Platform Competition under Network Effects: Piggybacking and Optimal Subsidization

Yifan Dou
School of Management, Fudan University, Shanghai, 200433, China
yfdou@fudan.edu.cn

D. J. Wu
Scheller College of Business, Georgia Institute of Technology, Atlanta, GA 30308, USA
dj.wu@scheller.gatech.edu

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A repeated challenge in launching a two-sided market platform is how to solve the "chicken-and-egg" problem. The solution often suggested in the literature is subsidizing one side of the market to jumpstart adoption of the platform. In this paper, using a game-theoretic framework, we study piggybacking – importing users from external networks – as a new approach to launching platforms. Our finding suggests that optimal use of the piggybacking strategy depends on the cross-side network effects. First, benchmarked with the case of no piggybacking, we find that the pricing impacts of piggybacking is non-trivial. It may help mitigate or avoid price competition. Second, we show that platform duopoly with piggybacking can become a "game of chicken" or even a prisoner's dilemma, which implies that platforms are not always better off (sometimes even worse off) with piggybacking. Finally, when piggybacking users are fabricated (e.g., zombies or fake users), the platform strategies differ greatly from the authentic piggybacking case. It also undermines both the competing platform's profit and the providers' surpluses. Managerial implications for platform practitioners are also discussed.

Key words: Analytical Modeling, Economics of IS, Network Effects, Piggybacking, Platform Competition, Pricing, Subsidization

1. Introduction

As more and more businesses (both physical and digital) search for multi-sided platform business models (i.e., intermediaries that connect two or more distinct groups of users and enable their direct interactions), a primary challenge is how to expand the user bases in view of the interdependence issue among different user groups – known as the "chicken-and-egg" problem (Caillaud and Jullien 2003). The most popular solution proposed in the extant literature is *subsidizing* one or more user groups to jumpstart adoption of the platform (e.g., Rochet and Tirole 2003, Parker and Van Alstyne 2005, Eisenmann et al. 2006, Bolt and Tieman 2008). Subsidizing strategies are widely used by practical platforms. For example, Microsoft took a total loss of over US$4 billion in the first four years after launching its Xbox gaming platform, primarily by allowing consumers to pay a market price below the cost of manufacturing.[1]

In addition to pricing controls (e.g., subsidies), platforms are increasingly embracing non-pricing controls to incentivize user adoptions. For example, Hagiu and Spulber (2013) suggest that platforms themselves could offer contents (i.e., first-party contents) to attract early adoptions. We contribute to this literature by examining a new non-pricing control – user traffic management in general and *piggybacking* in particular – importing external user traffic. By re-examining a platform's pricing/subsidizing decisions, we reveal a host of new insights into platforms' strategies when they are able to engage in piggybacking, with more complex characterizations on the interplay among traffic volume, cross-side network effects, and platform competition.

Piggybacking is defined as the ability of a platform to "connect with an existing user base from a different platform and stage the creation of value unit in order to recruit those users to participate" (Parker et al. 2016, pp. 91). Many examples of piggybacking exist in which platforms tap into external networks to import early traffic, rather than using subsidies alone to build an installed base from scratch. For example, in Google's early days, it served as the search engine for Yahoo!'s portal site, which helped Google gain access to Yahoo! users. Google also partnered

[1] https://venturebeat.com/2011/11/15/the-making-of-the-xbox-part-2/

with handset manufacturers to create the Open Handset Alliance (OHA) to counter the success of Apple's mobile operating system iOS. The OHA gave Google access to hardware partners' user bases and helped Google achieve a market share of more than 80% of the worldwide mobile market. In addition to this partnership-based piggybacking, some platforms take advantage of other platforms by accessing their users without existing collaborations or alliances, which is often known as the "growth hacking" or "wormhole" strategy. Airbnb, for example, used this tactic to boost its growth early on with the button "publish on Craigslist." By clicking on this button, Airbnb hosts could immediately publish their Airbnb listings on Craigslist, and anyone responding to the listing could still reach the host through Airbnb.[2] Parker et al. (2016) summarized more examples of piggybacking (Parker et al. 2016) such as YouTube (piggybacking on MySpace), PayPal (piggybacking on eBay), and JustDial (piggybacking on the local telephone directory).

A growing amount of evidence from practices suggests that piggybacking traffic on some digital platforms is mixed with fake profiles, automatic bots, and review frauds. For example, as documented in Parker et al. (2016), PayPal created bots that made purchases on eBay, thereby attracting sellers to the PayPal platform; dating services often simulate initial attraction by creating fake profiles and conversations; Reddit fakes profiles by posting links to the kind of content the founders wanted to see on the site over time; and editors on Quora ask questions and then answer the questions themselves, to simulate activity on the platform. Parker et al. (2016) call this "fake it until you make it." These fabricated users and activities contribute no monetary value to the platform directly but stimulate interactions on the digital platform. These fabricated accounts and activities are undoubtedly far from ethical. However, they have received little, if any, formal analysis in the literature to explore how they undermine user surplus and change the competition equilibrium among platforms. This paper aims to take the first step toward filling this gap.

We study two fundamental research questions in this paper: First, on the tactical level, *how does piggybacking affect the platforms' optimal pricing/subsidization strategy and profits?* Second,

[2] https://hbswk.hbs.edu/item/how-uber-airbnb-and-etsy-attracted-their-first-1-000-customers

on the strategic level, *when should a platform use piggybacking, especially when competing against other platforms?* Further, in our extension, we characterize the impacts of fabricated piggybacking in which external fabricated users (e.g., bots or fake users) generate network effects but no revenue. Our analytical model provides a general framework for studying platform competition in the presence of piggybacking. To the best of our knowledge, this paper is among the first to formally study optimal piggybacking strategies in the context of platform competition under network effects.

We address these research questions using backward induction. Our study yields many interesting findings. First, on the tactical level, benchmarked with the case of no piggybacking, we find that the pricing impacts of piggybacking is non-trivial. It may help mitigate or avoid the price competition, depending on the cross-side network effects. Second, on the strategic level, we find that piggybacking in the platform duopoly can become a "game of chicken" or even a prisoner's dilemma, which implies that platforms are not always better off (sometimes even worse off) with the piggybacking strategy. Finally, when piggybacking users are fabricated (e.g., zombies or fake users), the pricing strategies differ greatly from the authentic piggybacking case. More importantly, it undermines both the competing platform's profit and the providers' surplus.

The rest of this paper is organized as follows: Section 2 reviews the related literature, and Section 3 introduces our model. In Section 4, we show the results of our baseline model and extend it in Section 5. Section 6 outlines the managerial implications and concludes.

2. Related Literature

Our research is related to three research streams that we briefly review below. The first stream is the literature on launching a two-sided platform (e.g., Rochet and Tirole 2003, Parker and Van Alstyne 2005, Parker et al. 2016). A frequent challenge in launching a platform is the "chicken-and-egg" problem (Caillaud and Jullien 2003), discussed in Section 1. Subsidizing one side of the market, or the "seesaw principle," has been suggested as the solution (e.g., Wright 2004, Bhargava and Choudhary, 2004, Parker and Van Alstyne 2005, Rochet and Tirole 2006, Hagiu 2007, Bolt and Tieman 2008, Hagiu 2009). For example, Parker and Van Alstyne (2005) recommend giving away free access/products to either providers or consumers, depending on the cross-sided elasticities. Rochet and Tirole (2006) introduce the seesaw principle, in which a profit-maximizing platform charges a high price on one side and a low price on the other side. Although both papers (Parker and Van Alstyne 2005, Rochet and Tirole 2006) assume that users single-home, the seesaw principle is also shown to be optimal when users multi-home (i.e., they can join a platform as long as they pay for platform access and there is no price competition between platforms on the provider side). Armstrong (2006) shows that in the "competitive bottleneck" setting, when only one side multi-homes and the other side single-homes, the platform can charge a higher price on the multi-homing side because of its monopoly power in providing access to the single-homing side. The "competitive bottleneck" structure in Armstrong (2006) has become a standard setting in this literature (e.g., Rochet and Tirole 2003, Armstrong 2006, Economides and Tåg 2012, Hagiu and Hałaburda 2014). For example, Hagiu and Hałaburda (2014) consider different types of user expectations when they join a platform. They find that platforms might be better off when users are less informed and form their expectations passively. Using the "competitive bottleneck" setting, we formally study the strategic implications of piggybacking and optimal subsidization in launching a platform.

Prior studies related to piggybacking are mostly empirical observations (e.g., Boudreau 2010). This second line of research provides rich evidence that motivates formal modeling, as we attempt to do. Scenarios related to piggybacking include building market momentum (e.g., Gawer and Cusumano 2008), adding initial developers to the software platform (e.g., Boudreau 2012), attracting early users via single-sided features (e.g., Hagiu and Eisenmann 2007, Hagiu and Spulber 2013, Anderson Jr. et al. 2013) or advertising (e.g., Tucker and Zhang 2010), and integrating the user base with a complementary platform (e.g., Li and Agarwal 2016). In these studies, non-price controls other than piggybacking are often the driving force for additional user participation (e.g., extra functionalities, additional platform contents, and new developers). These platform controls trade-off between additional user participations and the associated costs to acquire them. In contrast, the role of piggybacking is richer. Piggybacking consumers might be either costly or beneficial for the platform, depending on whether platforms charge or subsidize the consumer side. In addition, piggybacking strategies involve managerial challenges for user participation not only on both sides of the market but also for the competing strategies. As we show later, it is non-trivial to address such challenges in the presence of cross-side network effects under platform competition.

We also extend our model to consider fabricated piggybacking, such as zombie users, fake profiles, and review frauds. Computer science and data-mining literature – the third related research stream – focuses mostly on the technical challenge in how to identify fabricated piggybacking (Chu et al. 2010, Hu et al. 2011, Hu et al. 2012, and Haustein et al. 2016). Economic analysis of fabricated piggybacking is rare other than documenting some suggestive empirical evidence. For example, Aral (2014) suggests that online review fraud is one of the prominent reasons for the J-shaped distribution of online consumer ratings discovered by Hu et al. (2009). Luca and Zervas (2016) provide empirical evidence from Yelp that a restaurant is more likely to commit review fraud when its reputation is weak and is more likely to receive unfavorable fake reviews under strong competition. This line of research has intrigued us to investigate formally the impact of fabricated piggybacking case, such as bots, compared with authentic piggybacking in which case piggybacking consumers are real. To the best of our knowledge, this paper is among the first to examine the issue, and we provide new insights on platforms to understand the economics of fabricated piggybacking, such as different strategic implications of fabricated versus authentic piggybacking for competing platforms.

3. Model Assumptions

Consider a duopoly between Platforms A and B in a two-sided market consisting of consumers (denoted by superscript c) and providers (denoted by superscript d). Consumers consist of those who are from the focal market (called "focal consumers") and those who are redirected from external networks as a result of piggybacking (called "piggybacking consumers"). We normalize the population of focal consumers to be 1. They are uniformly distributed in a linear city between 0 and 1. Focal consumers located at $x \in [0, 1]$ receive the following utility from adopting Platforms A and B, respectively:

$$U_A^c(x) = V - p_A^c - tx + \beta N_A^d, \tag{1}$$

$$U_B^c(x) = V - p_B^c - t(1-x) + \beta N_B^d,$$

where V represents the symmetric standalone value of adopting either Platform A or Platform B. p_k^c is the consumer-side fee to access Platform $k \in \{A, B\}$. tx $(t(1-x))$ is the transportation cost for consumers at x to adopt Platform A (Platform B). Following the classic Hotelling setup (Hotelling 1929), coefficient t measures the degree of horizontal differentiation between the two platforms: A higher t indicates that it is more difficult for platforms to attract focal consumers via low prices or subsidies. In other words, t can be viewed as a proxy for consumer-side price elasticity. $\beta \geq 0$ represents the degree of consumer-side network effects. N_k^d is the number of providers in equilibrium who participate in Platform $k \in \{A, B\}$.

