3}{\partial N_M}} > 0$.

Hence, we show Corollary 1.

■

Proof of Lemma 2

Proof. Taking the first derivative of the retailer's expect profit in Equation (17) with respect to N, we characterize the retailer's optimization problem in stage 1 in the case when reviews are costly:

$$\begin{aligned}\frac{dE(\pi_R^C)}{dN} &= \frac{1}{24}\epsilon\Big(\frac{12(h+1)^2(2\epsilon+1)}{(4\epsilon+1)^2(\theta^2+N+4\theta^2\epsilon)}+N\Big(-\frac{12(h+1)^2}{(4\epsilon+1)^2(\theta^2+N+4\theta^2\epsilon)^2}-\frac{1}{(N+\theta^2(\epsilon+1))^2} \\ &+\frac{44\epsilon+24}{(4\epsilon+3)(3N+\theta^2(4\epsilon+3))^2}\Big)+24\epsilon(N)^2\Big(-\frac{(h+1)^2}{(4\epsilon+1)^2(\theta^2+N+4\theta^2\epsilon)^3}-\frac{1}{(4\epsilon+3)(3N+\theta^2(4\epsilon+3))^3}\Big) \\ &+\frac{1}{N+\theta^2(\epsilon+1)}-\frac{4(3\epsilon+2)}{(4\epsilon+3)(3N+\theta^2(4\epsilon+3))}\Big)-c=0 \quad (45)\end{aligned}$$

The retailer's optimal number of reviews, N_R^* satisfies the following first order condition:

$$\begin{aligned}c &= \frac{1}{24}\epsilon\Big(\frac{12(h+1)^2(2\epsilon+1)}{(4\epsilon+1)^2(\theta^2+N_R^*+4\theta^2\epsilon)}+N_R^*\Big(-\frac{12(h+1)^2}{(4\epsilon+1)^2(\theta^2+N_R^*+4\theta^2\epsilon)^2}-\frac{1}{(N_R^*+\theta^2(\epsilon+1))^2} \\ &+\frac{44\epsilon+24}{(4\epsilon+3)(3N_R^*+\theta^2(4\epsilon+3))^2}\Big)+24\epsilon(N_R^*)^2\Big(-\frac{(h+1)^2}{(4\epsilon+1)^2(\theta^2+N_R^*+4\theta^2\epsilon)^3}-\frac{1}{(4\epsilon+3)(3N_R^*+\theta^2(4\epsilon+3))^3}\Big) \\ &+\frac{1}{N_R^*+\theta^2(\epsilon+1)}-\frac{4(3\epsilon+2)}{(4\epsilon+3)(3N_R^*+\theta^2(4\epsilon+3))}\Big) \quad (46)\end{aligned}$$

Since $E(\pi_R^C) = E(\pi_R) - cN$, $\frac{d^2E(\pi_R^C)}{dN^2} = \frac{d^2E(\pi_R)}{dN^2} < 0$ (from proof of Proposition 1).

Since $\frac{d^2E(\pi_R)}{dN^2} < 0$, $\frac{dE(\pi_R)}{dN}$ is decreasing in N: when $N = 0$, $\frac{dE(\pi_R)}{dN}$ has its maximum value, when $N = \bar{N}$, $\frac{dE(\pi_R)}{dN}$ has its minimum value. Substituting $N = \bar{N}$ and $N = 0$ into the

Equation (34), the range of $\frac{dE(\pi_R)}{dN}$ is the following:

$$\frac{1}{72}\theta^2\epsilon\left(\frac{72(h+1)^2\theta^2\epsilon}{\left(\bar{N}+\theta^2(4\epsilon+1)\right)^3}+\frac{8\theta^2\epsilon(4\epsilon+3)}{\left(3\bar{N}+\theta^2(4\epsilon+3)\right)^3}+\frac{3(\epsilon+1)}{\left(\bar{N}+\theta^2(\epsilon+1)\right)^2}-\frac{4(5\epsilon+3)}{\left(3\bar{N}+\theta^2(4\epsilon+3)\right)^2}\right)$$
$$<\frac{dE(\pi_R)}{dN}<\frac{1}{24}\epsilon\left(\frac{12(h+1)^2(2\epsilon+1)}{\theta^2(4\epsilon+1)^3}+\frac{1}{\theta^2(\epsilon+1)}-\frac{4(3\epsilon+2)}{\theta^2(4\epsilon+3)^2}\right) \quad (47)$$

