# A Model for Markets with Cross-Producer Bundles: Analyzing Value Co-Creation in Platforms

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**ABSTRACT**

This paper develops a model for an economic form in which value is co-created multiple producers and their outputs are collected and sold as a common bundle, often by a separate and independent firm, a retailer. This structure is found in many markets, especially for technology goods and services, e.g., software platforms such as *Slack* and in-home video entertainment. Producers in these markets are not competitors in the usual sense, because output of each casts an externality on production decisions of others. Given the typical challenges in formulating bundle demand under complex settings, this paper creates an abstract measure, *value units*, of bundle quality, represents market-level demand for the bundle with a reduced-form model that obeys the theory of bundling, and develops a production system consistent with this measure. Unlike traditional forms of competition, we find that equilibrium production quantities of competitors are strategic complements. However, competition between producers does manifest itself, e.g., if one acquires better production technology (i.e., makes value units at lower cost) then the equilibrium production levels of other producers are reduced. Insights are also derived for alternative market structures, e.g., production levels would be higher under a consortium of producers than when they must sell through a retailer. Further, while producers and the retailer would each like a higher share of bundle revenue, we show that the retailer's claim is bounded because outputs (and revenue) would fall if producers' share were too low; conversely, a very high share for producers would eventually hurt them by motivating the retailer to enter production. In sum, the formulation enables us to understand and specify equilibrium outcomes on the demand and supply sides, revenue-sharing between producers and the retailer, and also analyze market dynamics and alternative market structures.

# 1 Introduction

*What is common to in-home video entertainment, software platforms* `Slack`, `Intuit` *and* `Dropbox`, *and season or ground passes for events (sports tournaments, auto shows, technology exhibitions, county fairs, music festivals, art exhibitions)?*

A common thread in these examples is that individual outputs from multiple ($K$) independent producers are bundled into a "product" by a separate actor (e.g., retailer, community organizer, or a consortium, see the left panel of Fig. 1) who sets prices, collects revenues, and shares revenues with producers, who compete as well as cooperate in supplying value to consumers. Unlike the more common form of competition where producers compete for customers in a zero-sum game (e.g., automobile industry), here every producer "serves" all customers who purchase the bundle, hence the output of one benefits the others. Yet, producers are not just "team players," they do compete for a share of revenues, and their production and revenue-sharing decisions are governed by selfish interest. This paper develops an economic model for analyzing this market setting, equilibrium outcomes and strategies under different market structures, and the drivers of changes in market structure.



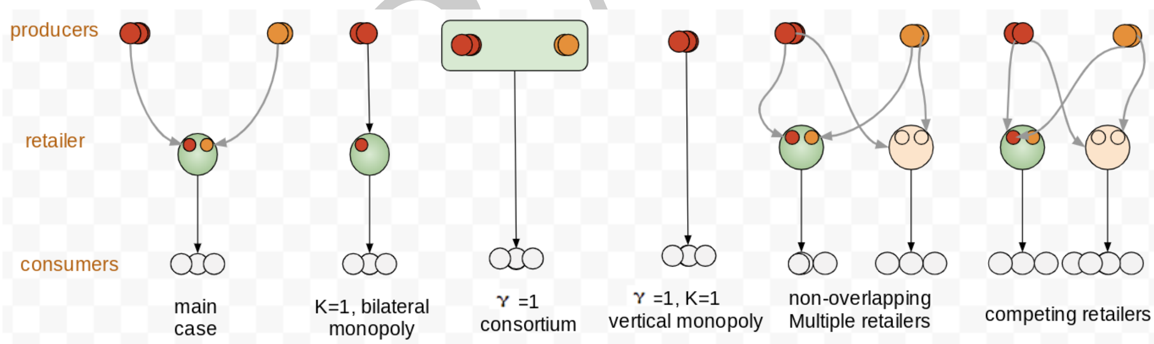



Figure 1: Some market structure variations for multi-producer bundle goods. $K$ is the number of independent producers, $\gamma$ is the total revenue share of producers, while $1-\gamma$ that of the retailer.

This paper was inspired by the market for in-home video entertainment (movies and TV shows consumed on TVs and other personal devices) where cable TV providers (and more recently, streaming providers such as Netflix) offer a bundle of entertainment content sourced from an oligopoly of multiple content providers such as studios and programming networks. Such value co-creation is a defining characteristic of platforms, a business architecture that has vaulted to prominence in the last decade. For example, the team productivity tool *Slack* contains dozens of "integrations" or features (covering capabilities such as polling, task management, graphic communication, etc.) that are sourced from various software developers and made available to *Slack* buyers under a collective single price. A special case of this setting is *Adobe Creative Cloud*, where also multiple apps are offered for one bundle price, except that all apps are owned by the same company, *Adobe*.

What is a suitable model for analyzing economic questions in markets that feature cooperative production and bundling, ranging from pricing, revenue sharing, and production levels in a static setting, to market dynamics covering both the causes and effects of changes in industry structure? One perspective which is fundamental to the analysis of these markets is the nature of competition amongst producers, and between producers and the retailer. There is a vast literature that covers many forms of competition: quantity competition with homogeneous goods (Bertrand, Cournot), horizontal product differentiation (Hoteling line, Salop circle, Chen and Riordan's Spokes), complementary and composite goods, and decisions by teams, co-operatives, and conglomerates; however, none of these reflects the economics of bundling in a meaningful way. We elaborate on this perspective in §2.1.

A second perspective for studying these markets, due to their collective-sales aspect, is product bundling. While the literature on bundling covers both the mechanics of bundling and its optimal design, it primarily employs a micro-level analysis of bundling which does not scale up to a market or industry-level analysis (§2.3 elaborates on this point). This is because the derivation of bundle demand faces deep mathematical complexities from the need to convolve demand distributions for multiple bundle components, either with or without correlation, super- or sub-additivity in valuations across components, and asymmetric demand profiles of bundle components. Even the simplest bundle setting (two products, no correlation and no super- or sub-additivity in valuations) is analytically intractable, and micro-level models stand little chance of addressing broader industry-level questions. An alternative and higher industry-level model, required for addressing these questions, is the quest of this paper.

This paper develops and applies a method for analyzing markets that involve co-production of goods, i.e., where market demand is defined over a combination of multiple outputs from multiple producers (e.g., in software platforms, art festivals, etc.). After discussing the relevant literature and challenges (§2), §3 develops a reduced-form specification for bundle demand which fits and respects the characteristics of bundling across a wide spectrum of bundling scenarios, and which is computationally tractable in terms of computing optimal bundle policies under different market structures. Next, §4 examines market outcomes under alternative market structures and also the drivers and consequences of changes in market structures.

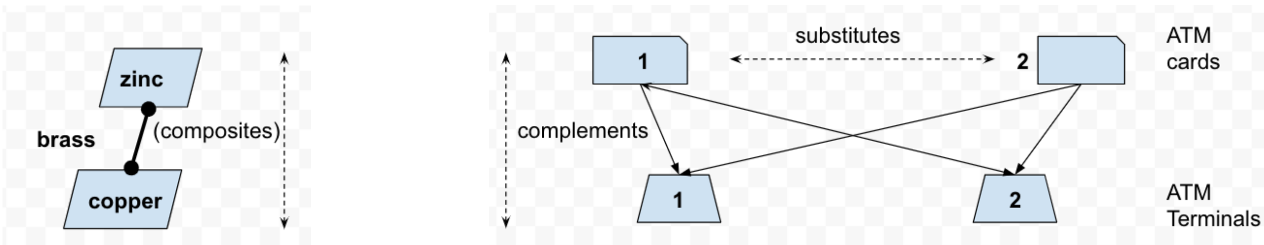
# 2 Perspectives from Related Literature

The literature on competition covers many forms including those where firms compete directly and fully, and others where competitors' payoffs have a collective component. We cover models of competition in §2.1, then value co-creation in platforms in §2.2, and finally examine the literature on bundling in §2.3.

## 2.1 Competition

Competing firms that make a homogeneous good can compete by choosing quantity (*Cournot* model) and/or price (*Bertrand* model), and market shares and profits of the firms are interdependent on all their choices because market price depends on total output (Varian, 1992, Ch. 16). In both cases, the market equilibrium is such that the lower-cost firm gets higher output and profits. In both forms of competition, the strategic response functions $(\Omega_i, \Omega_j)$ that determine equilibrium output levels $(Q_i, Q_j)$ move in opposite directions (or, price responses move in the same direction),

$$\frac{\partial \Omega_j(Q_i)}{\partial Q_i} < 0; \quad \frac{\partial \Omega_j(p_i)}{\partial p_i} \geq 0. \tag{1}$$




(a) Perfect complements (composites). (b) Systems with multiple brands of complements.