Given the utility functions above, the number of focal consumers adopting Platform k, N_k^c, can be obtained by solving the indifferent customer type $\hat{x}$ with $U_A^c(\hat{x}) = U_B^c(\hat{x})$.

$$N_A^c = \hat{x} = \frac{1}{2} - \frac{p_A^c - p_B^c}{2t} + \frac{\beta(N_A^d - N_B^d)}{2t}, \tag{2}$$

$$N_B^c = 1 - \hat{x} = \frac{1}{2} - \frac{p_B^c - p_A^c}{2t} + \frac{\beta(N_B^d - N_A^d)}{2t}.$$

We allow each platform to acquire consumers from external networks through piggybacking. We assume that there is no additional competition outside our focal market. Thus the number of piggybacking consumers imported to a platform is not affected by the other platform's strategies: Each platform would have another n consumers if it engaged in piggybacking (and 0 if not). We consider $n \leq \frac{1}{2}$ such that the overall population of piggybacking consumers never exceeds those from the focal market. Engaging in piggybacking incurs an additional cost for platforms, which we discuss in Section 4.2. To capture platforms' decisions on piggybacking, we use the following indicator function $\mathbb{I}_k$:

$$\mathbb{I}_k = \begin{cases} 1, \text{ if Platform } k \text{ chooses piggybacking}, \\ 0, \text{ otherwise}. \end{cases} \tag{3}$$

On the provider side, we follow the literature to employ the "competitive bottleneck" setting (e.g., Rochet and Tirole 2003, Armstrong 2006, Economides and Tåg, 2012, Hagiu and Hałaburda 2014) in which providers multi-home. The equilibrium demand function on the provider side, as in Hagiu and Hałaburda (2014), is given by

$$N_k^d = \alpha(\mathbb{I}_k n + N_k^c) - p_k^d, \tag{4}$$

where $\alpha \geq 0$ represents the degree of provider-side network effects. Equation (4) suggests that adoptions on the provider side are driven purely by consumer adoptions because a platform has no standalone value for providers if it has no consumers. Following the literature (e.g., Hagiu and Hałaburda 2014), we assume $\frac{(\alpha+\beta)^2}{4} \leq t$ to maintain a reasonable degree of network effects for the profit function to be well-behaved.

The timeline of events has three stages, which are illustrated in Figure 1. In stage 0, both platforms simultaneously make the piggybacking decision. If platform k decides to piggyback, a group of n consumers is imported from external networks; in stage 1, two platforms simultaneously announce their prices on both sides; and in stage 2, focal consumers choose to join either Platform A or Platform B, while each provider decides whether to participate in Platform A, Platform B, or both.

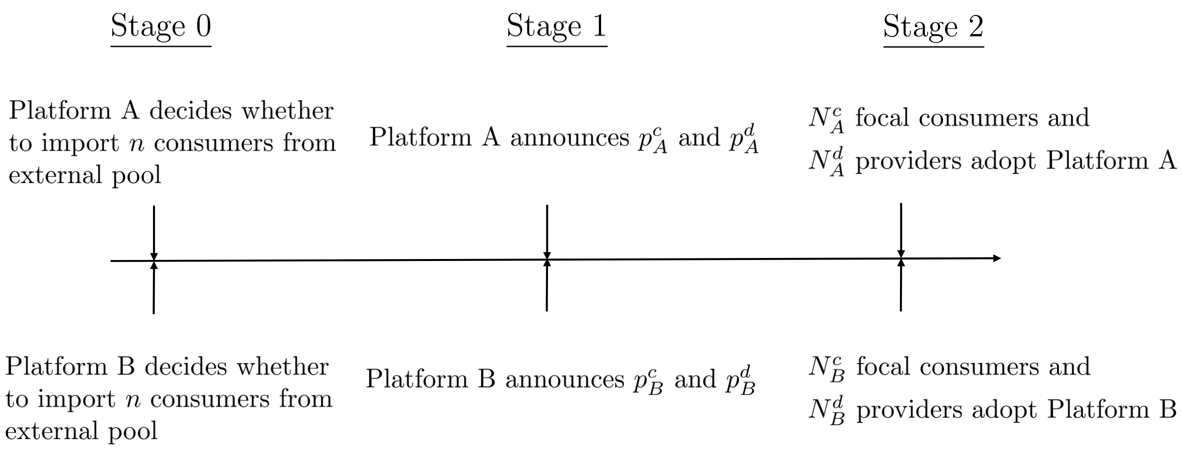

Figure 1 The timeline of events

4. Analysis and Findings

We use backward induction to examine the platform strategies. First, Section 4.1 investigates the sub-game perfect pricing equilibrium in stage 1. We split our discussion into three different scenarios, respectively: no piggybacking, asymmetric piggybacking (in which only one platform engages in piggybacking), and symmetric piggybacking. Note that, stage 1 analysis assumes that the piggybacking outcome is given and the costs associated with piggybacking are sunk. Later Section 4.2 discusses the strategic decision on piggybacking by examining the role of piggybacking costs.

4.1. Pricing Equilibrium in Stage 1

In stage 1, given the piggybacking outcome $\mathbb{I}_k$, Platform k determines optimal prices on both sides simultaneously to maximize its overall profit Π_k from both sides. A platform remains in the market as long as a non-negative number of focal consumers and providers adopts it in equilibrium, that is, equilibrium exists when $N_k^c \geq 0$ and $N_k^d \geq 0$ for $k \in \{A, B\}$. Additionally, it incurs a fixed cost c_k for Platform $k \in \{A, B\}$ to piggyback and obtain n consumers from an external pool. Specifically, each platform's profit maximization problem is the following:

$$\begin{aligned} \max_{p_k^c, p_k^d} \quad & \Pi_k(\mathbb{I}_k, \mathbb{I}_{-k}) = p_k^c(\mathbb{I}_k n + N_k^c) + p_k^d N_k^d - c_k \mathbb{I}_k, \\ \text{subject to} \quad & N_k^c = \frac{1}{2} - \frac{p_k^c - p_{-k}^c}{2t} + \frac{\beta(N_k^d - N_{-k}^d)}{2t}, \\ & N_{-k}^c = \frac{1}{2} - \frac{p_{-k}^c - p_k^c}{2t} + \frac{\beta(N_{-k}^d - N_k^d)}{2t}, \\ & N_k^d = \alpha(N_k^c + \mathbb{I}_k n) - p_k^d, \\ & N_k^c \geq 0,\ N_k^d \geq 0,\ \frac{(\alpha+\beta)^2}{4} \leq t,\ \text{and } n \leq \frac{1}{2}. \end{aligned} \tag{5}$$

The subscript $-k$ indicates the opponent platform for Platform k. We summarize our key notation in Table 2.

k	Platform index, $k \in \{A, B\}$,
t	Transportation cost, $t > 0$,
$\beta(\alpha)$	Degree of the consumer (provider) side network effects, $\frac{(\alpha+\beta)^2}{4} \le t$,
p_k^c (p_k^d)	Equilibrium price of Platform k on the consumer (provider) side,
N_k^c (N_k^d)	Number of adopting consumers (providers) of Platform k, N_k^c, $N_k^d \ge 0$,
Π_k	Equilibrium profit of Platform k,
$\mathbb{I}_k$	Platform k's decision on piggybacking, $\mathbb{I}_k = 1$ if Platform k chooses piggybacking,
$\mathbb{I}_k n$	Population of piggybacking consumers on Platform k, $n \in [0, \frac{1}{2}]$,
c_k	Piggybacking cost for Platform k, $c_k \ge 0$.

Table 1 Summary of Key Notation

4.1.1. No Piggybacking ($\mathbb{I}_A = 0, \mathbb{I}_B = 0$)

We start with the benchmark case in which neither of the platforms engages in piggybacking. Under $\{\mathbb{I}_A = 0, \mathbb{I}_B = 0\}$, the model is quickly reduced to the results in the prior studies without piggybacking consumers. Proposition 1 presents the optimal prices and profit.

Proposition 1. *When neither platform engages in piggybacking (i.e., $\mathbb{I}_A = 0, \mathbb{I}_B = 0$), the equilibrium prices are given by*

$$(p_A^c)^* = (p_B^c)^* = t - T_1,$$

$$(p_A^d)^* = (p_B^d)^* = \frac{\alpha - \beta}{4},$$

and equilibrium profits are

$$\Pi_k^* = \frac{t}{2} - \frac{T_2}{4},$$

where $T_1 = \frac{\alpha(\alpha+3\beta)}{4}$ and $T_2 = \frac{\alpha^2+6\alpha\beta+\beta^2}{4}$.

For a competitive-bottleneck market (i.e., one side single-homing and the other multi-homing), the consumer price is negative when $t < T_1$, indicating that strong network effects may drive consumer to switch, which provides more incentives for platforms to subsidize consumers. If $\alpha < \beta$, indicating that the network effect strength on the provider side is weaker, subsidizing providers becomes necessary. These results and their implications are consistent with the literature (e.g., see Section 4.1 in Hagiu and Hałaburda 2014). Next, we present our new results based upon these baseline findings.

4.1.2. Both Platforms Engage in Piggybacking ($\mathbb{I}_A = 1, \mathbb{I}_B = 1$)

Next we turn to the case in which both platforms engage in piggybacking. Again, the results are symmetric because the two platforms are identical. Specifically, we are interested in how n changes the baseline results in Proposition 1. We show the equilibrium results in the following Proposition 2.

Proposition 2. *When both platforms engage in piggybacking (i.e., $\mathbb{I}_A = 1, \mathbb{I}_B = 1$), the equilibrium prices are given by*

$$(p_A^c)^* = (p_B^c)^* = (1+2n)(t - T_1),$$

$$(p_A^d)^* = (p_B^d)^* = \frac{(1+2n)(\alpha - \beta)}{4},$$

and equilibrium profits are

$$\Pi_k^* = (1+2n)^2 \left(\frac{t}{2} - \frac{T_2}{4}\right) - c_k.$$

Compare Propositions 1 and 2. The piggybacking consumers have two roles. First, the introduction of piggybacking consumers increases the magnitude of prices on both sides. That being said, if platforms subsidize focal consumers in the case of no piggybacking (e.g., $t < T_1$ when network effects are strong), their subsidies would be even larger with piggybacking. This is unique to the two-sided market because platforms prefer to monetize the strong network effects on the provider side. Similarly, on the provider side, despite the absence of piggybacking traffic, platforms still prefer to subsidize more (when $\alpha < \beta$) or charge (when $\alpha > \beta$) more because the population of consumers has increased, which eventually affect providers through the cross-side network effects. Second, both platforms enjoy a marginally-increasing revenue growth from piggybacking. Clearly piggybacking, as its cost is sunk, is a favorable strategy and the piggybacking consumers help boost the virtuous circle between two sides of the market, leading to greater revenue.

4.1.3. Asymmetric Piggybacking ($\mathbb{I}_A = 1, \mathbb{I}_B = 0$)

Without loss of generality, we consider only the case in which Platform A engages in piggybacking. Unlike in the symmetric cases in Sections 4.1.1 and 4.1.2, under the asymmetric case, Platform B faces the threat of being pushed out of the market, such that $N_B^c < 0$. Proposition 3 identifies the conditions that ensure the existence of the duopoly equilibrium.