To ensure $0<N_R^*<\bar{N}$, the range of c should be that same as the range of $\frac{dE(\pi_R)}{dN}$:

$$\frac{1}{72}\theta^2\epsilon\left(\frac{72(h+1)^2\theta^2\epsilon}{\left(\bar{N}+\theta^2(4\epsilon+1)\right)^3}+\frac{8\theta^2\epsilon(4\epsilon+3)}{\left(3\bar{N}+\theta^2(4\epsilon+3)\right)^3}+\frac{3(\epsilon+1)}{\left(\bar{N}+\theta^2(\epsilon+1)\right)^2}-\frac{4(5\epsilon+3)}{\left(3\bar{N}+\theta^2(4\epsilon+3)\right)^2}\right)$$
$$<c<\frac{1}{24}\epsilon\left(\frac{12(h+1)^2(2\epsilon+1)}{\theta^2(4\epsilon+1)^3}+\frac{1}{\theta^2(\epsilon+1)}-\frac{4(3\epsilon+2)}{\theta^2(4\epsilon+3)^2}\right) \quad (48)$$

which implies that $\frac{dE(\pi_R^C)}{dN}=\frac{dE(\pi_R)}{dN}-c>0$ if
$c<\frac{1}{72}\theta^2\epsilon\left(\frac{72(h+1)^2\theta^2\epsilon}{\left(\bar{N}+\theta^2(4\epsilon+1)\right)^3}+\frac{8\theta^2\epsilon(4\epsilon+3)}{\left(3\bar{N}+\theta^2(4\epsilon+3)\right)^3}+\frac{3(\epsilon+1)}{\left(\bar{N}+\theta^2(\epsilon+1)\right)^2}-\frac{4(5\epsilon+3)}{\left(3\bar{N}+\theta^2(4\epsilon+3)\right)^2}\right)=\underline{c}$;
whereas, $\frac{dE(\pi_R^C)}{dN}=\frac{dE(\pi_R)}{dN}-c<0$ if
$c>\frac{1}{24}\epsilon\left(\frac{12(h+1)^2(2\epsilon+1)}{\theta^2(4\epsilon+1)^3}+\frac{1}{\theta^2(\epsilon+1)}-\frac{4(3\epsilon+2)}{\theta^2(4\epsilon+3)^2}\right)\bar{c}$.

Hence, we show Lemma 2. ■

Proof of Proposition 2

Proof. Move the LHS of the first order condition characterizing N_R^* Equation (46) to the RHS, and we define F as follows

$$F=\frac{1}{24}\epsilon(\frac{12(h+1)^2(2\epsilon+1)}{(4\epsilon+1)^2(\theta^2+N_R^*+4\theta^2\epsilon)}+N_R^*(-\frac{12(h+1)^2}{(4\epsilon+1)^2(\theta^2+N_R^*+4\theta^2\epsilon)^2}-\frac{1}{(N_R^*+\theta^2(\epsilon+1))^2}$$
$$+\frac{44\epsilon+24}{(4\epsilon+3)(3N_R^*+\theta^2(4\epsilon+3))^2})+24\epsilon(N_R^*)^2t(-\frac{(h+1)^2}{(4\epsilon+1)^2(\theta^2+N_R^*+4\theta^2\epsilon)^3}-\frac{1}{(4\epsilon+3)(3N_R^*+\theta^2(4\epsilon+3))^3})$$
$$+\frac{1}{N_R^*+\theta^2(\epsilon+1)}-\frac{4(3\epsilon+2)}{(4\epsilon+3)(3N_R^*+\theta^2(4\epsilon+3))})-c \quad (49)$$

Note that $F=-c+\frac{dE(\pi_R)}{dN}|_{N=N_R^*}$, thus $\frac{\partial F}{\partial N_R^*}=\frac{d^2E(\pi_R)}{dN^2}|_{N=N_R^*}<0$.

Taking the first derivative of $\frac{dE(\pi_R)}{dN}$ in Equation (34) with respect to θ:

$$\begin{aligned}\frac{d^2E(\pi_R)}{dNd\theta} &= \frac{1}{12}\theta\epsilon(72N^2\epsilon(\frac{(h+1)^2}{(4\epsilon+1)(N+\theta^2(4\epsilon+1))^4}+\frac{1}{(3N+\theta^2(4\epsilon+3))^4}) \\ &- \frac{12(h+1)^2(2\epsilon+1)}{(4\epsilon+1)(N+\theta^2(4\epsilon+1))^2}+2N(\frac{12(h+1)^2}{(4\epsilon+1)(N+\theta^2(4\epsilon+1))^3}+\frac{\epsilon+1}{(N+\theta^2(\epsilon+1))^3} \\ &- \frac{4(11\epsilon+6)}{(3N+\theta^2(4\epsilon+3))^3})-\frac{\epsilon+1}{(N+\theta^2(\epsilon+1))^2}+\frac{12\epsilon+8}{(3N+\theta^2(4\epsilon+3))^2})\end{aligned} \tag{50}$$

We notice $\frac{d^2E(\pi_R)}{dNd\theta}<0$ since $\epsilon>\frac{\theta^2+\bar{N}}{\theta^2}$. It follows that $\frac{\partial F}{\partial\theta}=\frac{d^2E(\pi_R)}{dNd\theta}|_{N=N_R^*}<0$.