Figure 2: Production and competition with composite goods and systems.

The identity always holds under *Bertrand* competition and $\frac{\partial \Omega_j(Q_i)}{\partial Q_i} < 0$ holds in *Cournot* competition if the inverse demand curve is concave or not too convex. In other words, output levels of firms $i, j$ are *strategic substitutes*. This fundamental property is also satisfied under variations of *Bertrand* or *Cournot* competition, including when the goods are differentiated. In contrast, we will see that the natural behavior under value co-creation is for output levels to be *strategic complements*, i.e., $\frac{\partial \Omega_j(Q_i)}{\partial Q_i}{>}0$, setting up a contrast against these common forms of competition.

While the above forms of competition consider substitute goods, some markets feature complements, with the extreme case being that of *composite goods* markets where outputs of multiple producers are fused to make a useful product. Cournot (1929) provided the example of brass as a composite of zinc and copper, both vital for making brass and obtained from multiple producers (see Fig. 2a). Cournot demonstrated that equilibrium input prices are higher under such disaggregated co-production (vs. a single producer). In this extreme case, producers are not really engaged in competition, but rather are co-producers of the composite good, although their profits do have interdependence (price increase by one firm weakens demand and profits for the other). A generalization of this structure appears in "systems competition" (Fig. 2b) where components have a complementary relationship but there are multiple brands of each component good (i.e., they compete directly), for instance ATM cards that require, and interoperate on, ATM machines. Economides and Salop (1991) examined price and quantity under alternative market structures (e.g, vertical integration) with two component types and two producers of each. In this structure, the component goods are offered directly on the market, in contrast to our setting where the composite good or bundle is the only one that is offered and priced.

The market setting of this paper has both competition (multiple brands of components) and cooperation (composite good effect): component providers are co-producers (e.g., in a TV bundle, crime thrillers and live news act as composites in a multi-genre bundle), but some subsets of components are also imperfect and competing substitutes (e.g., crime thrillers from multiple producers). Unlike quantity competition, where market price with combined outputs is lower than with outputs of one firm (i.e., $p(Q_i + Q_j) < \{p(Q_i), p(Q_j)\}$), composite goods markets have the property that market price increases as multi-producer outputs are combined (i.e., $p(Q_i + Q_j) > \{p(Q_i), p(Q_j)\}$, although $p(Q_i + Q_j) - p(Q_i) < p(Q_j)$). Bhargava (2012) examined pricing equilibria in this setting and showed that Cournot's over-pricing result holds even when the composite relationship is weak, i.e., not all components are necessary (although demand does increase with more components). This paper aims to go beyond price formation to consider issues of provision (production decision), and the consequences and drivers of alternative market structures.

## 2.2 Platforms and Value Co-Creation

Technology-enabled platform marketplaces facilitate multiple groups of trading partners (say, shoppers and merchants) to congregate, discover, and transact with each other (Choudary et al., 2016). Platforms have had tremendous impact in industries ranging from transportation (e.g., *Uber*) to advertising technology (e.g., *Google*), media (e.g., *Twitter*), healthcare (e.g., *Kyruus*), social networking (e.g., *Facebook*), retail (e.g., *Amazon*), and banking (e.g., *CreditKarma*). Platform firms like *Apple*, *Google*, *Facebook* and *Microsoft* now surpass traditional giants like *General Motors*, the *Coca-Cola Company* and *General Electric*, not only in terms of brand value, but also in terms of shareholder value (**Bhargava-Rubel-2019**).

Platforms focus on *enabling* value creation and exchange, rather than value production itself (e.g., *Facebook* users enjoying connecting with their friends; *OpenTable* diners get value when they book affiliated restaurants, and restaurants derive value from outreach to potential diners). Value co-creation has recently been discussed in the context of platforms (see e.g., Choudary et al. (2016)). These markets often feature co-opetition when firms that make the platform also produce complements, and must decide whether to allow complements from competing platforms (or whether to offer their complements on competing platforms). Adner et al. (2016) build a model for such "frenemies" motivated by the case of Apple's iPad and Amazon's Kindle. Foerderer et al. (2018) examine the effect on complement-provision and innovation when platform owners also make complements. Ceccagnoli et al. (2012) examine the effect on small producers' performance when they participate in a platform's value co-creation ecosystem. While these papers examine important issues in platforms and value co-creation, their goals and results are distinct from those of this paper. More generally, although there is a substantial and growing literature on platforms, existing papers have primarily considered micro-level decisions (e.g., pricing, product line expansion, salesforce compensation). In contrast the present paper aims to model the entire platform economy, explaining the platform's pricing as well as producers' output decisions, and exploring the effects of alternate market structures.

## 2.3 Bundling

Product bundling, one of the simplest and widely practiced business strategies, improves seller profits with little extra effort especially when component goods have low marginal costs. There is a vast literature on bundling, across marketing, economics and information systems. The earliest papers noted that bundling increases profits by reducing dispersion in product valuations across consumers (Stigler, 1963; Adams and Yellen, 1976; Schmalensee, 1984; McAfee et al., 1989). Other advantages of bundling include supply-side economies of scope (Evans and Salinger, 2005; Suroweicki, 2010), lower consumer transaction costs or other demand-side conveniences and network effects (Lewbel, 1985; Prasad et al., 2010), and strategic leverage across products (Burstein, 1960; Carbajo et al., 1990; Eisenmann et al., 2011; Stremersch and Tellis, 2002). For a discussion of emerging issues and past literature, see Rao et al. (2018), Kobayashi (2005) and Venkatesh and Mahajan (2009).

While bundling is beneficial across many business settings, analysis of bundle choice and bundle pricing is challenging. The basic approach—deriving bundle demand from known demand for individual components—becomes complicated due to possible correlation between valuations of individual components, sub- or super-additivity of valuations, and asymmetry in demand profiles across components, forcing simplifying assumptions on each dimension. Even when the research goal is only to identify conditions where bundling is profitable (rather than solve for optimal bundle pricing), this challenge restricts the analysis to two-item bundles, e.g., Schmalensee (1984) considered two goods with non-zero correlation, additive valuations, and symmetric demand profiles; Armstrong (2013) develops a general model of two-component bundles with correlation and non-additive valuations.

To convey the challenge in modeling bundle demand, consider the case of two component goods $i{=}1,2$ for which consumer valuations are distributed uniformly in $[0, b_1]$ and $[0, b_2]$ respectively, with the distributions being independent, and where bundle valuation $v_{iB}$ for consumer $i$ is simply $v_{i1}{+}v_{i2}$. The distribution of the $v_{iB}$'s then is simply the convolution of the two uniform distributions, either triangular or trapezoidal depending on $b_1$ and $b_2$, leading to a corresponding demand curve (see Fig. 3). Next, however, consider the possibility that valuations are not additive. Sub-additivity can be captured by writing bundle valuations as $v_{iB} = v_{i1}{+}\phi_i v_{i2}$ (if $v_{i1}{>}v_{i2}$, or $v_{iB} = v_{i2}{+}\phi_i v_{i1}$ otherwise), with each $\phi_i{\in}[0, 1]$ (or $>1$ for super-additive valuations) being distributed heterogeneously across consumers. This alone makes it difficult to write expressions for the distribution of valuations or the demand curve, and the problem goes out of bounds when considering correlation and $k{>}2$ products.