Proposition 3. *Denote* $T_3 = \frac{\alpha^2+4\alpha\beta+\beta^2}{4}$*. Platform B stays in the market in equilibrium if and only if (iff) Platform A's piggybacking traffic is bounded, i.e.,* $n \leq \bar{n} = \frac{t}{2(T_3-t)} - 1$.

Proposition 3 has two implications: First, when Platform A manages to acquire huge traffic from outside, it generates not only immediate consumer-side profits but also provider-side participation. Both of them collectively reinforce Platform A's overall advantages which eventually push Platform B out of the market. Second, the threshold $\bar{n}$ is monotonically decreasing in the strength of network effects (i.e., $\frac{\partial \bar{n}}{\partial T_3} < 0$), indicating that the threat of exit for Platform B is greater in a market where two sides are connected with each other more closely. This is consistent with the conventional wisdom of "winner takes all" in the prior literature on the two-sided market.

When both platforms stay in the market, Proposition 4 gives the equilibrium results.

Proposition 4. *When both platforms stay in the market and only Platform A engages in piggybacking, Table 2 summarizes the equilibrium results: optimal pricing strategies, market size, and the profit of each platform.*

As shown in Table 2, the equilibrium results are much more complex under asymmetric piggybacking, compared to those symmetric cases in Sections 4.1.1 and 4.1.2. Specifically, we are interested in how piggybacking traffic n affects the equilibrium results. We depict the comparative statics of Platform A's prices over n in Figure 2 and the comparative statics of Platform B's prices over n in Figure 3. All the shaded (white) regions represent the parameter space of network effects.

Proposition 5. *In equilibrium, the pricing impacts of piggybacking are summarized in Table 3.*

	Market Side	Platform A	Platform B
Pricing	consumer	$(t-T_1)\left(1+\frac{2(2t-T_3)}{3t-2T_3}\times n\right)$	$(t-T_1)\left(1+\frac{2(t-T_3)}{3t-2T_3}\times n\right)$
	provider	$\frac{\alpha-\beta}{4}\left(1+\frac{2(2t-T_3)}{3t-2T_3}\times n\right)$	$\frac{\alpha-\beta}{4}\left(1+\frac{2(t-T_3)}{3t-2T_3}\times n\right)$
Market Size	consumer	$\frac{1}{2}-\frac{t-T_3}{3t-2T_3}\times n$	$\frac{1}{2}+\frac{t-T_3}{3t-2T_3}\times n$
	provider	$\frac{\alpha+\beta}{4}\left(1+\frac{4(t-2T_3)}{3t-2T_3}\times n\right)$	$\frac{\alpha+\beta}{4}\left(1+\frac{2(t-2T_3)}{3t-2T_3}\times n\right)$
Profit		$\frac{(2t-T_2)[3t-2T_3+2(2t-T_3)\times n]^2}{4(3t-T_3)^2}-c_A$	$\frac{(2t-T_2)[3t-2T_3+2(t-T_3)\times n]^2}{4(3t-T_3)^2}$

Table 2 Equilibrium Results of Platform Duopoly

Region	Consumer Side ($n\uparrow$)	Provider Side ($n\uparrow$)	
I ($\alpha \geq \beta$ and $t < T_1$)	$(p_A^c)^*\downarrow$, $(p_B^c)^*\uparrow$	$(p_A^d)^*\uparrow$, $(p_B^d)^*\downarrow$	
II ($\alpha \geq \beta$ and $t \in [T_1, T_3)$)	$(p_A^c)^*\uparrow$, $(p_B^c)^*\downarrow$	$(p_A^d)^*\uparrow$, $(p_B^d)^*\downarrow$	Competition Avoidance
III ($\alpha < \beta$ and $t < T_3$)	$(p_A^c)^*\uparrow$, $(p_B^c)^*\downarrow$	$(p_A^d)^*\downarrow$, $(p_B^d)^*\uparrow$	
IV ($\alpha \geq \beta$ and $t \geq T_3$)	$(p_A^c)^*\uparrow$, $(p_B^c)^*\uparrow$	$(p_A^d)^*\uparrow$, $(p_B^d)^*\uparrow$	Competition Mitigation
V ($\alpha < \beta$ and $t \geq T_3$)	$(p_A^c)^*\uparrow$, $(p_B^c)^*\uparrow$	$(p_A^d)^*\downarrow$, $(p_B^d)^*\downarrow$	

Table 3 Comparative Statics on n

Table 3 summarizes our key findings on the pricing impacts of piggybacking. We illustrate regions in Figure 2 and 3. First, we note that, in some regions – namely, Regions IV and V – that is, on each side, consumer-side prices go up for both platforms, which indicates that platform competition is mitigated by the piggybacking traffic. Second, we note that, although it is optimal for both platforms to follow the seesaw principle (i.e., the price goes up on one side and down on the other) in Region V, the seesaw principle is no longer optimal in Region IV. In this case, on the consumer side, even the network effects are weaker (indicating smaller profitability for platforms – the "non-money side"), both platforms can raise prices, thanks to the increased number of piggybacking consumers on both platforms. Although these findings are different and new, intuitively, they are driven by an increased external pool. Third, in Regions I, II, and III, platform competition is more intense, even though the external pool has become larger. These regions have interesting dynamics, as we briefly discuss below. Platform A's strategies are intuitive in Regions I and III, that is, the seesaw principle remains optimal. In Region II, however, the seesaw principle is no longer optimal. In this region, while Platform A raises price on the money side (the provider side), which is not surprising, it can take advantage of the increase in piggybacking customers, by raising (rather than decreasing) prices on the consumer side. In all three regions, rather than competing head on with Platform A, Platform B avoids competition by changing prices in the opposite direction on each side. In each case, Platform B reduces subsidies on the non-money side because of its increased disadvantages in harvesting on the money side. In other words, Platform B scales back on both sides to minimize profit loss in response to the rival's piggybacking advantage. The findings in Table 3 are unique in the setting of a two-sided market in which platforms have the flexibility to choose which market side, or both, to monetize.

Next we investigate the impact of piggybacking on platform profits. It is intuitive that Platform A is always better off. However, Corollary 1 suggests that Platform B's profit is affected by piggybacking in a non-trivial way.

Corollary 1. *The following hold true in equilibrium under asymmetric piggybacking:*

(a) Platform A's overall profit and number of adopters on both sides are always increasing in n;

(b) Platform B's overall profit and number of adopters on both sides are increasing in n iff $t > T_3$;

(c) The profit gap between the two platforms is increasing and convex in n.

Interestingly, as illustrated in Figure 4, when network effects are not very strong, such that $T_3 < t$, Platform B can be better off as the external pool increases. Here is the intuition. In this case, Platform A tends to increase the price on the consumer side, leaving a larger consumer base for Platform B to serve. Thus Platform B has more pricing flexibility. But when network effects

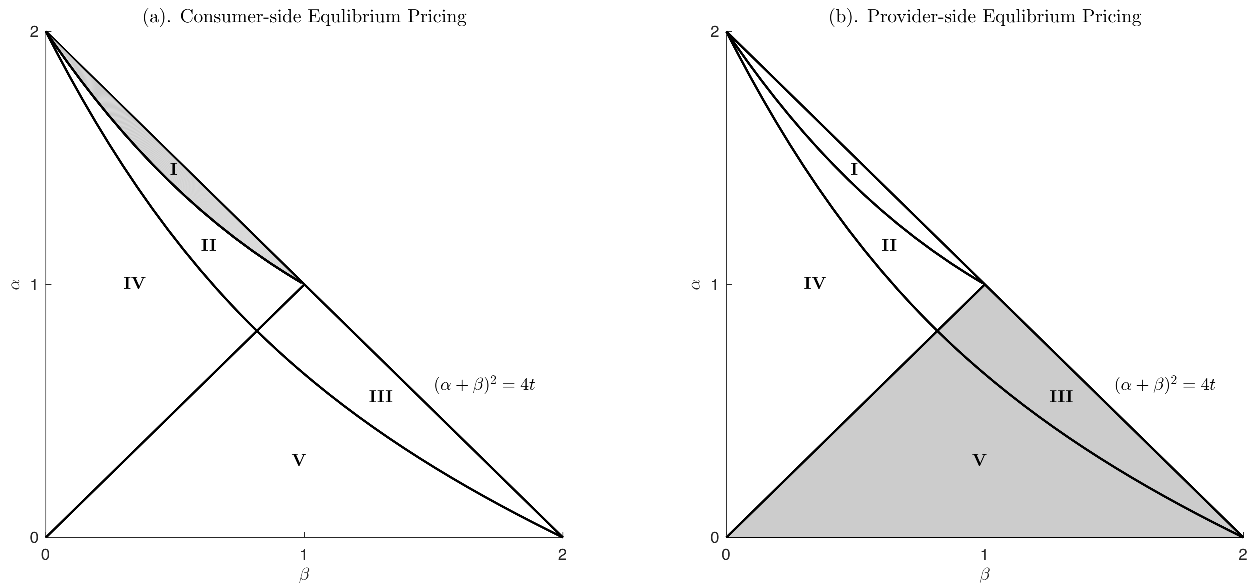

Figure 2 **Comparative Statics for Platform A's Equilibrium Prices. The shaded area represents regions where the price is decreasing in n. Regions are defined in Table 4.**

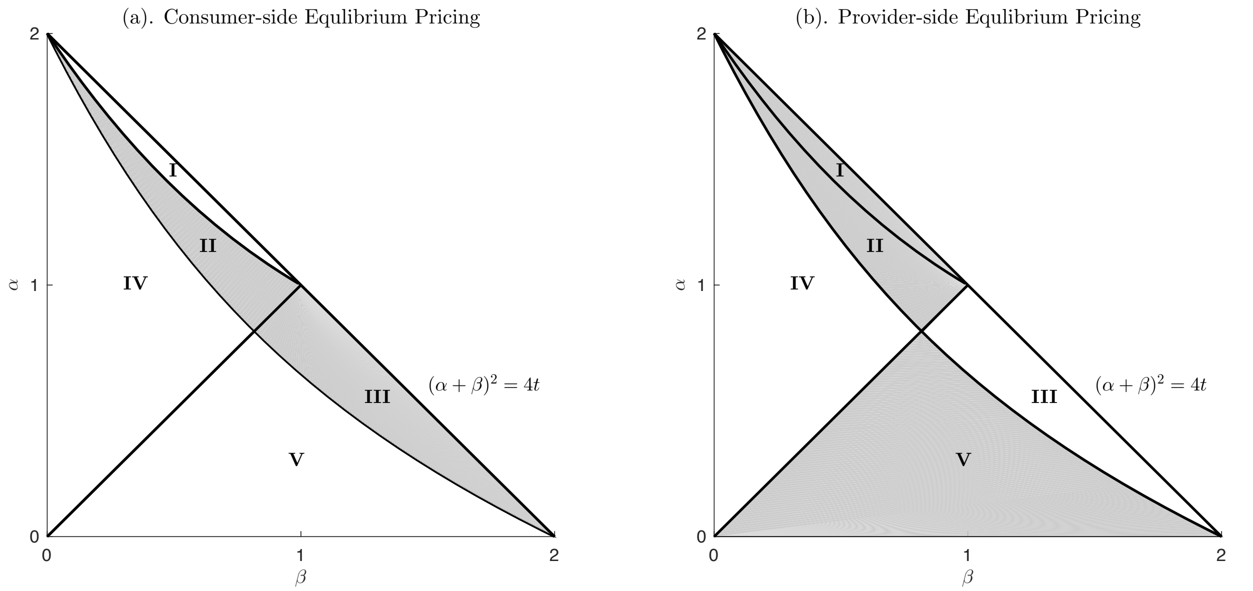

Figure 3 **Comparative Statics for Platform B's Equilibrium Prices. The shaded area represents regions where the price is decreasing in n. Regions are defined in Table 4.**

are strong enough to exceed a threshold (i.e., $T_3 \geq t$), Platform A finds it optimal to lower prices to leverage network effects. In this case, Platform B has little room for price adjustment. This, coupled with limited platform service differentiation (i.e., a small t), leads to a decrease in profit.