By implicit function theorem, we have $\frac{dN_R^*}{d\theta}=-\frac{\frac{\partial F}{\partial\theta}}{\frac{\partial F}{\partial N_R^*}}<0$.

Taking first derivative of F with respect to c, we get that $\frac{\partial F}{\partial c}=-1$.

By implicit function theorem, we have $\frac{dN_R^*}{dc}=-\frac{\frac{\partial F}{\partial c}}{\frac{\partial F}{\partial N_R^*}}<0$.

So that N_R^* decreases in θ and decreases in c.

Hence, we show Proposition 2.

■

Proof of Lemma 3

Proof. Taking the first derivative of the industry profit in Equation (19) with respect to N, we characterize the industry profit maximizing problem with the following first order derivative:

$$\begin{aligned}\frac{dE(\pi_{Industry})}{dN} &= \frac{1}{24}\epsilon(24N^2\epsilon(\frac{(h+1)^2}{(4\epsilon+1)^2(\theta^2+N+4\theta^2\epsilon)^3}+\frac{1}{(4\epsilon+3)(3N+\theta^2(4\epsilon+3))^3}) \\ &+ \frac{24(h+1)^2\epsilon}{(4\epsilon+1)^2(\theta^2+N+4\theta^2\epsilon)}+N(-\frac{48(h+1)^2\epsilon}{(4\epsilon+1)^2(\theta^2+N+4\theta^2\epsilon)^2}-\frac{1}{(N+\theta^2(\epsilon+1))^2} \\ &+ \frac{4(\epsilon+3)}{(4\epsilon+3)(3N+\theta^2(4\epsilon+3))^2})+\frac{1}{N+\theta^2(\epsilon+1)}-\frac{4(\epsilon+1)}{(4\epsilon+3)(3N+\theta^2(4\epsilon+3))})-c=0\end{aligned} \tag{51}$$

Thus, the industry efficient number of reviews, N^* satisfies the following equality:

$$c = \frac{1}{24}\epsilon(\frac{24(h+1)^2\epsilon}{(4\epsilon+1)^2(N^*+\theta^2(4\epsilon+1))}+N^*(-\frac{48(h+1)^2\epsilon}{(4\epsilon+1)^2(N^*+\theta^2(4\epsilon+1))^2}-\frac{1}{(N^*+\theta^2(\epsilon+1))^2}$$
$$+\frac{4(\epsilon+3)}{(4\epsilon+3)(3N^*+\theta^2(4\epsilon+3))^2})+24\epsilon(N^*)^2(\frac{(h+1)^2}{(4\epsilon+1)^2(N^*+\theta^2(4\epsilon+1))^3}+\frac{1}{(4\epsilon+3)(3N^*+\theta^2(4\epsilon+3))^3})$$
$$+\frac{1}{N^*+\theta^2(\epsilon+1)}-\frac{4(\epsilon+1)}{(4\epsilon+3)(3N^*+\theta^2(4\epsilon+3))}) \quad (52)$$

Since $E(\pi_{Industry}) = E(\pi_R)+E(\pi_A)+E(\pi_B)-cN$, and, $\frac{d^2E(\pi_R)}{dN^2} < 0$, $\frac{d^2E(\pi_A)}{dN^2} = \frac{d^2E(\pi_B)}{dN^2} < 0$, so we have $\frac{d^2E(\pi_{Industry})}{dN^2} = \frac{d^2E(\pi_R)}{dN^2} + \frac{d^2E(\pi_A)}{dN^2} + \frac{d^2E(\pi_B)}{dN^2} < 0$.

Since $\frac{d^2E(\pi_{Industry})}{dN^2} < 0$, when $N = 0$, $\frac{dE(\pi_{Industry})}{dN}$ has its maximum value, when $N = \bar{N}$, $\frac{dE(\pi_{Industry})}{dN}$ has its minimum value. Substituting $N = \bar{N}$ and $N = 0$ into $\frac{dE(\pi_{Industry})}{dN}$ in RHS of Equation (52):