Several authors have employed numerical simulations to create a better understanding of bundle demand or bundling strategy (see e.g., Olderog and Skiera (2000)). While expressions for bundle demand are not available except for the simplest two-item settings, the literature does establish a few basic properties of bundle demand: that across-consumer valuations for the bundle exhibit less

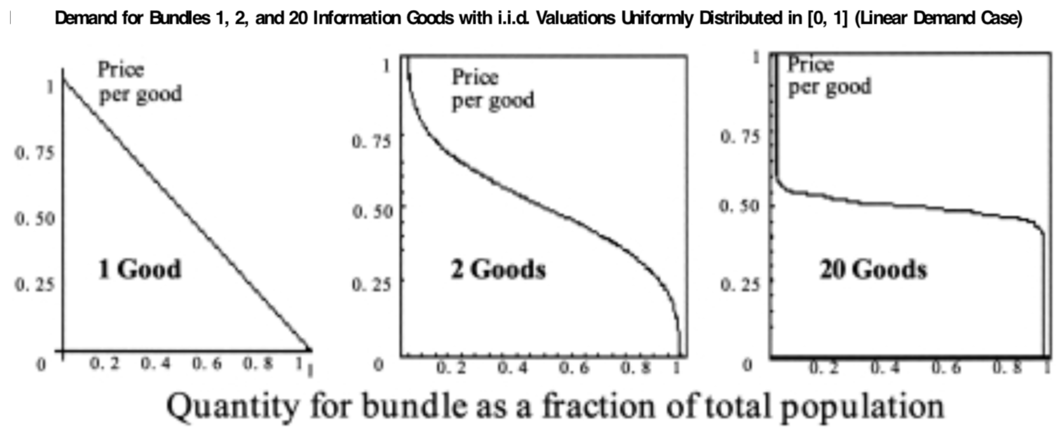



Figure 3: Bundle demand gets more "flatter in the middle" as bundle size increases. Reproduced from Bakos and Brynjolfsson (2000, Fig. 1).

variation (relative to mean) than for individual components; that demand is "flatter in the middle"; and that these properties are, *ceteris paribus*, amplified as bundle size increases. This behavior is vividly described in Bakos and Brynjolfsson (2000, Fig. 1), reproduced as Fig. 3.

The complexity of modeling bundle demand and optimizing bundle design has led to several innovative proposals. Y. Chen and Riordan (2013) use copula functions (which capture density of the joint distribution, hence allowing arbitrary levels of correlation) to provide a rigorous treatment of correlation in two-good bundles with additive demand. Bakos and Brynjolfsson (1999) and Ibragimov and Walden (2010) leverage asymptotic properties to analyze bundles of enormous (or infinite) size comprising individual items with identical demand profiles. Hitt and P. Chen (2005) show that customized bundling, where the firm specifies a price for $n$ items and customers pick the items, is computationally tractable and can capture most of the gains from bundling. Bhargava (2013) develops a closed-form approximate solution for two-item bundles by deriving and demonstrating closeness between lower- and upper-bound functions of bundle price.

The literature discussed above considers only bundles where the component goods are sourced from a single firm, which is also the firm that forms the bundle (e.g., a *MS Office* bundle). Bhargava (2012), and subsequent papers, have examined the multi-producer bundle setting described in this paper, but limited to two-item bundles. Moreover, like the remaining literature, these papers have not examined the provision of bundle components, the effect of market structure on provision, producer participation in the bundle, or the inter-dependencies between producers. Doing so requires a closed-form expression of bundle demand and a richer consideration of bundle settings. The next section examines this task.

# 3 Modeling Value Co-Creation and Bundling

This section develops an aggregate model of demand, supply, and revenue-sharing, for a co-produced bundle created by a retailer using outputs from multiple producers. To lend some clarity to model design considerations, the discussion employs a concrete setting which inspired this paper, that of "TV bundles" that are offered to consumers by communications firms (cable operators, telecom, satellite service providers) using content sourced from multiple studios and programming networks. The industry structure in the initial model setup represents the "cable era" of in-home entertainment where most market regions had a single dominant provider. A historical review of the entertainment industry over the last 150 years suggests that in-home entertainment has evolved across multiple eras, each characterized by new innovative technologies which influence the equilibrium market structure and its dynamics across the eras. Later sections of this article explore effects and causes of changes in market structure.



## 3.1 Modeling Bundle Demand

Consumer demand for the bundle reflects a desire to have quantity, variety (many genres and appealing across many moods, age groups, tastes etc., e.g., for a TV bundle buyers want a mix of movies, TV shows, political thrillers, children-oriented content, comedy, and so on), and quality (creative aspects, star talent, production quality etc.). The first element in the model is to define producer output in terms of canonical "value units" which represent a combination of quantity, variety, and quality. The bundle, measured by its magnitude of $Q$ value units, is offered to consumers under an unlimited-use price. Overall market demand can then be represented as $D(P, Q)$,

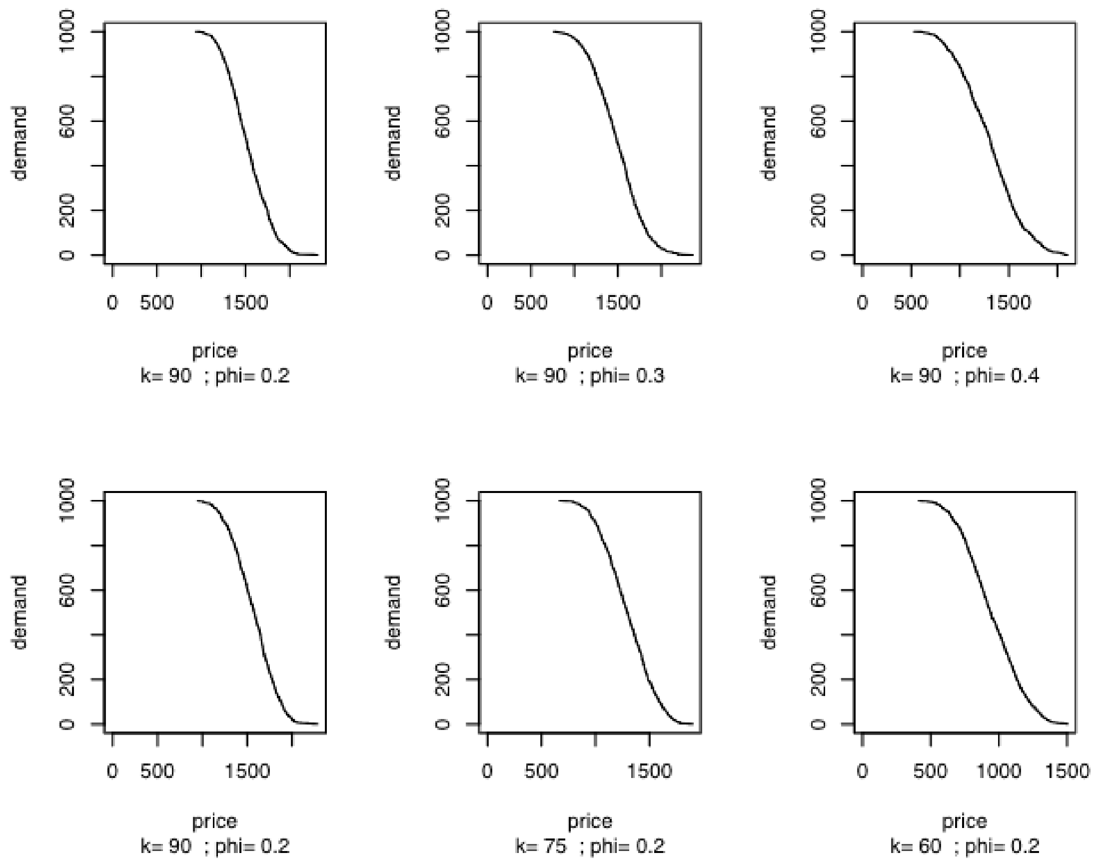



Figure 4: Simulations of bundle demand with $k$ products, with varying levels of sub-additivity in valuations ($\phi$) and asymmetry in demand profiles.

a function of bundle price and magnitude, which encapsulates a soupbox of heterogeneity in consumer valuations, and dependencies and correlations in valuations for pairs of items. The essence of this idea is that rather than individual-level consumers preferences, or specific choices of producers regarding the specific products they supply, the model will instead consider the aggregate characteristics that arise with a large number of individuals (consumers or producers) who are heterogeneous on multiple dimensions. The focus will be to ensure that this concept of "value units" is used in a consistent way across aggregate demand, marginal demand, total supply, and marginal supply.

First consider a few important characteristics of demand for a "value units" bundle. Write $\hat{D}(Q) = D(P{=}0, Q)$ to represent saturation demand for bundle of $Q$ value units. We shall consider how $D$ varies with $P$, with $Q$ generally and specifically at $P{=}0$, and jointly with $P$ and $Q$.