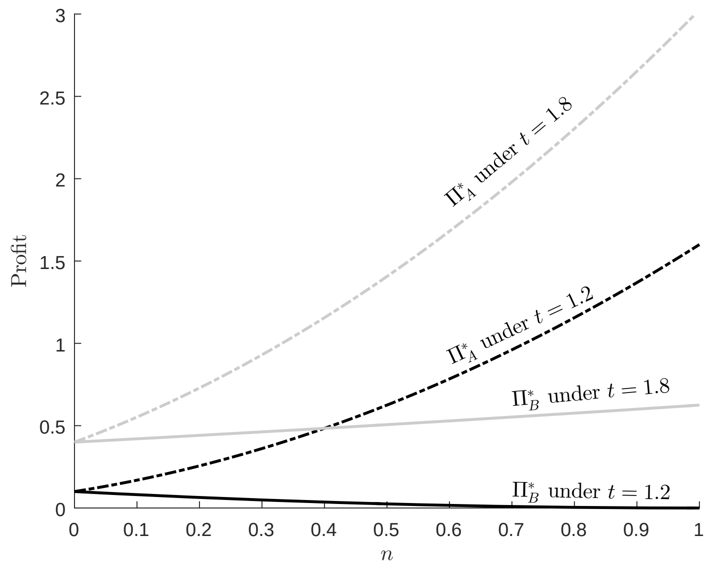

Figure 4 **Profit Impacts of n ($\alpha = \beta = 1$)**

We extend the baseline model in two directions in this section: endogenous piggybacking, in which platforms need to decide whether to engage in piggybacking; and fabricated piggybacking, in which a platform uses "zombie" consumers to generate network effects.

4.2. Endogenous Piggybacking in Stage 0

Next, we follow the sequence of backward induction to analyze the platforms' strategic decisions on piggybacking in Stage 0. Specifically, we let each platform choose $\mathbb{I}_k$ in equation (5). Platforms tradeoff between the costs and benefits of piggybacking. We study a general case in which c_A is not necessarily symmetric to c_B: For example, if $c_A \leq c_B$, it means that Platform A is relatively more capable (or smarter) of finding an external network to hack or to establish a partnership. Lemma 1 characterizes the existence of market equilibria when $c_B \geq c_A$. The results under $c_B < c_A$ are symmetric. All possible equilibria are illustrated in Figure 5.

Lemma 1. *If $c_B \geq c_A$, the piggybacking equilibrium is given by Table 4 below. The results under $c_B < c_A$ are symmetric.*

Among all equilibrium results, we highlight the region where multiple equilibria are possible (shown in yellow in Figure 5b). When network effects are sufficiently strong and c_A, $c_B \in (\tilde{c}_1, \tilde{c}_2)$,

	Region	$\{\mathbb{I}_A\ \mathbb{I}_B\}$	Piggybacking Platform
weak NE ($t \geq T_3$)	$c_B < \tilde{c}_1$ and $c_A < \tilde{c}_1$	$\{1,1\}$	both
	$c_B \geq \tilde{c}_1$ and $c_A \geq \tilde{c}_2$	$\{0,0\}$	neither
	$c_B \geq \tilde{c}_1$ and $c_A < \tilde{c}_2$	$\{1,0\}$	only Platform A
strong NE ($t < T_3$)	$c_A < \tilde{c}_1$ and $c_B < \tilde{c}_1$,	$\{1,1\}$	both
	$c_A \geq \tilde{c}_2$ and $c_B \geq \tilde{c}_2$	$\{0,0\}$	neither
	$c_A \in [\tilde{c}_1, \tilde{c}_2)$ and $c_B \in [\tilde{c}_1, \tilde{c}_2)$	$\{1,0\}$ or $\{0,1\}$	either Platform A or B
	all other cases	$\{1,0\}$	only Platform A

where $\tilde{c}_1 = \frac{n(2t-T_3)(2t-T_2)(n(4t-3T_1)+3t-2T_1)}{(3t-2T_3)^2}$ and $\tilde{c}_2 = \frac{n(2t-T_1)(2t-T_3)((n+3)t-(n+2)T_1)}{(3t-2T_3)^2}$.

Table 4 Equilibrium Results of Endogenous Piggybacking

both $\{\mathbb{I}_A = 1, \mathbb{I}_B = 0\}$ and $\{\mathbb{I}_A = 0, \mathbb{I}_B = 1\}$ are possible, and the platform with the cost advantage (i.e., Platform A) is better off without piggybacking if the opponent platform has decided to engage in piggybacking. This is known in the literature as a "game of chicken" or a "hawk-dove game" (Fudenberg and Tirole 1991, pp. 18) which suggests both players are better off if they act differently. If the platforms are similar enough and the cost of piggybacking is neither too large or too small (i.e, $c_A \in [\tilde{c}_1, \tilde{c}_2)$ and $c_B \in [\tilde{c}_1, \tilde{c}_2)$), Platform A, although it has an advantage in piggybacking cost, is forced to choose no piggybacking (i.e., "chicken").

In such a "game of chicken," as the literature suggests, if the information is transparent, platforms may signal their intentions creditably before the game actually begins (i.e., pre-commitment). For example, Platform B may announce a partnership with a giant firm, showing a strong tendency toward piggybacking and forcing Platform A to conform to an asymmetric equilibrium. Therefore our finding implies that platforms may prefer an early move on piggybacking, not only for profit but also to preempt the piggybacking by the opponent platforms. In such cases, even platforms that lack an advantage from piggybacking (in this case, Platform B) are motivated to search for opportunities to do so.

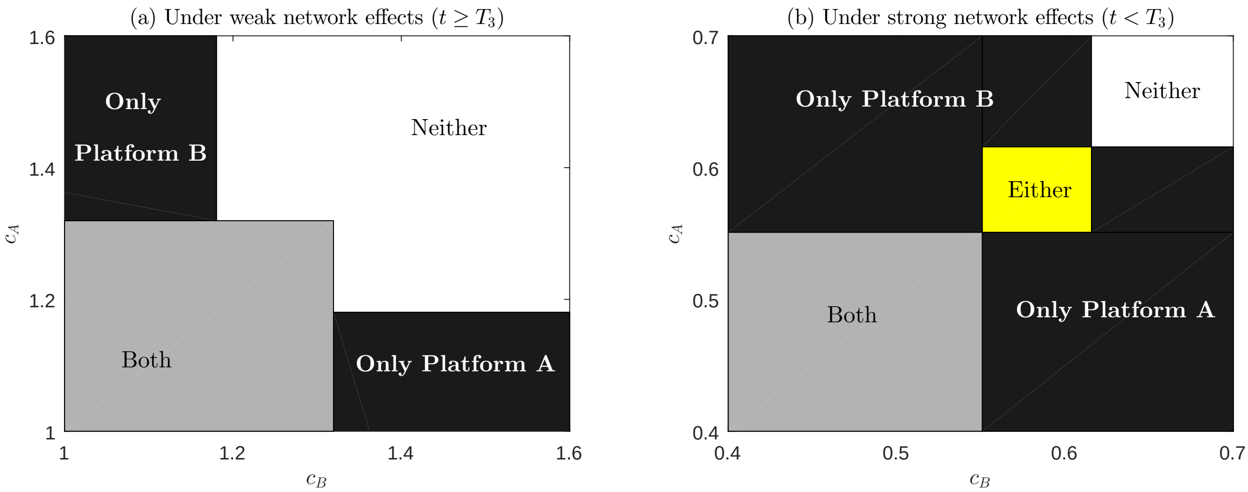

Figure 5 **Piggybacking Equilibria under Different Levels of Network Effects**

By taking a closer look at the equilibrium profits, we also discover a case in which piggybacking creates a prisoner's dilemma. See Proposition 6.

Proposition 6. *When the strength of the network effects is strong* ($t < T_3$) *and* $\{c_A, c_B\} \in (\tilde{c}_3, \tilde{c}_1)$*, the competition has a unique Nash equilibrium* $\{\mathbb{I}_A = 1, \mathbb{I}_B = 1\}$*, which is a prisoner's dilemma* (*where* $\tilde{c}_3 = n(1+n)(2t - T_1)$).

The region of the prisoner's dilemma defined in Proposition 6 is shown in purple in Figure 6. This suggests that platforms should have second thoughts about piggybacking. The key rule for platforms when making a decision on piggybacking, surprisingly, is that network effects are strong. The intuition is that, if network effects are strong, consumers in the piggybacking traffic contributes network effects rather than direct profits to both platforms, which intensifies the competition in the focal market. This is consistent with our baseline model in which consumer prices are negative under strong network effects, and the subsidy keeps growing in the presence of piggybacking (e.g., when $t < T_1$ in Proposition 2). In brief, strong network effects do not imply that a piggybacking

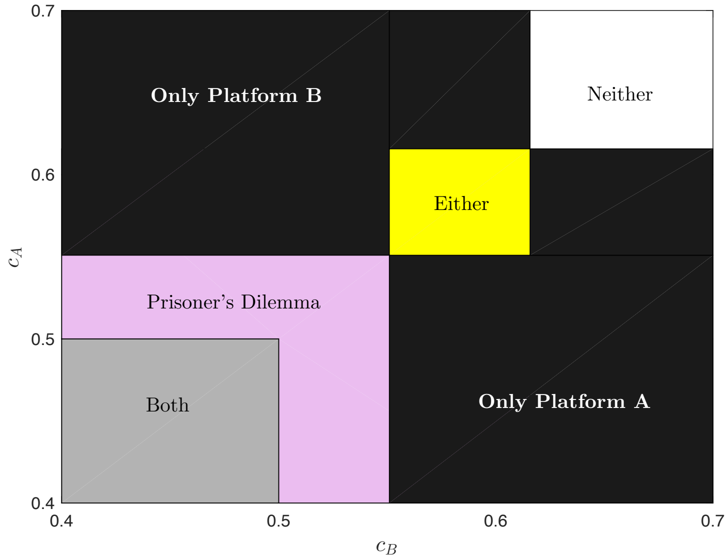

Figure 6 **Region of the Prisoner's Dilemma**

strategy is necessary. In fact, as Proposition 6 suggests, platforms need to be even more cautious about piggybacking because it can backfire.

5. Extension: Fabricated Piggybacking

Many platform businesses are found to use zombies or bots to fake platform participation, mostly in their early days after being launched. This seems similar to the piggybacking strategy we studied in our baseline model (called authentic piggybacking), but with fundamental differences. The authentic piggybacking traffic contributes either profits or network effects, or both, to the platforms, depending on the platform strategies. In sharp contrast, the fabricated piggybacking activities generate no revenue but fake platform activities that lure focal users to adopt them. To capture the essence of fabricated piggybacking, we remove n from the objective functions in Equations (5). Everything else remains the same as in our baseline setup. Proposition 7 examines the platforms' strategies when Platform A uses fabricated piggybacking to add n zombie consumers. To facilitate comparison with our earlier results, we refer to the piggybacking strategy in the baseline model as "authentic piggybacking."