$$\frac{1}{24}\epsilon(24\bar{N}^2\epsilon(\frac{(h+1)^2}{(4\epsilon+1)^2(\theta^2+\bar{N}+4\theta^2\epsilon)^3}+\frac{1}{(4\epsilon+3)(3\bar{N}+\theta^2(4\epsilon+3))^3})$$
$$+\frac{24(h+1)^2\epsilon}{(4\epsilon+1)^2(\theta^2+\bar{N}+4\theta^2\epsilon)}+\bar{N}(-\frac{48(h+1)^2\epsilon}{(4\epsilon+1)^2(\theta^2+\bar{N}+4\theta^2\epsilon)^2}-\frac{1}{(\bar{N}+\theta^2(\epsilon+1))^2}$$
$$+\frac{4(\epsilon+3)}{(4\epsilon+3)(3\bar{N}+\theta^2(4\epsilon+3))^2})+\frac{1}{\bar{N}+\theta^2(\epsilon+1)}-\frac{4(\epsilon+1)}{(4\epsilon+3)(3\bar{N}+\theta^2(4\epsilon+3))})$$
$$< \frac{dE(\pi_{Industry})}{dN} < \frac{1}{24}\epsilon\left(\frac{24(h+1)^2\epsilon}{(4\epsilon+1)^2\left(\theta^2+4\theta^2\epsilon\right)}+\frac{1}{\theta^2(\epsilon+1)}-\frac{4(\epsilon+1)}{\theta^2(4\epsilon+3)^2}\right) \quad (53)$$

For $0 < N^* < \bar{N}$, the range of c should be that same as the range of $\frac{dE(\pi_{Industry})}{dN}$, so we have,

$$\frac{1}{24}\epsilon(24\bar{N}^2\epsilon(\frac{(h+1)^2}{(4\epsilon+1)^2(\theta^2+\bar{N}+4\theta^2\epsilon)^3}+\frac{1}{(4\epsilon+3)(3\bar{N}+\theta^2(4\epsilon+3))^3})$$
$$+\frac{24(h+1)^2\epsilon}{(4\epsilon+1)^2(\theta^2+\bar{N}+4\theta^2\epsilon)}+\bar{N}(-\frac{48(h+1)^2\epsilon}{(4\epsilon+1)^2(\theta^2+\bar{N}+4\theta^2\epsilon)^2}-\frac{1}{(\bar{N}+\theta^2(\epsilon+1))^2}$$
$$+\frac{4(\epsilon+3)}{(4\epsilon+3)(3\bar{N}+\theta^2(4\epsilon+3))^2})+\frac{1}{\bar{N}+\theta^2(\epsilon+1)}-\frac{4(\epsilon+1)}{(4\epsilon+3)(3\bar{N}+\theta^2(4\epsilon+3))})$$
$$< c < \frac{1}{24}\epsilon\left(\frac{24(h+1)^2\epsilon}{(4\epsilon+1)^2\left(\theta^2+4\theta^2\epsilon\right)}+\frac{1}{\theta^2(\epsilon+1)}-\frac{4(\epsilon+1)}{\theta^2(4\epsilon+3)^2}\right) \quad (54)$$

Nonetheless, the range of c for N_R^* in the proof of Lemma 2 is tighter. Thus, for N_R^* and N^*

both to exist and finite, we need the range of c to be:

$$\frac{1}{72}\theta^2\epsilon\left(\frac{72(h+1)^2\theta^2\epsilon}{\left(\bar{N}+\theta^2(4\epsilon+1)\right)^3}+\frac{8\theta^2\epsilon(4\epsilon+3)}{\left(3\bar{N}+\theta^2(4\epsilon+3)\right)^3}+\frac{3(\epsilon+1)}{\left(\bar{N}+\theta^2(\epsilon+1)\right)^2}-\frac{4(5\epsilon+3)}{\left(3\bar{N}+\theta^2(4\epsilon+3)\right)^2}\right)$$
$$< c < \frac{1}{24}\epsilon\left(\frac{12(h+1)^2(2\epsilon+1)}{\theta^2(4\epsilon+1)^3}+\frac{1}{\theta^2(\epsilon+1)}-\frac{4(3\epsilon+2)}{\theta^2(4\epsilon+3)^2}\right) \quad (55)$$

Hence, we show Lemma 3. ■

Proof of Proposition 3

Proof. Move LHS of the first order condition characterizing N^* in Equation (52) to RHS, and we define G as follows:

$$G=\frac{1}{24}\epsilon(\frac{24(h+1)^2\epsilon}{(4\epsilon+1)^2(N^*+\theta^2(4\epsilon+1))}+N^*(-\frac{48(h+1)^2\epsilon}{(4\epsilon+1)^2(N^*+\theta^2(4\epsilon+1))^2}-\frac{1}{(N^*+\theta^2(\epsilon+1))^2}$$
$$+\frac{4(\epsilon+3)}{(4\epsilon+3)(3N^*+\theta^2(4\epsilon+3))^2})+24\epsilon(N^*)^2(\frac{(h+1)^2}{(4\epsilon+1)^2(N^*+\theta^2(4\epsilon+1))^3}+\frac{1}{(4\epsilon+3)(3N^*+\theta^2(4\epsilon+3))^3})$$
$$+\frac{1}{N^*+\theta^2(\epsilon+1)}-\frac{4(\epsilon+1)}{(4\epsilon+3)(3N^*+\theta^2(4\epsilon+3))})-c \quad (56)$$