As discussed in §2.3 (Fig. 3), bundle demand is “flatter in the middle” of the price region. While Bakos and Brynjolfsson (2000) demonstrated this using goods with independent and additive valuations and identical demand profiles, essentially the same behavior is observed when any or all of these conditions are relaxed (see Fig. 4, and Supplement for details). Note that Fig. 4 shows demand as a function of price in order to visually facilitate this relationship (rather than the usual *inverse* demand curve). The key characteristic here is that not only is demand decreasing in price (which is standard for any downward-sloping demand function), but also that the rate of decline increases rapidly as price increases, up to a point where price is quite high (and demand very low). Beyond this point demand reduces more gradually, however this shape is not relevant because bundling leads to a high-demand rather than low-demand high price equilibrium. Hence we can specify the following requirements on demand behavior against price:

**Assumption 1 (Demand response to price)** *Bundle demand declines more rapidly as price increases.*

$$\frac{\partial D(P,Q)}{\partial P} < 0, \frac{\partial^2 D(P,Q)}{\partial P^2} < 0. \tag{2}$$

Next, consider how $D(P,Q)$ varies with $Q$ and particularly how $D(0,Q)$ varies with $Q$. As discussed in §2.1, unlike the case of traditional competition, the saturation level of bundle demand (and also the bundle demand at any given price) is “elastic” in total quantity across all producers. That is, $D(P,Q)$ increases as $Q$ increases, but it increases slowly when $Q$ is already high (because of consumers’ finite time and other bio-costs of consuming content). Moreover, the higher-demand effect of $Q$ should be greater at higher levels of price than at lower levels where more of the market is already captured. These requirements are encompassed in our next assumption.

**Assumption 2** *Bundle demand increases as $Q$ increases, but at slower rates for higher $Q$, and*

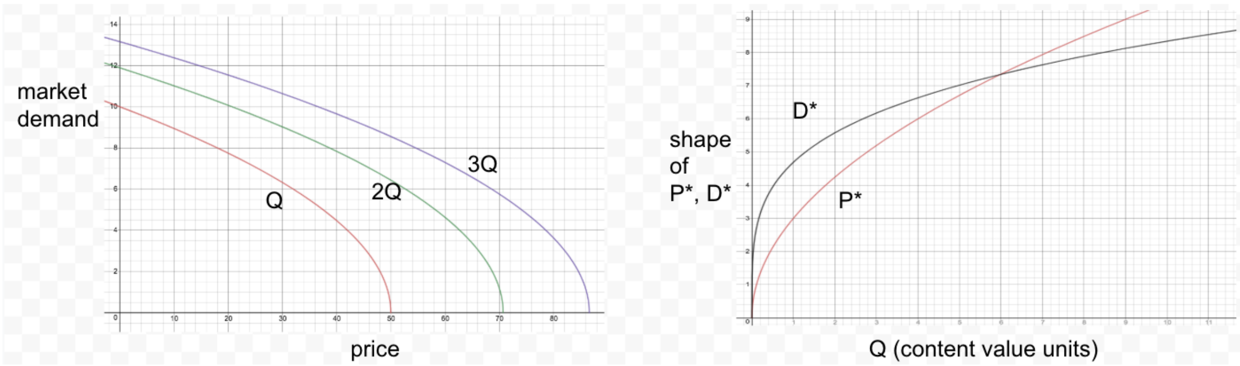



Figure 5: Ideal shape of demand curve for content bundle, and optimal price and demand level as a function of content value units.

*more rapidly for higher prices.*

$$\frac{\partial D}{\partial Q}>0, \frac{\partial^2 D}{\partial Q^2}<0, \frac{\partial \hat{D}(Q)}{\partial Q}>0, \frac{\partial^2 \hat{D}(Q)}{\partial Q^2}<0, \frac{\partial^2 D}{\partial P \partial Q}>0. \tag{3}$$

The requirements laid out above are visualized with demand functions in Fig. 5 (left panel). Finally, the literature on bundling suggests additional requirements on equilibrium outcomes (right panel of Fig. 5): (i) the equilibrium demand level should increase with $Q$ even though (ii) the equilibrium market price also increases in $Q$, and (iii) the *price per value unit* should decrease in $Q$. This is because repeated increases in $Q$ are subject to diminishing gains in consumer value from variety and quality, hence provide successively lower monetization power. We combine these requirements into the following assumption.

**Assumption 3 (Equilibrium Requirements)** *Market demand $D = D(P, Q)$ should exhibit the following equilibrium-price behavior.*

$$\frac{\partial D(P^*, Q)}{\partial Q}>0, \frac{\partial (P^*/Q)}{\partial Q}<0. \tag{4}$$

Table 1 lists properties of various demand models that were examined, covering linear and exponential demand and multiple ways to incorporate $Q$ into the demand function. This evaluation suggests the most suitable demand model from the various alternatives as $D(P, Q) = \sqrt{AQ^\theta - b\,P}$, (with $\theta \in [0, 2/3]$), with $AQ^\theta$ representing the market saturation level (demand at price 0) for a content bundle with $Q$ value units. Also (as we show next) a useful retailing cost

| $D(P,Q)$ | unit cost | $P^*$ | $D(P^*,Q)$ | $\frac{\partial^2 D}{\partial P^2}$ >0? | $\frac{\partial(\frac{P^*}{Q})}{\partial Q}<0$ at c=0 | $\frac{\partial D(P^*,Q)}{\partial Q}$ >0? | $\frac{\partial^2 D}{\partial P \partial Q}$ >0? |
|---|---|---|---|---|---|---|---|
| $A - b\,\frac{\mathbf{1}}{\mathbf{Q}}\;P$ | $c \ln Q$ | $\frac{AQ}{2b} + \frac{c \ln Q}{2}$ | $\frac{A}{2} - \frac{bc}{2}\frac{\ln Q}{Q}$ | **No** | **No** | Yes | Yes |
| $A - b\,\frac{\mathbf{1}}{\ln \mathbf{Q}}\;P$ | | $\frac{A \ln Q}{2b} + \frac{c \ln Q}{2}$ | $\frac{A}{2} - \frac{bc}{2}$ | **No** | Yes | **No** | Yes |
| $A\,\mathbf{Q}^\theta - b\,P$ | $c \ln Q$ | $\frac{AQ^\theta}{2b} + \frac{c \ln Q}{2}$ | $AQ^\theta - \frac{bc}{2} \ln Q$ | **No** | Yes | **?** | **?** |
| $A \ln \mathbf{Q} - b\,P$ | | $\frac{A \ln Q}{2b} + \frac{c \ln Q}{2}$ | $\left(A - \frac{bc}{2}\right) \ln Q$ | **No** | Yes | Yes | **?** |
| $A \ln \mathbf{Q} - b\,P^2$ | | $\frac{A \ln Q}{2b} + \frac{c \ln Q}{2}$ | $\left(A - \frac{bc}{2}\right) \ln Q$ | Yes | Yes | Yes | **?** |
| $\sqrt{A \ln \mathbf{Q} - b\,P}$ | $c \ln Q$ | $\frac{2A+bc}{3b}(\ln Q)$ | $\sqrt{\frac{A-bc}{3}(\ln Q)}$ | Yes | Yes | Yes | Yes |
| $\sqrt{A\mathbf{Q}^\theta - b\,P}$ | $cQ^\theta$ | $\frac{2A+bc}{3b}Q^\theta$ | $\sqrt{\frac{A-bc}{3}Q^\theta}$ | Yes | Yes | Yes | Yes |
| $A\,\mathbf{Q}^\theta e^{\frac{-P}{b}}$ | $c \ln Q$ | $b + c \ln Q$ | $AQ^\theta e^{-\frac{b + c \ln Q}{b}}$ | **No** | **No** | Yes | **No** |
| $A \ln \mathbf{Q} e^{\frac{-P}{b}}$ | | $b + c \ln Q$ | $A \ln Q e^{-\frac{b + c \ln Q}{b}}$ | **No** | **No** | Yes | **No** |
| $Ae^{\frac{-P}{b\,\mathbf{Q}^\theta}}$ | $c \ln Q$ | $bQ^\theta + c \ln Q$ | $Ae^{-\frac{bQ^\theta + c \ln Q}{b}}$ | **No** | Yes | Yes | **No** |

Table 1: Alternate ways to model demand for content at bundle price $P$. The exponent $\theta$ above is assumed to lie in $(0, 1)$ and further restricted to $(0, 2/3)$ in some cases.

function is $c(Q) = cQ^\theta$. While $Q$ itself is abstract (i.e., value units), it can be measured as its effect on aggregate market demand, i.e., $Q{=}(D^{-1}(0))^2/A$.

The equation can be generalized to $D(P,Q) = (AQ^\theta - b\;P)^\alpha$ (where $\alpha \in [0,1]$). For convenience in displaying the computations, we employ $\alpha{=}\frac{1}{2}$, i.e., $D(P,Q) = \sqrt{AQ^\theta - b\;P}$. However, our results apply more generally to $\alpha \in [0,1]$.