The goal of this extension is not to quantify the potential profit from fake platform activities. It is not surprising that a platform would be better off with such an unethical business activity. Our goal is to measure the potential damage that fabricated piggybacking could generate and whether the opponent platform could avoid the negative impacts (such as in Corollary 1b).

Proposition 7. *Assuming that only Platform A employs fabricated piggybacking, the following hold true in equilibrium:*

(a) Platform B stays in the market when $n < \frac{2(3t-2T_3)}{\alpha(\alpha+\beta)}$*;*

(b) Platform A's subsidization conditions are affected by piggybacking;

(c) Platform B's subsidization conditions are not affected by piggybacking.

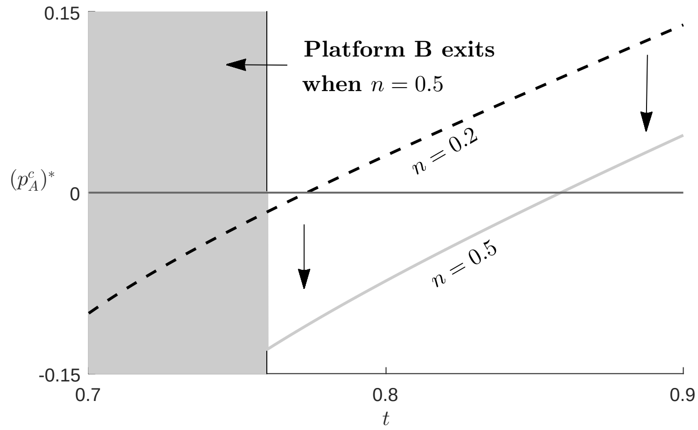

Figure 7 **Platform A's Consumer-Side Equilibrium Price under Fabricated Piggybacking. The shaded area represents regions where Platform B exits the market when n increases from 0.2 to 0.5 ($\alpha = 1$, $\beta = 0.6$)**

Proposition 7 reveals different platform strategies under fabricated piggybacking than under authentic piggybacking. Both Propositions 2 and 4 suggest that, an increase in n changes only the scale of the platform's price. The sign of the price is determined purely by the strength of network effects. This insight no longer holds under fabricated piggybacking for Platform A. Figures 7 and 8 illustrate further how n affects Platform A's subsidization strategy under fabricated piggybacking. Here Platform A finds it optimal to expand the region of subsidization as n increases (see Figure 7), and subsidizing providers might no longer be optimal even if $\beta > \alpha$ (see Figure 8). The key message is that managing zombie consumers is non-trivial because doing so requires the platform

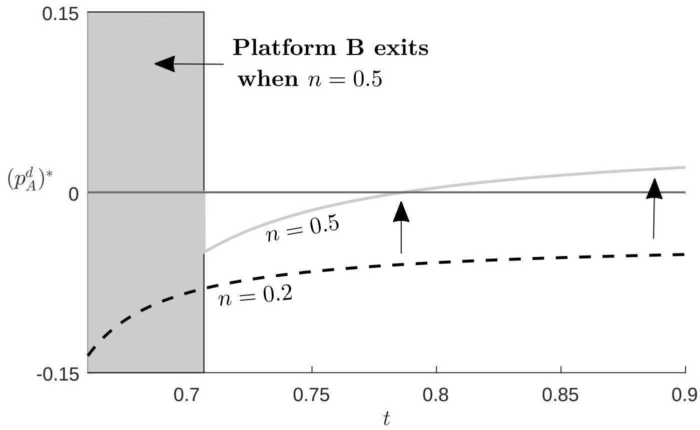

Figure 8 **Platform A's Provider-Side Equilibrium Price under Fabricated Piggybacking. The shaded area represents regions where Platform B exits the market when n increases from 0.2 to 0.5 ($\alpha = 0.6$, $\beta = 1$)**

to adjust pricing/subsidization strategies accordingly on both sides. The comparative statics of n on platform pricing are summarized in Proposition 8.

Proposition 8. *Assuming that only Platform A employs fabricated piggybacking, the following hold true in equilibrium:*

(a) $\frac{\partial (p_A^c)^*}{\partial n} < 0$ *iff* $\alpha(2T_1 - \beta^2) + 2t(\beta - 2\alpha) < 0$; $\frac{\partial (p_B^c)^*}{\partial n} < 0$ *iff* $t > T_2$;

(b) $\frac{\partial (p_A^d)^*}{\partial n} < 0$ *iff* $t < T_4 = \frac{\alpha^2 + 8\alpha\beta + 3\beta^2}{12}$; $\frac{\partial (p_B^d)^*}{\partial n} < 0$ *iff* $\alpha > \beta$.

We illustrate the insights from Proposition 8 in Figures 9 and 10. We note another sharp contrast between the platforms' pricing strategies under authentic and fabricated piggybacking, compared to Figures 2 and 3. Two immediate differences, for example, are that Platform A by and large raises the price on the provide side (unlike Figure 2b), even if it is not the money side (i.e., when $\alpha < \beta$), but Platform B decreases the price (see Figure 9a) on the consumer side, even if it is the money side (i.e., when $\alpha < \beta$). Intuitively, this is because fabricated piggybacking consumers do not contribute any profit (when the price is positive) or loss (when the price is negative) to Platform A. Therefore, it may no longer be optimal to give away provider-side profit to increase the price on the consumer side — the seesaw principle no longer holds here.

Not surprisingly, the pricing impact under fabricated piggybacking on the rival Platform B's consumer side is largely negative (see Figure 10a). This is because Platform B is under greater

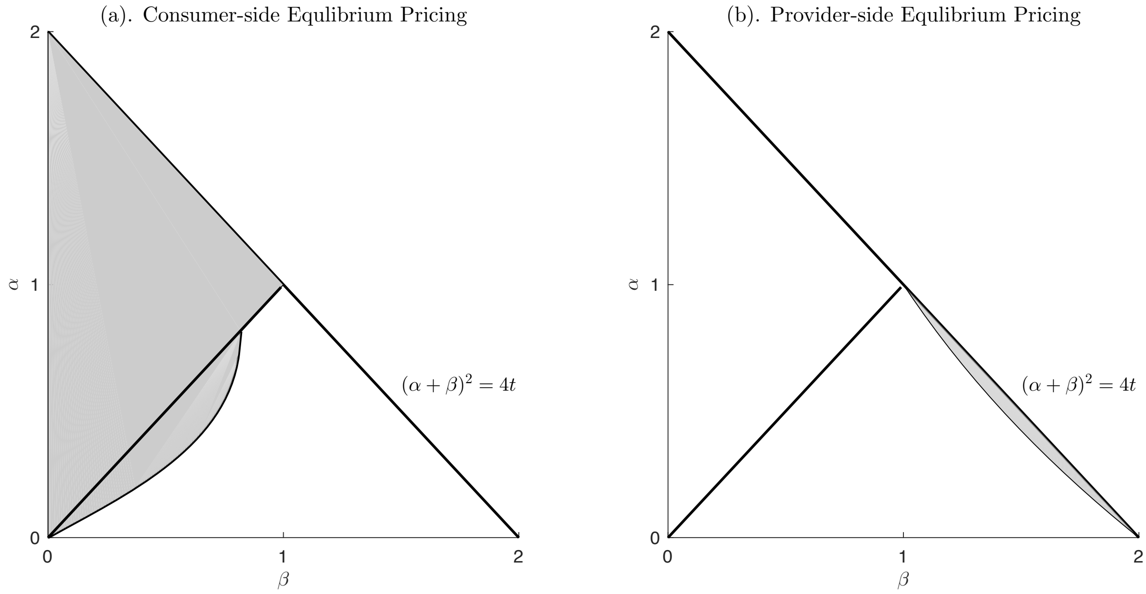

Figure 9 **Comparative Statics for Platform A's Equilibrium Prices under Fabricated Piggybacking. The shaded area represents regions where the price is decreasing in n.**

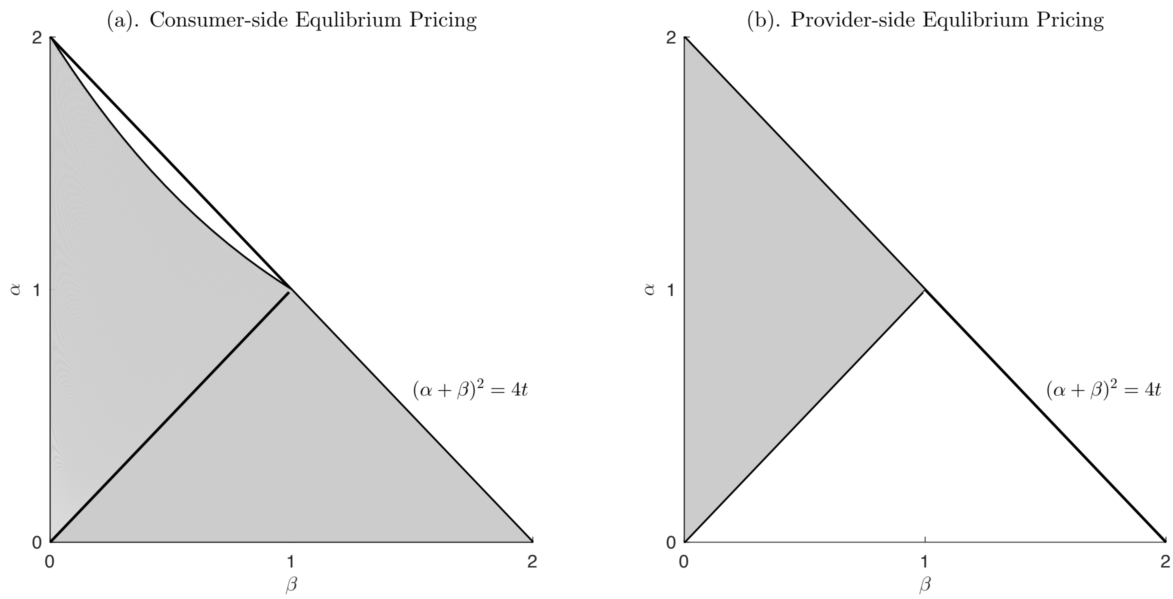

Figure 10 **Comparative Statics for Platform B's Equilibrium Prices under Fabricated Piggybacking. The shaded area represents regions where the price is decreasing in n.**

pricing pressure from Platform A on the consumer side, and this pressure might be greater on the provider side (see Figure 10b) when $\alpha > \beta$. In other words, when $\alpha > \beta$, indicating that the provider side is the money side, although $(p_B^d)^* > 0$ holds, fabricated piggybacking undermines Platform B's ability to monetize – Platform B is forced to reduce prices to avoid direct competition. This is further reflected in the profits, which we summarize in Proposition 9.

Proposition 9. *Assuming that only Platform A employs fabricated piggybacking, the following hold true in equilibrium:*

(a) Platform A is always better off as n increases;

(b) Platform B is always worse off as n increases;

(c) The profit gap between the platforms is increasing and convex in n.