Note that $G=\frac{dE(\pi_{Industry})}{dN}|_{N=N^*}=\frac{dE(\pi_R)}{dN}|_{N=N^*}+\frac{dE(\pi_A)}{dN}|_{N=N^*}+\frac{dE(\pi_B)}{dN}|_{N=N^*}-c$. Thus, $\frac{\partial G}{\partial N^*}=\frac{d^2E(\pi_R)}{dN^2}|_{N=N^*}+\frac{d^2E(\pi_A)}{dN^2}|_{N=N^*}+\frac{d^2E(\pi_B)}{dN^2}|_{N=N^*}<0$, since $\frac{d^2E(\pi_R)}{dN^2}<0$, $\frac{d^2E(\pi_A)}{dN^2}<0$ and $\frac{d^2E(\pi_B)}{dN^2}<0$.

Taking the partial derivative of G with respect to θ:

$$\frac{\partial G}{\partial\theta}=\frac{1}{12}\theta\epsilon(72N^{*2}\epsilon(-\frac{(h+1)^2}{(4\epsilon+1)(N^*+\theta^2(4\epsilon+1))^4}-\frac{1}{(3N^*+\theta^2(4\epsilon+3))^4})$$
$$-\frac{24(h+1)^2\epsilon}{(4\epsilon+1)(N^*+\theta^2(4\epsilon+1))^2}+2N^*(\frac{48(h+1)^2\epsilon}{(4\epsilon+1)(N^*+\theta^2(4\epsilon+1))^3}+\frac{\epsilon+1}{(N^*+\theta^2(\epsilon+1))^3}$$
$$-\frac{4(\epsilon+3)}{(3N^*+\theta^2(4\epsilon+3))^3})-\frac{\epsilon+1}{(N^*+\theta^2(\epsilon+1))^2}+\frac{4(\epsilon+1)}{(3N^*+\theta^2(4\epsilon+3))^2}) \quad (57)$$

We notice $\frac{\partial G}{\partial\theta}<0$ since $\epsilon>\frac{\theta^2+\bar{N}}{\theta^2}$.

Using the implicit function theorem, we have $\frac{dN^*}{d\theta}=-\frac{\frac{\partial G}{\partial\theta}}{\frac{\partial G}{\partial N^*}}<0$, Taking first derivative of G with respect to c: $\frac{\partial G}{\partial c} = -1$.

Using the implicit function theorem, we have $\frac{dN^*}{dc} = -\frac{\frac{\partial G}{\partial c}}{\frac{\partial G}{\partial N^*}} < 0$.

So that N^* decreases when θ increases or when c increases.

Hence, we show Proposition 3. ■

Proof of Proposition 4

Proof. When $c = \hat{c}$, $N_R^* = N^*$, which implies that N^* satisfies the first order condition in Equation (45): $\frac{dE(\pi_R^C)}{dN}|_{N=N^*,c=\hat{c}} = 0$. Thus, we characterize $\hat{c}$ as follows:

$$\hat{c} = \frac{1}{24}\epsilon(\frac{12(h+1)^2(2\epsilon+1)}{(4\epsilon+1)^2(\theta^2+N^*+4\theta^2\epsilon)} + N^*(-\frac{12(h+1)^2}{(4\epsilon+1)^2(\theta^2+N^*+4\theta^2\epsilon)^2} - \frac{1}{(N^*+\theta^2(\epsilon+1))^2} + \frac{44\epsilon+24}{(4\epsilon+3)(3N^*+\theta^2(4\epsilon+3))^2}) + 24\epsilon(N^*)^2(-\frac{(h+1)^2}{(4\epsilon+1)^2(\theta^2+N^*+4\theta^2\epsilon)^3} - \frac{1}{(4\epsilon+3)(3N^*+\theta^2(4\epsilon+3))^3}) + \frac{1}{N^*+\theta^2(\epsilon+1)} - \frac{4(3\epsilon+2)}{(4\epsilon+3)(3N^*+\theta^2(4\epsilon+3))}) \quad (58)$$