## 3.2 Modeling Production and Distribution

There are multiple producers ($i = 1...\mathcal{I}$) each of whom makes content which has some value to consumers. Let $Q_i{\geq}0$ indicate total content from producer $i$, at cost $c_i(Q_i)$. Because $Q$ already has a diminishing returns effect on demand, we employ a linear cost function $c_i(Q_i) = c_i Q_i$ (with $c_i{\geq}0$) which should yield an interior solution for content production (i.e., even high-cost or low-value or niche producers have positive production) rather than a corner solution in which the most advantaged producers secure the entire market.

Lacking direct reach into the consumer market, producers must rely on a specialist firm, a retailer, to sell to consumers. As a special case, the "retailer" could be a consortium of producers (e.g., Hulu, see §5.1 and Proposition 3), but for the main model we consider the retailer as an independent firm. Further, market conditions are such that the retailer's business model is to combine content from all producers into a single bundle (e.g., economies of scope in distribution, and consumer preferences for size, variety, flexibility), rather than sell each producer's goods separately (e.g., as in a grocery store).[2] The retailer's per-consumer cost of offering this content is $cQ^{\theta}$, implying both that the retailer enjoys economies of scale and also that as $Q$ increases, cost increases more rapidly than demand. Later we extend the model to account for other costs of setting up distribution deals with the retailer and executing these details across the market.

## 3.3 Revenue-Sharing

Market power is distributed between producers and the retailer. For example, for in-home entertainment producers have power because ultimately consumer demand is for content (as expressed in the oft-stated maxim "content is king") and the retailer has expertise and technology for delivering content (e.g., content delivery firms such as cable or satellite, who hold the conduit to deliver content into homes). The revenue-share between producers and the retailer will vary, e.g., based on the level of concentration within each layer. We start by assuming that the retailer is a monopolist. Let $(1-\gamma)$ denote the retailer's market power. With this, the total revenue available to producers is $\gamma$ fraction of net surplus created by the retailer ($S=(P-cQ^{\theta})D(P,Q)$). Producer split this revenue proportional to the value-units they provide, i.e., supplier $i$ receives $\pi_i = (\gamma \frac{Q_i}{Q} S)$. This framing is consistent with how $Q_i$'s affect bundle demand $D(P,Q)$.

[2] The model also works reasonably well under some variations. For instance what if, besides the oligopoly aspect of a few large and powerful content provider, there are hundreds of tiny or niche content elements? This wouldn't affect the main model because the tiny niche content doesn't alter the retailer's demand curve in a substantial way, and moreover the providers rely primarily on advertising rather than revenue-share from the retailer. Alternately, consider if there's extremely high-value or premium content: this can also be separated from the mail model, because such high-value content (e.g., HBO) is usually placed into an "add-on" or partial bundling framework outside the main content bundle (see, e.g., Bhargava (2013)).

# 4 Equilibrium Analysis

Producers are heterogeneous in production technology or efficiency. Producer $i$'s cost for making $Q_i$ value units is $c_iQ_i$, with producers indexed in ascending order of the cost parameter. Producers decide whether to produce and how much ($Q_i$). The retailer collects and sells total output $Q{=}\sum_i Q_i$ as a bundle, at price $P$, facing a market demand function $D(P,Q) = \sqrt{AQ^\theta - bP}$. The retailer incurs additional cost $cQ^\theta$ in selling the bundle to consumers, and pays producer $i$ a fee $F_i$. [3] As explained earlier, $F_i{=}\gamma\frac{Q_i}{Q}S(P,Q)$, where $\gamma$ measures market power of producers, with the retailer keeping fraction $1(-\gamma)$ of the surplus.

The sequence of the game is that producers choose the $Q_i$'s (which together determine $Q$), transfer prices $F_i$ are determined, and then the retailer sets market price $P$. We solve the problem in backward sequence, first identifying optimal $P$ which maximizes the retailer's profit given $Q$, then determining $Q_i$'s while satisfying the aggregation constraint ($Q{=}\sum_i Q_i$) and producer's participation constraints ($\pi_i(P^*(Q), Q_i, Q_{-i}){\geq}0$, where $Q_{-i}$ is the vector of all $Q_i$'s except $Q_i$.

The worth of this modeling framework is in the results it produces: ease of generating them, what they cover, how meaningful they are, and their credibility. Lemma 1 starts by describing the industry equilibrium solution, which we develop and explain in the rest of this section.

**Lemma 1 (Equilibrium Solution)** *With producers' costs $c_i$ per value unit arranged in ascending order, the equilibrium numbers of producers $i = 1...K$ who make content, their magnitude of value units produced, and the market price set by the retailer are*

$$K = \max\{i:\ c_i \leq \frac{c_1 + ... + c_i}{i - (1 - 3\theta/2)}\} \tag{5}$$

$$\forall\, i = 1...K: \quad Q_i = \left(2 - \frac{c_i}{\bar{c}}\frac{(2-3\theta)}{K}\right)\frac{Q}{(2-3\theta)} \tag{6}$$

$$\textit{with} \quad Q = \sum_{i=1}^{K} Q_i = \left[\frac{\gamma}{\bar{c}}\left(2 - \frac{(2-3\theta)}{K}\right)\right]^{2/(2-3\theta)}\left(\frac{A{-}bc}{3}\right)^{3/(2-3\theta)} \tag{7}$$

$$P^* = \frac{2A{+}bc}{3b}Q^\theta. \tag{8}$$

[3] With the equilibrium market coverage of the retailer ($D^*$) one could interpret $F_i$ as a per-subscriber fee of $\frac{F_i}{D^*}$, however the treatment as a fixed fee helps to avoid the inefficiency caused by double marginalization (Bhargava, 2012).

*When Eq. 5 yields $K{=}1$ (i.e., $c_2 > \frac{2c_1}{3\theta}$, only the lowest-cost producer survives), then*

$$\textit{when K=1}, \quad Q_1 = Q = \left(\frac{3\gamma\theta}{bc_1}\right)^{\frac{2}{2-3\theta}} \left(\frac{A-bc}{3}\right)^{\frac{3}{2-3\theta}}. \tag{9}$$

## 4.1 Pricing

Given the total available content $Q$, the retailer sets the optimal price to maximize profit, which is its content revenues less the fixed sunk cost of sourcing content from producers,

$$P^* = \arg\max_P \ \Pi_R(P,Q) :$$

$$\Pi_R(P,Q) = (1-\gamma)(P-cQ^\theta)D(P,Q) = (1-\gamma)(P-cQ^\theta)\sqrt{AQ^\theta - bP} \tag{10a}$$

$$\text{which yields} \qquad P^* = \frac{2A+bc}{3b}Q^\theta \tag{10b}$$

$$D^* = \sqrt{\frac{A-bc}{3}Q^\theta} \tag{10c}$$

$$\text{and} \qquad \Pi_R^* = \frac{2}{b}(1-\gamma)\left(\frac{A-bc}{3}Q^\theta\right)^{3/2} \tag{10d}$$

$$\text{with} \qquad S^*(Q) = \frac{2}{b}\left(\frac{A-bc}{3}Q^\theta\right)^{3/2}. \tag{10e}$$

The final term $S^*(Q)$ is the overall industry surplus when a content bundle of magnitude $Q$ is offered to consumers at $P^*$. Notice that the surplus and profit terms increase more than linearly when demand $D(P,Q)$ does not diminish too rapidly in $Q$ ($\theta \in [\frac{1}{3}, \frac{2}{3}]$), while the increase is less than linear when demand diminishes more rapidly ($\theta \in [0, \frac{1}{3}]$). The equilibrium level of demand i.e., market coverage increases with $Q$ but at a diminishing rate. It is trivial to confirm that the model meets all the requirements specified above in the Assumptions. The analysis would be the same if the retailer's profit function were set up (instead of Eq. 10a) as a constant fraction of net revenues (with producers getting the rest).