Proposition 9 reveals that the profit impact on Platform B is different under fabricated piggybacking than authentic piggybacking. Under authentic piggybacking, per Corollary 1b, Platform B might benefit from n when network effects are not very large. This is because Platform A will raise prices, giving Platform B greater pricing flexibility. However, under fabricated piggybacking, Platform A does not have such strong incentives for raising prices because zombie consumers do not contribute any profit directly. Consequently, Platform B suffers from Platform A's fabricated piggybacking because its pricing flexibility is narrowed.

In addition to the profit impacts, we are also interested in how fabricated piggybacking affects the welfare of platform participants. On the one hand, the existence of fake users induces more providers to join, which in return brings extra surpluses for focal consumers; on the other hand, all the providers would eventually suffer from zombie users because zombies do not contribute any profit. We denote CS the focal consumers' surplus and PS the providers' surplus. Proposition 10 summarizes the welfare implications when Platform A uses fabricated piggybacking.

Proposition 10. *Assume only Platform A employs fabricated piggybacking, the following hold true in equilibrium:*

(a). Consumer surplus is always increasing in n, i.e., $\frac{\partial CS}{\partial n} > 0$;

(b). Provider surplus is decreasing in n except when both the strength of network effects and fabricated traffic are large (i.e., $\frac{\partial PS}{\partial n} > 0$ iff $T_5 > t$ and $n > \hat{n}$). Both T_5 and $\hat{n}$ are defined in the appendix.

Proposition 10, together with Proposition 9, suggests that the damages caused by using zombies unethically harms its opponent platform (i.e., Platform B is always worse off) and the provider communities, but not the focal consumers. This is because more providers are lured to participate in the first place, which generates additional network effects for Platform A's focal consumers and impose more competing pressure on Platform B. For example, if a dating website uses fake profiles of females, then more male users sign up, and all authentic female users benefit. Eventually, only all the authentic male users are worse off because they can never access the fake female accounts. Our model does not consider any same-side peer effects among consumers. Therefore the potential damages caused by fabricated piggybacking could be much greater if same-side peer effects are taken into consideration (e.g., buyers are annoyed by fake reviews on the product page of the two-sided online marketplaces).

6. Discussions and Conclusion

Piggybacking, as perhaps the most prominent traffic-based strategy, has become a critical competitive strategy in practical platform launches. We explore formally the strategic implications of piggybacking and subsidization in a competitive setting with network effects. We reveal many insights, briefly summarized below. We identify conditions under which piggybacking can mitigate platforms' price competition. We show that, compared to the absence of piggybacking, a platform could either increase or decrease the scale of consumer subsidization, depending on the cross-side network effects. We then study piggybacking as a competitive strategy for platforms. We find that platforms, even without the cost advantage of piggybacking, may still have the incentive to use piggybacking as early as possible because it helps differentiate themselves from the rival platform. We also discover a region of the prisoner's dilemma in which piggybacking backfires and undermines the platform's profit. Lastly, we extend the model to examine fabricated piggybacking such as zombies and bots. Our finding reveals that, compared with results under authentic piggybacking, the platform's subsidizing strategies are largely different under fabricated piggybacking. In addition, it always hurts not only the rival platform's profit but also the provider-side surpluses, both of which are unique to fabricated piggybacking.

Practically, our results suggest that the cross-side network effects, central to the two-sided market, are the key to the success of piggybacking strategies. First, we show with Proposition 2 and 4 that the subsidizing conditions are not affected by the strength of piggybacking traffic and are determined only by the strength of network effects. Second, Proposition 6 posits that the existence of a prisoner's dilemma requires strong network effects. Both propositions highlight the strategic importance of understanding cross-side network effects when platforms consider using piggybacking strategies.

Our model results also predict interesting divergence in platform pricing competition in the presence of piggybacking. For example, in the video-streaming market, in which YouTube subsidizes content creators by sharing advertising revenue. Vimeo uses a business model by charging content creators for uploading[3] contents. In this way, as measured by its traffic record, Vimeo has remained very popular in recent years.[4] Our model provides a theoretical framework for analyzing Vimeo's strategy.

In addition, by looking into piggybacking strategy in particular, this paper also contributes to the new framework to examine the user traffic management in general. A wide range of user traffic controls are increasingly popular among practical platforms. Our insights into piggybacking suggest that managing user traffic is essentially different from traditional pricing problems and requires further investigations. For example, firms undergoing a business model transformation often migrate consumer traffic from old products to new platforms (Zhu and Furr 2016). OpenTable, as it transformed from a restaurant customer relationship management (CRM) software vendor to an online reservation platform, leveraged its existing restaurant clients to attract consumers seeking to book a table. Similar strategies were employed when Valve expanded from a game developer to the leading gaming platform (i.e., Steam) as well as when Qihoo 360 launched the largest software marketplace in China (i.e., 360 Software Manager) based on its success with anti-virus software. Moreover, many social media websites open their traffic to startups through social logins (e.g.,

[3] https://vimeo.com/upgrade

[4] http://www.alexa.com/siteinfo/vimeo.com

Facebook Connect[5] and Google Sign-In[6]), which calls for further research on traffic-based strategies in launching a platform.

Finally, our model extension also sheds light on fake user profiles and review fraud in digital platform competition. We find that pricing competition is more intense under fabricated piggybacking than under authentic piggybacking, because platforms have fewer incentives to raise prices under fabricated piggybacking. We leave it to future research to empirically test our model's predictions.

[5] https://www.similartech.com/technologies/facebook-connect

[6] https://developers.google.com/identity/

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Appendix

Proofs

Proof of Proposition 1. The equilibrium conditions are

$$N_A^c = \frac{1}{2} - \frac{p_A^c - p_B^c}{2t} + \frac{\beta(N_A^d - N_B^d)}{2t},$$
$$N_B^c = \frac{1}{2} - \frac{p_B^c - p_A^c}{2t} + \frac{\beta(N_B^d - N_A^d)}{2t},$$
$$N_A^d = \alpha N_A^c - p_A^d, \text{ and } N_B^d = \alpha N_B^c - p_B^d.$$

Solving the system of equations gives the equilibrium market size.

$$\begin{aligned}
N_A^c &= \frac{1}{2} + \frac{p_B^c - p_A^c + \beta\left[p_B^d - p_A^d\right]}{2(t-\alpha\beta)}, \\
N_B^c &= \frac{1}{2} - \frac{p_B^c - p_A^c + \beta\left[p_B^d - p_A^d\right]}{2(t-\alpha\beta)}, \\
N_A^d &= \frac{\alpha}{2} + \frac{\alpha\left[p_B^c - p_A^c + \beta(p_A^d + p_B^d)\right] - 2tp_A^d}{2(t-\alpha\beta)}, \\
N_B^d &= \frac{\alpha}{2} + \frac{\alpha\left[p_A^c - p_B^c + \beta(p_A^d + p_B^d)\right] - 2tp_B^d}{2(t-\alpha\beta)}.
\end{aligned} \tag{A.1}$$

Insert Equation (A.1) into the objective functions in Equation (5). Take Platform A as an example. The profit function is

$$\begin{aligned}
\Pi_A = &-\frac{1}{2(t-\alpha\beta)} \times (p_A^c)^2 + \left(\frac{1}{2} - \frac{(\alpha+\beta)p_A^d - p_B^c - \beta p_B^d}{2(t-\alpha\beta)}\right) \times p_A^c \\
&+\alpha\left(\frac{1}{2} + \frac{p_B^c + \beta p_B^d}{2(t-\alpha\beta)}\right) \times p_A^d - \left(1 + \frac{t}{t-\alpha\beta}\right) \times (p_A^d)^2.
\end{aligned}$$

If we take the second-order derivatives to p_A^c and p_A^d, respectively, we have

$$\frac{\partial^2 \Pi_A}{\partial (p_A^c)^2} = -\frac{1}{t-\alpha\beta},$$
$$\frac{\partial^2 \Pi_A}{\partial (p_A^d)^2} = -\left(1+\frac{t}{t-\alpha\beta}\right),$$

both of which are negative because $(\alpha+\beta)^2 < 4t$. The determinant of the Hessian matrix is

$$\det(\mathbf{H}_{p_A^c, p_A^d}(\Pi_A)) = \frac{8t-\alpha^2-6\alpha\beta-\beta^2}{4(t-\alpha\beta)^2},$$

which is positive when $(\alpha+\beta)^2 < 4t$ holds. Therefore, Π_A is jointly concave in p_A^c and p_A^d. Similarly, we can show that Π_B is jointly concave in p_B^c and p_B^d, which implies that a unique equilibrium exists.

$$\frac{\partial \Pi_A}{\partial p_A^c} = \frac{1}{2} + \frac{p_B^c + \beta p_B^d - \alpha\beta - 2p_A^c - (\alpha+\beta)p_A^d}{2(t-\alpha\beta)},$$
$$\frac{\partial \Pi_A}{\partial p_A^d} = \frac{\alpha(t-\alpha\beta+p_B^c+\beta p_B^d) - (\alpha+\beta)p_A^c - 2(2t-\alpha\beta)p_A^d}{2(t-\alpha\beta)},$$
$$\frac{\partial \Pi_B}{\partial p_B^c} = \frac{1}{2} + \frac{p_A^c + \beta p_A^d - \alpha\beta - 2p_B^c - (\alpha+\beta)p_B^d}{2(t-\alpha\beta)},$$
$$\frac{\partial \Pi_B}{\partial p_B^d} = \frac{\alpha(t-\alpha\beta+p_A^c+\beta p_A^d) - (\alpha+\beta)p_B^c - 2(2t-\alpha\beta)p_B^d}{2(t-\alpha\beta)},$$

The equilibrium in Proposition 1 can be obtained by solving all four first-order conditions (FOCs) above simutaneously. Both platforms remain in the market, and the equilibrium is symmetric. □

Proof of Proposition 2. Following an approach similar as in the proof of Proposition 1, we can show that the profit functions for both platforms are jointly concave in the consumer-side and provider-side prices. Again it suggests a unique equilibrium. The FOCs are

$$\frac{\partial \Pi_A}{\partial p_A^c} = \frac{1}{2} + \frac{(2t-\alpha\beta)n + p_B^c + \beta p_B^d - \alpha\beta - 2p_A^c - (\alpha+\beta)p_A^d}{2(t-\alpha\beta)},$$

$$\frac{\partial \Pi_A}{\partial p_A^d} = \frac{\alpha(\alpha(2t-\alpha\beta)n + t - \alpha\beta + p_B^c + \beta p_B^d) - (\alpha+\beta)p_A^c - 2(2t-\alpha\beta)p_A^d}{2(t-\alpha\beta)},$$

$$\frac{\partial \Pi_B}{\partial p_B^c} = \frac{1}{2} + \frac{(2t-\alpha\beta)n + p_A^c + \beta p_A^d - \alpha\beta - 2p_B^c - (\alpha+\beta)p_B^d}{2(t-\alpha\beta)},$$

$$\frac{\partial \Pi_B}{\partial p_B^d} = \frac{\alpha(2t-\alpha\beta)n + t - \alpha\beta + p_A^c + \beta p_A^d) - (\alpha+\beta)p_B^c - 2(2t-\alpha\beta)p_B^d}{2(t-\alpha\beta)}.$$

The equilibrium results in Proposition 2 can be obtained by solving all four FOCs above together. Both platforms remain in the market and the equilibrium is symmetric. □

Proof of Proposition 3. Under asymmetric piggybacking, we can show that joint concavity holds. However, the interior solution of equilibrium may not hold because the market size is not symmetric if only Platform A chooses piggybacking. Therefore we find the region where the focal market share is non-negative on both sides for both platforms. This helps us rule out the infeasible cases where the equilibrium market size is negative.