Since $E(\pi_{Industry}) = E(\pi_R)+E(\pi_A)+E(\pi_B)-cN$ and $E(\pi_R^C) = E(\pi_R)-cN$, $\frac{dE(\pi_{Industry})}{dN} = \frac{dE(\pi_R^C)}{dN} + \frac{dE(\pi_A)}{dN} + \frac{dE(\pi_B)}{dN}$. When $c = \hat{c}$, $\frac{dE(\pi_R^C)}{dN}|_{N=N^*} = 0$ and $\frac{dE(\pi_{Industry})}{dN}|_{N=N^*} = 0$, thus, $\frac{dE(\pi_A)}{dN}|_{N=N^*} = \frac{dE(\pi_B)}{dN}|_{N=N^*} = 0$. This implies that when $c = \hat{c}$, $N^* = N_R^* = N_M$. Moreover, from Propositions 2 and 3, N^* and N_R^* decrease when c increases, whereas N_M is independent of c. Thus, when $c > \hat{c}$, $N^* < N_M$ and $N_R^* < N_M$.

Moreover, from Proposition 1, when $N < N_M$, $\frac{dE(\pi_A)}{dN} = \frac{dE(\pi_B)}{dN} > 0$, thus, $\frac{dE(\pi_{Industry})}{dN} > \frac{dE(\pi_R^C)}{dN}$; and when $N > N_M$, $\frac{dE(\pi_A)}{dN} = \frac{dE(\pi_B)}{dN} < 0$, thus $\frac{dE(\pi_{Industry})}{dN} < \frac{dE(\pi_R^C)}{dN}$.

When $c > \hat{c}$, $N^* < N_M$, thus, $\frac{dE(\pi_{Industry})}{dN}|_{N=N^*} > \frac{dE(\pi_R^C)}{dN}|_{N=N^*}$. Morevoer, per definition, $\frac{dE(\pi_{Industry})}{dN}|_{N=N^*} = 0$, thus $\frac{dE(\pi_R^C)}{dN}|_{N=N^*} < 0$. Furthermore, per definition, $\frac{dE(\pi_R^C)}{dN}|_{N=N_R^*} = 0$, and also since $\frac{d^2E(\pi_R^C)}{dN^2} < 0$, $N_R^* < N^* < N_M$.

Conversely, When $c < \hat{c}$, $N^* > N_M$, thus, $\frac{dE(\pi_{Industry})}{dN}|_{N=N^*} > \frac{dE(\pi_R^C)}{dN}|_{N=N^*}$. Since $\frac{dE(\pi_{Industry})}{dN}|_{N=N^*} = 0$, $\frac{dE(\pi_R^C)}{dN}|_{N=N^*} > 0$. Moreover, since $\frac{dE(\pi_R^C)}{dN}|_{N=N_R^*} = 0$ and $\frac{d^2E(\pi_R^C)}{dN^2} < 0$, $N_R^* > N^*$. Thus, $N_R^* > N^* > N_M$ when $c < \hat{c}$.

Hence, we show Proposition 4. ■

Proof of Corollary 2

Proof. When $c = \hat{c}$, from proof of Proposition 4, $N^* = N_R^* = N_M$. We rewrite the expression of $\hat{c}$ in Equation (58) as follows: $\hat{c} = \frac{dE(\pi_R^C)}{dN}|_{N=N_M}$. Taking the first derivative of $\hat{c}$ with respect to θ: $\frac{d\hat{c}}{d\theta} = \frac{dN_M}{d\theta}\frac{\partial^2 E(\pi_R^C)}{\partial N^2}|_{N=N_M} + \frac{d^2 E(\pi_R^C)}{dNd\theta}|_{N=N_M}$.

Since $\frac{d^2E(\pi_R^C)}{dNd\theta} = \frac{d^2E(\pi_R)}{dNd\theta}$, and $\frac{d^2E(\pi_R)}{dNd\theta} < 0$ from Equation (50), $\frac{d^2E(\pi_R^C)}{dNd\theta} < 0$. Morevoer, $\frac{\partial^2E(\pi_R^C)}{\partial N^2} = \frac{\partial^2E(\pi_R)}{\partial N^2} < 0$, and also $\frac{dN_M}{d\theta} > 0$ from Corollary 1, $\frac{d\hat{c}}{d\theta} = \frac{dN_M}{d\theta}\frac{\partial^2E(\pi_R^C)}{\partial N^2}|_{N=N_M} + \frac{d^2E(\pi_R^C)}{dNd\theta}|_{N=N_M} < 0$.

Hence, we show Corollary 2.