## 4.2 Content Provision

Each producer's content production level is chosen to maximize the profit function,

$$Q_i^* = \underset{Q_i \geq 0}{\arg\max}\ \pi_i : \quad \pi_i(P, Q_i, Q_{-i}) = \left[\frac{2}{b}\left(Q^\theta \frac{A-bc}{3}\right)^{3/2}\right]\frac{Q_i}{Q}\gamma - c_i Q_i \tag{11a}$$

$$\text{set } \frac{\partial \pi_i}{\partial Q_i} = 0 : \qquad c_i = \frac{2\gamma}{bQ^2}\left(Q^\theta \frac{A-bc}{3}\right)^{3/2}\left(Q - \frac{(2-3\theta)Q_i}{2}\right) \tag{11b}$$

$$\Leftrightarrow \qquad Q_i = \left[2 - \frac{bc_i Q}{\gamma}\left(\frac{1}{Q^\theta}\frac{3}{A-bc}\right)^{3/2}\right]\frac{Q}{(2-3\theta)} \tag{11c}$$

$$\text{share of production} \qquad \frac{Q_i}{Q} = \left[2 - \frac{bc_i Q}{\gamma}\left(\frac{3}{AQ^\theta - bc}\right)^{3/2}\right]\frac{1}{(2-3\theta)} \tag{11d}$$

$$\text{IR constraint} : \forall\, i \qquad Q_i \geq 0 \ \Leftrightarrow c_i \leq \frac{2\gamma}{bQ^{\frac{2-3\theta}{2}}}\left(\frac{A-bc}{3}\right)^{3/2} \tag{11e}$$

$$\Leftrightarrow Q \leq \left(\frac{2\gamma}{bc_{\hat{K}}}\right)^{\frac{2}{2-3\theta}}\left(\frac{A-bc}{3}\right)^{\frac{3}{2-3\theta}} \tag{11f}$$

where $\hat{K}$ is the highest $i$ for which the RHS of Eq. 11e holds.

Eq. 11c provides the condition for optimal content supply for producers $i$=1..$\hat{K}$, however it is an implicit condition stated in terms of $Q=\sum_i Q_i$. Solving this system jointly (which we do below) requires also ensuring each producers' individual rationality (IR, or participation) constraint which ensures positive profit in equilibrium (Eq. 11e). With $c_i Q_i$ being the only cost considered so far (Eq.11a), this constraint corresponds to $Q_i \geq 0$ in Eq. 11c. However, a full consideration of this constraint needs a fuller characterization of the industry equilibrium for $Q$ (total content across all producers who satisfy the IR constraint), specified later in Eq. 12c. For now, given that $c_{\hat{K}}$ is the highest cost, we can reduce the series of equations 11e to one Eq. 11f, which is needed to ensure that the optimization does not settle into infeasible maximum points (i.e., negative $Q_i$'s are produced for high $c_i$'s). Trivially, content producers who can produce value units at lower unit cost will supply a greater amount of content to the retailer (i.e., $\frac{\partial Q_i}{\partial c_i} < 0$).

Next, to figure out the optimal levels of content produced, repeat the final equation for each $i$, then add up all equations (using $Q = \sum_i Q_i$), and let $\bar{c} = \sum_i c_i$ be the average cost parameter for content production. For convenience let this summation be performed over the equilibrium set of participating producers (i.e., $1...\hat{K}$). This yields

$$Q = \frac{\hat{K}Q}{(2-3\theta)}\left[2 - \frac{b\bar{c}}{\gamma}Q^{\frac{2-3\theta}{2}}\left(\frac{3}{A{-}bc}\right)^{3/2}\right] \tag{12a}$$

$$\Rightarrow \quad Q^{\frac{2-3\theta}{2}} = \frac{\gamma}{b\bar{c}}\left(2 - \frac{(2-3\theta)}{\hat{K}}\right)\left(\frac{A{-}bc}{3}\right)^{3/2} \tag{12b}$$

$$Q = \left[\frac{\gamma}{b\bar{c}}\left(2 - \frac{(2-3\theta)}{\hat{K}}\right)\right]^{2/(2-3\theta)}\left(\frac{A{-}bc}{3}\right)^{3/(2-3\theta)} \tag{12c}$$

Plugging Eq. 12b into Eq. 11c yields the optimal $Q_i$ in the form given in Lemma 1 Eq. 6. Other outcome variables (demand, profit, surplus) can similarly be obtained by substitution. Finally, consistency of this solution requires that $K$ be such that IR constraints are satisfied for all participating producers (the rest have $Q_i{=}0$). Combining Eq. 12c with Eq. 11f, for each $i$, yields that participation is limited to producers with the following cost parameters.

$$\text{feasible cost vector} : (c_1, ..., c_{\hat{K}}) \text{ such that} \quad c_{\hat{K}} \leq \frac{\bar{c}\cdot\hat{K}}{\hat{K}-(1-3\theta/2)}. \tag{13}$$

Among all the feasible vectors, the optimal one trivially is the highest $\hat{K}$ for which the feasibility condition is satisfied, denoted as $K$ in Lemma 1. Procedurally, $K$ can be identified by starting with all $\mathcal{I}$ producers, then checking the condition successively; if it fails then the highest $c_i$ is removed from the vector, successively, until the condition is satisfied. The first vector that achieves the condition is a feasible set of producers and each will then have non-negative $Q_i$. Once this is done, Eq. 12c describes the overall content bundle, given the various parameters of the problem, and combining this with Eq. 11c produces $Q_i$ the content supplied by each producer. Given that producers with index higher than $K$ have no production, henceforth $K$ will represent the number of producers in the market.

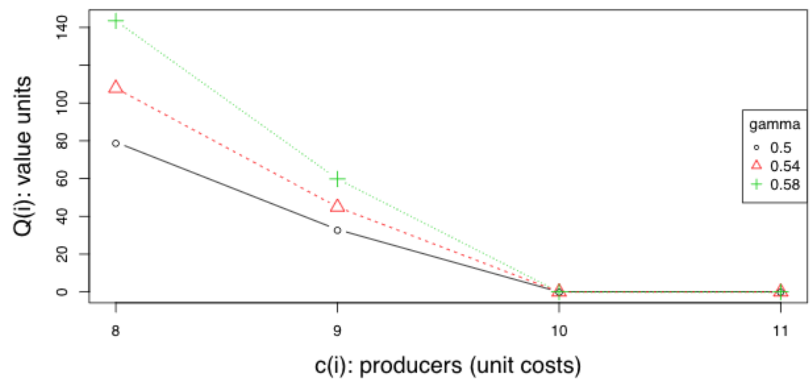



Figure 6: Impact of cost structure and revenue-sharing on number of producers and production levels. Other parameters are $\theta$=0.5, and $A, b, c$ such that $\frac{A-bc}{3}$=10.

### 4.3 Equilibrium Properties

Plugging Eq. 12b into Eq. 11d produces a simplified value for the fraction of content made by each producer, specifically $\frac{Q_i}{Q} = \left(2 - \frac{c_i}{\bar{c}}\right)\left(\frac{2}{2-3\theta} - \frac{1}{K}\right)$. Equilibrium values of $Q_i, P^*, D^*$ and the profits of retailer and producers can be derived similarly. Fig. 6 illustrates simulation results that demonstrate intuitive properties of the equilibrium solution: producers make more content as their share of revenue $\gamma$ increases (each producer's share $\frac{Q_i}{Q}$ is unchanged), producers with better production technology (i.e., lower cost per value unit) have a competitive advantage, and not all producers with high enough cost structure cannot sustain positive production.

The linear cost structure for producers ($c_i Q_i$) raises the possibility of a bang-bang equilibrium solution in which only the most efficient or lowest-cost producer has positive production level. This is because producers "draw from the same well" for revenue, and marginal revenue is linked to total bundle size rather than how much each producer has made, whereas the lowest-cost producer always has the better cost for every unit. The first result below is therefore reassuring, both for the intuitive first part (that lower-cost producers make more content) but mainly for the second part that the equilibrium does *not* feature a bang-bang solution.

**Proposition 1 (Production levels vs. costs)** *Lower-cost (i.e., more efficient) producers make more content, however in general, higher-cost producers remain in the market with positive content levels. Formally,* $(c_i{<}c_j)$ *implies* $Q_i{>}Q_j$.

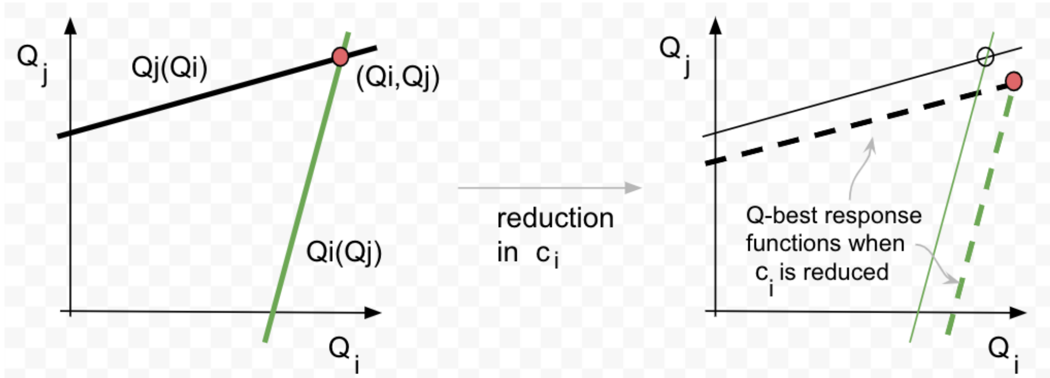



Figure 7: Content provision strategy, and effect of change in one producer's technology.