We first solve for interior equilibrium prices and reinsert them in Equation (A.1). Both the equilibrium prices and market size can be found in Table 4.

Under our assumptions of $(\alpha+\beta)^2 < 4t$, it can be shown that N_A^c and N_A^d are both always non-negative. We next find the regions of $N_B^c \geq 0$ and $N_B^d \geq 0$.

Region 1: $N_B^c = \frac{1}{2} + \frac{t-T_3}{3t-2T_3} \times n \geq 0$:

N_B^c can be either increasing or decreasing in n, depending on the sign of $t - T_3$. When $t \geq T_3$, N_B^c is increasing in n. Thus $N_B^c \geq 0$ always holds because $N_B^c = \frac{1}{2} > 0$ when $n = 0$. Otherwise when $t < T_3$, N_B^c is decreasing in n. A threshold of n exists, above which $N_B^c < 0$. In this case, we solve for n from $N_B^c = 0$:

$$\frac{1}{2} + \frac{(t-T_3)}{3t-2T_3} \times n = 0 \rightarrow n = \frac{3t-2T_3}{2(T_3-t)}.$$

Therefore, when $t < T_3$, $N_B^c \geq 0$ requires $n \leq \frac{3t-2T_3}{2(T_3-t)}$.

Region 2: $N_B^d = \frac{\alpha+\beta}{4}\left(1 + \frac{2(t-T_3)}{3t-2T_3} \times n\right) \geq 0$:

Similarly, we can show that $N_B^d \geq 0$ requires the same condition, $n \leq \frac{3t-2T_3}{2(T_3-t)}$. Therefore, the equilibrium exists everywhere except when $t < T_3$ and $n > \frac{3t-2T_3}{2(T_3-t)}$. $\square$

Proof of Proposition 4. Based on Proposition 3, when equilibrium exists, we can obtain the equilibrium by solving the following systems of FOCs:

$$\frac{\partial \Pi_A}{\partial p_A^c} = -\frac{p_A^c}{t-\alpha\beta} + \left(\frac{1}{2} - \frac{(\alpha+\beta)p_A^d - p_B^c - \beta p_B^d - (2t-\alpha\beta)n}{2(t-\alpha\beta)}\right) = 0,$$

$$\frac{\partial \Pi_A}{\partial p_A^d} = -\left(1 + \frac{t}{t-\alpha\beta}\right) p_A^d + \frac{\alpha\left[p_B^c + \beta p_B^d + n(2t-\alpha\beta) + (t-\alpha\beta)\right] - (\alpha+\beta)p_A^c}{2(t-\alpha\beta)} = 0,$$

$$\frac{\partial \Pi_B}{\partial p_B^c} = -\frac{p_B^c}{t-\alpha\beta} + \left(\frac{1}{2} - \frac{(\alpha+\beta)p_B^d - p_A^c - \beta p_A^d + \alpha\beta n}{2(t-\alpha\beta)}\right) = 0,$$

$$\frac{\partial \Pi_B}{\partial p_B^d} = -\left(1 + \frac{t}{t-\alpha\beta}\right) p_B^d + \frac{\alpha\left[p_A^c + \beta p_A^d + \alpha\beta n + (t-\alpha\beta)\right] - (\alpha+\beta)p_B^c}{2(t-\alpha\beta)} = 0.$$

The equilibrium market size and the corresponding profits can be obtained by inserting equilibrium prices in Equation (5). $\square$

Proof of Proposition 5. Define parameter regions I through IV as in Table 4. We take the first-order derivatives with respect to n in equilibrium prices. Note that $T_3 > T_1$ always holds.

$$\frac{\partial (p_A^c)^*}{\partial n} = \frac{2\,(2t-T_3)\,(t-T_1)}{3t-2T_3},$$

$$\frac{\partial (p_B^c)^*}{\partial n} = \frac{2\,(t-T_3)\,(t-T_1)}{3t-2T_3},$$

$$\frac{\partial (p_A^d)^*}{\partial n} = \frac{(\alpha-\beta)\,(2t-T_3)}{2(3t-2T_3)},$$

$$\frac{\partial (p_B^d)^*}{\partial n} = \frac{(\alpha-\beta)\,(t-T_3)}{2(3t-2T_3)}.$$

(a) Region I satisfies $t < T_1$ and $\alpha > \beta$. Therefore we have $\frac{\partial (p_A^c)^*}{\partial n} < 0$, $\frac{\partial (p_A^d)^*}{\partial n} > 0$, $\frac{\partial (p_B^c)^*}{\partial n} > 0$, and $\frac{\partial (p_B^d)^*}{\partial n} < 0$. In contrast, Region III satisfies $t < T_3$ and $\alpha < \beta$, which result in $\frac{\partial (p_A^c)^*}{\partial n} > 0$, $\frac{\partial (p_A^d)^*}{\partial n} < 0$, $\frac{\partial (p_B^c)^*}{\partial n} < 0$, and $\frac{\partial (p_B^d)^*}{\partial n} > 0$. Region II satisfies $t \in [T_1, T_3)$ and $\alpha \geq \beta$. Therefore we have $\frac{\partial (p_B^c)^*}{\partial n} < 0$ and $\frac{\partial (p_B^d)^*}{\partial n} < 0$.

(b) Regions IV and V satisfy $t \geq T_3 \geq T_1$. Note that $3t > 2T_3$ always holds. Therefore we have $\frac{\partial (p_A^c)^*}{\partial n} > 0$ and $\frac{\partial (p_B^c)^*}{\partial n} > 0$. $\square$

Proof of Corollary 1.

(a) Take the first- and second-order derivatives with respect to n in Π_A^*.

$$\frac{\partial(\Pi_A)^*}{\partial n} = \frac{(2t-T_3)\,(2t-T_2)\,((4n+3)\,t-2\,(n+1)\,T_3)}{(3t-2T_3)\,^2} > 0,$$

$$\frac{\partial^2(\Pi_A)^*}{\partial n^2} = \frac{2\,(T_3-2t)\,^2\,(2t-T_2)}{(3t-2T_3)\,^2} > 0,$$

because $2t-T_2 \geq 0$ always holds when $4t > (\alpha+\beta)^2$. Then $(\Pi_A)^*$ is increasing and convex in n.

(b) Take the first- and second-order derivatives with respect to n in Π_B^*.

$$\frac{\partial(\Pi_B)^*}{\partial n} = \frac{(t-T_3)\,(2t-T_2)\,[(2n+3)\,t-2\,(n+1)\,T_3]}{(3t-2T_3)^2},$$

$$\frac{\partial^2(\Pi_B)^*}{\partial n^2} = \frac{2\,(t-T_3)\,^2\,(2t-T_2)}{(3t-2T_3)^2} > 0.$$

In the parameter region defined by Proposition 3, $(2n+3)\,t-2\,(n+1)\,T_3 \geq 0$ always holds. Therefore, it requires $t-T_3>0$ for $\frac{\partial(\Pi_B)^*}{\partial n}$ to be positive, and $(\Pi_B)^*$ is convex in n.

(c) The profit gap between the two platforms is given by

$$(\Pi_A)^* - (\Pi_B)^* = \frac{n\,(n+1)\,t\,(2t-T_2)}{3t-2T_3}.$$

Take the first- and second-order derivatives with respect to n.

$$\frac{\partial[(\Pi_A)^*-(\Pi_B)^*]}{\partial n} = \frac{(2n+1)\,t\,(2t-T_2)}{3t-2T_3} > 0,$$

$$\frac{\partial^2[(\Pi_A)^*-(\Pi_B)^*]}{\partial n^2} = \frac{2t\,(2t-T_2)}{3t-T_3} > 0.$$

Then the profit gap is increasing and convex in n. □

Proof of Lemma 1. When piggybacking becomes an endogenous decision for platforms, we incorporate the piggybacking costs, c_A and c_B, for platform A and B, respectively. Without loss of generality, we focus on the case of $c_B \geq c_A$. The scenario of $c_A > c_B$ is symmetric.

Note that $\tilde{c}_1 \geq \tilde{c}_2$ iff $t \geq T_3$. We find conditions for each possible equilibrium as the following:

(a) $c_A \geq \tilde{c}_2$ and $c_B \geq \tilde{c}_2 \rightarrow \{\mathbb{I}_A = 0, \mathbb{I}_B = 0\}$,

(b) $c_A < \tilde{c}_1$ and $c_B < \tilde{c}_1 \rightarrow \{\mathbb{I}_A = 1, \mathbb{I}_B = 1\}$,

(c) $c_A < \tilde{c}_2$ and $c_B \geq \tilde{c}_1 \rightarrow \{\mathbb{I}_A = 1, \mathbb{I}_B = 0\}$,

(d) $c_A \geq \tilde{c}_1$ and $c_B < \tilde{c}_2 \rightarrow \{\mathbb{I}_A = 0, \mathbb{I}_B = 1\}$.

$\tilde{c}_1$ and $\tilde{c}_2$ are both functions of network effect strength. Therefore we need to discuss over different regions of c_A, c_B, and network effect strength to determine the type of equilibria. We start by splitting the discussion into two cases: $t \geq T_1$ and $t < T_1$.

Case 1: $t \geq T_1$**.** When $t \geq T_1$, $\tilde{c}_1 \geq \tilde{c}_2$ always holds. Note that $c_B \geq c_A$ always holds, which implies that $c_A \geq \tilde{c}_1$ and $c_B < \tilde{c}_2$ is impossible. There are six sub-cases in all.

Subcase 1-1: $c_A < \tilde{c}_2 \leq \tilde{c}_1$ and $c_B < \tilde{c}_2 \leq \tilde{c}_1$, then $c_A < \tilde{c}_1$ and $c_B < \tilde{c}_1$ is true and the equilibrium is $\{\mathbb{I}_A = 1, \mathbb{I}_B = 1\}$;

Subcase 1-2: $c_A < \tilde{c}_2 \leq \tilde{c}_1$ and $\tilde{c}_2 \leq c_B < \tilde{c}_1$, then $c_A < \tilde{c}_1$ and $c_B < \tilde{c}_1$ is true and the equilibrium is $\{\mathbb{I}_A = 1, \mathbb{I}_B = 1\}$;

Subcase 1-3: $c_A < \tilde{c}_2 \leq \tilde{c}_1$ and $\tilde{c}_1 \leq c_B$, then $c_A < \tilde{c}_2$ and $\tilde{c}_1 \leq c_B$ is true and the equilibrium is $\{\mathbb{I}_A = 1, \mathbb{I}_B = 0\}$;

Subcase 1-4: $\tilde{c}_2 \leq c_A < \tilde{c}_1$ and $\tilde{c}_2 \leq c_B < \tilde{c}_1$, then $\{c_A < \tilde{c}_1, c_B < \tilde{c}_1\}$ and $\{c_A \geq \tilde{c}_2, c_B \geq \tilde{c}_2\}$ are both true and there are two possible pure-strategy Nash equilibria. Apply Pareto principle to refine, we find that both platforms are better off under $\{\mathbb{I}_A = 1, \mathbb{I}_B = 1\}$. Thus the unique feasible equilibrium is $\{\mathbb{I}_A = 1, \mathbb{I}_B = 1\}$;

Subcase 1-5: $\tilde{c}_2 \leq c_A < \tilde{c}_1$ and $\tilde{c}_1 \leq c_B$, then $c_A \geq \tilde{c}_2$ and $c_B \geq \tilde{c}_2$ is true and the equilibrium is $\{\mathbb{I}_A = 0, \mathbb{I}_B = 0\}$;

Subcase 1-6: $\tilde{c}_2 \leq \tilde{c}_1 \leq c_A$ and $\tilde{c}_2 \leq \tilde{c}_1 \leq c_B$, then $c_A \geq \tilde{c}_2$ and $c_B \geq \tilde{c}_2$ is true and the equilibrium is $\{\mathbb{I}_A = 0, \mathbb{I}_B = 0\}$.