■

Proof of Proposition 5

Proof. Taking first order derivative of $p^*N_p^* = E(\pi_i|N = N^*) - E(\pi_i|N = N_R^*)$ with respect to c:

$$\frac{dp^*N_p^*}{dc} = \frac{dN^*}{dc}\frac{dE(\pi_i)}{dN}|_{N=N^*} - \frac{dN_R^*}{dc}\frac{dE(\pi_i)}{dN}|_{N=N_R^*} \tag{59}$$

From Propositions 2 and 3, $\frac{dN_R^*}{dc} < 0$ and $\frac{dN^*}{dc} < 0$.

From Proposition 4, when $c < \hat{c}$, $N_M < N^* < N_R^*$, thus $0 > \frac{dE(\pi_i)}{dN}|_{N=N^*} > \frac{dE(\pi_i)}{dN}|_{N=N_R^*}$. Since when $c = \hat{c}$, $N^* = N_R^*$, if we assume $\frac{dN^*}{dc} \leq \frac{dN_R^*}{dc}$<0, then when c decreases from $\hat{c}$, N^* should increase no slower than N_R^*, which implies $N^* \geq N_R^*$. However, we know $N^* < N_R^*$, thus by contradiction, $0 > \frac{dN^*}{dc} > \frac{dN_R^*}{dc}$. Thus, $0 < \frac{dN^*}{dc}\frac{dE(\pi_i)}{dN}|_{N=N^*} < \frac{dN_R^*}{dc}\frac{dE(\pi_i)}{dN}|_{N=N_R^*}$, so $\frac{dN^*}{dc}\frac{dE(\pi_i)}{dN}|_{N=N^*} - \frac{dN_R^*}{dc}\frac{dE(\pi_i)}{dN}|_{N=N_R^*} < 0$, and we have $\frac{dp^*N_p^*}{dc} < 0$.

Conversely, when $c > \hat{c}$, $N_R^* < N^* < N_M$, thus $\frac{E(\pi_i)}{\partial N}|_{N=N_R^*} > \frac{dE(\pi_i)}{dN}|_{N=N^*}$>0. If we assume $\frac{dN^*}{dc} \leq \frac{dN_R^*}{dc} < 0$, then when c increases from $\hat{c}$, N^* should decrease no slower than N_R^*, which implies $N^* \leq N_R^*$. However, $N_R^* < N^*$, thus, by contradiction, $0 > \frac{dN^*}{dc} > \frac{dN_R^*}{dc}$. Thus, $0 > \frac{dN^*}{dc}\frac{\partial E(\pi_i)}{dN}|_{N=N^*} > \frac{dN_R^*}{dc}\frac{dE(\pi_i)}{dN}|_{N=N_R^*}$, so $\frac{dN^*}{dc}\frac{\partial E(\pi_i)}{dN}|_{N=N^*} - \frac{dN_R^*}{dc}\frac{dE(\pi_i)}{dN}|_{N=N_R^*} > 0$ and we have $\frac{dp^*N_p^*}{dc} > 0$.

Hence we show Proposition 5 part (a).

Taking the derivative of $p^*N_p^*$ with respect to θ:

$$\frac{dp^*N_p^*}{d\theta} = \frac{dN^*}{d\theta}\frac{dE(\pi_i)}{dN}|_{N=N^*} + \frac{dE(\pi_i)}{d\theta}|_{N=N^*} - \frac{dN_R^*}{dd}\frac{dE(\pi_i)}{dN}|_{N=N_R^*} - \frac{dE(\pi_i)}{d\theta}|_{N=N_R^*} \tag{60}$$

When $c < \hat{c}$, $N_M < N^* < N_R^*$, thus $0 > \frac{dE(\pi_i)}{dN}|_{N=N^*} > \frac{dE(\pi_i)}{dN}|_{N=N_R^*}$.

From Propositions 2 and 3, and Corollary 1, $\frac{dN_R^*}{d\theta} < 0$, $\frac{dN^*}{d\theta} < 0$ and $\frac{dN_M}{d\theta} > 0$. N_M is the profit maximizing N for manufacturers, N_R^* is the profit maximizing N for retailer, and N^* is the profit maximizing N for the industry. Then it must be $\frac{dN_R^*}{d\theta} < \frac{dN^*}{d\theta} < 0$. Therefore, $0 < \frac{dN^*}{d\theta}\frac{dE(\pi_i)}{dN}|_{N=N^*} < \frac{dN_R^*}{d\theta}\frac{dE(\pi_i)}{dN}|_{N=N_R^*}$.