Why does it work out that higher-cost producers can survive despite the production function being linear in cost? The intuition for this illustrates the distinctive aspect of competition in cross-producer bundling, which is that the marginal revenue of each producer $i$ is enhanced by the production of others. In traditional Cournot competition, higher production by one firm hurts its competitor because it lowers marginal revenue for all firms. In the case of cross-producer bundling, however, content produced by any producer improves market demand for the bundle (which is common to all producers), and this provides partial benefit to "competing" producers. This can formally be derived from the behavior of the $Q_j$ best-response against $Q_i$.

**Corollary 1 (Collaborative Producers)** *Increased production by producer $i$ motivates,* ceteris paribus*, higher production by producer $j$.*

Corollary 1 illustrates and confirms why "competitors" are in fact "collaborators" in this market form, and that their outputs are *strategic complements*, unlike traditional competition where production quantities are strategic subsititutes. Notably, this result holds with other parameters being the same, and because $Q_i$'s are themselves endogenous the result is more usefully interpreted as describing the mechanism leading to the equilibrium (see the depiction of the best-response functions in Fig. 7). Consider now why the premise of the Corollary, i.e., an increase in $Q_i$ might occur. The most intuitive and simple reason is that producer $i$ acquires some new technology or assets that reduce its production cost $c_i$. How does a change in $c_i$ affects participation and production decisions of other producers? The previous result would indicate a positive effect, but a formal consideration reveals the opposite and sheds further light on the nature of competition between producers.

**Corollary 2** *Reduction (improvement) in one producer's cost can move some producers out of profitability and forces others to reduce production. Formally,* $\frac{\partial Q_j}{\partial c_i}{>}0$.

Suppose producer $i$ is able to reduce $c_i$, then intuitively it wants to produce higher $Q_i$ for any given choices of other producers. Because $i$ produces more, other producers $j$ observe a lower marginal revenue at the existing level $Q_j$ (because of diminishing marginal gains from higher $Q$) and must ramp production down to the point where marginal revenue equals marginal cost (this can be confirmed by computing $\frac{\partial Q_j}{\partial c_i}{>}0$). Hence, their best-response functions in response to the lower $c_i$ shift towards lower $Q_j$ (see the right panel in Fig. 7). In equilibrium, with lower $c_i$, producer $i$ makes more and other producers make less. This result holds when $K$ remains the same, i.e., no $Q_j$ crosses the boundary $Q_j{\geq}0$ (at the boundary, if some producers are driven out of the market, then it is possible for other producers to have higher $Q_j$ than before.) The result further illuminates the intuitive understanding that while producers are engaged in collaborative production in this setting, they do in fact compete for a shared resource (consumer spending).

# 5 Market Dynamics and Other Extensions

The previous two sections have proposed a reduced-form model for describing markets with multi-producer bundled goods and developed an equilibrium solution that describes the essential market outcomes in this setting. Collectively, the results presented thus far confirm the distinctive aspects of this market structure and also validate the reduced-form demand structure employed in setting up the model. Next we demonstrate that the model can be deployed to address a series of additional questions, further corroborating the value of the model and also producing useful insights about how such markets evolve.

## 5.1 Market Structure Variations

The main model considered a setting with multiple producers and a separate single retailer (shown in the left panel of Fig. 1). Setting $K{=}1$ in the model generates the case of a bilateral monopoly comprising a single producer and single retailer, setting just $\gamma{=}1$ (with $K{>}1$) represents a production consortium where the consortium makes pricing decisions (rather than a separate retailer who shares revenues), and setting both $K{=}1$ and $\gamma{=}1$ corresponds to a vertically integrated monopoly. The parameter settings for each case yield the total industry content that would be produced in those settings.

First, consider how the number of producers affects total content (Eq. 11c). Intuitively, given that market demand $D(P, Q)$ is responsive to $Q$ rather than $K$, a single producer should produce more total content than if production and profits were shared among multiple producers. This outcome is also indicated by the nature of competition implied in Corollary 2. However, computing the expression $\frac{\partial Q}{\partial K}$ using Eq. 12c, we find that higher K leads to greater supply of content. In other words, a single producer (with cost equal to $\bar{c}$) would produce less content than the total made by multiple producers whose average cost parameter is $\bar{c}$.

**Proposition 2 (# Producers)** Ceteris paribus*, more producers leads to higher supply of content,* $\frac{\partial Q}{\partial K}{>}0$.

This unusual result is obtained due to the effects of bundling and revenue-sharing. Generally, a producer's content production level is one where the marginal cost of making more content (here, a constant $c_i$) equals the marginal revenue. However, under multi-producer bundling, producer $i$'s benefit from every dollar spent on production (cost $c_i$) gets amplified. Producer $i$ benefits from the higher market demand and price that arises due to the $Q_j$'s of other producers, but can enjoy these gains only proportional to its share of content. Together, Corollary 2, and Proposition 2 explain the interplay of collaboration-competition in this market structure: one, producers do compete because higher production by one crowds out others, but conversely each producers production also creates some gains for others.

**Proposition 3 (Consortium vs. a Retailer)** *Producers supply more content when selling content bundles as a consortium (e.g., Hulu) rather than through a separate retailer. Generally, producers make more content when they can get higher share of content subscription revenues, i.e.,* $\frac{\partial Q}{\partial \gamma} > 0$.

The $\gamma$=1 extreme hastwo branches, a market structure in which producers create a consortium for selling (with $K$>1) and a vertically integrated monopoly ($K$=1). Higher $\gamma$ motivates higher output levels because producers capture more of the gains they create from production. Marginal gain to producer $i$ equals marginal cost $c_i$ at a higher level of $Q_i$, and the output level is highest under vertical integration.

Third, consider how $K$ affects the profit-sharing level $\gamma$ between the retailer and content producers. With a high $K$, a retailer can afford losing a producer with whom it can't reach a profit sharing agreement, and this threat of being shut out forces producers to give a high share to the retailer. In contrast, a small $K$ (and the extreme, $K$=1) makes the producer(s) more consequential to the retailer's survival, and the retailer must surrender a higher level of content revenues to the producer(s). Hence, we see that content mergers and acquisitions (which cause lower $K$) have a doubly harmful effect on the retailer, who earns lower revenues on account of lower $Q$ and higher $\gamma$. This result is another peculiarity of the bundling structure inherent in selling in-home entertainment content. It contrasts a typical industry where producers compete for individual customers (through a retailer), and consolidation among producers generally leads to higher prices and higher margins for both producers and the retailer. We summarize the result below.

**Proposition 4 (Horizontal mergers among content producers)** *Content mergers and acquisitions, and other actions that reduce $K$, reduce the retailer's profits,* $\frac{\partial \Pi_R}{\partial K} > 0$.

## 5.2 Partial Forward Integration

Consider the nature of competition and the equilibrium when the retailer doubles as content producer. This has been a common thread in the entertainment industry, and indeed both broadcast networks and cable firms have traditionally had substantial content production or licensing. Let $(c_R, Q_R)$ represent the retailer's unit cost and number of content value units produced, with total production cost $c_R Q_R$. Let $Q$ still denote the total value units made by other producers, so that bundle size is now $Q{+}Q_R$. Then, trivially, Eq. 6 remains valid for the optimal bundle price, except replacing $Q$ with $Q{+}Q_R$. For the content production decision, let's consider the retailer's decision first and then that of other producers.