Lemma 1-1 can be obtained by summarizing the analysis above.

Case 2: $t < T_1$**.** When $t < T_1$, $\tilde{c}_1 < \tilde{c}_2$ always holds. There are also six sub-cases in all.

Subcase 2-1: $c_A < \tilde{c}_1 \leq \tilde{c}_2$ and $c_B < \tilde{c}_1 \leq \tilde{c}_2$, then $c_A < \tilde{c}_1$ and $c_B < \tilde{c}_1$ is true and the equilibrium is $\{\mathbb{I}_A = 1, \mathbb{I}_B = 1\}$;

Subcase 2-2: $c_A < \tilde{c}_1 \leq \tilde{c}_2$ and $\tilde{c}_1 \leq c_B < \tilde{c}_1$, then $c_A < \tilde{c}_2$ and $c_B \geq \tilde{c}_1$ is true and the equilibrium is $\{\mathbb{I}_A = 1, \mathbb{I}_B = 0\}$;

Subcase 2-3: $c_A < \tilde{c}_1 \leq \tilde{c}_2$ and $\tilde{c}_2 \leq c_B$, then $c_A < \tilde{c}_2$ and $c_B \geq \tilde{c}_1$ is true and the equilibrium is $\{\mathbb{I}_A = 1, \mathbb{I}_B = 0\}$;

Subcase 2-4: $\tilde{c}_1 \leq c_A < \tilde{c}_2$ and $\tilde{c}_1 \leq c_B < \tilde{c}_2$, then $\{c_A < \tilde{c}_2,\ c_B \geq \tilde{c}_1\}$ and $\{c_A \geq c_1,\ c_B < c_2\}$ are both true and there are two possible pure-strategy Nash equilibria, neither of which constitutes Pareto improvement;

Subcase 2-5: $\tilde{c}_1 \leq c_A < \tilde{c}_2$ and $\tilde{c}_2 \leq c_B$, then $c_A < \tilde{c}_2$ and $c_B \geq \tilde{c}_2$ is true and the equilibrium is $\{\mathbb{I}_A = 1, \mathbb{I}_B = 0\}$;

Subcase 2-6: $\tilde{c}_1 \leq \tilde{c}_2 \leq c_A$ and $\tilde{c}_2 \leq c_B$, then $c_A \geq \tilde{c}_2$ and $c_B \geq \tilde{c}_2$ is true and the equilibrium is $\{\mathbb{I}_A = 0, \mathbb{I}_B = 0\}$.

Lemma 1-2 can be obtained by summarizing the analysis above. $\square$

Proof of Proposition 6. This proposition can be shown by comparing the symmetric profits of $\Pi_k(\mathbb{I}_A = 0, \mathbb{I}_B = 0)$ and $\Pi_k(\mathbb{I}_A = 1, \mathbb{I}_B = 1)$ shown in the proof of Lemma 1. $\square$

Proof of Proposition 7. As in the proof for Proposition 3, we first show that the equilibrium, if any, is unique. Note that equilibrium numbers of focal market adopters are identical to those of Equation (A.1) because fabricated piggybacking does not change the role of network effects. However, the objective functions do not contain n because fabricated piggybacking does not contribute any direct profit. Following an approach similar to the one we used in the proof for Proposition 4, we can derive the equilibrium prices as follows:

$$
\begin{aligned}
(p_A^c)^* &= t - T_1 + \frac{\alpha n\left(\alpha(2T_3 - \beta^2) + 2t(\beta - 2\alpha)\right)}{2(3t - 2T_3)}, \\
(p_A^d)^* &= \frac{1}{4}\left(\alpha - \beta + \frac{(12 - \alpha^2 - 8\alpha\beta - 3\beta^2)\,\alpha n}{2(3t - 2T_3)}\right), \\
(p_B^c)^* &= (t - T_1)\left(1 - \frac{\alpha(\alpha + \beta)n}{2(3t - 2T_3)}\right), \\
(p_B^d)^* &= \frac{1}{4}(\alpha - \beta)\left(1 - \frac{\alpha(\alpha + \beta)n}{2(3t - 2T_3)}\right).
\end{aligned}
$$

(a) Reinsert equilibrium prices in the equilibrium market size in Equation (A.1). $N_B^c \geq 0$ and $N_B^d \geq 0$ require an identical condition: $1 \geq \frac{\alpha(\alpha+\beta)n}{2(3t-2T_3)}$.

(b) It can be observed from the equilibrium prices above that the signs of prices are sensitive to n, meaning that subsidization conditions are affected by the magnitude of piggybacking.

(c) It can be observed from the equilibrium prices above that when the equilibrium exists (i.e.,$1 \geq \frac{\alpha(\alpha+\beta)n}{2(3t-2T_3)}$), the signs of $(p_B^c)^*$ and $(p_B^d)^*$ are not affected by n. □

Proof of Proposition 8. If we take the first-order derivatives to equilibrium prices above, we have

$$\frac{\partial (p_A^c)^*}{\partial n} = \frac{\alpha\left(\alpha(2T_3-\beta^2)+2t(\beta-2\alpha)\right)}{2(3t-2T_3)},$$

$$\frac{\partial (p_A^d)^*}{\partial n} = \frac{3\alpha\,(t-T_4)}{2(3t-2T_3)},$$

$$\frac{\partial (p_B^c)^*}{\partial n} = -\frac{\alpha(t-T_1)(\alpha+\beta)}{2(3t-2T_3)},$$

$$\frac{\partial (p_B^d)^*}{\partial n} = -\frac{\alpha(\alpha-\beta)(\alpha+\beta)}{2(3t-2T_3)}.$$

(a) The sign of $\frac{\partial (p_A^c)^*}{\partial n}$ depends on the sign of $\alpha(2T_3-\beta^2)+2t(\beta-2\alpha)$. The sign of $\frac{\partial (p_B^c)^*}{\partial n}$ depends on the sign of $t-T_1$.

(b)The sign of $\frac{\partial (p_A^d)^*}{\partial n}$ depends on the sign of $t-T_4$. The sign of $\frac{\partial (p_B^d)^*}{\partial n}$ depends on the sign of $\alpha-\beta$.

□

Proof of Proposition 9. (a) & (b) This can be obtained by taking the first- and second-order derivatives with respect to n in equilibrium profits.

(c) The profit gap between the two platforms is

$$(\Pi_A)^* - (\Pi_B)^* = \frac{1}{4}\alpha n\left(\alpha n + \frac{(\alpha+\beta)\,(\alpha^2+6\alpha\beta+\beta^2-8t)}{\alpha^2+4\alpha\beta+\beta^2-6t}\right),$$

which is increasing and convex in n. □

Proof of Proposition 10.

(a). Consumer surplus is

$$CS = \int_0^{N_A^c}\left(V-p_A^c+\beta N_A^d-tx\right)dx + \int_{N_A^c}^{1}\left(V-p_B^c+\beta N_B^d-t(1-y)\right)dy.$$

Insert p_A^c, p_B^c, N_A^c, N_A^d, and N_B^d from Table 2 in CS. We get

$$CS = \frac{1}{4}\left(\alpha^2 + 4\alpha\beta + \beta^2 + \alpha n(\alpha+\beta)\left(\frac{\alpha n t(\alpha+\beta)}{(\alpha^2+4\alpha\beta+\beta^2-6t)^2} + 1\right) - 5t + 4V\right),$$

which is increasing in n.

(b). Provider surplus is

$$PS = \int_0^{N_A^d}\left(\alpha N_A^c - p_A^d - z\right)dz + \int_0^{N_B^d}\left(\alpha N_B^c - p_B^d - z\right)dz.$$

Note that fake consumers do not contribute actual surplus to providers. Insert p_A^d, p_B^d, N_A^d, N_A^c, and N_B^c from Table 2 to PS. We get

$$\begin{aligned} PS &= \frac{1}{16}(\alpha+\beta)^2 - \frac{1}{8}\alpha n(\alpha+\beta) \\ &- \frac{\alpha^2 n^2\left(3\alpha^4 + 32\alpha^3\beta + 82\alpha^2\beta^2 + 32\alpha\beta^3 + 3\beta^4 + 216t^2 - 12t\left(5\alpha^2 + 22\alpha\beta + 5\beta^2\right)\right)}{16\left(\alpha^2+4\alpha\beta+\beta^2-6t\right)^2}. \end{aligned}$$

Take the first-order derivative with respect to n.

$$\frac{\partial PS}{\partial n} = -\frac{1}{8}\alpha(\alpha+\beta) - \frac{\alpha^2 n\left(3\alpha^4 + 32\alpha^3\beta + 82\alpha^2\beta^2 + 32\alpha\beta^3 + 3\beta^4 + 216t^2 - 12t\left(5\alpha^2 + 22\alpha\beta + 5\beta^2\right)\right)}{8\left(\alpha^2+4\alpha\beta+\beta^2-6t\right)^2}.$$

Denote $\Phi = 3\alpha^4 + 32\alpha^3\beta + 82\alpha^2\beta^2 + 32\alpha\beta^3 + 3\beta^4 + 216t^2 - 12t\left(5\alpha^2 + 22\alpha\beta + 5\beta^2\right)$. If $\Phi \geq 0$, then $\frac{\partial PS}{\partial n} < 0$ always holds. Otherwise if $\Phi < 0$, then $\frac{\partial PS}{\partial n} \geq 0$ becomes possible if n is large enough. Note that Φ is convex and quadratic in t and $t > \frac{(\alpha+\beta)^2}{4}$ always hold. We insert the lower bound of t into the first-order derivative of t.

$$\frac{\partial \Phi}{\partial t}\Big|_{t=\frac{(\alpha+\beta)^2}{4}} = 48(\alpha^2 - \alpha\beta + \beta^2) > 0,$$

which implies that $\frac{\partial \Phi}{\partial t} \geq 0$ for all feasible t. Therefore, $\Phi < 0$ requires that t is small enough (alternatively, network effects are strong enough). Specifically, we need t to be smaller than the bigger root of $\Phi = 0$. Solve $\Phi = 0$. We have two roots.

$$t_1 = \frac{1}{36}\left(5\alpha^2 + 22\alpha\beta - \sqrt{7}(\alpha+\beta)^2 + 5\beta^2\right),\ t_2 = \frac{1}{36}\left(5\alpha^2 + 22\alpha\beta + \sqrt{7}(\alpha+\beta)^2 + 5\beta^2\right).$$

t_2 is bigger than t_1. Then $\Phi < 0$ requires $t < t_2$. Rename t_2 as T_5. Note that $T_5 < T_3$ always holds for all positive α and β, indicating that the region for $\Phi < 0$ is very restricted. The threshold of n, $\hat{n}$ can be solved when $t < T_5$.

$$\frac{\partial PS}{\partial n}|_{n=\hat{n}} = 0 \rightarrow \hat{n} = -\frac{(\alpha+\beta)(\alpha^2+4\alpha\beta+\beta^2-6t)^2}{\alpha\Phi}.$$

$\frac{\partial PS}{\partial n} \geq 0$ requires both $t < T_5$ and $n > \hat{n}$. $\square$