Taking cross partial derivative of $E(\pi_i)$ with respect to N and θ:

$$\frac{d^2E(\pi_i)}{dNd\theta} = \frac{1}{6}\theta\epsilon(36N^2\epsilon(-\frac{(h+1)^2}{(4\epsilon+1)(N+\theta^2(4\epsilon+1))^4} - \frac{1}{(3N+\theta^2(4\epsilon+3))^4}) \\ + \frac{3(h+1)^2}{(4\epsilon+1)(N+\theta^2(4\epsilon+1))^2} + \frac{6(h+1)^2N(4\epsilon-1)}{(4\epsilon+1)(N+\theta^2(4\epsilon+1))^3} + \frac{-2\epsilon-1}{(3N+\theta^2(4\epsilon+3))^2} + \frac{2N(10\epsilon+3)}{(3N+\theta^2(4\epsilon+3))^3}) \tag{61}$$

We notice $\frac{d^2E(\pi_i)}{dNd\theta} > 0$ when $h > \bar{h}$ and $N > N_M$. Since $N_M < N^* < N_R^*$ when $c < \hat{c}$, we know from the proof of proposition 1 that when $N > N_M$, $\frac{dE(\pi_i)}{dv_R} > 0 \Rightarrow \frac{dE(\pi_i)}{d\theta} = \frac{dE(\pi_i)}{dv_R}\cdot\frac{dv_R}{d\theta} > 0$, so we have $0 < \frac{\partial E(\pi_i)}{d\theta}|_{N=N^*} < \frac{dE(\pi_i)}{\partial\theta}|_{N=N_R^*}$.

Therefore, $\frac{dp^*N_p^*}{d\theta} = \frac{dN^*}{d\theta}\frac{dE(\pi_i)}{dN}|_{N=N^*} + \frac{dE(\pi_i)}{d\theta}|_{N=N^*} - \frac{dN_R^*}{d\theta}\frac{dE(\pi_i)}{dN}|_{N=N_R^*} - \frac{dE(\pi_i)}{d\theta}|_{N=N_R^*} < 0$, when $c < \hat{c}$.

Hence, we show Proposition 5.

■

Proof of Proposition 6

Proof. We can rewrite the retailer's expected profit when the manufacturers accept the offer in the first stage in the review monetization case as follows:

$$E(\pi_R^M) = E(\pi_R(N_p)) + 2[E(\pi_i(N_p)) - E(\pi_i(N_R^*))] - cN_p \tag{62}$$

Taking the first derivative of the retailer's expected profit $E(\pi_R^M)$ with respect to N_p:

$$\frac{dE(\pi_R^M)}{dN_p} = \frac{1}{24}\epsilon(24N_p^2\epsilon(\frac{(h+1)^2}{(4\epsilon+1)^2(\theta^2+N_p+4\theta^2\epsilon)^3} + \frac{1}{(4\epsilon+3)(3N_p+\theta^2(4\epsilon+3))^3}) + \frac{24(h+1)^2\epsilon}{(4\epsilon+1)^2(\theta^2+N_p+4\theta^2\epsilon)} + N_p(-\frac{48(h+1)^2\epsilon}{(4\epsilon+1)^2(\theta^2+N_p+4\theta^2\epsilon)^2} - \frac{1}{(N_p+\theta^2(\epsilon+1))^2} + \frac{4(\epsilon+3)}{(4\epsilon+3)(3N_p+\theta^2(4\epsilon+3))^2}) + \frac{1}{N_p+\theta^2(\epsilon+1)} - \frac{4(\epsilon+1)}{(4\epsilon+3)(3N_p+\theta^2(4\epsilon+3))}) - c = 0 \tag{63}$$

If we compare equation (63) to equation (51), we will find that the RHS of both equations are exactly the same after we replace N_p in equation (63) to N, this is because the term $E(\pi_i(N_R^*))$ in equation (63) is independent of N_p, so it disappears once we take first derivative of $E(\pi_R^M)$ with respect to N_p.

This suggests the solution to equation (51), N^*, is exactly the solution to equation (63), N_p^*, so the efficient number of reviews N^* maximize the industry total profit as well as the retailer's own profit in review monetization case, that is, $N_p^* = N^*$.

We have shown that N^* is finite, thus so is N_p^*. First we assume there exists a cost $c \leq 2p^*$. Then taking derivative of the retailer's expected profit $E(\pi_R^M)$ with respect to N_p, we have $\frac{dE(\pi_R^M)}{dN_p} = \frac{\partial E(\pi_R(N_p))}{\partial N_p} + 2p^* - c$. We know that when reviews are costless, the retailer's profit can always be benefited from more reviews, so $\frac{\partial E(\pi_R(N_p))}{\partial N_p} > 0$, and by the assumption $2p^* - c \geq 0$, so $\frac{\partial E(\pi_R(N_p))}{\partial N_p} + p^* - c > 0$, which means that $\frac{dE(\pi_R^M)}{dN_p} > 0$ and there is no finite solution of N_p^*. However, N_p^* is finite, thus by contradiction, it should be that $2p^* < c$.

Hence, we show Proposition 6. ■