The retailer's content production incentives are, however, different from those of other producers because it can coordinate its content production and bundle-selling decisions. With total content level $\hat{Q}$ the retailer keeps only a $(1{-}\gamma)$ fraction of surplus due to content of other producers (i.e., it gets $(1{-}\gamma)\frac{Q}{\hat{Q}}S^*(\hat{Q})$) but keeps the remaining share arising from its own content $Q_R$ (i.e., $1 \cdot \frac{Q_R}{\hat{Q}}S^*(\hat{Q})$). Therefore, rather than Eq. 11a, $Q_R$ is chosen to maximize the retailer's total profit, variant of Eq. 10a.

$$Q_R^* = \underset{Q_R \geq 0}{\arg\max}\ \Pi_R(Q_R): \quad \Pi_R(Q_R) = \frac{2}{b}\left(\frac{A-bc}{3}\right)^{\frac{3}{2}} \hat{Q}^{\frac{3\theta}{2}}\left((1{-}\gamma)\frac{Q}{\hat{Q}} + \frac{Q_R}{\hat{Q}}\right) - c_R Q_R$$

$$= \frac{2}{b}\left(\frac{A-bc}{3}\right)^{\frac{3}{2}} \frac{1}{\hat{Q}^{\frac{2-3\theta}{2}}}\left((1{-}\gamma)Q{+}Q_R\right) - c_R Q_R \quad (14a)$$

$$\text{FOC}: \quad \frac{b}{2}c_R\left(\frac{3}{A-bc}\right)^{\frac{3}{2}} = \left(\frac{1}{\hat{Q}^{\frac{2-3\theta}{2}}} - \frac{2{-}3\theta}{2}\frac{(1{-}\gamma)Q{+}Q_R}{\hat{Q}^{\frac{4-3\theta}{2}}}\right) \quad (14b)$$

$$bc_R\left(\frac{3}{A-bc}\right)^{\frac{3}{2}} = \frac{1}{\hat{Q}^{\frac{4-3\theta}{2}}}\left(3\theta\hat{Q} + (2{-}3\theta)\gamma Q\right) \quad (14c)$$

The final equation above can be solved to produce optimal values of $Q_R$, however a few things are evident from comparative statics.

1. $\frac{\partial Q_R}{\partial c_R} < 0$. This is intuitive. Higher cost makes production less attractive, and the retailer would rather make money by bundling other producers' content.

2. $\frac{\partial Q_R}{\partial \gamma} > 0$. When producer-sourced content requires a bigger share of content revenues to producers, the retailer has a stronger motivation to make its own content. More on this in the next section.

3. $\frac{\partial Q_R}{\partial Q} > 0$. For any given $c_R$ and $\gamma$, if other producers make more, making more content

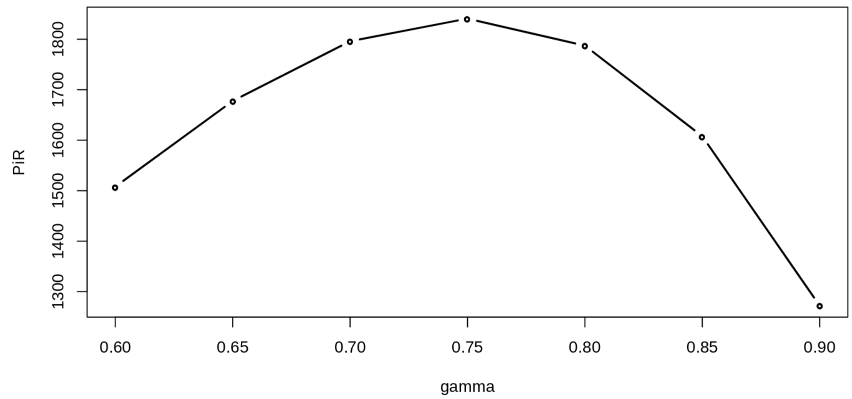



Figure 8: Effect of $\gamma$ (fraction of net revenue shared with producers) on the retailer's profit.

enables the retailer to reduce the fraction of content revenue that is subject to revenue sharing, giving it incentive to produce more content.

### 5.3 Revenue-Sharing between Producers and Retailer

So far, the revenue-sharing arrangement has been described with a constant exogenous parameter $\gamma$, as we explored pricing, provision and profits (and how these relate to various parameters in the environment). Now let's consider the effect that $\gamma$ has on the profits of the retailer and producers, and see how those consequences shed some light on the setting of $\gamma$. Naturally, producers benefit when $\gamma$ is higher: they not only make more content, but they also keep a higher share of revenues from it. For the retailer, therefore, $\gamma$ has two contrasting effects of motivating more (or less) output and keeping a lower (or higher) share of the revenues. If $\gamma$ is too low, the resulting low $Q$ can cause lower overall industry profits (including for the retailer), which should motivate the retailer to be more generous in profit-sharing with producers. Fig. 8 illustrates how the retailer's profit varies with $\gamma$. Although tensions will arise between producers and the retailer on the level of revenue-sharing, it is useful to note that it is not in the retailer's interest to always reduce the revenue-share of producers.

Conversely, consider how the vertical relationship affects producers' demand for a higher level of revenue-sharing. If the retailer is unable to make content or if producers have a huge competitive advantage in doing so, they can demand higher $\gamma$. But unless prevented by regulation, the retailer can always forward integrate and produce content, and a high vs. low value of $c_R$ represents, respectively, a weak or strong motivation to do so. Crucially, at any $c_R$, the retailer will choose to produce more content when $\gamma$ is higher. This is detrimental to producers because it not only makes some part of revenue not subject to revenue-sharing, but also because the higher $Q$ crowds out producers' own economic gain from making content. Moreover, when $\gamma$ is too high, the retailer will take actions (such as investing in technology to reduce $\gamma$, acquisitions, etc.) that threaten producers' business model and profits. This is the sort of industry dynamic that led to Netflix's move into content production around 2010.

## 5.4 Multiple Competing Retailers

Next consider the market outcomes when there are multiple (say, $J$) retailers who can reach all consumers and who compete based on some exogenous factor of differentiation (e.g., a Salop circle model of competition). For simplicity in notation and to focus on the effects of competition among retailers, suppose that retailers have identical capabilities consumer preferences are distributed identically across them, and therefore we consider only symmetric outcomes and can avoid indexing the retailers. In equilibrium there are $J$ sub-markets of equal size. Assume that producers multi-home, i.e., offer their content across all retailers, and hence total output level $Q$ is the same in all sub-markets. Let $D_J(P, Q)$ represent the bundle demand function for each retailer. To derive $D_J(P, Q)$, consider the identity $D_J(0, Q) = \frac{1}{J}D(0, Q)$, which yields

$$D_J(P,Q) = \sqrt{\frac{A}{J^2}Q^\theta - bP} \tag{15}$$

Equilibrium market price, given bundle level $Q$, will be lower than the consolidated case for two reasons. First is the inward shift in the bundle demand function in each sub-market, implied by Eq. 15. The second is the effect of competition between retailers which would cause them to offer lower price in equilibrium, for any given level of $Q$. Denote this equilibrium price as $P^J < P^*$.

This lower price function therefore implies that producers will make lower output in equilibrium, i.e., where producer $i$'s marginal gain from output equals marginal cost. Thus, $Q_i^J < Q_i$ and $Q^J < Q$, the latter casting a further reduction in equilibrium price $P^J$. Hence fragmentation has all-around negative consequences thus far. For producers, there could be one positive effect if competition amongst retailers causes them to surrender a higher revenue-share $\gamma^J > \gamma$. For retailers, however, all three factors (lower price, lower $Q$, and higher $\gamma$) have a negative effect on profits.

Hence we see that competition between retailers has more dire consequences for them when their business involves a multi-producer bundle. As noted earlier, this should create higher motivation for retailers to enter the production layer, i.e., increase their unit cost threshold $c_R$ for doing so. Thus, an increase in competition between retailers would force some upstream integration. That, in turn, is likely to motivate producers to look for alternative ways of reaching consumers. In the entertainment industry, this manifests itself as direct-streaming by content firms such as *HBO* and *Disney*, and such direct streaming can be explained as a reaction to retailers' entry into production (e.g., *Netflix*'s and *AT&T*'s increased presence in the production layer), which in turn is a reaction to the proliferation of "over-the-top (OTT) bundle retailers" made possible by digital convergence between telecom and cable.

# 6 Conclusion

This paper has developed a model for analyzing markets in which a retailer firm offers a bundle of outputs sourced from multiple producers. Such markets have existed for a long time (e.g., art and music festivals, county fairs, sports tournaments, and other events with a ground or seasons pass). They have become more prominent with the popularity of platforms, where value co-creation is a defining characteristic and the platform often becomes the base for distribution of outputs from multiple producers. In contrast to the existing platform literature, our goal is to model the entire platform economy, explaining the platform's pricing as well as producers' output decisions, modeling revenue-sharing between them, and exploring the effects of alternate market structures.

Given the well-known challenges in representation and analysis of bundle demand, we achieve our goals by first developing a reduced-form specification for bundle demand which fits and respects the characteristics of bundling across a wide spectrum of bundling scenarios, and then deriving market outcomes under alternative market structures as well as the drivers and consequences of changes in market structures.

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